Microscopic description of four-body excitations in heavy nuclei

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Under the assumption that the ground state of a many-body system is formed by a condensate of quartets, the structure of these quartets, as well as the equations that describe the excitations of such a system, are obtained. It is shown that such a description can be applied to well deformed nuclei. In particular, it is found that the one- and three-body excitations will have a description that is similar to the one provided by the Nilsson model. Some experimental differences are predicted.

I. INTRODUCTION

The problem of alpha clustering in nuclei has been one of the most challenging ones in the field of nuclear physics. In light nuclei the large stability of even-even nuclei with the same number of protons and neutrons suggested, since the early times of nuclear physics,[1] models considering the alpha particle as the elementary building block needed for the description of the ground states and some special excited states of those nuclei.

Danos and Gillet[2] found that the quartet model[3] has not only a reasonable success in the description of the low-energy features of nuclear spectra for light nuclei but that it was consistent with the behavior of nuclear masses throughout the Periodic Table. In heavy nuclei there are many indications of the relevance of the alphalike clustering and we will review some of the most noticeable.

The first one was related to the study of the alpha preformation coefficients by Bonetti and Millazzo-Colli.[4] They found that the preformation coefficients evaluated from (n,α) and (p,α) reactions[5] in heavy nuclei and the values obtained from a statical approach to alpha radioactivity are in excellent agreement. This coefficient has a rather smooth A dependence showing a strong dip at the magic number $N=126$ and the value of the coefficient is almost 1 at its maximum.

A second indication was provided by Becchetti *et al.*[6,7] by the study of the (d,^{6}Li) reaction systematics in heavy nuclei. They found that the (d,^{6}Li) ground state cross section has a general trend and that it decreases with the target mass $A_t \propto 1/A_t^3$. Superimposed on this general behavior are systematic variations with local maxima and minima. Doing distorted-wave Born approximation calculations they were able to separate an "alpha-spectroscopic factor" that has a remarkably similar dependence with A to the mass variation of the alpha preformation coefficient when the data overlap. Moreover, this alpha spectroscopic factor has well pronounced dips near the magic nuclei ^{208}Pb, ^{132}Sn, and ^{56}Ni.

A third indication comes from several studies of the binding energies. As we already have said, Danos and Gillet found that the quartet picture is consistent with the behavior of nuclear masses throughout the Periodic Table and they discussed also the implications of the idea with many particle—many holes states in the region of $A=100$ and 150. Later,[8] the pairing and symmetry terms of the semi-empirical mass formulae were replaced by one of the Casimir operators of SU(4). This quarteting scheme imposes a very special dependence on isospin that the fit to the experimental binding energies seems to favor.

In Ref. 9 the stability of the internal structure of alpha clusters in nuclei was investigated through the study of the regularities of the two-nucleon separation energies up to $A=72$.

In Ref. 10 the binding energies were studied to explore in a systematic way all possible modes of excitation based upon clusters of several nucleons. It was found that if one considers clusters with more than two particles, only alphalike clusters qualify as elementary modes of excitation. It was also found that the alpha-particle separation energies suggested that deformed heavy nuclei can be described as an alpha-particle condensate. This comes from the fact that for a *boson condensate*, when the interaction between the bosons is attractive, the ground state energy will depend quadratically[11] on the number of bosons, and therefore the separation energy will depend linearly on the same number, as is the case in Fig. 6 of Ref. 10.

Gambhir, Ring, and Schuck[12] made a description of deformed nuclei as a superfluid condensate of alpha particles. They assumed that these particles were formed in the framework of the IBM model, as a superposition of S and D bosons. In this model they were able to reproduce, at least qualitatively, the even-odd staggering of the binding energies. It has been pointed out[13,14] that this staggering can be understood in terms of the symmetry term of the semi-empirical mass formulae. In Ref. 15 it has been stressed that when there are many particles outside closed shells, one must take care of the symmetry energy in the form of correlations and that the importance of the alpha correlations and their relation to the symmetry term have already been noted in Ref. 8, where the symmetry energy

follows from the SU(4) symmetry originated in the quartets.

A fourth indication came from the study of ^{212}Po done by Dodig-Crnkovic, Januch, and Liotta.[16,17] For the calculation of absolute alpha-decay widths in ^{212}Po, two states were considered. One was the product $|\text{g.s.}^{210}\text{Po}\rangle\otimes|\text{g.s.}^{210}\text{Pb}\rangle$ that takes into account the pairing interaction between protons and between neutrons. The other state was $|^{210}\text{Bi}\rangle\otimes|^{210}\text{Bi}\rangle$ that takes into account the proton-neutron interaction.

It was found that the presence of the Bi part of the wave function has two appealing effects. On the one hand, the absolute alpha-decay rate is between 10 and 20 times bigger, and on the other hand, an alpha cluster on the surface appears. In Ref. 18 it was shown that the pairing interaction localizes the four particles in the surface, but not in a small volume, as they are essentially distributed throughout the surface.

These calculations[16,17] suggest the existence of two different types of systems as one moves away from the closed shell. One will be formed by the usual pairing vibrations or an equivalent type of description. The interaction between the proton and neutron pairs can be known from the study of ^{212}Po. The other type of description will be based on the $|^{210}\text{Bi}\rangle\otimes|^{210}\text{Bi}\rangle$ basis. In Ref. 19 it is shown that for nuclei heavier than ^{208}Pb the energy of the second type of system is equal to the one corresponding to the first type around the transition region between spherical and deformed nuclei. In this oversimplified picture the ground state in the deformed region is a coherent mixture of many $T=0$ pairs and the four body cluster has a collective structure. This very simple calculation can be considered as an indication that the assumption that the ground state is a condensate of quartets may be connected with real nuclei.

The treatments of Refs. 12 and 19 provided a phenomenological description in terms of bosons. In one case,[12] the alphalike cluster is formed by a mixture of S and D bosons. In the other case[19] it is a coherent mixture of many $T=0$ pairs and the alphalike cluster will therefore have a collective origin. Both treatments were related to the work of Nozieres and Saint James,[20] where the competition between boson condensation and coherent wave functions of pairs of bosons is discussed.

An alternative phenomenological treatment based on the interaction boson model was developed by Iachello and Jackson.[21] They suggested that the clustering aspects may be taken into account by a dipole degree of freedom of the IBM. This degree of freedom is originated by the oscillation of the alpha particle relative to the remaining part of the nucleus.[22]

With regard to the microscopic description of the alpha-clustering phenomena in terms of fermions, there are few antecedents. The first one is the work of Bremond and Valatin,[23] where they used for the ground state a coherent state, changing the usual BCS structure

$$\prod_{\alpha}(U_{\alpha}+V_{\alpha}a^{\dagger}_{\alpha}a^{\dagger}_{\bar{\alpha}})|0\rangle \tag{1.1}$$

to

$$\prod_{\alpha}(S_{\alpha}+V_{\alpha p}a^{\dagger}_{\alpha p}a^{\dagger}_{\bar{\alpha}p}+V_{\alpha n}a^{\dagger}_{\alpha n}a^{\dagger}_{\bar{\alpha}n}+T_{\alpha}a^{\dagger}_{\alpha p}a^{\dagger}_{\bar{\alpha}p}a^{\dagger}_{\alpha n}a^{\dagger}_{\bar{\alpha}n})|0\rangle\ . \tag{1.2}$$

The operators $a^{\dagger}_{\alpha p}$ and $a^{\dagger}_{\alpha n}$ create, respectively, a proton (or a neutron) with quantum numbers α. The coefficients S, V, and T were determined, as usual, by minimizing the Hamiltonian. This wave function contains, as a particular case the usual BCS one, when $S_{\alpha}=[(1-V^2_{\alpha n})(1-V^2_{\alpha p})]^{1/2}$ and $T_{\alpha}=V_{\alpha n}V_{\alpha p}$.

