Proton scattering and the interacting boson model in the *s-d* shell

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The scattering cross sections of 29.4 MeV protons to the first five levels of the ^{32}S and of 40 MeV protons to the first seven levels of ^{24}Mg, have been analyzed. The calculations have been performed with a coupled-channel program using nuclear matrix elements obtained within the IBM approximation including g-boson configurations.

I. INTRODUCTION

Many calculations of elastic and inelastic scattering cross sections of particles off s-d shell nuclei have been performed using a description of collective states in terms of the geometrical model.[1] However, in the last few years, an alternative approach to the description of collective states has been developed, i.e., the interacting boson model (IBM) approximation of Arima and Iachello.[2]

In this paper we attempt to investigate whether or not this collective model can be used to describe the elastic and inelastic scattering of protons by s-d shell nuclei.

Although light nuclei are not the best suited for this investigation, since collective features are not so well developed, we have studied this region because of the considerable amount of available data and the detailed shell-model calculations which have been made through it. Some nuclei in the s-d shell have already been treated in the IBM, using s and d boson configurations, and the model has been found to describe, in an acceptable manner, the observed spectra and the $E2$ transitions.[3–5] Unfortunately, the $E4$ transitions cannot be explained so well, as is pointed out in Ref. 3.

This situation could be improved by the introduction of basis states with higher angular momentum (g bosons), as for heavier nuclei.[6–8] We will explicitly introduce this additional degree of freedom in our analysis. The details are given in Sec. II.

Our calculations concern a nucleus of the first half of the sd shell (^{24}Mg, four boson particles out of the ^{16}O core) and a nucleus of the second half (^{32}S, four boson holes out of the ^{40}Ca core). The proton energies considered are 40 MeV (Ref 9) for ^{24}Mg and 29.4 MeV (Ref. 10) for ^{32}S, high enough to permit the optical model to work well in this mass region.

Successful preliminary results had previously been presented for protons of 20 MeV.[11]

The description of the observed spectra and of the $E0$, $E2$, and $E4$ transitions is presented in Sec. III. Comparison between theory and experiment permits fixing of the parameter values in the IBM Hamiltonian and the effective boson charges of the electromagnetic transition operators.

The comparison between the calculated and experimental proton scattering cross sections is shown and discussed in Sec. IV. Finally, our conclusions are presented in Sec. V.

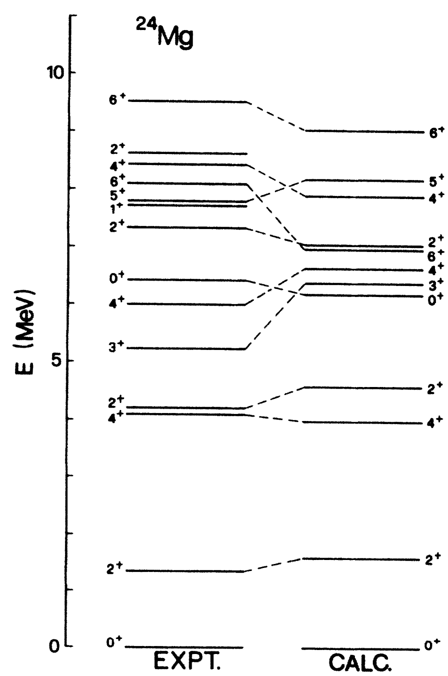

FIG. 1. Low-lying energy spectrum of ^{24}Mg: on the left, experimental levels (Ref. 23); on the right, calculated levels with the IBM parameters of Table III.

II. INTERACTING BOSON MODEL FORMALISM

The energy spectra and the $B(E2)$ values of some nuclei in the *sd* shell have already been quite well described with different versions of the interacting boson model in terms of *s* and *d* bosons.[3–5] The experimental $B(E4)$ values suggest, however, the explicit introduction of a *g*-boson degree of freedom. We work with the simplest version of the model (IBM-1), extended to include *g*-boson excitations.

