Density matrix expansion for microscopic optical model potential

Taksu Cheon
Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824
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An approximation scheme for evaluating the folding optical model potential for nucleon-nucleus scattering is developed. The microscopic optical model potential is expressed as a simple algebraic function of nuclear densities. Nonlocality and finite-range effects are properly incorporated. A few numerical examples are given.

I. INTRODUCTION

While there have been intensive studies of the microscopic optical model potential for nucleon-nucleus scattering,[1] most of the proton scattering data are still analyzed with conventional phenomenological potentials and its parameters are adjusted in each experiment in *ad hoc* manner. The reason for its unpopularity may lie in that (1) the microscopic optical model involves a cumbersome folding procedure, and (2) there are several unexamined assumptions which make the theoretical prediction uncertain. We do not discuss the local density approximation which we need to resort to when we apply the nuclear matter scattering matrix to a finite system. This cannot be termed an approximation in a strict sense, since it only has phenomenological justification, if any at all. The effective two-nucleon interaction obtained in this or another way is, in general, a nonlocal operator. The validity of commonly used localization procedure has not been checked. The relation between various approximations to handle the exchange nonlocality is also not clear. These approximations need to be controlled before *quantitative* success (or failure) of the microscopic optical model potential is claimed.

In this paper, we develop an approximation scheme for the evaluation of the microscopic folding potential with a general nonlocal effective two nucleon force. Our method is closely related to the density matrix expansion theory of Negele and Vautherin[2] which was very successful in the calculation of the bound state properties of finite nuclei. The resulting optical model potential is expressed as the algebraic function of nuclear densities and its radial derivatives. Nonlocality and finite-range correction terms are separated from the bulk volume effect. We hope our formalism helps to make the microscopic optical model potential more easily accessible.

Though our treatment is based on the nonrelativistic theory, presently popular relativistic dynamics[3–5] can be easily incorporated, since it is known that Dirac dynamics can be simulated or reinterpreted in terms of a density-dependent repulsive force[6] or Lorentz-Lorenz correction to the momentum-dependent effective interaction.[7]

The formalism is described in Sec. II. Illustrative numerical examples are given in Sec. III. The last section contains concluding remarks.

II. DENSITY MATRIX EXPANSION OF FOLDING POTENTIAL

We assume that the effective nucleon-nucleon interaction v to be used for the nucleon-nucleus scattering is, as is the case in all calculable theories, local with respect to the center-of-mass coordinate:

$$v(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_1',\mathbf{r}_2')=v(\mathbf{r},\mathbf{r}';R)\cdot\delta(\mathbf{R}-\mathbf{R}')\ , \tag{1}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the coordinates of two nucleons in the initial states, and $\mathbf{r}_1'$ and $\mathbf{r}_2'$ those of two nucleons in the final states. The relative and center-of-mass coordinates $\mathbf{r}$, $\mathbf{r}'$, $\mathbf{R}$, and $\mathbf{R}'$ are defined as

$$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2\ , \tag{2a}$$

$$\mathbf{r}'=\mathbf{r}_1'-\mathbf{r}_2'\ , \tag{2b}$$

$$\mathbf{R}=\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\ , \tag{2c}$$

$$\mathbf{R}'=\frac{\mathbf{r}_1'+\mathbf{r}_2'}{2}\ . \tag{2d}$$

The impulse approximation[8] gives an effective force which is independent of $\mathbf{R}$. Nuclear matter g-matrix theory[9,10] gives a density dependent force $v(\mathbf{r},\mathbf{r}';\rho)$ which is usually used combined with the local density assumption $\rho=\rho(R)$. There is ambiguity regarding at which point the density should be evaluated. We deliberately leave this point open for later consideration, and hence suppress the variable $\mathbf{R}$ in the effective force $v(\mathbf{r},\mathbf{r}';\mathbf{R})$.

We define a matrix element M which represents the scattering between projectile nucleon and target nucleus,

$$M=\int d\mathbf{r}_1\int d\mathbf{r}_2\int d\mathbf{r}_1'\int d\mathbf{r}_2'\delta(\mathbf{R}-\mathbf{R}')\psi^*(\mathbf{r}_1)\times\sum_b\phi_b^*(\mathbf{r}_2)v(\mathbf{r},\mathbf{r}')\phi_b(\mathbf{r}_2')\psi(\mathbf{r}_1')\ , \tag{3}$$

where ψ is the projectile wave function and the ϕ_b's are the target wave functions, the sum running over the filled target states. The knock-on exchange process is taken into account by the antisymmetrization of the effective force,

$$v(\mathbf{r},\mathbf{r}')=v^d(\mathbf{r},\mathbf{r}')+v^{\mathrm{ex}}(\mathbf{r},-\mathbf{r}')\ . \tag{4}$$

The optical model potential can be defined as an operator U which satisfies the equation

$$M=\int d\mathbf{r}_1\psi^*(\mathbf{r}_1)U(\mathbf{r}_1)\psi(\mathbf{r}_1)\ . \tag{5}$$

The term microscopic optical model potential designates the potential U, which is expressed in terms of effective force v. One cannot write the explicit expression for U, however, from Eqs. (3) and (5) as they are. We look for an approximation scheme to this end which is tractable, accurate, and hopefully gives us insight into the underlying physics. For pedagogical reasons, we first restrict the effective force to be local and then generalize to the nonlocal cases.

