# Real part of the interaction potential between two $^{238}$U nuclei

H. M. M. Mansour, M. Ismail, M. Osman, and Kh. A. Ramadan
*Physics Department, Faculty of Science, Cairo University, Cairo, Egypt*
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In the present work we use the energy density formalism derived from the Skyrme density dependent interaction to study the orientation and energy dependence of the real part of the ion-ion potential for the $^{238}$U+$^{238}$U system. It is found that both the nuclear part and the total potential (nuclear plus Coulomb) depend strongly on the relative orientation of the two interacting nuclei. Pockets are predicted in the total potential whose depths are found to depend on the orientation angle.

## I. INTRODUCTION

The static and dynamic deformations give rise to significant changes in the Coulomb and nuclear energies. For example, the form factors describing the coupling between elastic and inelastic channels in peripheral heavy ion reactions depend sensitively on the static and dynamic properties of the matter distribution.[1] Moreover, in recent experiments spontaneous positron emission in heavy ion collision has been measured.[2] The analysis of the data indicated that the two nuclei stick together for a time of about $10^{-19}$ sec or longer. Greiner[3] has suggested that the prolonged nuclear sticking time might be caused by a potential energy minimum in the nuclear surface—a "pocket"—depending on orientation which can capture the nuclei.

Several attempts have been made to derive the potential between two deformed nuclei starting from an effective two-body interaction. For example, such potentials have been investigated in the proximity approximation[4] and the double folding procedure.[5,6] As is well known in the case of spherical nuclei, although the double folding model predicts qualitative features of the elastic scattering data, it has been found that a renormalization of the strength of the real potential to about one-half of its value at the strong absorption radius is essential to get quantitative agreement.[7] The discrepancy may be due to the neglect of the saturation effects or exchange effects due to antisymmetrization. Saturation effects can be taken into account by deriving a folded potential form a density dependent effective interaction.[8]

Another method for calculating the ion-ion potential is by using a Hamiltonian energy density derived from density dependent effective interaction.[9] This method has been applied successfully to derive the real part of the interaction potential between several pairs of spherical nuclei.[10] Besides the fact that the above mentioned method takes into account the effect of saturation of nuclear forces and the exchange effects due to antisymmetrization, it can be modified easily to include energy dependence of the ion-ion potential.[11]

A natural extension of the work done in Ref. 9 is to use the energy density formalism derived from density dependent effective interaction to study the energy and orientation dependence of the heavy ion potential between two deformed nuclei. This study will be done in the present paper. For this purpose, we use the Skyrme interaction with parameter set SIII (Ref. 12) to investigate the real part of the interaction potential between two $^{238}$U nuclei. For the $^{238}$U nucleus we use deformed Fermi type matter density with static quadrupole and hexadecapole deformations.[13] We calculated both the real part of the optical-model potential and the Coulomb potential. We study both the energy dependence and orientation dependence of the $^{238}$U + $^{238}$U potential. In the next section we briefly describe the theory. In Sec. III the results are presented and discussed.

## II. THEORY

The interaction potential between two nuclei separated by a distance $R$ is given by the difference between their energies at separation distance $R$ and their energies at separate distance infinity. Explicitly,

$$V(K,R)=E(K,R)-E(K,\infty)\ , \tag{1}$$

where $E$ is obtained from the Hamiltonian energy density $H$ according to

$$E(K,R)=\int d\mathbf{r}\,H(\mathbf{r},K,R)\ . \tag{2}$$

The potential $V$ between the two ions is also characterized by the average relative momentum per projectile nucleon, $K$, which is related to the laboratory energy of the projectile, $E_{\rm lab}$, according to

$$K=(1/\hbar)\sqrt{2mE_{\rm lab}/A_P}\ , \tag{3}$$

where $A_P$ is the mass number of the projectile and $m$ is the nucleon mass.

The Hamiltonian energy density $H$ consists of the kinetic energy density $(\hbar^2/2m)\tau$ and the potential energy density $\pi$. For the potential energy density we take the Skyrme form.

