Quasi-elastic electron scattering from ^{238}U

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Rosenbluth separations of electron scattering response functions for uranium targets were made with data using measurements from five laboratory angles (60°, 90°, 134.5°, 140°, and 160°) at three-momentum transfers ranging from 280 to 500 MeV/c. The separations were made to determine if previously reported quenching of the longitudinal response function persisted in targets with large atomic number. The results are compared to relativistic Fermi gas calculations and calculations following the formalism of Rosenfelder. The measured transverse response functions agree quite well with the Rosenfelder-type calculations and almost as well with the relativistic Fermi gas calculations. The measured longitudinal response functions show significant quenching at low q. This quenching diminishes with increasing q but is never quite overcome.

INTRODUCTION

Results of deep inelastic electron scattering experiments in the quasi-elastic region involving separation via Rosenbluth plots of the longitudinal and transverse response functions for various nuclei including ^{56}Fe, ^{48}Ca, and ^{40}Ca (Refs. 1–3) have in the recent past all displayed a common feature: The experimental longitudinal response function agrees rather poorly when compared with relativistic Fermi gas model (FGM) calculations, while the transverse response function agrees quite well. This poor agreement of the longitudinal response function with the FGM is characterized by a suppression of the experimental response function relative to the calculations at low three-momentum transfer (q). As q increases, this suppression diminishes but is never completely overcome. This discrepancy is quite striking in light of the previous success of the FGM in predicting differential cross sections for electron scattering in the quasi-elastic region.[4,5]

Explanations put forward to account for this phenomenon include manifestations of quark substructure resulting in so-called "medium modified nucleon form factors,"[6,7] relativistic effects causing a breakdown of the weak mean field approximation,[8] and final state interaction effects.[9] Thus far, where the comparison has been made, no single explanation has succeeded in accounting for the differences between experiment and theory over the full range of q. Some explanations are indeed not supported by the full body of available data.[10]

The present experiment adds ^{238}U to the list of nuclei whose separated quasi-elastic response functions have been measured over a significant range of q. ^{238}U was chosen principally because it represents a part of the periodic table previously unstudied in this kinematic regime. ^{238}U is also the closest available approximation to infinite nuclear matter, where one can expect subtleties of single particle states to be minimized.

As an alternative to the familiar FGM calculation,[11] ^{238}U response functions will also be compared to calculations that rigorously follow the formalism of Rosenfelder.[12] The Rosenfelder calculation has the advantage that it uses the more realistic results of density dependent Hartree-Fock (DDHF) calculations[13] instead of the Fermi gas model to describe the momentum distribution of nucleons in a nucleus. To account for the interaction of the ejected nucleon with the nucleus, the Rosenfelder calculation uses an effective mass, M^*. This M^* derives its justification from a relativistic field theoretic description of nuclear matter.[14] The nucleon is treated as a Dirac particle in constant scalar and vector-meson fields. In that framework the Dirac equation used for the nucleon is identical to that used to describe a free nucleon except that the nucleon mass is replaced everywhere by $M^*=M-g_s\varphi_0$, where M is the free nucleon mass, g_s is the scalar-meson coupling constant, and φ_0 is the mean scalar-meson field. The calculation we employ uses the full nucleon electromagnetic form factors in the parametrization of Höhler *et al.*[15]

DATA ANALYSIS

Rosenbluth separations of the data taken at the Bates Linear Accelerator Center at five laboratory angles (60°, 90°, 134.5°, 140°, and 160°) for electron scattering from ^{238}U were performed at q's from 280 to 500 MeV/c. Incident electron energies ranged from 100 to 690 MeV. Scattered electrons were detected using the 900 MeV/c energy loss spectrometer[16] with standard focal plane detectors and a silica aerogel Cerenkov detector ($n=1.05$). The standard uranium target thickness was ≈ 100 mg/cm^2. However, periodically targets as thin as ≈ 20 mg/cm^2 were used. The data from the thinner targets served as a check on the radiative corrections applied to the data. For more details on the experimental procedure see Refs. 2, 3, and 23.

Several corrections were applied to the measured cross sections prior to extracting the separated response functions. The purpose of these corrections was to remove the effects of radiative processes and Coulomb distortions, both of which are aggravated by the large atomic number of the target nucleus. These corrections are generally small, particularly in the vicinity of the quasi-elastic peak and at higher values of q. The results of the Rosenbluth separations in the region of the quasi-elastic peak are essentially unchanged with no corrections.