A. Local force

If the nucleon-nucleon force is local with respect to the relative coordinate, i.e.,

$$v(\mathbf{r},\mathbf{r}')=v(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')\ , \tag{6}$$

Eq. (3) takes the form

$$M=\int d\mathbf{r}_1\int d\mathbf{r}_2\psi^*(\mathbf{r}_1)\sum_b\phi_b^*(\mathbf{r}_2)v^d(\mathbf{r}_1-\mathbf{r}_2)\phi_b(\mathbf{r}_2)\psi(\mathbf{r}_1)+\int d\mathbf{r}_1\int d\mathbf{r}_2\psi^*(\mathbf{r}_1)\sum_b\phi_b^*(\mathbf{r}_2)v^{\rm ex}(\mathbf{r}_1-\mathbf{r}_2)\phi_b(\mathbf{r}_1)\psi(\mathbf{r}_2)\ . \tag{7}$$

We define the displacement operator $e^{\mathbf{r}\cdot\nabla}$ which, for example, acts on wave function as

$$\psi(\mathbf{r}_1+\mathbf{r})=e^{\mathbf{r}\cdot\nabla}\psi(\mathbf{r}_1)\ .$$

This operator can be expanded to its multipoles in the form

$$\begin{aligned}e^{\mathbf{r}\cdot\nabla}&=\sum 4\pi i^{-l}j_l(ir\nabla)Y_{lm}^*(\hat{\mathbf{r}})Y_{lm}(\nabla)\\&\approx j_0(ir\nabla)+\frac{3}{ir\nabla}j_1(ir\nabla)\mathbf{r}\cdot\nabla+\cdots\\&=f_0(ir\nabla)+f_1(ir\nabla)\mathbf{r}\cdot\nabla\ ,\end{aligned} \tag{8}$$

where the last line defines the operator f_0 and f_1, which can again be expanded in terms of r and ∇ as

$$f_0(ir\nabla)=j_0(ir\nabla)\approx 1+\tfrac{1}{6}r^2\nabla^2+\tfrac{1}{120}r^4\nabla^4+\cdots\ , \tag{9a}$$

$$f_1(ir\nabla)=\frac{3}{ir\nabla}j_1(ir\nabla)\approx 1+\tfrac{1}{10}r^2\nabla^2+\tfrac{1}{280}r^4\nabla^4+\cdots\ . \tag{9b}$$

With a change of integration variables, the direct part (first term) in Eq. (7) is rewritten as

$$M^d=\int d\mathbf{r}_1\int d\mathbf{r}\,\psi^*(\mathbf{r}_1)\psi(\mathbf{r}_1)\rho(\mathbf{r}_1-\mathbf{r})v^d(\mathbf{r})\ , \tag{10}$$

where the target density ρ is defined by

$$\rho(r)=\sum_b\phi_b^*(r)\phi_b(r)\ . \tag{11}$$

Multipole expansion of the displacement operator, Eq. (8) can be applied to the $\rho(\mathbf{r}_1-\mathbf{r})$ in Eq. (10),

$$\rho(\mathbf{r}_1-\mathbf{r})=e^{-\mathbf{r}\cdot\nabla}\rho(\mathbf{r}_1)=(f_0-f_1\mathbf{r}\cdot\nabla)\rho(r_1)\ . \tag{12}$$

Noticing that integration by $\mathbf{r}$ eliminates all the terms in Eq. (10) except the first, for example,

$$\int d\mathbf{r}\,\mathbf{r}\cdot\nabla A(r)=0\ ,$$

one obtains

$$U^d(r_1)=\int d\mathbf{r}f_0(ir\nabla)\rho(r_1)v(r)\ . \tag{13}$$

This is still an exact equation. If expansion (9a) is applied to second order, one has

$$U^d(r)=\rho(r)\int d\mathbf{r}'v(r')+\nabla^2\rho(r)\tfrac{1}{6}\int d\mathbf{r}'r'^2v(r')\ . \tag{14}$$

For the exchange part of Eq. (7), a similar technique leads to the equation

$$\begin{aligned}M^{\rm ex}&=\int d\mathbf{r}_1\int d\mathbf{r}\,\psi^*(\mathbf{r}_1)\psi(\mathbf{r}_1-\mathbf{r})v(r)\sum_b\phi_b^*(\mathbf{r}_1-\mathbf{r})\phi_b(\mathbf{r}_1)\\&\approx\int d\mathbf{r}_1\int d\mathbf{r}\,\psi^*(\mathbf{r}_1)(f_0-f_1\mathbf{r}\cdot\nabla)\psi(\mathbf{r}_1)v(r)\sum_b(f_0-f_1\mathbf{r}\cdot\nabla)\phi_b^*(\mathbf{r}_1)\phi_b(\mathbf{r}_1)\\&=\int d\mathbf{r}_1\int d\mathbf{r}\left[\psi^*(\mathbf{r}_1)f_0\psi(\mathbf{r}_1)v(r)\sum_b f_0\phi_b^*(\mathbf{r}_1)\phi_b(\mathbf{r}_1)+\tfrac{1}{3}r^2\psi^*(\mathbf{r}_1)f_1\nabla\psi(\mathbf{r}_1)v(r)\cdot\sum_b f_1\nabla\phi_b^*(\mathbf{r}_1)\phi_b(\mathbf{r}_1)\right]\ .\end{aligned} \tag{15}$$

