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Stability of the ground state of finite nuclei against neutral pion condensation

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The possible occurrence of abnormal nuclear states associated with neutral pion condensation has been studied in ^{12}C and ^{16}O within a Jastrow-type variational approach using an effective interaction to simulate Δ mixing into the nucleon states. The role played by short range correlations and Δ excitations in producing π^0 condensation is discussed.

In recent years, several theoretical works have been devoted to the possible occurrence of abnormal nuclear states, associated with neutral pion condensation, both in infinite matter[1-3] and in atomic nuclei.[4-6] In the π^0 condensate the nucleons are assumed to be arranged in a one-dimensional, solidlike spin-ordered configuration, produced by the periodic potential associated with the pionic standing wave. The resulting ground state of infinite matter is characterized by a localization of the nucleons in parallel layers. Protons and neutrons in the same layer have opposite spins and the spin directions change alternately layer by layer. Within the finite size of atomic nuclei, such a state reduces to a single layer, i.e., to a strongly oblate portion of nuclear matter made up of spin-up protons and spin-down neutrons only (or *vice versa*).

In principle, the stability of the condensed state with respect to the standard one depends upon the balance between the additional kinetic energy coming from the localization and the spin ordering of the particles and the potential energy associated with the tensor component of the one-pion-exchange (OPE) interaction between nucleons, which is strongly enhanced in the π^0 condensate. However, the results of numerical studies performed within the potential approach, using realistic nucleon-nucleon (NN) forces,[1-3,6] clearly indicate that, beside this driving mechanism, two different effects play a crucial role in determining the occurrence of neutral pion condensation and must be carefully taken into account: short range correlations and the mixing of the Δ resonance into the nucleon states.

In this paper we extend to the nucleus ^{16}O the variational study on the stability of the standard nuclear ground state performed in Ref. 6. Furthermore, using an effective interaction built up following the prescription of Ref. 3, we give an estimate of the effect of including processes involving Δ excitations.

We assume a standard description of nuclear systems in terms of nonrelativistic pointlike nucleons interacting through a two-body force reasonably accounting for deuteron properties and NN scattering phase shifts. Therefore, the A-body Hamiltonian is given by

$$H(1,\ldots,A)=\sum_{i=1}^{A}\frac{p_i^2}{2m}+\frac{1}{2}\sum_{i\neq j=1}^{A}v(ij)\;, \tag{1}$$

where p_i and m are the momentum of the ith nucleon and the nucleon mass, respectively, and the interaction $v(ij)$ can be written in the form

$$v(ij)=\sum_{k=1}^{6}v^{(k)}(r_{ij})O_{ij}^{(k)}\;, \tag{2}$$

with $O_{ij}^{(k)}=1$, $(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)$, $(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)$, $(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)$ $(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)$; $S_{ij}=3(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}}_{ij})(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ij})-(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)$, $S_{ij}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)$. The potential employed in numerical calculations is defined as

$$v(ij)=v_{\rm RSC}(ij)+[(f_{\rm eff}/f)^2-1]\tilde{v}_{\rm OPE}(ij)\;, \tag{3}$$

where $v_{\rm RSC}$ is the V6 version of the Reid soft core (RSC) interaction[7] and the modified OPE potential ($x=0.7r_{ij}$)

$$\tilde{v}_{\rm OPE}(ij)=\frac{1}{3}m_\pi\frac{f^2}{4\pi}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\{(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)e^{-x}/x+S_{ij}[(1+3/x+3/x^2)e^{-x}/x-(12/x+3/x^2)e^{-4x}/x]\}\;, \tag{4}$$

m_π and f being the pion mass and the coupling constant of the πNN vertex, roughly describes the enhancement of the OPE interaction associated with Δ isobar mixing. The effective coupling constant $f_{\rm eff}$ in Eq. (4) is defined in terms of the Δ percentage v^2 and of the coupling constants of the $\pi N\Delta$ and $\pi\Delta\Delta$ vertices g and h:

$$f_{\rm eff}=fu^2+(4/3)guv+(1/4)hv^2\;,$$

with $u^2=1-v^2$. The values of g and h employed in numerical calculations have been taken so as to reproduce the width of the Δ resonance: $g=2f$ and $h=(4/5)f$.