The radiative tails from elastic scattering were subtracted using the formalism of Tsai,[17] modified to include Thomas-Fermi atomic form factors[18] instead of using screening approximations for calculating bremsstrahlung. The Schwinger term was modified to include radiation by the recoiling nucleus, and the radiation length was calculated using a formula[18] which is accurate for large atomic numbers. The elastic cross sections used in the calculation were determined using phase shift codes employing charge distributions from Creswell.[19] The accuracy of the radiative tail correction procedure in targets with large atomic number was recently confirmed in tungsten.[20] The unfolding procedure followed that of Miller,[21] but was similarly modified for screening and radiation length to accommodate large atomic numbers.

Over most of the quasi-elastic region these calculated radiative corrections were negligible. For the lowest incident energy spectra, however, some of the largest energy loss points extended into a region where the elastic radiative tail was substantial. Points for which the radiative tail exceeded 30% of the measured cross section were excluded from the final analysis and played no part in calculating the Rosenbluth separations.

Corrections required due to the distortion of the electron wave in the vicinity of the deep nuclear Coulomb potential were calculated by assuming that the quasi-elastic scattering process is best approximated by elastic scattering from a single nucleon but using electron waves which include distortion effects caused by penetration into the nucleus.

These distortions were treated in two parts. The dominant effect, taken up first, is the fact that the effective momentum transfer at the point of scattering is increased to q_{eff} according to the prescription[22]

$$q_{\text{eff}}=q\left[1+\frac{3Z_\alpha}{2RE_0}\right],$$

where R is the equivalent nuclear radius; this causes a reduction in cross section.

The second effect[23] is a partial restoration of the cross section due to the focusing of the electron wave in the nuclear Coulomb field. This additional Coulomb correction was calculated by adapting a phase shift code[24,25] for finding the elastic scattering cross section for an extended charge distribution. Such codes first calculate phase shifts in expanded Dirac plane waves due to a point charge at a specified "matching radius" located at the edge of the charge distribution. The remaining radial phase shifts due to the extended charge distribution are then calculated for the real and imaginary parts of each partial wave in discrete steps inward from the matching radius where the solutions are required to join smoothly.

The Coulomb and radial phases are then added together using trigonometric identities to form total phase shifts which are used to calculate scattering probabilities for specified scattering angles. The incident energy, scattering angle, and charge distribution in principle determine the calculated cross section, although there can also be some sensitivity to the choice of the matching radius[25] and number of partial waves used in the expansion.

The objective in developing this procedure was to calculate the phase shifts produced by a single proton, but to use incident partial waves which have already been distorted by the potential of the uranium nucleus. This means using the total phase shifts for a uranium charge distribution to find the additional phase shifts produced by a single proton superimposed at rest at the center of the nucleus. These additional phase shifts will differ from those produced by a free proton because each partial wave at the bottom of the uranium potential well will effectively differ in energy as well as in starting phase from the partial waves of an incident plane wave. The set of "extra" phase shifts produced by the added proton will then include the interference effects of the Coulomb field of the nucleus.

To find the Coulomb distorted phase shifts for a single proton, the radial phase shifts for elastic scattering from uranium were subtracted from those produced by the same charge distribution but assuming a net charge of $Z=91$ plus the additional potential of a single proton at the center. Because these two distributions have the same total charge, the Coulomb phase shifts are the same. The difference between the radial solutions then reflects the entire effect of concentrating one unit of charge in the uranium charge distribution into a single proton at the center. By comparing scattering cross sections calculated from these derived phase shifts with those of a free proton, the magnitudes of the Coulomb distortion effects are seen.

For these comparisons, the energy used for the calculations must be adjusted for the recoil of a single nucleon rather than the whole nucleus. The effective binding energy was also incorporated in the recoil factor, but otherwise the scattering was treated as elastic. The lab energy was adjusted using the q_{eff} prescription to allow comparisons between the free and bound cases. The matching radius used was such that it contained all but 10^{-5} of the total charge.

The effect of Coulomb distortion was found to completely vanish at higher incident energies and amounts to a maximum restoration of 10% at the lowest incident energy (100 MeV).

Because this *ad hoc* distortion calculation was based on several simplifying assumptions (elastic approximation, scattering from a proton at rest, charge scattering only) it was checked using the DWBA code HEIMAG. Transition currents for a single outer shell nucleon scattered into the continuum were arbitrarily constructed for this test. Scattering probabilities for each of several multipolarities were calculated at small increments of scattering angle in the neighborhood of the experimental scattering angle to determine which multipolarity was the first to participate at a diffraction peak. The distorted cross section at this multipole was then compared to the corresponding plane wave solution to find a representative correction for the entire cross section.