To obtain the third line, we used

$$\int d\mathbf{r}\,\mathbf{r}\cdot\mathbf{A}\,\mathbf{r}\cdot\mathbf{B}=\tfrac{1}{3}\int d\mathbf{r}\,r^2\mathbf{A}\cdot\mathbf{B}\ .$$

If we expand f_0 and f_1 by Eqs. (9) and truncate them at the second order, we get an equation very similar to Eq. (5.1) of Ref. 2. However, this causes a problem concerning the convergence of the expansion when applied to the scattering kinematics. Expansion equations (9) applied to Eq. (15) amount to expanding f_0 and f_1 in the parameter (r_0k_0), where r_0 is the range of force and k_0 the momentum of the projectile. This parameter is a quantity of the order of 10 and convergence becomes poorer as the projectile energy increases. We propose instead a different expansion which suits the kinematics of intermediate energy proton scattering. The differential operator acting on the

projectile wave function can be replaced by the local momentum of the projectile,

$$i\nabla\psi(r)=\mathbf{k}\psi(r) . \tag{16}$$

The energy-momentum relation for the projectile nucleon is

$$k_0^2=k^2+2mU , \tag{17}$$

where k_0 is the incident momentum given by

$$E_p=k_0^2/2m .$$

In Eq. (17), U is the optical model potential itself. When the projectile incident energy is sufficiently high compared to the potential energy ($E_p > 100$ MeV), it is advantageous to expand k around k_0.[11] Using the relation

$$j_0(z)\approx j_0(z_0)+(z-z_0)\frac{d}{dz}j_0(z_0)$$
$$\approx j_0(z_0)-\frac{z^2-z_0^2}{2z_0}j_1(z_0) , \tag{18}$$

one obtains

$$f_0(kr)\approx f_0(k_0r)+\tfrac{1}{6}2mU(r)r^2f_1(k_0r) . \tag{19}$$

If one applies Eq. (19) to f_0 standing before ψ in Eq. (15), one has

$$U^{\rm ex}(r_1)=\int d\mathbf{r}\sum_b[f_0(ir\nabla)\phi_b^*(\mathbf{r}_1)]\phi_b(\mathbf{r}_1)v(r)\left[f_0(k_0r)+\frac{2m}{6}U(r)r^2f_1(k_0r)\right] . \tag{20}$$

The second term in Eq. (15) can be neglected, since

$$\int d\mathbf{r}_1\mathbf{k}\cdot\nabla\rho(r)\approx\int dr\,\mathbf{k}_0\cdot\nabla\rho(r)=0 .$$

Expanding $f_0(ir\nabla)$ before ϕ using Eq. (9a), one gets

$$\sum_b[f_0(ir\nabla)\phi_b^*(\mathbf{r})]\phi_b(\mathbf{r})\approx\sum_b\phi_b^*(\mathbf{r})\phi_b(\mathbf{r})+\sum\tfrac{1}{6}[\nabla^2\phi_b^*(\mathbf{r})]\phi_b(\mathbf{r})=\rho(r)+\tfrac{1}{12}\nabla^2\rho(r)-\tfrac{1}{6}\tau(r) , \tag{21}$$

where τ is the momentum density of the target defined by

$$\tau(r)=\sum_b\nabla\phi_b^*(r)\cdot\nabla\phi_b(r) . \tag{22}$$

We now obtain a recursive equation:

$$U^{\rm ex}(r_1)=\rho(r_1)\int d\mathbf{r}\,v(r)f_0(k_0r)+\rho(r_1)\frac{2m}{6}\int d\mathbf{r}\,r^2v(r)f_1(k_0r)U(r)+[\nabla^2\rho(r_1)-2\tau(r_1)]\tfrac{1}{12}\int d\mathbf{r}\,r^2v(r)f_0(k_0r) . \tag{23}$$

If we combine Eqs. (14) and (23), and replace U in the second term of Eq. (23) by the leading terms, we arrive at the following expression for the microscopic optical model potential,

$$U(r)=(1+\gamma)A\rho(r)+B\nabla^2\rho(r)+C\tau(r) , \tag{24}$$

where the coefficients A, B, C, and γ are given, respectively, by

$$A=\int d\mathbf{r}\,v^d(r)+\int d\mathbf{r}\,v^{\rm ex}(r)f_0(k_0r) , \tag{25a}$$

$$B=\tfrac{1}{6}\int d\mathbf{r}\,v^d(r)r^2+\tfrac{1}{12}\int d\mathbf{r}\,v^{\rm ex}(r)r^2f_0(k_0r) , \tag{25b}$$

$$C=-\tfrac{1}{6}\int d\mathbf{r}\,v^{\rm ex}(r)r^2f_0(k_0r) , \tag{25c}$$

and

$$\gamma=\tfrac{1}{6}2m\rho_0\int d\mathbf{r}\,v^{\rm ex}(r)r^2f_1(k_0r) . \tag{25d}$$