Flowers and Vujicic[24] noted that the energy gained by the difference between the BCS four body term and the one corresponding to the usual factorization was very small, of the same order of magnitude as the Hartree-Fock corrections of the pairing interaction, and that therefore, that sort of wave function was not able to produce large or collective effects. They were interested in a method to take into account charge independent pairing correlations in light nuclei where the advantages that one can get from the Wigner supermultiplet symmetry are very rich. To be able to use this advantage they worked in L-S coupling with a wave function of the type

$$\Phi=\prod_{ll'mm'}{\prod_{pqrs}}'(U_l+V_{ll'}\epsilon_{pqrs}a^{\dagger}_{lmp}a^{\dagger}_{l-mq}a^{\dagger}_{l'm'r}a^{\dagger}_{l'-m's})|0\rangle\ , \tag{1.3}$$

where ϵ is the antisymmetry tensor. The prime in the second product merely signifies that the product is to be taken only over the 24 elements of ϵ that are nonzero.

They found that the quasiparticles have a simple structure when $l=l'$, namely,

$$\alpha^{\dagger}_{kp}=U_k a^{\dagger}_{kp}-\frac{1}{\sqrt{12}}V_k\sum_{qrs}\epsilon_{pqrs}a_{-ks}a_{kr}a_{-kq}\ . \tag{1.4}$$

This is a natural extension of the BCS structure since now the *Cooper pairs* are indeed formed by four particles and the *hole* term in the quasiparticle must be replaced by a particular mixture of three holes.

When $l\neq l'$ they found it necessary to make an expansion in powers of V, retaining only the lowest orders. They found that the description was quite complicated and almost intractable.

We can say therefore that there are papers that attempt to describe deformed nuclei as being due to the presence of alpha particles (see, for instance, Refs. 21 and 22) or as a condensate of pairs of nucleons (for instance, Refs. 15 and 16). On the other hand, the Nilsson model provides the standard explanation of these nuclei, giving a very precise description of those nuclei. As both ways of looking at the problem of deformed nuclei seem to be antagonistic, our main purpose will be to prove that it is possible to reconciliate both approaches, by showing that the Nilsson model also follows from the assumption that the ground state is a condensate of quartets.

We will develop a method to study excitations of a system under the assumption that the ground state is a condensate of quartets. It will have some similarities with BCS, where the g.s. is considered as a condensate of pairs.

The assumption that the g.s. is quartet condensate will permit the classification of different processes and the

selection of diagrams that are of the same importance. This is done in Sec. II.

In Sec. III we obtain diagramatically the equations that give in a self-consistent way the excitations of the system as well as the structure of the ground state. Their interpretation and some general characteristics are discussed.

In Sec. IV we compared the results obtained with the standard description of deformed nuclei.

II. CLASSIFICATION SCHEME

The nuclear field theory[25] (NFT), that assumes that the matrix elements of relevant operators have an analytic behavior, is an approach that seems suitable for understanding the relative importance of different processes in heavy nuclei, when it can be considered that the interaction between the nucleons contains only one and two body terms.

If the ground state of the system is normal, i.e., nondeformed and nonsuperconductive, the NFT uses a two-level model,[26] where each shell has a degeneracy N, to classify the relative importance of the different contributions. Due to the assumption of analyticity, it is possible to write the matrix elements of an operator $\hat{F}$ between an initial state $|i\rangle$ and a final one $|f\rangle$ as

$$\langle f|F|i\rangle=\alpha\sum_n f_n\left[\frac{1}{N}\right]^n, \tag{2.1}$$

where α and f_n depend upon the operator $\hat{F}$ under consideration.

These constants f_n correspond usually to the sum of an infinite perturbative series in the interaction strength and in the NFT each diagram corresponds to a particular power of $1/\sqrt{N}$. In particular, when the collective excitations are described in the random-phase approximation (RPA) the full $n=0$ part of (2.1) is taken into account by the RPA diagrams.

The characteristic of the NFT that allows for this classification scheme, is that the amplitudes defined by $\langle$*particle-hole*$|$*relevant collective state*$\rangle$ are of order $1/\sqrt{N}$. All the dependences of a diagram are given by the number of amplitudes and the number of fermionic loops that a diagram has.

When the ground state of the system is supposed to be a condensate of K pairs,[27] the NFT uses a one level model with $2K$ particles to classify the processes. In this case, one can write

$$\langle f|F|i_m\rangle=\beta_m\sum_n f_{mn}\left[\frac{1}{N}\right]^n\left[\frac{K}{N}\right]^m. \tag{2.2}$$

The factor K appears because every time that one pair belonging to the condensate is created or annihilated one gets a factor $\sqrt{K}$.

In Refs. 27 and 28 an approximation scheme was developed that permits the isolation of the processes that contribute to the $n=0$ series in even and odd systems. This procedure was called the principal series (PS) approximation, as it retains the principal series (i.e., the one corresponding to $n=0$) in the development (2.2). We will try to develop a similar approximation when the ground state of the system is a condensate of quartets. Unfortunately, in this case, there is not any simple model on which one can rely to see the worth of the classification scheme. We will then need to *extrapolate* the concepts used by the NFT in the two previously discussed cases.

If, as is desirable, the interaction between the quartets is not of the same order of magnitude as the terms incorporated in their definition, it is necessary that the zero order (in $1/N$) description of the four particle excitations be not the product of two two-body structures. If that is not the case, the concept of four-body clusters is not going to be useful.[19]

To obtain a different zero order description, we use a scheme where we start with two-body excitations evaluated in the RPA. From these two-body excitations we isolate the R that have a collective character. This number R is of the same order of magnitude as N. If the interaction is attractive, these states will correspond to the lowest energy solution of the RPA equations for different spins and isospins.

We will construct a four body collective excitation as a coherent mixture of pairs of these two-body *collective excitations*. This approximation amounts to the assumption that the amplitudes of two pairs in a quartet is of order $1/\sqrt{R}$. To simplify the notation we will represent by $a,b,\ldots,c$ the one particle states, by $\alpha,\beta,\ldots,\gamma$ the two particle states, by $r,s,\ldots,t$ the three particle states, and by $L,M,\ldots,Q$ the four particle states.

Now we come to the problem of determining how many three-particle states will be relevant. If we only consider the R collective two-particle states we will have $RN=L$ three-particle states. We will assume that of these L states there are only S (of the same order of magnitude as N and R) that are relevant. There are several reasons for this assumption. One is that in this way the two amplitudes [Figs. 1(c) and 1(d)] are of the same order of magnitude as those of Figs. 1(a) and 1(b). This will mean that each particle state has a small number of partners to form the quartet. This is also suggested by the framework used by Flowers and Vujicic[24] where for light nuclei there was only a single three-hole partner for each one-particle state.

The ground state wave function will have two different types of contributions. One will be the *coherent sum* of pairs [corresponding to Fig. 1(d)], and a second part will be given by a "coherent sum" of particles coupled to trios [corresponding to Fig. 1(c)]. If one considers all the states

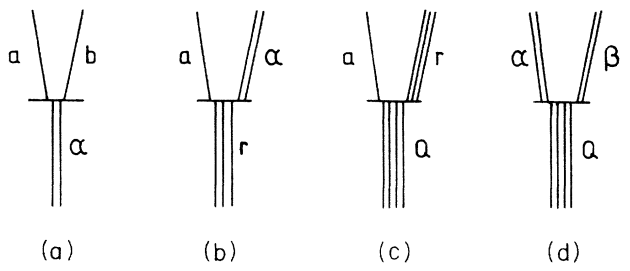

FIG. 1. Amplitudes into different states that are assumed of order $1/\sqrt{N}\approx 1/\sqrt{R}\approx 1/\sqrt{S}$.

formed by two pairs, they will form a complete basis for the system with four particles and there will be N^4 states of this type. Similarly one can form a complete basis with the N^4 states formed by coupling particles to all the possible trios. As we are only considering N states of each type, there will not be any problem related with overcompleteness of the basis to this order. The double counting of processes that will be done will be of order $1/N$ compared with the processes retained, as follows from inspection.

It must be noted that in the NFT the one[27] and two[26] level models were used to recognize that the amplitudes shown in Fig. 1(a) are of order $1/\sqrt{N}$ and that anytime an external pair is created or annihilated a factor $\sqrt{K}$ appears. The assumptions displayed in Fig. 1 allow the classification of the diagrams contributing to any process and in particular to the ground state energy. It is important to remember that as we will work with diagrams containing composite excitations we must consider the structure of these excitations only when they interact with other excitations. For example, the diagram shown in Fig. 2(a) contains the diagram shown in Fig. 2(b). In Fig. 2(b) we have decomposed the quartet before it interacts with the other excitations, and as the diagram has been already counted, it must not be considered.