Although the results of the DWBA procedure did not agree exactly with the quasi-elastic approach, the test did confirm that the focusing effects become vanishingly small at moderate incident energies.[23] The maximum increase in cross section at 100 MeV predicted by this method was 18% compared to the 10% stated above for the quasi-elastic scattering method. The DWBA procedure does have the advantage that it is presumably equally applicable to parts of the spectrum where the process is clearly not quasi-elastic scattering off the single nucleons, but only provided that a reasonable model for determining transition currents is available. The DWBA method is also cumbersome and time consuming since the result is very sensitive to the selection of the lowest multipole at the diffraction peak and this determination must be made for each point. Even after great care is taken in this selection, the sensitivity of the result to this choice of multipole leaves serious doubts about the final accuracy, even if the transition currents could be trusted. As a result, this method was only used as a general check on the quasi-elastic approach.

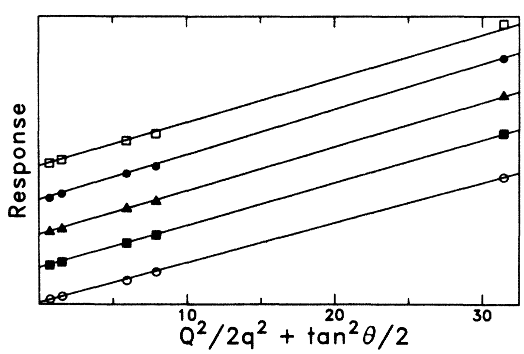

FIG. 1. Representative Rosenbluth plots, after Coulomb corrections, used for response function separations in the vicinity of the quasi-elastic peak. (In order to show all the plots on the same graph the vertical scale is necessarily arbitrary.) Open squares are for $q=300$ MeV/c; the fitted line has a linear correlation coefficient of 0.998 and a χ^2/(degree of freedom) of 0.7. Solid circles, $q=360$ MeV/c, linear correlation coefficient=0.999, and χ^2/(degree of freedom)=0.34. Solid triangles, $q=400$ MeV/c, linear correlation coefficient=0.994, χ^2/(degree of freedom)=2.19. Solid squares, $q=400$ MeV/c, linear correlation coefficient=0.990, χ^2/(degree of freedom)=3.81. Open circles, $q=480$ MeV/c, linear correlation coefficient=0.996, χ^2/(degree of freedom)=1.12. Experimental errors are smaller than the plotted points.

The Rosenbluth plots obtained after making corrections for all distortion effects were nearly linear, as shown in Fig. 1. These are typical fits at the peak of the quasi-elastic response using interpolated cross sections from all five angles. Although the linearity of the Rosenbluth plots does not by itself constitute proof that the Coulomb distortions have been properly removed, we remark that the focusing effect becomes more significant as the incident energy becomes smaller. This means that if the focusing had been underestimated, the error would have preferentially effected the backward angles, reducing the transverse and increasing the longitudinal response.[26]

COMPARISON WITH CALCULATION

The extracted transverse and longitudinal response functions were compared to two calculations: (1) the rela-