The optical model potential equation (24) is written as a linear combination of nuclear densities. The leading term $A\rho$ represents the bulk volume contribution, $B\nabla^2\rho$ the finite range correction, $C\tau$ the contribution from the momentum density, and γ gives the measure of exchange nonlocality. The coefficients A, B, C, and γ are numbers if the two-nucleon force v is density independent, while they can be functions of density if v depends on density. The most convenient choice of density dependence is

$$v(r)=v(r;\rho(r_1)) , \tag{26}$$

which yields $A=A(\rho(r_1))$, etc. Though the momentum density τ is calculable from the target wave functions, it may be useful in the simplified calculations to express τ in terms of ρ using the Thomas-Fermi approximation, namely

$$\tau(r)\approx\tfrac{3}{5}k_F^2\rho(r)=\tfrac{3}{5}c_k[\rho(r)]^{5/3} , \tag{27}$$

where

$$c_k=\left[\frac{3\pi^2}{2}\right]^{2/3} .$$

Once coefficients A, B, C, and γ are calculated from whatever effective interaction v, the microscopic optical model potential equation (24) is ready to use. The other more important advantage of Eqs. (24) and (25) is that the various corrections to the bulk volume effect are disentangled and expressed as separate terms. We will see in the next section that, in actual proton scattering of a few hundred MeV, the main corrections to the leading volume term turn out to be the finite range effect and the nonlocality effect, and momentum density correction is small.

At this point, we want to comment on the relation between our treatment and that of the original density matrix expansion of Negele and Vautherin.[2] Even when k_0

is set equal to zero in Eqs. (25), we do not get the equations found in the power of Negele and Vautherin, which, in our notation, read, for example,

$$B=\tfrac{1}{6}\int d\mathbf{r}\, v^d(r)r^2 f_3(k_f r)+\tfrac{1}{12}\int d\mathbf{r}\, v^{\rm ex}(r)r^2 f_3(k_0 r)\ , \tag{25b'}$$

where

$$f_3(x)=1-\tfrac{1}{18}x^2+\cdots\ .$$

This difference is actually a reflection of the ambiguity of the r^4 and higher power contributions associated with the truncation of fourth and higher derivatives of nuclear density. The choice of the coefficient of r^4 and higher power terms is, as noted as Negele and Vautherin, a matter of convenience and can be adjusted to achieve better convergence in the truncated series. In our treatment, however, this does not cause any practical problem since the factors f_0 and f_1 in Eqs. (25) already ensure fast convergence.

B. L-S force

The two-body L-S force induces a spin-orbit term in the optical model potential. We only consider the spin-saturated nucleus. We write the L-S coupling part of the nucleon-nucleon effective interaction in the form

$$V(r_1,r_2)=v_{ls}(r_1-r_2)\frac{i}{2}(\mathbf{r}_1-\mathbf{r}_2)\times(\nabla_1-\nabla_2)\cdot(\mathbf{s}_1-\mathbf{s}_2)\ . \tag{28}$$

The direct piece of the scattering matrix equation (7) is

$$\begin{aligned}M_{ls}^d&=\int d\mathbf{r}_1\int d\mathbf{r}\,\psi(r_1)v_{ls}^d(r)\rho(r_1-r)\frac{i}{2}\mathbf{r}\times\nabla\psi(r_1)\cdot\mathbf{s}_1\\&=-\int d\mathbf{r}_1\int d\mathbf{r}\,\psi(r_1)v_{ls}^d(r)f_1\mathbf{r}\cdot\nabla\rho(r_1)\frac{i}{2}\mathbf{r}\times\nabla\psi(r_1)\cdot\mathbf{s}_1\\&=-\frac{i}{6}\int d\mathbf{r}_1\int d\mathbf{r}\,r^2\psi(r_1)v_{ls}^d(r)f_1\mathbf{r}\cdot\nabla\rho(r_1)\times\nabla\psi(r_1)\cdot\mathbf{s}_1\ .\end{aligned} \tag{29}$$

To get the third line, we used the relation

$$\int d\mathbf{r}\,\mathbf{r}\cdot\mathbf{A}\,\mathbf{r}\times\mathbf{B}=\tfrac{1}{3}\int d\mathbf{r}\,r^2\mathbf{A}\times\mathbf{B}\ .$$

Comparing Eqs. (5) and (29), we obtain the direct part of the spin-orbit potential,

$$U_{ls}^d(r_1)=-\frac{1}{6}\frac{1}{r_1}\frac{d}{dr_1}\rho(r_1)\int d\mathbf{r}\,r^2 v_{ls}^d(r)\mathbf{l}_1\cdot\mathbf{s}_1\ . \tag{30}$$

We replaced $f_1(ir\nabla)$ in Eq. (29) by 1, since Eq. (29) already contains the r^2, which makes this a consistent level of truncation to other expressions for central force. It can be justified by the short-range nature of the L-S force.