Clearly, the modified NN interaction (3) can be written in the V6 form (2). Moreover, as $v^2\to 0$, $v(ij)\to v_{\rm RSC}(ij)$ and the pure NN potential is recovered.

The variational wave function has been written in a Jastrow-type product form

$$\Psi(1,\ldots,A)=F(1,\ldots,A)\Phi(1,\ldots,A)\;, \tag{5}$$

where Φ provides an independent particle model (IPM) description of the system, whereas the correlation factor F is a symmetrized product of two-body operators suitable to describe the correlation structure induced by the interaction (2):

$$F(1,\dots,A)=S\left\{\prod_{j>i=1}^{A}\left[\sum_{k=1}^{6} f^{(k)}(r_{ij})O_{ij}^{(k)}\right]\right\}. \tag{6}$$

The IPM functions employed in numerical calculations for ^{12}C and ^{16}O are Slater determinants of harmonic oscillator (HO) single particle (s.p.) states. In the oblate configurations corresponding to the π^0 condensate and to the standard ground state of ^{12}C (see Ref. 6) different oscillator lengths have been allowed for in the xy plane ($b_x=b_y$) and along the z axis (b_z), whereas in the standard phase of ^{16}O one has $b_x=b_y=b_z=b_s$. The requirement for the volume of the system to be the same in the two phases clearly implies the constraint $(b_x)^2b_z=(b_s)^3$. The quantum numbers of the s.p. states employed in building up the Φ's are listed in Table I.

The expectation value $\langle H\rangle=\langle\Psi|H|\Psi\rangle/\langle\Psi|\Psi\rangle$ has been evaluated within the same approximations employed in Ref. 6: (i) the two-body correlation functions $f^{(k)}$ have been taken from nuclear matter calculations[7] assuming $f^{(4)}=f^{(6)}$ and $f^{(2)}=f^{(3)}=f^{(5)}=0$; (ii) a cluster expansion of $\langle H\rangle$ has been performed, retaining only the leading term in A^{-1} in the two-body cluster contribution.[8]

For any given value of the Δ percentage and of the volume parameter b_s, $\langle H\rangle$ has been evaluated in the spherical standard ground state of ^{16}O, whereas in the oblate configurations corresponding to the π^0 condensate and to the standard ground state of ^{12}C a further energy minimization, with respect to the Nilsson parameter

$$\epsilon=-3(1-b_z/b_x)/(1+2b_z/b_x)\ ,$$

has been performed. The resulting minima in the standard ground states at $v^2=0$ are $\langle H\rangle/A=-4.2$ MeV at $b_s=b_0=1.4$ fm and $\epsilon=-0.36$ for ^{12}C and $\langle H\rangle/A=-5.1$ MeV at $b_s=b_0=1.45$ fm for ^{16}O.

Since the cluster expansion of $\langle H\rangle$ rapidly converges in the density region relevant to the study of standard nuclear ground states, the underbinding resulting from our calculations, whose value in ^{16}O is very close to that obtained in Ref. 9 using the coupled cluster approach and the full RSC interaction, seems to be mainly due to the inadequacy of the NN interaction. However, it should be pointed out that the RSC V6 potential incorporates both the strong short range repulsion and the OPE tensor attraction, which are expected to play the major role in determining the critical density for neutral pion condensation, so that using a more realistic interaction would probably not change the conclusions of the present work appreciably.

TABLE I. Quantum numbers (n_z,n,m) of the HO s.p. states $\varphi_{n_z nm}(\mathbf{r})=R_{n_z}(z)R_{nm}(r_\perp)$ employed to build up the IPM wave function of ^{12}C and ^{16}O, both in the standard and in the π^0 condensed phases.