For calculating the kinetic energy density, we used the following prescription.[14] Having two densities $\rho_P$ (projectile density) and $\rho_T$ (target density) at position $\mathbf{r}$, one may relate them to Fermi momenta in nuclear matter using the relation

$$\rho_{P,T}=\left[\frac{2}{3\pi^2}\right]K_{f_{P,T}}^3\ ; \tag{4}$$

 

since each projectile nucleon has an average momentum $K$, we obtain two Fermi spheres whose centers are separated by a distance $K$ in momentum space. If the two Fermi spheres overlap, i.e., at high densities or low energies, the kinetic energy density is given by[14]

$$\tau=\tau^{(2)}+K_G^2\rho \ , \tag{5}$$

where the average momentum $\mathbf{K}_G$ and the intrinsic kinetic energy density $\tau^{(2)}$ are given by

$$\mathbf{K}_G=\int_F \mathbf{K}\,d\mathbf{K}\Big/\int_F d\mathbf{K} \ , \tag{6}$$

$$\tau^{(2)}=\frac{4}{(2\pi)^3}\int_F d\mathbf{K}(\mathbf{K}-\mathbf{K}_G)^2 \ . \tag{7}$$

$\rho$ is the total density given by

$$\rho=\rho_P+\rho_T \ . \tag{8}$$

In case of nonoverlapping Fermi spheres, Eq. (5) reduces to

$$\tau=\tau_{\rm TF}^{(2)}+K^2\rho \ , \tag{9}$$

where $\tau_{\rm TF}^{(2)}$ is the Thomas-Fermi approximation for the intrinsic kinetic energy density of the considered system.

For overlapping Fermi spheres, a double occupancy region appears in the momentum space which is forbidden by the Pauli principle. Therefore, we have to increase the Fermi radii to become $K'_{F_P}$ and $K'_{F_T}$, so as to keep the overall density. In this case a simple expression for the kinetic energy density in terms of $\rho_P$, $\rho_T$, and $K$ can no longer be given. The new radii have to be calculated numerically using an iterative procedure.

The Coulomb potential between the two $^{238}$U nuclei can be calculated using the double folding model,

$$V_C(R)=(Ze)^2\int d\mathbf{r}_1 d\mathbf{r}_2 \rho_P^C(\mathbf{r}_1)\frac{1}{|\mathbf{R}+\mathbf{r}_2-\mathbf{r}_1|}\rho_T^C(\mathbf{r}_2) \ , \tag{10}$$

where $Z$ is the charge number of $^{238}$U nucleus. $\rho_P^C$ and $\rho_T^C$ denote the charge distribution of the nucleons in the ground state of the projectile and the target, respectively. The above integral is a six dimensional integral, which is difficult to solve. However, if one uses the Fourier transformation technique, it can be transformed to a sum of terms each term contains one dimensional integrals. This procedure—which is outlined in Ref. 6—has been used in the present work to calculate the Coulomb interaction for $^{238}$U+$^{238}$U system.

## III. NUMERICAL CALCULATIONS AND DISCUSSION

In the present work, we have calculated both the real part of the optical-model potential (nuclear potential) and the Coulomb potential for the $^{238}$U+$^{238}$U system.

Equation (1) has been used to calculate the nuclear potential at different values of projectile energy $E_{\rm lab}$. The Hamiltonian energy density has been derived from Skyrme nucleon-nucleon interaction. The kinetic energy density was calculated as described in Sec. II. This produces the volume part of the intrinsic kinetic energy density, $\tau_v^{(2)}$. The total intrinsic kinetic energy density $\tau^{(2)}$ is calculated using the modified Thomas-Fermi approximation[15] given by

$$\tau^{(2)}=\alpha\tau_v^{(2)}+\beta\frac{|\nabla\rho|^2}{\rho} \ , \tag{11}$$

where the values of $\alpha$ and $\beta$, for different nuclei, are presented in Ref. 15. This approximation was found to be suited for ion-ion scattering processes.

The matter distribution of either the target or projectile was approximated by the Fermi shape,

$$\rho(\mathbf{r})=\rho_0/(1+e^{[r-R(\theta)]/a}) \ . \tag{12}$$

We assumed static deformation for the $^{238}$U nucleus and we expanded the radius parameter $R(\theta)$ in the usual way,

$$R(\theta)=R_0[1+\delta_2 Y_{20}(\theta)+\delta_4 Y_{40}(\theta)] \ , \tag{13}$$

where $\theta$ is the angle between the vector $\mathbf{r}$ and the symmetry axis of the $^{238}$U nucleus. We used the recent experimentally determined results (13), $\delta_2=0.261$, $\delta_4=0.087$, $R_0=6.8054$ fm, and $a=0.6049$ fm. The value of $\rho_0$ is determined from the condition

$$\int d\mathbf{r}\rho(\mathbf{r})=\text{mass number} \ .$$

The charge distribution of the nucleons, appearing in Eq. (10), was assumed to have the same form as Eq. (12), but scaled by $Z/A$, where $Z$ is the charge number and $A$ is the mass number of the nucleus.