The exchange piece of Eq. (7) is given by

$$\begin{aligned}M_{ls}^{\rm ex}&=\int d\mathbf{r}_1\int d\mathbf{r}_2\psi^*(r_1)\sum_b\phi_b^*(r_2)v_{ls}^{\rm ex}(r)\frac{i}{2}(\mathbf{r}_1-\mathbf{r}_2)\times(\nabla_1-\nabla_2)\phi_b(r_1)\psi(r_2)\cdot\mathbf{s}_1\\&=-\frac{i}{6}\int d\mathbf{r}_1\int d\mathbf{r}\,r^2\psi(r_1)v_{ls}^{\rm ex}(r)\sum_b[f_0\phi_b(r_1)\nabla\phi_b(r_1)\times f_1\nabla\psi(r_1)-f_1\nabla\phi_b(r_1)\times\phi_b(r_1)f_0\nabla\psi(r_1)]\cdot\mathbf{s}_1\ .\end{aligned} \tag{31}$$

Again neglecting $f_0(ir\nabla)$ and $f_1(ir\nabla)$ before ϕ_b, we obtain

$$\begin{aligned}U_{ls}^{\rm ex}(r_1)=&\frac{1}{6}\frac{1}{r_1}\frac{d}{dr_1}\rho(r_1)\\&\times\int d\mathbf{r}\,r^2 v_{ls}^{\rm ex}(r)\tfrac{1}{2}[f_1(kr)+f_0(kr)]\mathbf{l}_1\cdot\mathbf{s}_1\ ,\end{aligned} \tag{32}$$

which has a form slightly different from the "standard" result,[10] but, of course, reduces to the formula of Blin-Stoyle[12] in the short-range limit. The full expression for the spin-orbit optical model potential is

$$U_{ls}(r)=D\frac{1}{r}\frac{d}{dr}\rho(r)\mathbf{l}\cdot\mathbf{s}\ , \tag{33}$$

where

$$\begin{aligned}D=&-\tfrac{1}{6}\int d\mathbf{r}\,r^2 v_{ls}^d(r)\\&+\tfrac{1}{6}\int d\mathbf{r}\,r^2 v_{ls}^{\rm ex}(r)\tfrac{1}{2}[f_1(kr)+f_0(kr)]\ .\end{aligned} \tag{34}$$

C. Nonlocal force

The advantage of the density matrix expansion approach becomes obvious when we deal with the general nonlocal effective force. Though straightforward folding calculations are within reach of the modern computer, and are actually now in progress,[13] we believe it is useful to make a well defined approximation scheme to achieve a comprehensive understanding of underlying physics.

We introduce the new coordinate variable s with the equation

$$\mathbf{s}=2(\mathbf{r}_1-\mathbf{r}_1')\ . \tag{35}$$

The following relations between coordinate variables hold:

$$\mathbf{r}_2=\mathbf{r}_1-\mathbf{r}\ , \tag{36a}$$

$$\mathbf{r}_1'=\mathbf{r}_1-\mathbf{s}/2\ , \tag{36b}$$

$$\mathbf{r}_2'=\mathbf{r}_1-\mathbf{r}+\mathbf{s}/2\ . \tag{36c}$$

The last relation is ensured by the locality assumption of the effective force with respect to the center-of-mass coordinate. We take $\mathbf{r}_1$, $\mathbf{r}$, and $\mathbf{s}$ for the new set of integral variables,

$$\int d\mathbf{r}_1\int d\mathbf{r}_2\int d\mathbf{r}_1'\int d\mathbf{r}_2'\delta(\mathbf{R}-\mathbf{R}')=\int d\mathbf{r}_1\int d\mathbf{r}\int d\mathbf{s}\ . \tag{37}$$

The scattering matrix element equation (6) now reads

$$M=\int d\mathbf{r}_1\int d\mathbf{r}\int d\mathbf{s}\,\psi^*(\mathbf{r}_1)\sum_b\phi_b^*(\mathbf{r}_1-\mathbf{r})v(\mathbf{r},\mathbf{r}-\mathbf{s})\times\phi_b\left[\mathbf{r}_1-\mathbf{r}+\frac{\mathbf{s}}{2}\right]\times\psi\left[\mathbf{r}_1-\frac{\mathbf{s}}{2}\right]. \tag{38}$$

The definition of the optical model potential equation (5) then yields

$$U=\int d\mathbf{r}\int d\mathbf{s}\,v(\mathbf{r},\mathbf{r}-\mathbf{s})\sum_b e^{-\mathbf{r}\cdot\nabla}\phi_b^*(r_1)e^{(-\mathbf{r}+\mathbf{s}/2)\cdot\nabla}\times\phi_b(r_1)e^{-\mathbf{s}/2\cdot\nabla}$$

$$\approx\int d\mathbf{r}\int d\mathbf{s}\,v(\mathbf{r},\mathbf{r}-\mathbf{s})(a+b+c)\,, \tag{39}$$

where

$$a=\sum_b f_0(ir\nabla)\phi_b^*(r_1)f_0\left[i\left|\mathbf{r}-\frac{\mathbf{s}}{2}\right|\nabla\right]\phi_b(r_1)f_0(is\nabla)\,, \tag{40a}$$