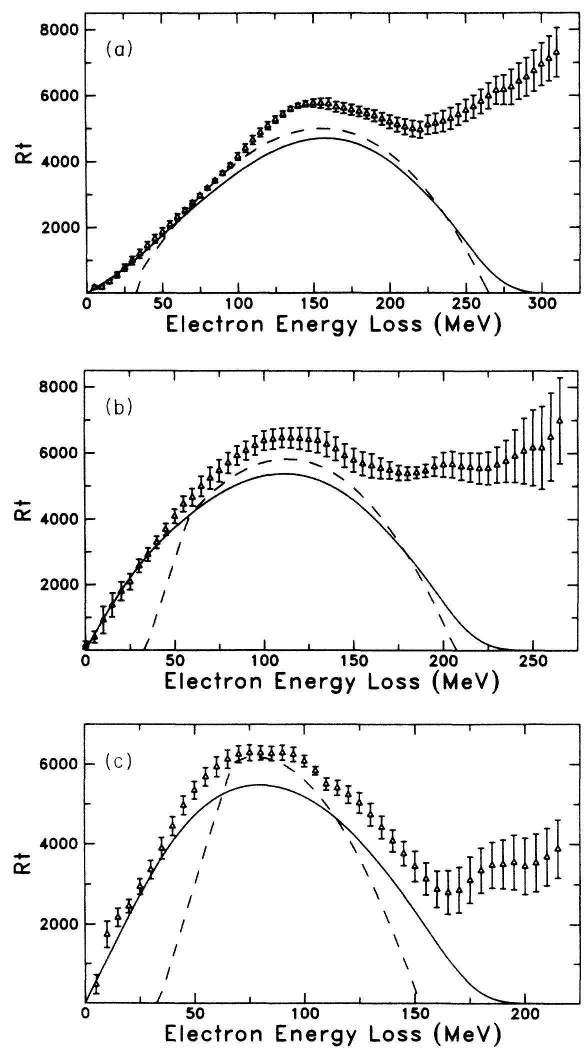

FIG. 2. Measured transverse response functions versus energy loss in MeV for ^{238}U. Indicated errors represent combined statistical and systematic errors. The dashed line is the FGM calculation. The solid line is the Rosenfelder calculation. (a) $q=500$ MeV/c, $M^*=0.76M$; (b) $q=400$ MeV/c, $M^*=0.69M$; (c) $q=300$ MeV/c, $M^*=0.54M$.

tivistic Fermi gas model of Van Orden,[11] which uses fully relativistic dynamics, a Fermi gas nucleon momentum distribution, and free nucleon form factors in the parametrization of Höhler *et al.*,[15] and (2) the Rosenfelder calculation[12] mentioned earlier.

Representative plots of the extracted transverse and longitudinal response functions are shown in Figs. 2 and 3. The transverse response agrees with the FGM reasonably well at all q's observed, although the peak and integrated strength are consistently higher than the FGM prediction. The corresponding longitudinal response functions approach the FGM predictions as q increases, but never quite reach them.

In Figs. 2 and 3 results of the Rosenfelder calculation are also displayed. In order to achieve a reasonable agreement between the Rosenfelder calculation and the data, it was necessary to use M^* as an adjustable parameter. The values of M^* used in Figs. 2(a), 2(b), 3(a), and 3(b) are the best-fit values for the longitudinal response functions. Except at 300 MeV/c, where it was impossible to determine a best-fit value of M^* from the longitudinal response function, the transverse response functions were not used to determine M^* because of possible contributions to the transverse response from π production, meson exchange currents, and delta production, which are not accounted for in the calculation. Figure 4 shows the best-fit values of M^* as a function of q. These results are quite similar to those reported by Hotta *et al.*[27] for quasi-elastic electron scattering from ^{56}Fe at 180°. This is somewhat in contradiction with the spirit of Rosenfelder's calculation which suggests that M^* should have one fixed value that is characteristic of the nucleus. The agreement between calculation and experiment for the transverse response is nonetheless quite good.

In comparing the results of the Rosenfelder calculation with experiment, the advantage of using a more realistic nucleon momentum density is readily evident. The calculated transverse response function agrees with the data quite well and has a shape that resembles the experimentally determined shape more closely than the FGM. Some excess strength is observed in the transverse response, but it is probable that this is due to known inelastic processes

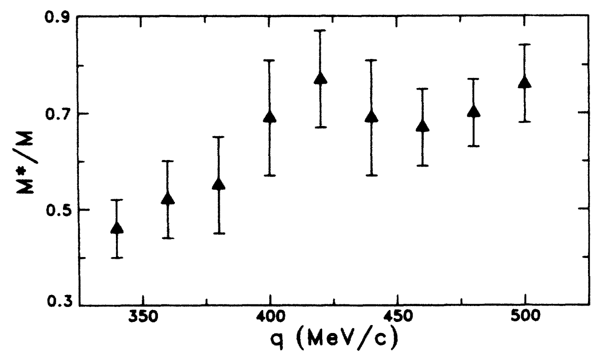

FIG. 4. Best-fit values of M^*/M vs q. Fit to the measured longitudinal response functions.