Hence, for a Hamiltonian describing a system composed of $|(sd)^N\rangle$ and $|(sd)^{N-1}\otimes g\rangle$ configurations, we can write, following the notations of Ref. 8,

$$\hat{H}=\hat{H}_{sd}+\epsilon_g g^{\dagger}\tilde{g}+\hat{H}_{\rm int}\ , \qquad (1)$$

where $\hat{H}_{sd}$ is the usual IBM-1 Hamiltonian.[2] In the multipole expansion notation,

$$\hat{H}_{sd}=\epsilon_d\hat{n}_d+a_0\hat{P}^{\dagger}\hat{P}+a_1\hat{L}\hat{L}+a_2\hat{Q}\hat{Q}+a_3\hat{T}_3\hat{T}_3+a_4\hat{T}_4\hat{T}_4\ . \qquad (2)$$

TABLE I. $E\lambda$ matrix elements $\langle f||T(E\lambda)||i\rangle$ and static quadrupole moments for ^{24}Mg.

$I_i \rightarrow I_f$	Expt.[a]	IBA-1[b]
	$\langle f\|\|T(E0)\|\|i\rangle$ in $e\,\mathrm{fm}^2$	
$0_2\rightarrow 0_1$	6.8 ± 1.1[c]	7.28
	$\langle f\|\|T(E2)\|\|i\rangle$ in $e\,\mathrm{fm}^2$	
$2_1\rightarrow 0_1$	20.8 ± 0.5[c]	24.61
$4_1\rightarrow 2_1$	29.2 ± 2.5[c]	36.98
$2_2\rightarrow 0_1$	4.5 ± 0.2[c]	4.96
$2_2\rightarrow 2_1$	6.4 ± 0.3[c]	-5.57
$3_1\rightarrow 2_1$	7.6 ± 0.6[c]	5.09
$4_2\rightarrow 2_1$	6.1 ± 0.6[c]	-11.36
$4_2\rightarrow 2_2$	25.8 ± 2.9[c]	-14.74
$0_2\rightarrow 2_1$	1.5 ± 0.2[c]	4.73
$0_2\rightarrow 2_2$	6.0 ± 0.7[c]	4.38
	$\langle f\|\|T(E4)\|\|i\rangle$ in $e\,\mathrm{fm}^4$	
$4_1\rightarrow 0_1$	44.7 ± 3.4[d]	-14.00
$4_2\rightarrow 0_1$	207.4 ± 14.5[d]	-236.90
$4_3\rightarrow 0_1$	26.08[e]	-26.81
$4_4\rightarrow 0_1$	17.29[e]	6.38
Q_{2_1} ($e\,\mathrm{fm}^2$)	-18 ± 2[f]	-28.96
Q_{2_2} ($e\,\mathrm{fm}^2$)	16 ± 10[g]	18.92

[a]Experiments give the absolute value of the matrix elements.
[b]The effective boson charges for the IBM-1 results are, for $\lambda=0$, $e_{dd}^{(0)}=17.5$ $e\,\mathrm{fm}^2$, $e_{ss}^{(0)}=0$ $e\,\mathrm{fm}^2$, and $e_{gg}^{(0)}=0$ $e\,\mathrm{fm}^2$; for $\lambda=2$, $e_{ds}^{(2)}=3.42$ $e\,\mathrm{fm}^2$, $e_{dd}^{(2)}=-6.63$ $e\,\mathrm{fm}^2$, $e_{dg}^{(2)}=-3.21$ $e\,\mathrm{fm}^2$, and $e_{gg}^{(2)}=0$ $e\,\mathrm{fm}^2$; for $\lambda=4$, $e_{gs}^{(4)}=28.3$ $e\,\mathrm{fm}^4$, $e_{dd}^{(4)}=-1.84$ $e\,\mathrm{fm}^4$, $e_{gd}^{(4)}=243$ $e\,\mathrm{fm}^4$, and $e_{gg}^{(4)}=4480$ $e\,\mathrm{fm}^4$.
[c]Reference 13.
[d]Reference 14.
[e]Reference 15 (shell model calculations; see the text).
[f]Reference 16.
[g]Reference 3.