At this point we will discuss how to calculate the matrix elements needed to evaluate the amplitudes shown in Fig. 1. Let us assume, for simplicity, that we know the amplitudes for the quartet, the trios, and pairs, i.e., that we know

$$\langle Q \mid a_1^\dagger a_2^\dagger a_3^\dagger a_4^\dagger \mid 0\rangle,\quad \langle t \mid a_1^\dagger a_2^\dagger a_3^\dagger \mid 0\rangle,\quad \langle \alpha \mid a_1^\dagger a_2^\dagger \mid 0\rangle .$$

Then, from the knowledge of the two-body part of the Hamiltonian

$$V=\tfrac{1}{4}\sum_{1234} V_{12,34}a_1^\dagger a_2^\dagger a_4 a_3 , \tag{2.3}$$

we can evaluate the different coupling constants as in the NFT (Ref. 26), i.e.,

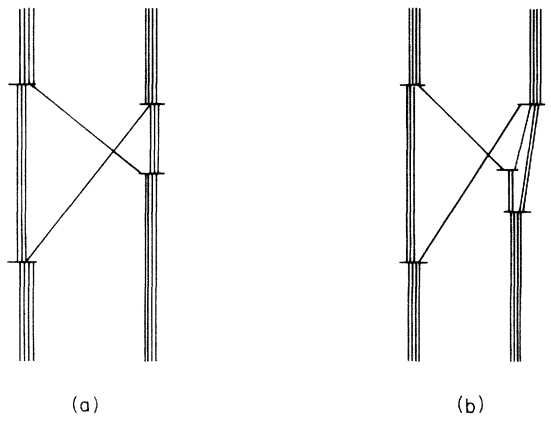

FIG. 2. Different contributions to the ground state energy. (a) Illustrates a diagram that must be considered, while (b) shows a process that as been incorporated in (a).

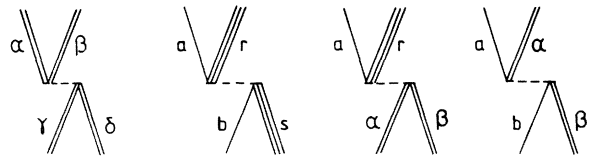

FIG. 3. Illustrations of different effective matrix elements between dressed structures.

$$\Lambda^{\alpha}_{ab}=\sum_{12} V_{ab,12}\langle \alpha \mid a_1^\dagger a_2^\dagger \mid 0\rangle , \tag{2.4a}$$

$$\Lambda^{t}_{\alpha\beta}=\sum_{1234} V_{\alpha 4,12}\langle t \mid a_1^\dagger a_2^\dagger a_3^\dagger \mid 0\rangle\langle 0 \mid a_3 a_4 \mid \beta\rangle , \tag{2.4b}$$

$$\Lambda^{Q}_{ar}=\sum_{12345} V_{a5,12}\langle Q \mid a_1^\dagger a_2^\dagger a_3^\dagger a_4^\dagger \mid 0\rangle\langle 0 \mid a_4 a_3 a_5 \mid r\rangle , \tag{2.4c}$$

$$\Lambda^{Q}_{\alpha\beta}=\sum_{123456} V_{56,23}\langle Q \mid a_1^\dagger a_2^\dagger a_3^\dagger a_4^\dagger \mid 0\rangle \times\langle 0 \mid a_1 a_5 \mid \alpha\rangle\langle 0 \mid a_6 a_2 \mid \beta\rangle . \tag{2.4d}$$

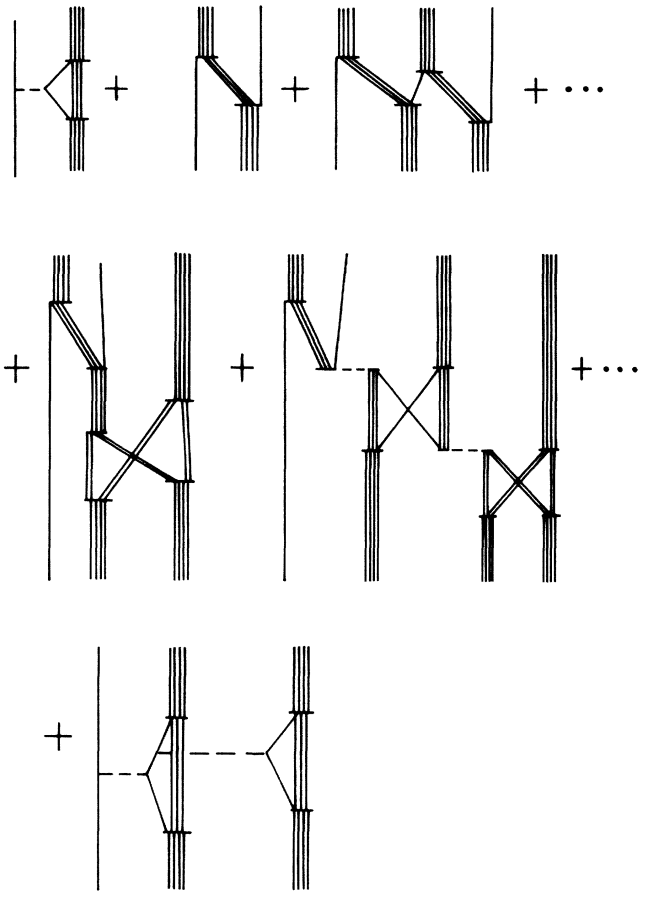

FIG. 4. Diagrams that contribute to the PS of the one-body excitations.

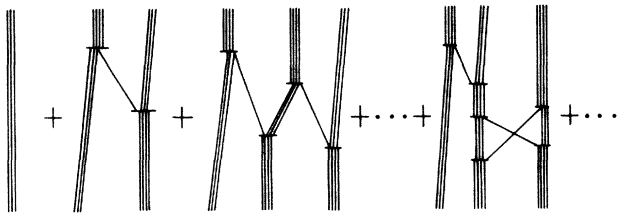

FIG. 5. Diagrams that contribute to the PS of the three-body excitations.

In a similar way we can define the matrix elements for the Hamiltonian that is effective between the composite structures, shown schematically in Fig. 3.

Another concept that in the present situation is ill defined is the concept of loop. We will assume, for classification purposes, that each one particle state (m) has as partner a single trio ($\bar{m}$) and that each one of the R two-particle states (α) has as partner only one two-particle state ($\bar{\alpha}$). These assumptions make clear which diagrams have loops and it will help us to decide if a diagram is of the same order of magnitude as another one or not.

In order to determine which diagrams are the most important it is necessary to isolate the factors introduced by the different parts of a diagram. A quartet that is created or annihilated can generate two types of contributions: either $\sqrt{K/N}$ if it belongs to the initial or final state (condensate) or $1/\sqrt{N}$, if it is one of the internal quartets that appears in intermediate states. All vertices corresponding to the opening of a pair or a trio will be of order $1/\sqrt{N}$ (we consider that $N \simeq S \simeq R$).

Each fermionic vertex yields a factor $1/N$ and each loop carries a factor N. It is therefore seen that the principal series for a given process will be given by the set of diagrams where the number of internal connections (internal bosons or fermionic couplings) is as close as possible to the number of loops.

Some of the diagrams corresponding to the PS (principal series) for the fermionic one (three) particle excitations are shown in Fig. 4 (5) and in a similar way for the boson excitations corresponding to two (four) particles in Fig. 6

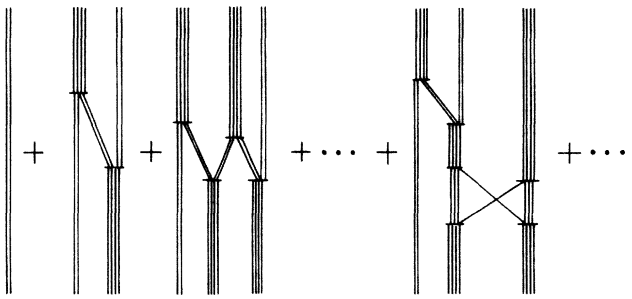

FIG. 6. Diagrams that contribute to the PS of the two-body excitations.