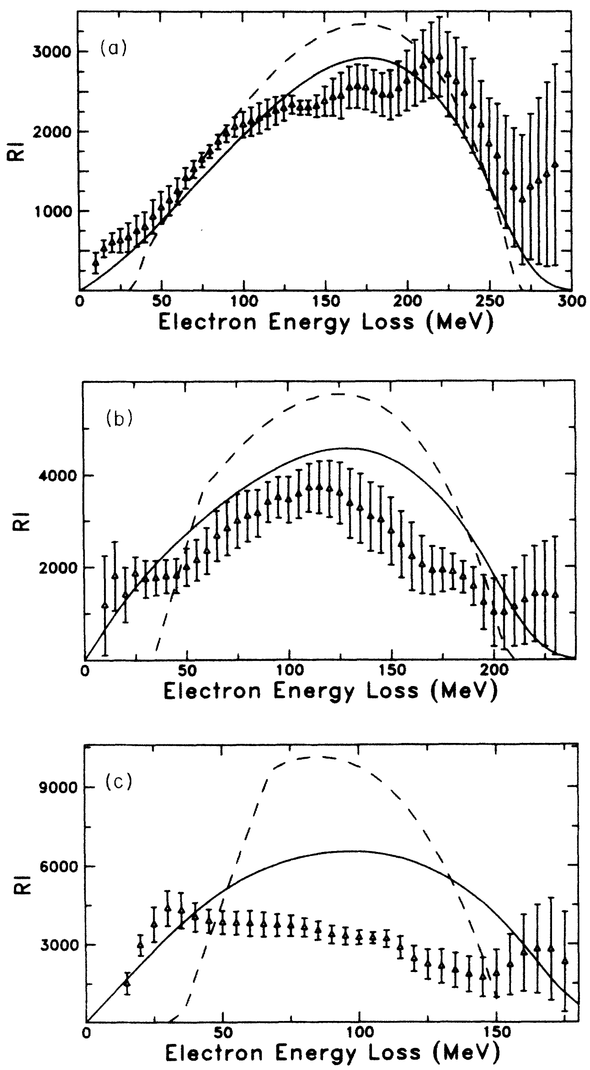

FIG. 3. Measured longitudinal response functions versus energy loss in MeV for ^{238}U. Indicated errors represent combined statistical and systematic errors. The dashed line is the FGM calculation. The solid line is the Rosenfelder calculation. (a) $q=500$ MeV/c, $M^*=0.76M$; (b) $q=400$ MeV/c, $M^*=0.69M$; (c) $q=300$ MeV/c, $M^*=0.54M$.

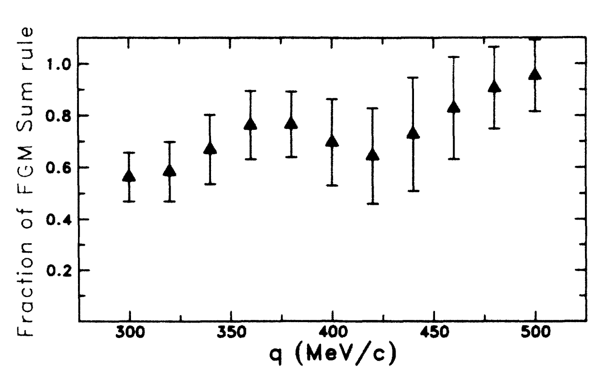

FIG. 5. Integral of the measured longitudinal response function divided by the predicted FGM value plotted vs q for ^{238}U. The integration of the measured response function was cut off at the point where the FGM prediction went to zero.

not included in the calculation. However, the strength discrepancy in the longitudinal response remains.

At the lower q values studied, the peak of the longitudinal response in ^{238}U is notably lower, but also broader than the FGM prediction. A comparison of the integrated longitudinal response, evaluated using points extending only to the end of the FGM peak, with the corresponding FGM sum rule is shown in Fig. 5. This method of cutting off the integrated longitudinal response at the end of the theoretical strength disregards any measured longitudinal strength that extends beyond the cutoff and thus underestimates the experimental value. The reported measured integrated strength is only a lower limit of the possible total integrated longitudinal strength.

CONCLUSION

The suppression of the longitudinal response as observed in ^{40}Ca, ^{48}Ca, and ^{56}Fe (Refs. 1–3) persists in ^{238}U. This suppression is not accounted for by the failure of the FGM to accurately describe the momentum distribution of nucleons in a nucleus. The Rosenfelder calculation, which uses the results of DDHF calculations to describe the momentum distribution, produces a better longitudinal response function shape but overestimates the overall strength just as badly as the FGM. At this time the most plausible explanation seems to be that of Pickelsimer *et al.*,[9] whose work on the (e,e′p) reaction indicates that for that reaction the longitudinal response is strongly suppressed by final state interaction effects, whereas the transverse response is much less sensitive.

It is also significant to note that at low q the longitudinal sum rule is not saturated while at $q=500$ MeV/c ($\approx 2k_F$) we see almost complete saturation. This occurs in a situation where the measured integrated longitudinal response is only a lower limit on the total possible integrated longitudinal response.

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