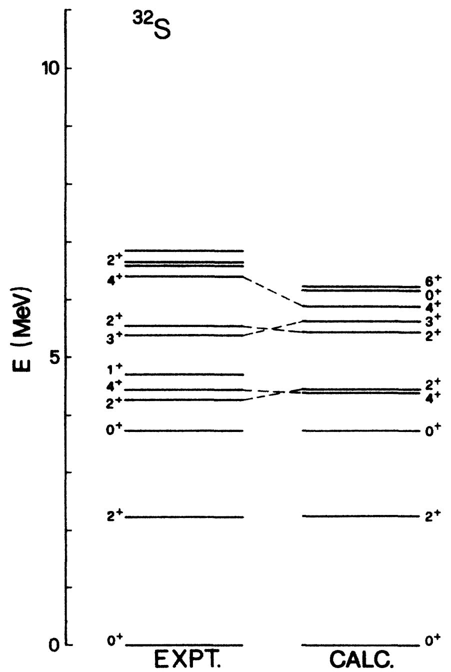

FIG. 2. Low-lying energy spectrum of ^{32}S: on the left, experimental levels (Ref. 23); on the right, calculated levels with the IBM parameters of Table III.

TABLE II. $E\lambda$ matrix elements $\langle f||T(E\lambda)||i\rangle$ and static quadrupole moments for ^{32}S.

$I_i \rightarrow I_f$	Expt.[a]	IBA-1[b]
	$\langle f\|\|T(E0)\|\|i\rangle$ in $e\,\mathrm{fm}^2$	
$0_2\rightarrow 0_1$	2.0 ± 0.2[c]	2.07
	$\langle f\|\|T(E2)\|\|i\rangle$ in $e\,\mathrm{fm}^2$	
$2_1\rightarrow 0_1$	17.4 ± 0.9[c]	17.62
$0_2\rightarrow 2_1$	9.2 ± 0.7[c]	-12.29
$2_2\rightarrow 0_1$	6.5 ± 0.5[c]	-7.02
$2_2\rightarrow 2_1$	16.8 ± 1.2[c]	-18.09
$4_1\rightarrow 2_1$	25.5 ± 2.1[c]	25.78
	$\langle f\|\|T(E4)\|\|i\rangle$ in $e\,\mathrm{fm}^4$	
$4_1\rightarrow 0_1$	-223.81[d]	-266.44
$4_2\rightarrow 0_1$	-148.95[d]	-177.22
$4_3\rightarrow 0_1$	38.44[d]	45.74
Q_{2_1} ($e\,\mathrm{fm}^2$)	-9 ± 4[e]	-18.07
Q_{2_2} ($e\,\mathrm{fm}^2$)	14 ± 10[f]	4.77

[a]Experiments give the absolute value of the matrix elements.
[b]The effective boson charges for the IBM-1 results are, for $\lambda=0$, $e_{dd}^{(0)}=79.1$ $e\,\mathrm{fm}^2$, $e_{ss}^{(0)}=-62.0$ $e\,\mathrm{fm}^2$, and $e_{gg}^{(0)}=0$ $e\,\mathrm{fm}^2$; for $\lambda=2$, $e_{ds}^{(2)}=4.90$ $e\,\mathrm{fm}^2$, $e_{dd}^{(2)}=-1.66$ $e\,\mathrm{fm}^2$, $e_{gd}^{(2)}=7.00$ $e\,\mathrm{fm}^2$, and $e_{gg}^{(2)}=0$ $e\,\mathrm{fm}^2$; for $\lambda=4$, $e_{gs}^{(4)}=125$ $e\,\mathrm{fm}^4$, $e_{dd}^{(4)}=102$ $e\,\mathrm{fm}^4$, $e_{gd}^{(4)}=972$ $e\,\mathrm{fm}^4$, and $e_{gg}^{(4)}=0$ $e\,\mathrm{fm}^4$.
[c]Reference 13.
[d]Reference 15 (shell model calculations; see the text).
[e]Reference 16.
[f]Reference 3.

TABLE III. IBM parameters (in MeV).