	Standard ground state			π^0 condensate		
	n_z	n	m	n_z	n	m
^{12}C	0	0	0	0	0	0
	0	1	1	0	1	1
	0	1	−1	0	1	−1
				0	2	2
				0	2	0
				0	2	−2
^{16}O	0	0	0	0	0	0
	0	1	1	0	1	1
	0	1	0	0	1	−1
	0	1	−1	0	2	2
				0	2	0
				0	2	−2
				0	3	3
				0	3	−3

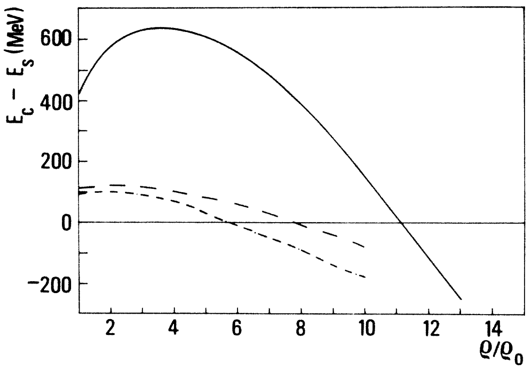

FIG. 1. Energy difference between the π^0 condensate and the standard ground state in ^{16}O at $v^2=0$ (no Δ mixing). The full curve represents the results of the present approach, whereas the dashed and dash-dot lines refer to the results obtained in Ref. 5 including π and $\pi+\rho$ exchange in the residual interaction, respectively.

In Fig. 1 the energy difference E_c-E_s between the condensed and the standard state of ^{16}O at $v^2=0$ (no Δ mixing) is plotted as a function of the ratio $(\rho_s/\rho_0)=(b_0/b_s)^3$, together with the corresponding results obtained by Tripathi and co-workers.[5] The critical density, in units of the equilibrium density of the standard phase ρ_0, is given by the value of the ratio $(b_0/b_s)^3$ yielding a vanishing energy difference. In Ref. 5 the binding energies have been evaluated within the IPM, using the Hartree approximation and a one-boson exchange potential, modified at short interparticle distance by a cutoff procedure, as a residual interaction. The results clearly indicate that, as in the case of ^{12}C (see Ref. 6), the inclusion of the full NN interaction produces a strong increase of the critical density ρ_c for the transition to the pion condensate.

The effect of allowing for a Δ mixing into the nucleon state in ^{16}O has been investigated by minimizing the total energy with respect to the Δ percentage v^2. The results are illustrated in Fig. 2, where the density dependence of the energy differences E_c-E_s corresponding to different Δ percentages are shown. The solid and dashed lines refer to the results obtained with the values $v^2=0.6\%$ and $v^2=1\%$, the latter corresponding to the minimum energy at the critical density. It clearly appears that, as in the case of infinite matter,[2,3] the inclusion of Δ excitations leads to a lowering of ρ_c by a factor of 2. It should be noticed that these results qualitatively agree with those of Ref. 5, where a similar effect has been found at $v^2=2.5\%$

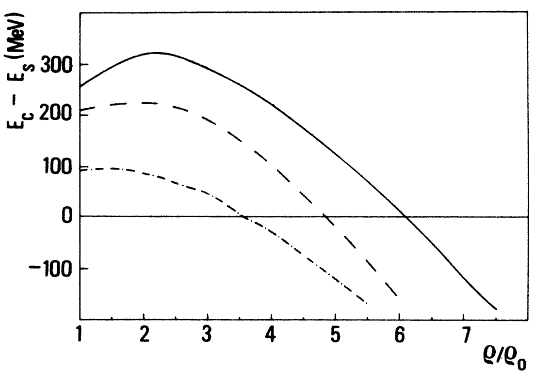

FIG. 2. Energy difference $E_c - E_s$ in ^{16}O obtained within the present approach at $v^2 = 0.6\%$ (solid line) and 1% (dashed line). The dash-dot line represents the results obtained in Ref. 5 including $\pi + \rho$ exchange in the residual interaction and taking $v^2 = 2.5\%$.