$$b=\frac{r^2}{3}\sum_b f_1(ir\nabla)\nabla\phi_b^*(r_1)f_0\left[i\left|\mathbf{r}-\frac{\mathbf{s}}{2}\right|\nabla\right]\cdot\nabla\times\phi_b(r_1)f_0(is\nabla)\,, \tag{40b}$$

and

$$c=-\sum_b f_1(ir\nabla)\mathbf{r}\cdot\nabla\,\phi_b^*(r_1)f_0\left[i\left|\mathbf{r}-\frac{\mathbf{s}}{2}\right|\nabla\right]\frac{\mathbf{s}}{2}\cdot\nabla\times\phi_b(r_1)f_0(is\nabla)\,. \tag{40c}$$

Expanding f_0 and f_1 before the target wave function and taking up to the order of r^2 as before, one has

$$a=\rho(r_1)+\frac{1}{6}\left[r^2-\tfrac{1}{2}\mathbf{r}\cdot\mathbf{s}+\tfrac{1}{8}s^2\right][\nabla^2\rho(r_1)-2\tau(r_1)]\,, \tag{41a}$$

$$b=\tfrac{1}{3}r^2\tau(r_1)f_0(is\nabla)\,, \tag{41b}$$

and

$$c=-\frac{1}{6}\mathbf{r}\cdot\mathbf{s}\,\tau(r_1)f_0(is\nabla)+\Delta\,, \tag{41c}$$

where

$$\Delta=\sum_i\sum_j r_i s_j(1-\delta_{ij})\nabla_i\phi_b^*(r_1)\nabla_j\phi_b(r_1)f_0(is\nabla)$$

is the traceless tensor part of momentum density which we neglect as being small because of its incoherence. The final expression for the microscopic optical model potential with a general nonlocal operator has the same formal structure as the local case,

$$U(r)=(1+\gamma)A\rho(r)+B\nabla^2\rho(r)+C\tau(r)\,. \tag{42}$$

The coefficients A, B, C, and γ are now given by double integrals

$$A=\int d\mathbf{r}\int d\mathbf{s}\,v(\mathbf{r},\mathbf{r}-\mathbf{s})f_0\left[\frac{k_0 s}{2}\right], \tag{43a}$$

$$B=\tfrac{1}{6}\int d\mathbf{r}\int d\mathbf{s}(r^2-\tfrac{1}{2}\mathbf{r}\cdot\mathbf{s}+\tfrac{1}{8}s^2)v(\mathbf{r},\mathbf{r}-\mathbf{s})f_0\left[\frac{k_0 s}{2}\right], \tag{43b}$$

$$C=-\tfrac{1}{6}\int d\mathbf{r}\int d\mathbf{s}\,\tfrac{1}{4}s^2 v(\mathbf{r},\mathbf{r}-\mathbf{s})f_0\left[\frac{k_0 s}{2}\right], \tag{43c}$$

$$\gamma=\tfrac{1}{6}2m\rho_0\int d\mathbf{r}\int d\mathbf{s}\,\tfrac{1}{4}s^2 v(\mathbf{r},\mathbf{r}-\mathbf{s})f_1\left[\frac{k_0 s}{2}\right]. \tag{43d}$$

One can easily check that Eqs. (43) reduce to Eqs. (25) by setting the effective force v local. The physical meaning of each term is identical to the local case. The nonlocal correction γ has a contribution from the "genuine" nonlocality of the effective force. We can perform the direct calculation of the folding optical model potential without resorting to the conventional intermediate step of localizing the effective force.

III. NUMERICAL EXAMPLES

A. Local force

We give two numerical examples to illustrate our method. Systematic calculations and comparison with experiments are left for future work.

The existing localized effective forces[14–16] can be easily transformed into the parameters A, B, C, D, and γ using Eqs. (25) and (34). With the Franey-Love t matrix[17] at $E_p=210$ MeV, one gets

$$A=-174.6-206.3i\ \text{MeV fm}^3\,, \tag{44a}$$

$$B=-118.7-76.0i\ \text{MeV fm}^5\,, \tag{44b}$$

$$C=4.1-3.9i\ \text{MeV fm}^5\,, \tag{44c}$$

$$D=22.5-4.7i\ \text{MeV fm}^5\,, \tag{44d}$$

$$\gamma=-0.07+0.03i\,. \tag{44e}$$

We apply this to the proton scattering on ^{40}Ca at $E_p=180$ MeV. The volume integral per nucleon K and root mean square radius R of the optical model potential, which are defined by

$$K=\begin{cases}\int d^3r\,U(r)/A & \text{(central potential)}\,, \quad (45a)\\ \int d^3r\,U(r)/A^{1/3} & \text{(spin-orbit potential)}\,, \quad (45b)\end{cases}$$

$$R=\left[\int d^3r\,r^2U(r)\Big/\int d^3r\,U(r)\right]^{1/2}, \tag{45c}$$

are known to be empirically well determined quantities. If the matter distribution of ^{40}Ca is given by a Woods-Saxon function