	^{24}Mg	^{32}S
ϵ_d	0.800	2.000
a_0	1.520	−0.420
a_1	0.225	0.040
a_2	−0.230	−0.230
a_3	0.097	−0.130
a_4	−0.160	0.044
ϵ_g	6.000	5.000
b_1	3.500	1.000
b_2	1.000	0

The general (sdg)-boson interaction Hamiltonian has been restricted to only two mixing terms:

$$\hat{H}_{\text{int}}=b_1[(d^\dagger d^\dagger)^{(4)}(\tilde{g}\tilde{s})^{(4)}+\text{H.c.}]^{(0)}+b_2[(d^\dagger d^\dagger)^{(4)}(\tilde{g}\tilde{d})^{(4)}+\text{H.c.}]^{(0)}\ . \tag{3}$$

In the same spirit, the electromagnetic transition operators are modified by including g-boson terms:

$$\hat{T}(E0)=\epsilon_{ss}^{(0)}(r)(s^\dagger\tilde{s})^{(0)}+\epsilon_{dd}^{(0)}(r)(d^\dagger\tilde{d})^{(0)}+\epsilon_{gg}^{(0)}(r)(g^\dagger\tilde{g})^{(0)}\ , \tag{4a}$$

$$\hat{T}(E2)=\epsilon_{ds}^{(2)}(r)(d^\dagger\tilde{s}+s^\dagger\tilde{d})^{(2)}+\epsilon_{dd}^{(2)}(r)(d^\dagger\tilde{d})^{(2)}+\epsilon_{gd}^{(2)}(r)(g^\dagger\tilde{d}+d^\dagger\tilde{g})^{(2)}+\epsilon_{gg}^{(2)}(r)(g^\dagger\tilde{g})^{(2)}\ , \tag{4b}$$

$$\hat{T}(E4)=\epsilon_{dd}^{(4)}(r)(d^\dagger\tilde{d})^{(4)}+\epsilon_{gs}^{(4)}(r)(g^\dagger\tilde{s}+s^\dagger\tilde{g})^{(4)}+\epsilon_{gd}^{(4)}(r)(g^\dagger\tilde{d}+d^\dagger\tilde{g})^{(4)}+\epsilon_{gg}^{(4)}(r)(g^\dagger\tilde{g})^{(4)}\ . \tag{4c}$$

The first term in (4a) is really nonessential; it can, in fact, be written as the sum of a c number and a term proportional to $(d^\dagger\tilde{d})^{(0)}$ because the total number N of bosons must be conserved. Therefore, formula (4a) can be written

$$\hat{T}(E0)=\epsilon_{ss}^{(0)}(r)N+[\epsilon_{dd}^{(0)}(r)-\sqrt{5}\epsilon_{ss}^{(0)}(r)](d^\dagger\tilde{d})^{(0)}+\epsilon_{gg}^{(0)}(r)(g^\dagger\tilde{g})^{(0)}\ , \tag{4d}$$

where the first term on the right-hand side of form (4d) has only diagonal matrix elements and thus does not contribute to $E0$ transitions.