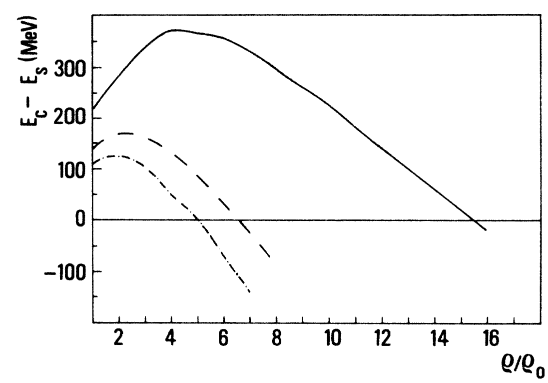

FIG. 3. Energy difference $E_c - E_s$ in ^{12}C obtained within the present approach at $v^2 = 0$ (solid line), 0.6% (dashed line), and 1% (dash-dot line).

within a coupled channel approach. As can be seen in Fig. 3, the energy differences calculated in ^{12}C exhibit the same behavior as those of ^{16}O.

On the basis of the results of the present work, it appears that both the short range part of the NN interaction and the excitation of the Δ resonance play a crucial role in determining the stabilization of the π^0 condensate in finite nuclear systems and have to be taken into account in a fully consistent way in order to get any reliable estimate of the critical density ρ_c. In this regard, it should be pointed out that our prescription to include Δ mixing effects in the two-body interaction is quite rough and that an accurate calculation of ρ_c within the potential model would probably require the explicit inclusion of the Δ degrees of freedom in the many-body wave function and the use of realistic transition interactions.

As far as the validity of the approximations employed in the energy calculation is concerned, it should be noticed that the inclusion of isobar mixing, moving the critical density toward lower values, produces an improvement of the convergence of the cluster series in the transition region. Although the so-called "wound parameter" $\varkappa$, whose value roughly indicates the rate of convergence,[8] is still quite sizable at $\rho \sim 5\rho_0$ ($\varkappa \sim 0.2$), the energy separation between the two phases seems to be larger than the error bars associated with the values of $\langle H \rangle$, so that the inclusion of higher order cluster contributions should not drastically change our results.

As a concluding remark, we would like to point out that it seems to be unlikely that a more refined model to include Δ mixing effects and a more accurate evaluation of the energy expectation values could bring the critical density for neutral pion condensation in atomic nuclei into a region ($\rho \sim 2-3\rho_0$) where the standard nonrelativistic potential model is expected to be meaningful. As a matter of fact, at densities $\rho \sim 5\rho_0$ the possible occurrence of abnormal nuclear phases other than the pion condensate and explicitly involving the degrees of freedom of the nucleon constituents, as in a quark-gluon plasma,[10] should also be taken into account.

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[1]D. J. Sandler and J. W. Clark, Phys. Lett. **100B**, 213 (1981).

[2]O. Benhar, Phys. Lett. **106B**, 375 (1981); **124B**, 305 (1983); Nucl. Phys. **A437**, 590 (1985).

[3]K. Tamiya and R. Tamagaki, Prog. Theor. Phys. **66**, 341 (1981).

[4]G. Do Dang, Phys. Rev. Lett. **43**, 1708 (1979).

[5]R. K. Tripathi, A. Faessler, and K. Shimizu, Z. Phys. A **297**, 275 (1980).

[6]O. Benhar, C. Ciofi degli Atti, and F. Cleri, Phys. Lett. **141B**, 33 (1984).

[7]O. Benhar, C. Ciofi degli Atti, S. Fantoni, and S. Rosati, Nucl. Phys. **A238**, 127 (1979).

[8]J. W. Clark, in *The Many-Body Problem. Jastrow Correlations Versus Brueckner Theory,* edited by R. Guardiola and J. Ros (Springer, Berlin, 1981), p. 184.

[9]H. Kummel and J. G. Zabolitzky, Phys. Rev. C **7**, 549 (1973).

[10]M. Jacob, Nucl. Phys. **A418**, 7 (1984).