$$\rho(r)=\rho_0\left[1+\exp\left[\frac{r-R}{a}\right]\right]^{-1}, \tag{46}$$

with

$$R_0=3.69 \text{ fm}, \quad a=0.53 \text{ fm}, \quad \text{and } \rho_0=0.158 \text{ fm}^{-3}, \tag{47}$$

the parameter set (44) gives

$$\begin{aligned} &K_R^C=-156 \text{ MeV fm}^3, \quad K_I^C=-197 \text{ MeV fm}^3, \\ &R_R^C=4.08 \text{ m}, \quad R_I^C=3.79 \text{ fm}, \\ &K_R^{SO}=-96 \text{ MeV fm}^3, \quad K_I^{SO}=20 \text{ MeV fm}^3, \end{aligned} \tag{48}$$

and

$$R^{SO}=4.05 \text{ fm}.$$

Superscripts C and SO denote central and spin-orbit, and subscripts R and I denote the real and imaginary parts, respectively. These values are to be compared to the empirical values[18]

$$\begin{aligned} &K_R^C=-195 \text{ MeV fm}^3, \quad K_I^C=-115 \text{ MeV fm}^3, \\ &R_R^C=4.17 \text{ fm}, \quad R_I^C=4.45 \text{ fm}, \\ &K_R^{SO}=-88 \text{ MeV fm}^3, \quad K_I^{SO}=44 \text{ MeV fm}^3 \end{aligned} \tag{49}$$

and

$$R^{SO}=4.14 \text{ fm}.$$

Spin observables calculated with the parameter set (44) are shown in Figs. 1 and 2 with the experimental values.[4,19]

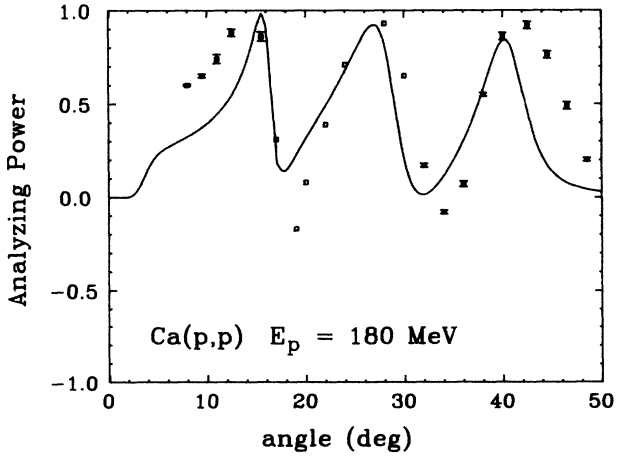

FIG. 1. Analyzing power for the 180 MeV proton scattered from ^{40}Ca. Solid line is the theoretical curve based on the optical model potential equations (24) and (33).

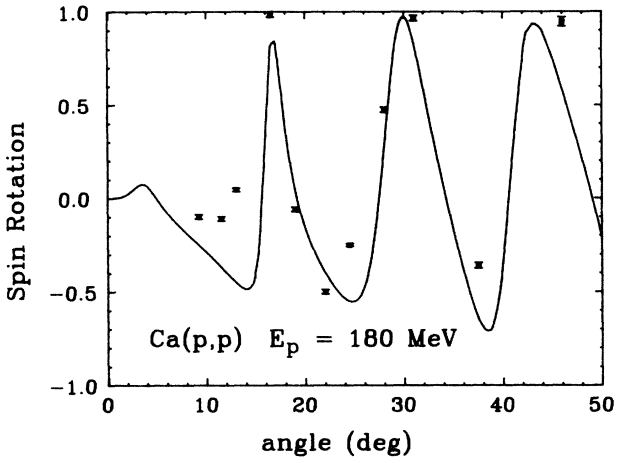

FIG. 2. Spin rotation for the 200 MeV proton scattered from ^{40}Ca.

The overall oscillatory structure is reproduced by the calculation, as could be anticipated by the correct values of K and R. For a better description, we need medium corrections and possibly the relativistic dynamics. A large portion of the contribution to B of Eq. (44b) comes from the direct part [i.e., the first term in Eq. (25b)], and B is as large as A. Actually, convergence of the expansion equation (14) is just marginal. In some cases, the exact folding with Eq. (13) might be necessary.

B. Separable force

To make calculations with nonlocal force feasible, one decomposes the interaction operator into partial waves,

$$v(\mathbf{r},\mathbf{r}')=\sum_{lm} Y_{lm}^*(\hat{\mathbf{r}})v_l(r,r')Y_{lm}(\hat{\mathbf{r}}'). \tag{50}$$

We can reduce the double integrals in Eqs. (43) to single integrals by (a) expanding the f_l's as

$$f_l(|\mathbf{r}-\mathbf{r}'|)\approx f_l(r)f_l(r')+\frac{1}{2l+3}\mathbf{r}\cdot\mathbf{r}'f_{l+1}(r)f_{l+1}(r'), \tag{51}$$

the function f_l being the generalization of f_0 and f_1 of Eqs. (9), which is defined by

$$f_l(x)=\frac{(2l+1)!}{l!}(2x)^{-l}j_l(x), \tag{52}$$

and (b) assuming a *separate form* of the interaction

$$v_l(r,r')=g_l(r)\lambda g_l(r'). \tag{53}$$

Taking only s and p waves, we have

TABLE I. Parameters for the separable potential equation (57). These values are obtained by the fitting of the N-N phase shift at $E_p = 25$—300 MeV.