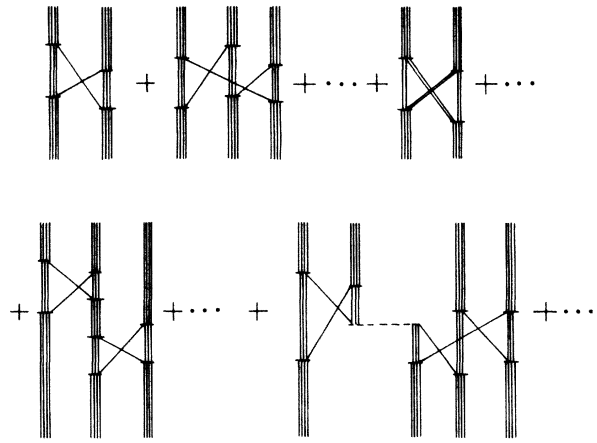

FIG. 7. Diagrams that contribute to the PS of the four-body excitations.

(7). To clarify this picture, a diagram that does not belong to the PS for the one particle excitation is shown in Fig. 8. This classification scheme then allows for the isolation of more important diagrams that will form the corresponding PS when the ground state of the system is a condensate of quartets.

III. DESCRIPTION OF THE EXCITATIONS

We have defined the principal series (PS) as being given by those diagrams with the smallest difference between the number of loops and the number of internal connections. A PS diagram is characterized by the fact that it becomes separated into two disconnected parts by cutting it through any connection.

To obtain the structure of the excitations and its energies it is simpler to study the poles of the one, two, three, and four body Green's functions than to sum explicitly, using the Raleigh-Schrödinger perturbative expansion, all the perturbative corrections of the type shown in Figs. 4–7.

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Feynman diagrams. One must also take care to include each process only once, i.e., one must be careful of equivalent lines, equivalent processes, etc. This method of transforming the Rayleigh-Schrödinger diagrams to the time dependent version is what we call by inspection, i.e., by making sure that all the diagrams contained in Figs. 3–7 are considered in the processes described by these multiparticle GF.

In this way we must obtain by inspection the set of cou-

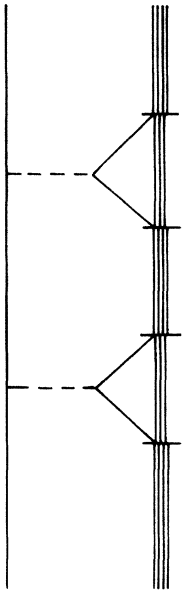

FIG. 8. Diagram that does not belong to the PS of the one-body excitations.

pled multiparticle Green's functions (CMPGF). For example, the types of diagrams that contribute to the one-body Green's function are shown in Fig. 9.

First, it must be noted that it is advantageous to replace the quartet belonging to the condensate by its *dressed version.* In this way one takes into account many diagrams and it diminishes largely the type of diagrams that one must consider. For example, the last diagram of the first line of Fig. 9 must not be considered, as it corresponds to the *dressing* of the quartet. The consideration of the processes that dress the one- and three-body excitations makes the equations much more compact.

As was done in the case when the ground state of the system was taken to be a condensate of pairs,[28] it is convenient to choose the zero of the single particle energies λ in such a way that the propagator of the quartets (QP) has its root at zero energy. This has the great advantage that one can avoid the problems associated with the asymptotic behavior of the QP at $t=+\infty$ and $t=-\infty$.

The QP corresponding to external lines can be omitted because they will not have any time dependence and their only contribution will be given by the vertices. Using these simplifications, the CMPGF are described by the diagrams shown in Fig. 10. One obtains seven coupled equations. Even if they look very complicated they have a rather simple structure.

The one, two, and three body GF can be dressed by two different mechanisms. The first one is related to the internal structure of the excitations. This is considered in Figs. 10(a), 10(c), and 10(e), respectively. Afterwards, one must consider the *sawlike* diagrams shown in Figs. 10(b), 10(d), and 10(f), that take into account their participation in the quartet. The last equation gives essentially the quartet wave function. As was discussed in Sec. II, it has two different types of contributions. One coming from the *opening* of the quartet in one- and three-body excitations and the other one coming from its *opening* in pairs. One is making here a double counting of processes but the error is of order $1/N$, as follows from inspection. One can then say that the equations associated with Figs. 10(a), 10(c), and 10(e) will give the *zero order* energies or, equivalently, that it will correspond to the independent particle part of the effective Hamiltonian for the corresponding propagators. We will now analyze in some detail the diagram 10(a). A two-body Hamiltonian admits two alternative descriptions, as a particle-particle or as particle-hole one. (A detailed discussion in a situation similar to the one present here is discussed in Ref. 29.) In one case the relevant coherent modes are of the pairing type while in the other case they will have a multipole character. For Fig. 10(a) the relevant description is the multipole-multipole one.

As is usual in deformed nuclei, to understand the main features of the excitations, we assume that the most important part of the Hamiltonian is the quadrupole-quadrupole part, i.e.,

$$H_{\text{int}}=-\frac{\chi}{2}\sum_{ij\mu} q_\mu^*(i)q_\mu(j)\ , \tag{3.1}$$

where $q_\mu(i)$ is the quadrupole operator acting on the ith particle and the factors introduced correspond to the usual parametrization.

The effect of the diagram shown in Fig. 10(a) will be to replace the single particle Hamiltonian by

$$H'_{\text{s.p.}}=H_{\text{s.p.}}-\chi\sum_{i\mu} q_\mu^*(i)\langle Q_\mu\rangle\ , \tag{3.2}$$

where $\langle Q_\mu\rangle$ is the expectation value of $\sum_i q_\mu(i)$ in the vacuum. This expectation value is evaluated as shown in

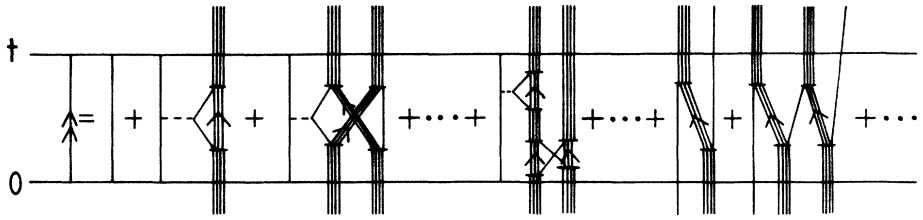

FIG. 9. Diagrams defining the PS for the one-body excitation.

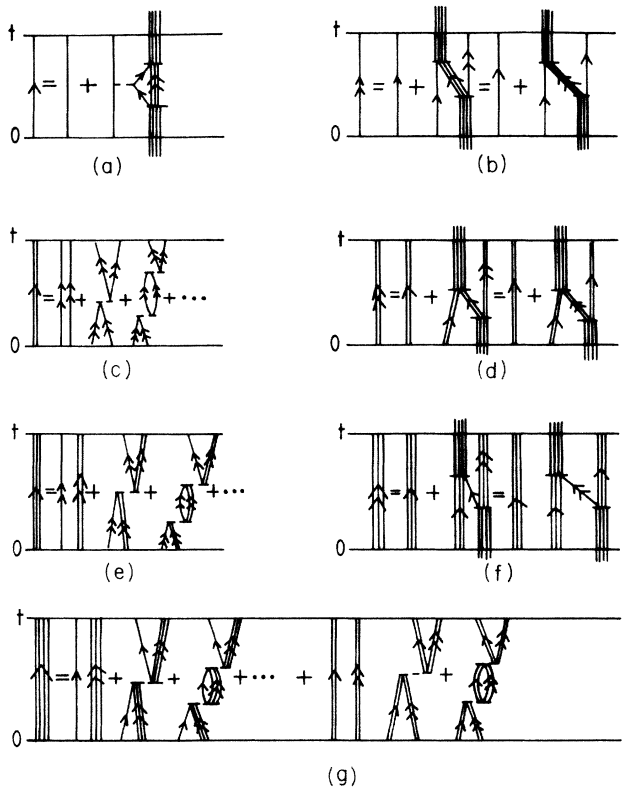

FIG. 10. Diagrams defining the PS for the CMPGF. (a) and (b) correspond to the one body part, (c) and (d) to the two-body part, (e) and (f) to the three-body part, and (g) to the QP.