In order to show the orientation dependence of the $^{238}$U+$^{238}$U potential, we considered three different relative orientation angles, $\beta_1=\beta_2=0°$, 45°, and 90° ($\beta_i$ is the

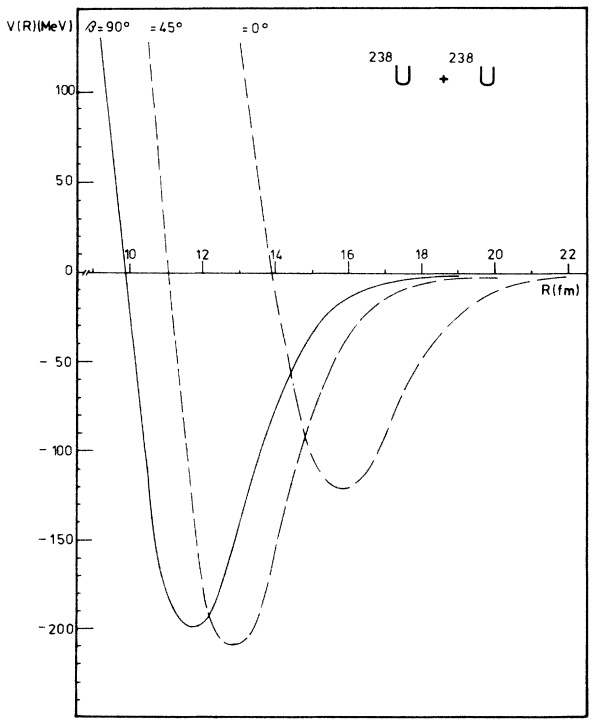


FIG. 1. The real part of the potential without the Coulomb interaction for the $^{238}$U+$^{238}$U system calculated at $K_r=0.537$ fm$^{-1}$ ($E_{\rm lab}/A=6.02$ MeV) for different orientation angles $\beta=0°$, 45°, and 90°.

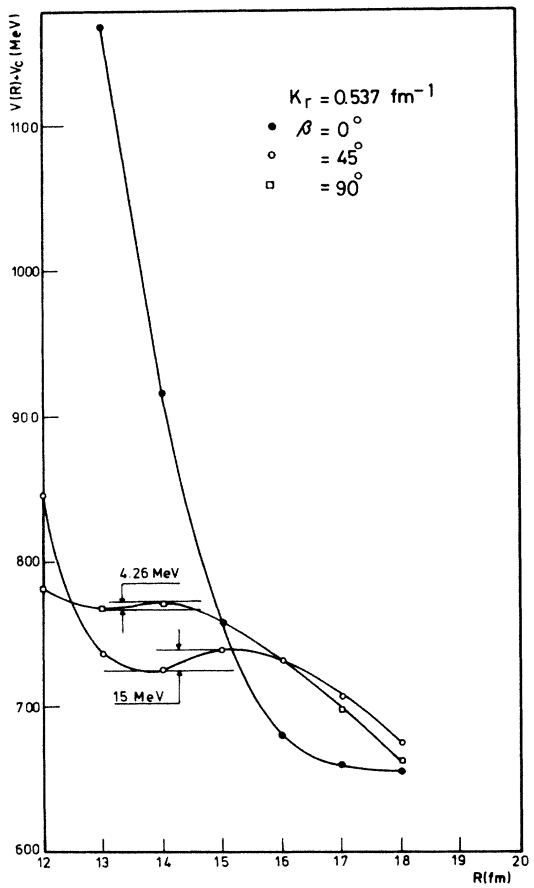


FIG. 2. Total real (nuclear plus Coulomb) potential for the same system at $K_r = 0.537$ fm$^{-1}$ for the three different orientation angles $\beta = 0°$, 45°, and 90°.