		^1S	^3S	^1P	^3P
Real	β^2	4.7 fm^{-2}	16.0 fm^{-2}	10.0 fm^{-2}	12.0 fm^{-2}
	λ	-1.03×10^5 MeV fm^2	-5.88×10^5 MeV fm^2	7.70×10^6 MeV	-2.47×10^6 MeV
Imag	β^2	1.6 fm^{-2}	2.2 fm^{-2}	25.0 fm^{-2}	50.0 fm^{-2}
	λ	$-2.64\times10^4 i$ MeV fm^2	$-6.73\times10^4 i$ MeV fm^2	$-3.57\times10^7 i$ MeV	$-2.29\times10^8 i$ MeV

$$A = 4\pi\lambda I(2,0,0)^2 + \frac{4\pi\lambda}{3}\frac{k_0^2}{4} I(3,1,1)^2 , \tag{54a}$$

$$B = \frac{4\pi\lambda}{8} I(2,0,0)I(4,0,0) + \frac{4\pi\lambda}{216}\frac{k_0^2}{4} I(4,0,1)^2 + \frac{4\pi\lambda}{24} I(3,1,0)^2 + \frac{4\pi\lambda}{24}\frac{k_0^2}{4} I(3,1,1)I(5,1,1) , \tag{54b}$$

$$C = -\frac{4\pi\lambda}{12} I(2,0,0)I(4,0,0) + \frac{4\pi\lambda}{108}\frac{k_0^2}{4} I(4,0,1)^2 + \frac{4\pi\lambda}{12} I(3,1,0)^2 - \frac{4\pi\lambda}{36}\frac{k_0^2}{4} I(3,1,1)I(5,1,1) , \tag{54c}$$

and

$$\gamma = 2m\rho_0 \left[\frac{4\pi\lambda}{12} I(2,0,1)I(4,0,1) - \frac{4\pi\lambda}{180}\frac{k_0^2}{4} I(4,0,2)^2 - \frac{4\pi\lambda}{12} I(3,1,1)^2 + \frac{4\pi\lambda}{60}\frac{k_0^2}{4} I(3,1,2)I(5,1,2) \right] , \tag{54d}$$

where I is defined by

$$I(n,l,m) = \int dr\, r^n g_l(r) f_m\left(\frac{k_0 r}{2}\right) . \tag{55}$$

To incorporate the spin and isospin degrees of freedom, one needs to sum up the contribution from different spin-isospin channels as

$$Z = \tfrac{2}{16}[3Z(1,0) + 3Z(0,1) + Z(0,0) + 9Z(1,1)] , \tag{56}$$

where Z stands for the coefficients A, B, C, and γ, and the indices (S,T) denote the spin and isospin. Our choice of the functional form of $g_l(r)$ is of standard Yamaguchi type,[20] namely

$$g_0(r) = \frac{1}{4\pi}\frac{e^{-\beta r}}{r} \tag{57a}$$

and

$$g_1(r) = \frac{1}{8\pi} e^{-\beta r} . \tag{57b}$$

The parameters—β's and λ's—are determined from the fit of N-N phase shifts in the energy range $E = 25$—300 MeV. They are listed in Table I. The optical model potential parameters obtained for $E_p = 200$ MeV are

$$A = -125.8 - 167.4i \text{ MeV fm}^3 , \tag{58a}$$

$$B = -7.81 - 11.8i \text{ MeV fm}^5 , \tag{58b}$$

$$C = 2.4 - 3.0i \text{ MeV fm}^5 , \tag{58c}$$

$$\gamma = -0.03 - 0.09i . \tag{58d}$$

These are fairly consistent with the values we get from the Franey-Love t matrix, except for B, the measure of the range of the force. These parameter sets give the following values for the volume integral and root mean square radius of the optical model potential,

$$K_R^C = -136 \text{ MeV fm}^3, \quad K_I^C = -151 \text{ MeV fm}^3 , \qquad R_R^C = 3.52 \text{ fm}, \quad R_I^C = 3.54 \text{ fm} . \tag{59}$$

Overly small values of R are the reflection of the small B's. This can be the result of either the simplistic parametrization equations (57) or the neglect of higher partial waves $l > 1$. We believe that this calculation, though schematic, gives a fair idea of how to treat nonlocal forces in the calculation of the folding potential.

IV. SUMMARY

We developed a formalism with which to calculate an optical model potential for nucleon-nucleus scattering without the intervening procedure of localizing the effective nucleon-nucleon force. We could separate out physical mechanisms which were not carefully treated in conventional approaches. Numerical examples show that the main corrections to the bulk volume contribution come from the finite range effect and from nonlocality. The simple algebraic structure of our expression may hopefully help in the understanding of the relation between microscopic and macroscopic optical potentials.

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