Fig. 11. Once the structure of the quartet is known, through the use of the diagrams shown in Fig. 10, the $\langle Q_\mu \rangle$ is evaluated with the help of Fig. 11. We can conclude that the diagrams shown in Fig. 10(a) will give as the Nilsson Hamiltonian for the one body excitations.

The treatment of the equation shown in Fig. 10(c) corresponds to a RPA between the quasiparticles. It is important to point out that to make the presentation simpler we have ignored the anomalous Green's function that will appears in a similar way as that in the theory of superconductivity, and that we show in Fig. 12. If they are included, one must do a treatment for the two-particle Green's function similar to the one done in Ref. 29. That will correspond to coupling the two-body Green's function with the GF corresponding to one quartet more and one quartet less than in the initial GF.

The treatment of Fig. 10(e) can be done in a similar way. It must be noted that the treatment will not be essentially different from the one in Ref. 30 for the three quasiparticle case. If the processes where the anomalous propagators appear are considered, then one will couple again the $3P$ GF to the ones with one quartet more and one quartet less than the $3P$ GF. We will assume that these three equations have been solved and that they provide us with the *zero order* energies and wave functions for the one, two, and three particle excitations.

To write down the remaining equations explicitly we have to write down the different GF in their Lehmann

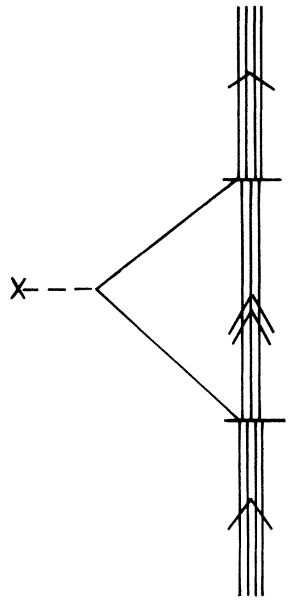

FIG. 11. Diagram representing the expectation value of Q_μ.

representations. It is convenient to start with the definition of the Green's functions[31]

$$G_n(BA,t)=\langle 0 \mid T[D_n(A,t)D_n^\dagger(B,0)] \mid 0 \rangle \ , \tag{3.3}$$

where T is the time ordering operator and $D_n^\dagger(A,t)$ creates n particles with quantum numbers A at time t. The Lehmann representation of this GF will be given, as usual, by

$$G_n(BA;E)=i\left[\sum_e \frac{\langle 0 \mid D_n(A) \mid e \rangle \langle e \mid D_n^\dagger(B) \mid 0 \rangle}{E-E_e} \pm \sum_f \frac{\langle 0 \mid D_n^\dagger(B) \mid f \rangle \langle f \mid D_n(A) \mid 0 \rangle}{E+E_f}\right], \tag{3.4}$$

where the $+$ $(-)$ sign is valid for the one and three (two and four) particle GF and the states $\mid e \rangle$ $(\mid f \rangle)$ form a complete set of eigenstates of the full Hamiltonian for the systems with n particles more (less) than the reference

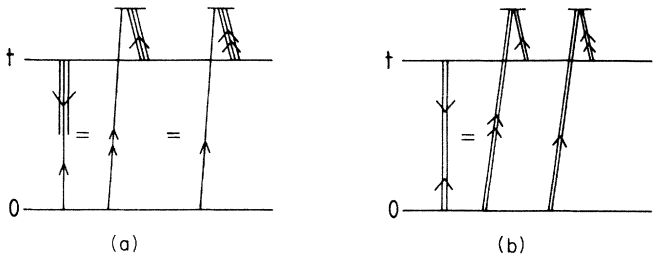

FIG. 12. Diagrams representing the anomalous Green's functions. (a) Corresponds to the odd systems, while (b) corresponds to the even ones.

state with $4K$ particles $|0\rangle$. The energy of the states $|e\rangle$ and $|f\rangle$ with respect to the reference state $|0\rangle$ are E_e and E_f, respectively.

It will be convenient to use a more compact notation for the residues. The state of the *e type* corresponding to one, two, and three particles will be denoted by a, α, and t as we said before. It must be noted that the states of the *f type* for one, two, and three particles will be therefore denoted by t, α, and a, as a complete basis for one particle is going to be the same as a complete basis for three holes, and *vice versa*. We will denote these as

$$U_{ai}=\langle a\,|\,D_1^\dagger(i)\,|\,0\rangle\ , \tag{3.5a}$$

$$V_{rj}=\langle 0\,|\,D_1^\dagger(j)\,|\,r\rangle\ , \tag{3.5b}$$

$$X_{\alpha l}=\langle \alpha\,|\,D_2^\dagger(l)\,|\,0\rangle\ , \tag{3.6a}$$

$$Y_{\alpha k}=\langle 0\,|\,D_2^\dagger(k)\,|\,\alpha\rangle\ , \tag{3.6b}$$

$$Z_{tm}=\langle t\,|\,D_3^\dagger(m)\,|\,0\rangle\ , \tag{3.7a}$$

$$T_{bn}=\langle 0\,|\,D_3^\dagger(n)\,|\,b\rangle\ , \tag{3.7b}$$

and the GF in the Lehmann representation can then be written explicitly as

$$G_1(ij;E)=i\left[\sum_a \frac{U_{ai}^* U_{aj}}{E-E_a}+\sum_r \frac{V_{ri}^* V_{rj}}{E+E_r}\right], \tag{3.8}$$

$$G_2(kl;E)=i\left[\sum_\alpha \frac{X_{\alpha k}^* X_{\alpha l}}{E-E_\alpha}-\frac{Y_{\alpha k}^* Y_{\alpha l}}{E+E_\alpha}\right], \tag{3.9}$$

$$G_3(mn;E)=i\left[\sum_r \frac{Z_{rm}^* Z_{rn}}{E-E_r}+\sum_a \frac{T_{am}^* T_{an}}{E+E_a}\right], \tag{3.10}$$

where we have used the fact that the complete set of states for one particle is the same as for three holes, and for three particles is the same as for one hole.

All the equations corresponding to Fig. 10 are, due to the trick that was used to omit the QP, of the convolution type. Therefore its Fourier transform can be written straightforwardly as the product of the Fourier transforms of the different parts of the diagram. The only care to be exercised is the use of $G(ab;-E)$ in the intermediate GF, due to the way they have been grouped.

The equations corresponding to Figs. 10(b), 10(d), and 10(f) can then be written as

$$G_1(ij;E)=G_1^{(0)}(ij;E)-\sum_{grsh} G_1^{(0)}(ig;E)\Lambda_{gr}^Q \times G_3^{(0)}(rs;-E)\Lambda_{sh}^Q G_1(hj;E)\ , \tag{3.11}$$

$$G_2(kl;E)=G_2^{(0)}(kl;E)-\sum_{mnpo} G_2^{(0)}(km;E)\Lambda_{mn}^Q \times G_2^{(0)}(np;-E)\Lambda_{po}^Q G_2(ol;E)\ , \tag{3.12}$$

$$G_3(mn;E)=G_3^{(0)}(mn;E)-\sum_{pijn} G_3^{(0)}(mp;E)\Lambda_{pi}^Q \times G_1^{(0)}(ij;-E)\Lambda_{jq}^Q G_3(qn;E)\ . \tag{3.13}$$