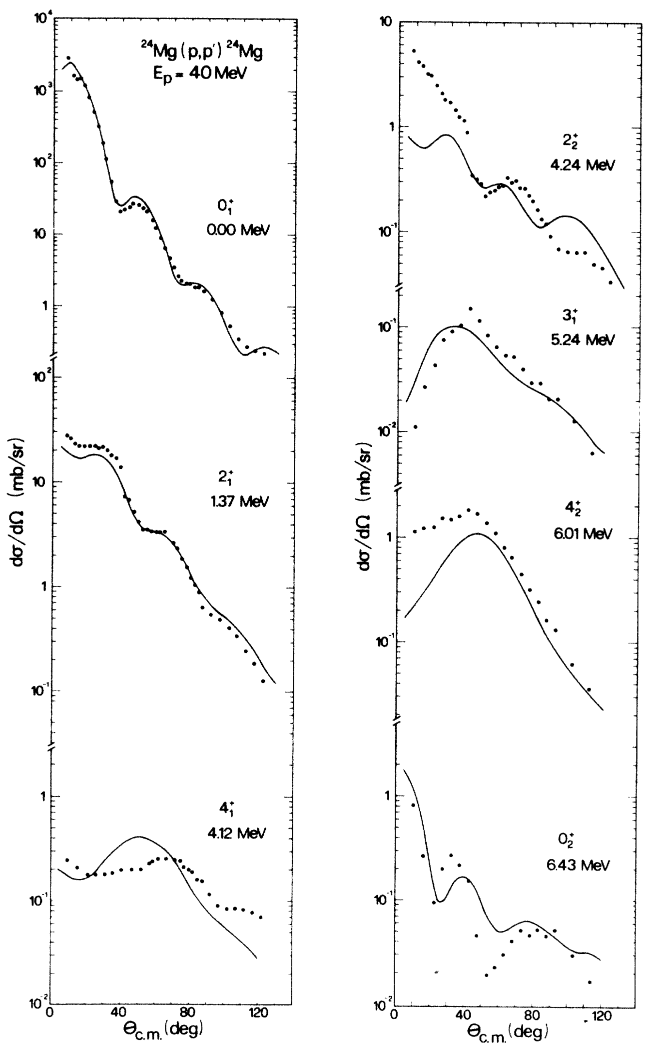

FIG. 3. Elastic and inelastic scattering cross sections of 40 MeV protons by ^{24}Mg; experimental data are taken from Ref. 9. Solid lines: coupled-channel calculations with the IBM transition matrix elements.

The related transition densities can be obtained by taking matrix elements of the transition operators (4b)—(4d) between initial and final states.

For example,

$$\rho_{I_i\to I_f}^{(2)}(r)=\epsilon_{ds}^{(2)}(r)\langle I_f||(d^\dagger\tilde{s}+s^\dagger\tilde{d})^{(2)}||I_i\rangle+\epsilon_{dd}^{(2)}(r)\langle I_f||(d^\dagger\tilde{d})^{(2)}||I_i\rangle+\epsilon_{gd}^{(2)}(r)\langle I_f||(g^\dagger\tilde{d}+d^\dagger\tilde{g})^{(2)}||I_i\rangle+\epsilon_{gg}^{(2)}(r)\langle I_f||(g^\dagger\tilde{g})^{(2)}||I_i\rangle\ . \tag{5}$$

The reduced matrix elements on the right-hand side of formula (5) are numbers which contain the nuclear structure information and can be calculated using the program PHINTL.[12] From the transition densities one obtains the $B(E\lambda)$ values for $\lambda=2$ and 4:

$$B(E\lambda;I_i\to I_f)=\frac{1}{2I_i+1}\left[\int_0^\infty r^{2+\lambda}\rho_{I_i\to I_f}(r)dr\right]^2\ , \tag{6}$$

and, for the monopole transition matrix element,

$$M(E0;0_i^+\to 0_f^+)=4\pi\int_0^\infty r^4\rho_{0_i^+\to 0_f^+}(r)dr\ . \tag{7}$$

We note that, for $\lambda=2$ and 4,

$$\int_0^\infty \epsilon_{ij}^{(\lambda)}(r)r^{2+\lambda}dr=e_{ij}^{(\lambda)}\ , \tag{8a}$$

and, for $\lambda=0$,

$$4\pi \int_0^\infty \epsilon_{ij}^{(0)}(r) r^4 dr = e_{ij}^{(0)} , \quad (8b)$$

where the $e_{ij}^{(\lambda)}$ coefficients are the effective boson charges of Ref. 8.

III. THE INTERACTING BOSON MODEL CALCULATIONS

Comparison between the calculated and experimental energy spectra of ^{24}Mg and ^{32}S is shown in Figs. 1 and 2, respectively. Tables I and II show the calculated and experimental[3,13–16] values of the $E0$, $E2$, and $E4$ matrix elements and of the quadrupole moments of the 2_1^+ and 2_2^+ states of ^{24}Mg and ^{32}S, respectively. The adopted parameter values in the Hamiltonian (1,2,3) are listed in Table III. In particular, for the unperturbed g-boson energies ϵ_g, values are taken which are about 1 MeV above 2Δ,[17] where Δ is the pairing energy, estimated to be $11A^{-1/2}$ MeV.

The effective boson charges shown in Tables I and II, for the different multipolarities, are determined by equating the transition strengths, calculated following Eqs. (6) and (7), with the corresponding experimental ones.

In detail, for $\lambda=0$, neglecting the small contribution of the term proportional to $(g^\dagger \tilde{g})^{(0)}$, we need $e_{dd}^{(0)}$ and $e_{ss}^{(0)}$ both for ^{24}Mg and ^{32}S. Since we have, for each nucleus, only one experimental value of the matrix element (for the transition $0_2^+ \to 0_1^+$), we have to determine the considered effective charges tentatively by fitting both the proton scattering cross sections and the observed monopole transition strength.