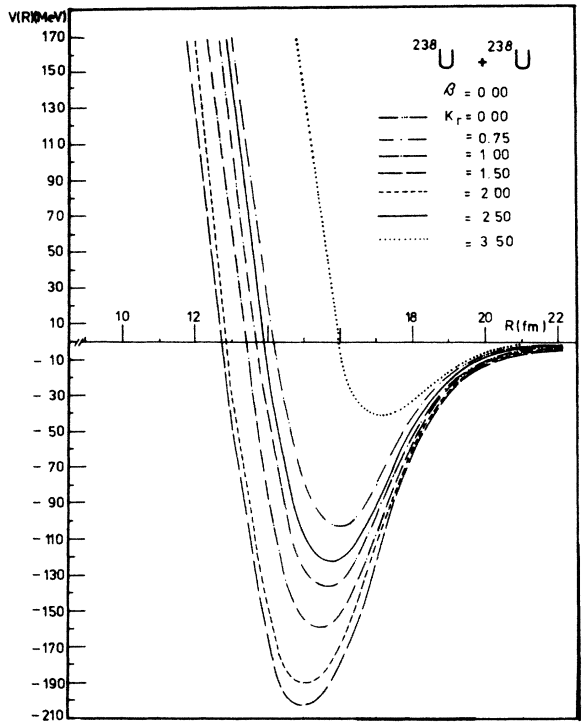


FIG. 3. The real part of the nuclear potential at different energies for $\beta_1 = \beta_2 = 0°$.

angle between the symmetry axis of nuclei $i$ and the vector joining the centers of mass of the two colliding nuclei). Our results are displayed in Figs. 1–3. Figures 1 and 2 show the results for $K_r = 0.537$ fm$^{-1}$, which corresponds to an incident energy per projectile nucleon of $E_{\text{lab}}/A = 6.02$ MeV. These figures show strong orientation dependence of the $^{238}U + ^{238}U$ potential. For each relative orientation, the nuclear part of the potential has a repulsive core at small separation distances followed by an attractive part. The range of the repulsive part increases as the relative orientation angle decreases from 90° to 0°. As pointed out in Ref. 10, the repulsive part is due to the lack of distortion in the single particle wave functions and of the approximations used for the densities. The attractive part has a range which increases as the relative orientation angle decreases from 90° to 0°. Also, the position of its minimum together with its depth depend strongly on the relative orientation of the interacting nuclei. This is because the position of the minimum is related to the sum of the half-density radii, while its depth depends on the volume of the overlap region of the two density distributions that happen at this position. For example, in case of $\beta_1 = \beta_2 = 0°$, the sum of the two half-density radii is large while the overlap region is small compared with that for the other relative orientations, and as a result we have a minimum in the nuclear potential located at larger separation distance which has a smaller depth. Another feature that affects the depth of $V(R)$ is that we included the hexadecapole deformations of the $^{238}U$ nucleus in our calculation. Consequently, the $^{238}U$ nucleus has flat surface areas (diamond shaped). For a certain orientation, two flat areas face each other and in this case the number of nucleons which come into nuclear contact is considerably increased in comparison with the situation of two curved surfaces. This explains the deeper minimum which is observed for the case $\beta_1 = \beta_2 = 45°$.

The total potential (nuclear plus Coulomb) calculated at $E_{\text{lab}}/A = 6.02$ MeV is shown in Fig. 2. It is seen from this figure that pockets appear for orientations $\beta_1 = \beta_2 = 45°$ and 90°. The depth of the pocket depends strongly on the relative orientation, the deeper one occurring at $\beta_1 = \beta_2 = 45°$ and with a depth of about 15 MeV. For $\beta_1 = \beta_2 = 90°$, the total potential shows a weaker pocket whose depth is 4.3 MeV. Thus our calculations support the positron emission from the $^{238}U + ^{238}U$ combined system, which has been observed at $E_{\text{lab}}/A \simeq 6$ MeV.

Figure 3 shows the energy dependence of the $^{238}U + ^{238}U$ nuclear potential for $\beta_1 = \beta_2 = 0°$. It is seen from this figure that when the projectile energy increases, the potential curve goes downwards until $E_{\text{lab}}/A = 46.97$ MeV and then it starts rising again as the energy increases beyond this value.

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