The zero order Green's function can be written as

$$G_1^{(0)}(ij;E)=i\frac{\delta_{ij}}{E-(\epsilon_i-\lambda)}\ , \tag{3.14}$$

$$G_2^{(0)\text{advanced}}(kl;E)=i\frac{\delta_{kl}}{E-(\omega_k-2\lambda)}\ , \tag{3.15a}$$

$$G_2^{(0)\text{retarded}}(kl;E)=-i\frac{\delta_{kl}}{E+(\omega_k-2\lambda)}\ , \tag{3.15b}$$

$$G_3^{(0)}(mn;E)=i\frac{\delta_{mn}}{E-(E_m-3\lambda)}\ . \tag{3.16}$$

We can write Eqs. (3.11)—(3.13) in an algebraic form as

$$\sum_a \frac{U_{ai}^* U_{aj}}{E-E_a}+\sum_r \frac{V_{ri}^* V_{rj}}{E+E_r}=\frac{\delta_{ij}}{E-(\epsilon_i-\lambda)}+\sum_{nk}\frac{\Lambda_{in}^Q}{[E-(\epsilon_i-\lambda)]}\frac{\Lambda_{nk}^Q}{[E-(E_n-3\lambda)]}\left[\sum_a \frac{U_{ak}^* U_{aj}}{E-E_a}+\sum_r \frac{V_{rk}^* V_{rj}}{E+E_r}\right], \tag{3.17}$$

$$\sum_a \frac{T_{am}^* T_{an}}{E+E_a}+\sum_r \frac{Z_{rm}^* Z_{rn}}{E-E_r}=\frac{\delta_{mn}}{E-(E_m-3\lambda)}+\sum_{is}\frac{\Lambda_{is}^Q}{[E-(\epsilon_i-\lambda)]}\frac{\Lambda_{mi}^Q}{[E-(E_m-3\lambda)]}\left[\sum_a \frac{T_{as}^* T_{an}}{E+E_a}+\sum_r \frac{Z_{rs}^* Z_{rn}}{E-E_r}\right], \tag{3.18}$$

$$\sum_\alpha \frac{X_{\alpha k}^* X_{\alpha l}}{E-E_\alpha}-\frac{Y_{\alpha k}^* Y_{\alpha l}}{E+E_\alpha}=\delta_{kl}\frac{2(\omega_k-2\lambda)}{E^2-(\omega_k-2\lambda)^2}-\sum_{nm}\frac{\Lambda_{kn}^Q}{[E+(\omega_n-2\lambda)]}\frac{\Lambda_{nm}^Q}{[E-(\omega_k-2\lambda)]}\left[\sum_\alpha \frac{X_{\alpha m}^* X_{\alpha l}}{E-E_\alpha}-\frac{Y_{\alpha m}^* Y_{\alpha l}}{E+E_\alpha}\right]. \tag{3.19}$$

If we use the second equality in Figs. 10(b), 10(d), and 10(f), we obtain instead of (3.17)—(3.19),

$$\sum_a \frac{U_{ai}^* U_{aj}}{E-E_a}+\sum_r \frac{V_{ri}^* V_{rj}}{E+E_r}=\frac{\delta_{ij}}{E-(\epsilon_i-\lambda)}+\sum_{nm}\frac{\Lambda_{im}^Q}{[E-(\epsilon_i-\lambda)]}\frac{\Lambda_{nj}^Q}{[E-(\epsilon_j-\lambda)]}\left[\sum_a \frac{T_{am}^* T_{an}}{E-E_a}+\sum_r \frac{Z_{rm}^* Z_{rn}}{E+E_r}\right], \tag{3.20}$$

$$\sum_a \frac{T_{am}^* T_{an}}{E-E_a}+\sum_r \frac{Z_{rm}^* Z_{rn}}{E+E_r}=\frac{\delta_{mn}}{E-(E_m-3\lambda)}+\sum_{ij}\frac{\Lambda_{mi}^Q}{[E-(E_n-3\lambda)]}\frac{\Lambda_{jn}^Q}{[E-(E_m-3\lambda)]}\left[\sum_a \frac{U_{ai}^* U_{aj}}{E+E_a}+\sum_r \frac{V_{ri}^* V_{rj}}{E-E_r}\right], \tag{3.21}$$

$$\sum_{\alpha}\frac{X^*_{\alpha k}X_{\alpha l}}{E-E_\alpha}-\frac{Y^*_{\alpha k}Y_{\alpha l}}{E+E_\alpha}=\delta_{kl}\frac{2(\omega_k-2\lambda)}{E^2-(\omega_k-2\lambda)^2}-\sum_{nm}\frac{\Lambda^Q_{kn}}{[E-(\omega_k-2\lambda)]}\frac{\Lambda^Q_{ml}}{[E-(\omega_l-2\lambda)]}\left[\sum_{\alpha}\frac{X^*_{\alpha n}X_{\alpha m}}{E+E_\alpha}-\frac{Y^*_{\alpha n}Y_{\alpha m}}{E-E_\alpha}\right]. \tag{3.22}$$

As usual the unknown residues and energies are obtained by studying (3.17)—3.22) at theirs poles. Making $E=E_a$ one obtains

$$[E_a-(\epsilon_i-\lambda)]U^*_{ai}=\sum_{nk}\frac{\Lambda^Q_{in}\Lambda^Q_{nk}}{E_a+(E_n-3\lambda)}U^*_{ak}\ , \tag{3.23a}$$

$$U_{aj}=\frac{\sum_n\Lambda^Q_{nj}T_{an}}{E_a-(\epsilon_j-\lambda)}\ , \tag{3.23b}$$

and from $E=-E_a$ we obtain

$$[E_a+(E_m-3\lambda)]T^*_{am}=\sum_{is}\frac{\Lambda^Q_{mi}\Lambda^Q_{is}}{E_a-(\epsilon_i-\lambda)}T^*_{as}\ , \tag{3.24a}$$

$$T_{an}=\frac{\sum_j\Lambda^Q_{jn}U_{aj}}{E_a+(E_n-3\lambda)}\ . \tag{3.24b}$$

In a similar way, when $E=\pm E_r$ we have

$$[E_r+(\epsilon_i-\lambda)]V^*_{ri}=\sum_{nk}\frac{\Lambda^Q_{in}\Lambda^Q_{nk}}{E_r-(E_n-3\lambda)}V^*_{rk}\ , \tag{3.25a}$$

$$V_{rj}=\frac{\sum_m\Lambda^Q_{mj}Z_{rm}}{E_r+(\epsilon_j-\lambda)}\ , \tag{3.25b}$$

$$[E_r-(E_m-3\lambda)]Z^*_{rm}=\sum_{is}\frac{\Lambda^Q_{mi}\Lambda^Q_{is}}{E_r+(\epsilon_i-\lambda)}Z^*_{rs}\ , \tag{3.26a}$$

$$Z_{rm}=\frac{\sum_j\Lambda^Q_{jn}V_{rj}}{E_r-(E_m-3\lambda)}\ . \tag{3.26b}$$

While for $E=\pm E_\alpha$,

$$[E_\alpha-(\omega_k-2\lambda)]X^*_{\alpha k}=-\sum_{nm}\frac{\Lambda^Q_{kn}\Lambda^Q_{nm}}{E_\alpha+(\omega_n-2\lambda)}X^*_{\alpha m}\ , \tag{3.27a}$$

$$X_{\alpha k}=\frac{\sum_n\Lambda^Q_{kn}Y_{\alpha n}}{E_\alpha-(\omega_k-2\lambda)}\ , \tag{3.27b}$$

$$[E_\alpha+(\omega_k-2\lambda)]Y^*_{\alpha k}=-\sum_{nm}\frac{\Lambda^Q_{kn}\Lambda^Q_{nm}}{E_\alpha-(\omega_n-2\lambda)}Y^*_{\alpha m}\ , \tag{3.27c}$$

$$Y_{\alpha k}=\frac{\sum_n\Lambda^Q_{kn}X_{\alpha n}}{E_\alpha+(\omega_k-2\lambda)}\ , \tag{3.27d}$$

and from $E=\pm(\epsilon_i+\delta-\lambda)$ the ortho-normalization conditions follow:

$$\delta_{ij}+\sum_{nk}\frac{\Lambda^Q_{in}\Lambda^Q_{nk}}{\epsilon_i+E_n-4\lambda}\left[\sum_a\frac{U^*_{ak}U_{aj}}{\epsilon_i-\lambda-E_a}+\sum_r\frac{V^*_{rk}V_{rj}}{\epsilon_i-\lambda+E_r}\right]=0\ , \tag{3.28a}$$

$$\delta_{ij}+\sum_{mn}\frac{\Lambda^Q_{im}\Lambda^Q_{nj}}{\epsilon_i-\epsilon_j+\delta}\left[\sum_a\frac{T^*_{am}T_{an}}{\epsilon_i-\lambda-E_a+\delta}+\sum_r\frac{Z^*_{rm}Z_{rn}}{\epsilon_i-\lambda+E_r+\delta}\right]=0\ . \tag{3.29a}$$