For $\lambda=2$, there are many experimental electric transition strengths. We determine the four effective charges $e_{ds}^{(2)}$, $e_{dd}^{(2)}$, $e_{gd}^{(2)}$, and $e_{gg}^{(2)}$ for each nucleus with a χ^2 fit to the available data.

For $\lambda=4$ two hexadecapole transition strengths are measured for ^{24}Mg and none for ^{32}S, while at least four of them are needed for each nucleus to fix its effective charges $e_{dd}^{(4)}$, $e_{gs}^{(4)}$, $e_{gd}^{(4)}$, and $e_{gg}^{(4)}$.

We replace the missing experimental values with the corresponding ones predicted by the shell-model calculations,[15] and by equating the calculated (IBM) and experimental (or shell-model deduced) hexadecapole strengths, we obtain the requested effective charges.

IV. PROTON SCATTERING ANALYSIS

The elastic and inelastic proton scattering cross sections are calculated by means of a coupled-channel program.[18] The parameter values of the optical potential used in this analysis are the same as Ref. 19.

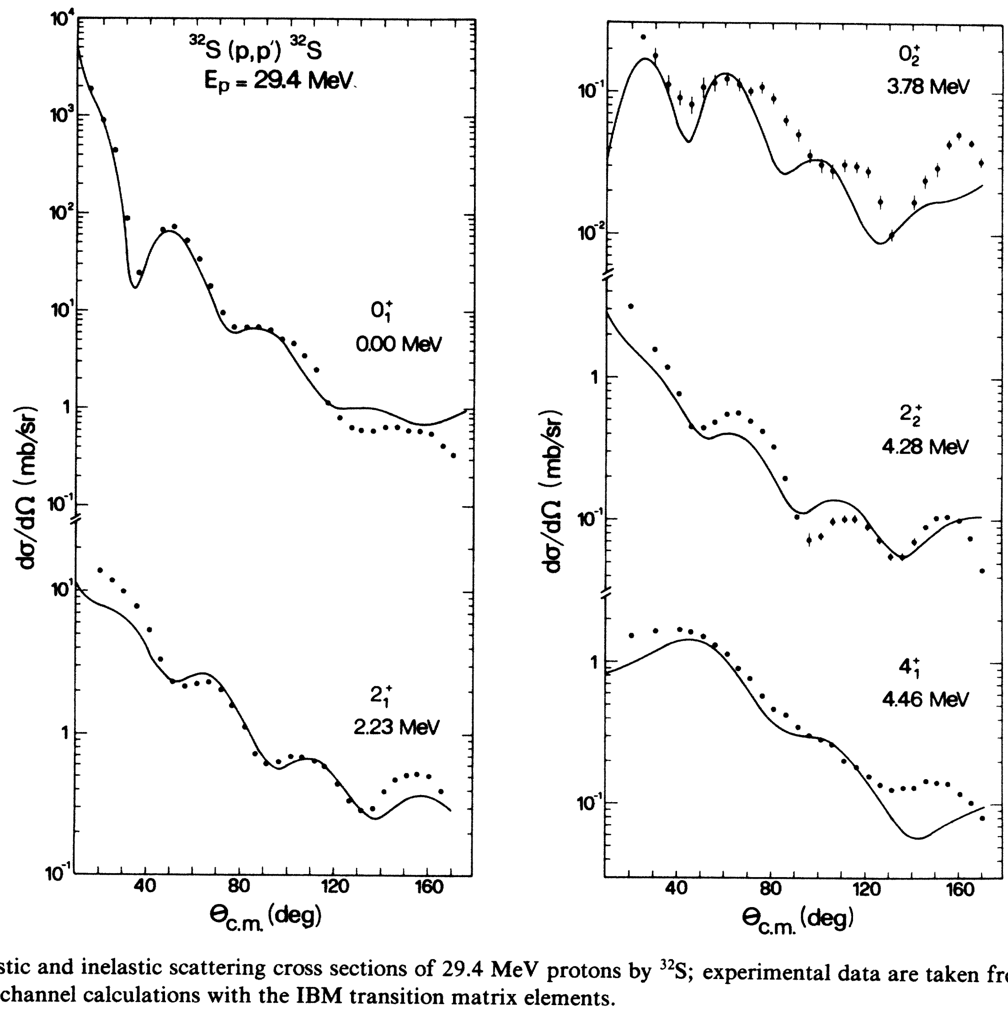

FIG. 4. Elastic and inelastic scattering cross sections of 29.4 MeV protons by ^{32}S; experimental data are taken from Ref. 10. Solid lines: coupled-channel calculations with the IBM transition matrix elements.