Similarly, if $E=\pm(E_m-3\lambda+\delta)$ the result is

$$\delta_{mn}+\sum_{ij}\frac{\Lambda^Q_{mi}\Lambda^Q_{jn}}{E_m-E_n+\delta}\left[\sum_a\frac{U^*_{ai}U_{aj}}{E_m+\delta-3\lambda+E_a}+\sum_r\frac{V^*_{ri}V_{rj}}{E_m+\delta-3\lambda-E_r}\right]=0\ , \tag{3.28b}$$

$$\delta_{mn}+\sum_{is}\frac{\Lambda^Q_{mi}\Lambda^Q_{is}}{\epsilon_i+E_m-4\lambda}\left[\sum_a\frac{T^*_{as}T_{an}}{E_m-3\lambda+E_a}+\sum_r\frac{Z^*_{rs}Z_{rn}}{E_m-3\lambda-E_r}\right]=0\ , \tag{3.29b}$$

while $E=\pm(\omega_k-2\lambda+\delta)$ yields

$$\delta_{kl}-\sum_{nm}\frac{\Lambda^Q_{kn}\Lambda^Q_{nm}}{(\omega_k+\omega_n-4\lambda)}\left[\sum_a\frac{X^*_{\alpha m}X_{\alpha l}}{(\omega_k-2\lambda)-E_\alpha}-\frac{Y^*_{\alpha m}Y_{\alpha l}}{(\omega_k-2\lambda)+E_\alpha}\right]=0\ , \tag{3.30a}$$

$$\delta_{kl}-\sum_{nm}\frac{\Lambda^Q_{kn}\Lambda^Q_{ml}}{(\omega_k-\omega_n+\delta)}\left[\sum_a\frac{X^*_{\alpha n}X_{\alpha m}}{(\omega_k-2\lambda)+E_\alpha+\delta}-\frac{Y^*_{\alpha n}Y_{\alpha m}}{(\omega_k-2\lambda)-E_\alpha+\delta}\right]=0\ . \tag{3.30b}$$

Equations (3.28b), (3.29a), and (3.30b) also can be written as

$$\delta_{ij}=\sum_a U^*_{ai}U_{aj}\frac{[E_a-(\epsilon_j-\lambda)]}{[E_a-(\epsilon_i-\lambda)]}+\sum_r V^*_{ri}V_{rj}\frac{[E_r+(\epsilon_j-\lambda)]}{[E_r+(\epsilon_i-\lambda)]}\ , \tag{3.29a'}$$

$$\epsilon_j \left[\sum_a U_{ai}^* U_{aj} + \sum_r V_{ri}^* V_{rj} \right] = \sum_a U_{ai}^* U_{aj} E_a - \sum_r V_{ri}^* V_{rj} E_r \ , \quad (3.29a'')$$

$$\delta_{mn} = \sum_a T_{am}^* T_{an} \frac{[E_a + (E_n - 3\lambda)]}{[E_a + (E_m - 3\lambda)]} + \sum_r Z_{rm}^* Z_{rn} \frac{[E_r - (E_n - 3\lambda)]}{[E_r - (E_m - 3\lambda)]} \ , \quad (3.28b')$$

$$E_n \left[\sum_a T_{am}^* T_{an} + \sum_r Z_{rm}^* Z_{rn} \right] = \sum_r Z_{rm}^* Z_{rn} E_r - \sum_a T_{am}^* T_{an} E_a \ , \quad (3.28b'')$$

$$\delta_{kl} = \sum_\alpha X_{\alpha k}^* X_{\alpha l} \frac{[E_\alpha + (\omega_l - 2\lambda)]}{[E_\alpha + (\omega_k - 2\lambda)]} - Y_{\alpha k}^* Y_{\alpha l} \frac{[E_\alpha - (\omega_l - 2\lambda)]}{[E_\alpha - (\omega_k - 2\lambda)]} \ , \quad (3.30b')$$

$$\omega_l \left[\sum_\alpha X_{\alpha k}^* X_{\alpha l} - Y_{\alpha k}^* Y_{\alpha l} \right] = \sum_\alpha (X_{\alpha k}^* X_{\alpha l} + Y_{\alpha k}^* Y_{\alpha l}) E_a \ . \quad (3.30b'')$$

Equations (3.17)—(3.30) allow the obtainment and normalization of X, Y, U, V, T, Z as well as the energies E_a, E_r, and E_α, if one assumes that all the needed Λ^Q coefficients as well as λ are known.

To understand the meaning of these conditions, one can study the particular case when each one-particle state has only one three-particle partner to form the quartet and when each two-particle state has a single two-particle partner.

That is equivalent to assuming that $\Lambda^Q_{r\bar{i}} = \delta_{r\bar{i}} \Lambda_{\bar{i}\bar{i}}$ and $\Lambda^Q_{\alpha\beta} = \delta_{\beta\bar{\alpha}} \Lambda_{\alpha\bar{\alpha}}$. All the sums in this case reduce to a single term (we will assume for simplicity that $\omega_{\bar{k}} = \omega_k$), and the energy equations reduce to

$$[E_a - (\epsilon_i - \lambda)][E_a + (E_{\bar{i}} - 3\lambda)] = |\Lambda_{i\bar{i}}|^2 \ , \quad (3.31a)$$

$$[E_r + (\epsilon_i - \lambda)][E_r - (E_{\bar{i}} - 3\lambda)] = |\Lambda_{i\bar{i}}|^2 \ , \quad (3.31b)$$

$$E_\alpha^2 - (\omega_k - 2\lambda)^2 = -|\Lambda_{k\bar{k}}|^2 \ , \quad (3.32)$$

and therefore the values for the energies are (E_a and E_r must be positive)

$$E_a = -\frac{(E_{\bar{i}} - \epsilon_i - 2\lambda)}{2} + \left[\left(\frac{E_{\bar{i}} + \epsilon_i - 4\lambda}{2} \right)^2 + |\Lambda_{i\bar{i}}|^2 \right]^{1/2} \ , \quad (3.33a)$$

$$E_r = (E_{\bar{i}} - \epsilon_i - 2\lambda) + E_a = \frac{(E_{\bar{i}} - \epsilon_i - 2\lambda)}{2} + R \ , \quad (3.33b)$$

$$E_\alpha = [(\omega_k - 2\lambda)^2 - |\Lambda_{k\bar{k}}|^2]^{1/2} \ . \quad (3.34)$$

In a similar way the residues are

$$V_i^2 = \frac{1}{2} \left[1 - \frac{(\epsilon_i + E_i - 4\lambda)}{2R} \right] \ , \quad (3.35a)$$

$$U_i^2 = \frac{1}{2} \left[1 + \frac{(\epsilon_i + E_i - 4\lambda)}{2R} \right] \ , \quad (3.35b)$$

$$T_i = V_i \ , \quad (3.35c)$$

$$Z_i = U_i \ , \quad (3.35d)$$

$$X_{\alpha k}^2 = \frac{1}{2} \left[1 + \frac{(\omega_k - 2\lambda)}{E_\alpha} \right] \ , \quad (3.36a)$$

$$Y_{\alpha k}^2 = \frac{1}{2} \left[-1 + \frac{(\omega_k - 2\lambda)}{E_\alpha} \right] \ , \quad (3.36b)$$