The main inputs in our program are the transition densities (5). The reduced matrix elements of the boson operators, appearing in formula (5), are calculated with the program PHINTL, as mentioned in Sec. III.

The functions $\epsilon_{ij}^{(\lambda)}(r)$, appearing in formula (5) too, cannot be easily calculated in the IBM framework. Some of them had been obtained in a number of IBM calculations, for other nuclei, by fitting the form factors. In a recent paper[5] $\epsilon_{ds}^{(2)}(r)$ and $\epsilon_{dd}^{(2)}(r)$ are calculated microscopically, explicitly including core polarization, for some nuclei of the *sd* shell.

We assume that the radial functions $\epsilon_{ij}^{(\lambda)}(r)$ can be approximated by the first or second derivatives of the Woods-Saxon potential well, following—roughly—the suggestions of the geometrical model, of the fitting the electron scattering form factors,[20,21] and of the microscopic shell model[5] and Hartree-Fock-Bogoliubov plus collective model[22] calculations, but the choice of the shape of the $\epsilon_{ij}^{(\lambda)}(r)$ depends, above all, on the goodness of the fits to the elastic and inelastic scattering cross sections they are able to give.

More precisely, for $\lambda=2$, $\epsilon_{ds}^{(2)}(r)$ is the first derivative and $\epsilon_{dd}^{(2)}(r)$, $\epsilon_{dg}^{(2)}(r)$, and $\epsilon_{gg}^{(2)}(r)$ the second derivatives of the Woods-Saxon well, with the same geometrical parameters of the central potential for ^{32}S, whereas for ^{24}Mg the second derivatives have a smaller radius $r_0=1.05$ fm. For $\lambda=4$, all the $\epsilon_{ij}^{(4)}(r)$ are the first derivatives of the Woods-Saxon well, with the same geometrical parameter values of the central potential for ^{24}Mg, and with a smaller radius $r_0=1.10$ fm for ^{32}S. For $\lambda=0$, $\epsilon_{dd}^{(0)}(r)$ is the second derivative of the Woods-Saxon well with the same geometrical parameters of the central potential for ^{32}S and with a smaller radius $r_0=1.07$ fm for ^{24}Mg; $\epsilon_{ss}^{(0)}(r)$ is the first derivative of the central well with a higher radius $r_0=1.45$ fm for ^{32}S, whereas for ^{24}Mg this function has been kept equal to zero because its contribution does not determine the quality of the fits to the scattering cross sections.

We note that all the functions $\epsilon_{ij}^{(\lambda)}(r)$ are normalized in order to satisfy relations (8a) and (8b). The calculated and experimental scattering cross sections are shown in Figs. 3 and 4 for ^{24}Mg and ^{32}S, respectively.

The comparison between theory and experiment shows that the present analysis gives results of comparable quality with those of the geometrical description in all the cases for which it accounts for the observed data, and is successful also when the geometrical approach failed (e.g., for the inelastic scattering to the 3_1^+ state of ^{24}Mg).[9,19]

We outline that in this analysis we have concentrated our attention on gross features; in fact, the functions $\epsilon_{ij}^{(\lambda)}(r)$, used in the calculations, are only empirical curves and the proton scattering cross sections are sensitive to their radial shapes.

V. CONCLUSIONS

The acceptable agreement between theory and experiment for energy spectra, electric transition strengths, and proton scattering cross sections allows us to conclude that the interacting boson model works in the *sd* shell too. Its main advantage lies in the fact that nuclei with completely different structure, such as ^{24}Mg and ^{32}S, can be treated within the same theoretical framework.

The detailed knowledge of the radial shape of the transition densities would improve the fits. In fact, it is the aim of the present approach to attempt a unified description of inelastic scattering of several projectiles, including protons, α particles, and electrons, in order to extract detailed information on the nuclear form factors.

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