We must now write the equations corresponding to Fig. 10(g). Our first task will be to evaluate the vertices shown in Fig. 3. One must be rather careful to use for each of the lines the appropriate *dressing* required by Fig. 10(g). For example, only the forward part of the propagator is effective when we have the one particle propagator [that is associated with Fig. 10(a)]. Then, the effective matrix elements of the Hamiltonian that are needed can be written as

$$A_{jt,ir} = \sum_{mn} Z_{rm}^* Z_{tn} \langle jn \mid H \mid im \rangle + T_{rm}^* T_{tn} \langle jm \mid H \mid in \rangle \ , \quad (3.37)$$

$$B_{n\beta,ir} = \sum_{ml} Z_{rm}^* X_{\beta l} \langle nl \mid H \mid im \rangle \ , \quad (3.38)$$

$$C_{m\beta,n\alpha} = \sum_{kl} X_{\alpha k}^* X_{\beta l} \langle ml \mid H \mid nk \rangle + Y_{\alpha k}^* Y_{\beta l} \langle nk \mid H \mid ml \rangle \ , \quad (3.39)$$

where the two-body Hamiltonian must destroy one particle in each of the two groups and create a new one in each of the two final groups. As the four particle GF can be decomposed in two different forms, it will be helpful to define two auxiliary functions

$$K(is,jr;t) = \langle 0 \mid T[D_1(j,t) D_3(r,t) D_1^\dagger(i,0) D_3^\dagger(s,0)] \mid 0 \rangle \ , \quad (3.40a)$$

$$S(n\alpha,m\beta;t) = \langle 0 \mid T[D_2(m,t) D_2(\beta,t) D_2^\dagger(n,0) D_2^\dagger(\alpha,0)] \mid 0 \rangle \ . \quad (3.40b)$$

To simplify the notation we define

$$\langle 0 \mid D_1^\dagger(i) D_3^\dagger(r) \mid 0 \rangle = \gamma_{ir} \ , \quad (3.41a)$$

$$\langle 0 \mid D_2^\dagger(m) D_2^\dagger(\alpha) \mid 0 \rangle = \eta_{m\alpha} \ . \quad (3.41b)$$

The functions K and S will be initially calculated using the series of diagrams shown in Fig. 13. It is possible to write their Fourier transforms as

$$K_1(is,jr;E) = K_0(is;E) \delta_{is,jr} - \sum_{kt} K_0(is,E) A_{kt,is} K_1(kt,jr;E) \ , \quad (3.42a)$$

$$S_1(n\alpha,m\beta;E)=S_0(n\alpha;E)\delta_{n\alpha,m\beta} - \sum_{p\gamma} S_0(n\alpha,E)C_{p\gamma,n\alpha}S_1(p\gamma,m\beta;E) , \tag{3.42b}$$

where $K_0(is,E)$ and $S_0(n\alpha,E)$ are given by

$$K_0(is;E)=\frac{i}{E-(\epsilon_i+E_s-\lambda)} , \tag{3.43a}$$

$$S_0(n\alpha;E)=i\frac{2(\omega_n-2\lambda+E_\alpha)}{E^2-(\omega_n-2\lambda+E_\alpha)^2} . \tag{3.43b}$$

How to label the external lines at time zero and t depends upon which type of residues one is interested in. In Fig. 14(a) are shown the diagrams to be summed to evaluate K, while in Fig. 14(b) are the ones corresponding to S. The matrices $L_{\rm I}$ and $L_{\rm II}$ that are shown in Fig. (14) will have the same poles. Using matrix notation, one can write for Fig. 14(a)

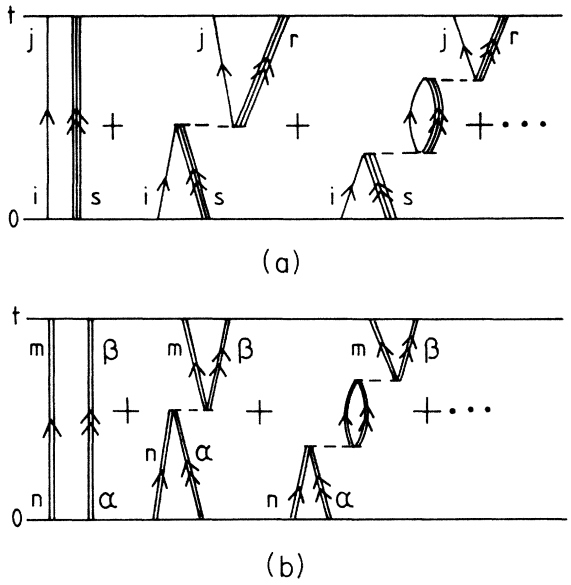

FIG. 13. Diagrams to be summed initially to simplify the evaluation of the QP corresponding to (a) $K_1(is,jr;E)$ and (b) $S_1(n\alpha,m\beta;E)$.

$$K=K_0+K_0BS_1BK_1+K_0BS_1BK_1BS_1BK_1+\cdots+K_0AK_1+K_0AK_1BS_1BK_1+\cdots=(K_0+K_0AK_1)\frac{1}{1-BS_1BK_1} . \tag{3.44}$$

We must remember that the relevant pole of K has $E=0$. It will be necessary to change λ until the matrix

$$B(E)S_1(E)B(E)K_1(E)-1 \tag{3.45}$$

has an eigenvalue equal to 0 for $E=0$. From the diagonalization of this matrix we will be able to evaluate γ_{iq} and $\eta_{n\alpha}$ except for a multiplicative constant. Afterwards we must go back to (2.4c) and (2.4d) and evaluate the coupling constants Λ^Q_{ir} and $\Lambda^Q_{n\beta}$, that in terms of γ_{iq} and $\eta_{n\alpha}$ can be written as

$$\Lambda^Q_{n\beta}=\sum_{m\alpha} C_{n\beta,m\alpha}\eta_{m\alpha} , \tag{3.46a}$$

$$\Lambda^Q_{ir}=\sum_{js} A_{ir,js}\gamma_{js} . \tag{3.46b}$$

For the normalization of the residues there are two possibilities. One is to use the same procedure that was used in Ref. 29. In this case it will be extremely complicated,

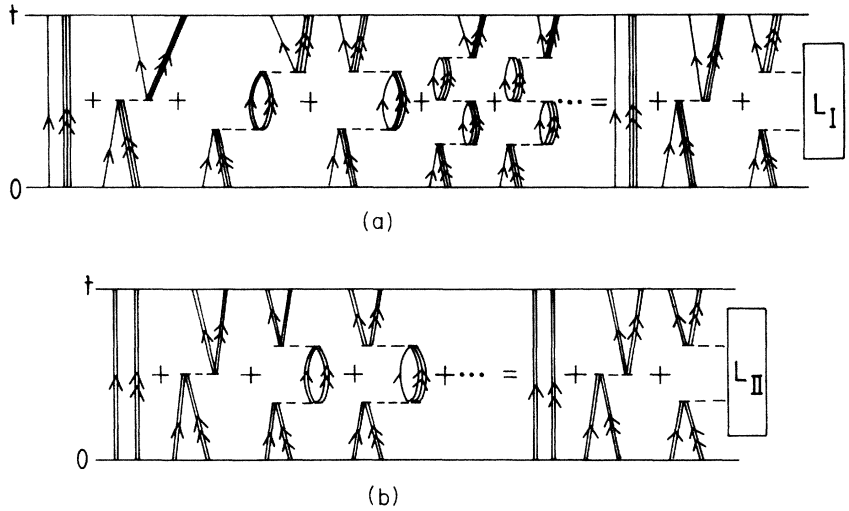

FIG. 14. Diagrams corresponding to the full (a) $K(ir,jt;E)$ and (b) $S(n\alpha,m\beta;E)$.

as it involves a series expansion around the pole at $E=0$. In Ref. 28 was found a simpler procedure to determine the arbitrary constant through the evaluation of the number of particles contained in the vacuum, i.e., to use Fig. 12 with the number operator. In this case one obtains

$$\sum_{ir} \gamma_{ir}^2 = 4K \ . \tag{3.47}$$

We have therefore closed the prescription. One can do a self-consistent determination of the structure of the excitations as well as for the quartets using Eqs. (3.11)—(3.13), (3.37)—(3.39), and (3.45)—(3.47).

IV. CONCLUSIONS

In the present paper we have derived, starting from a set of rules that permits the classification of the relative importance of different processes, a set of equations (CMPGF) that permit a microscopic description of a system formed by a condensate of quartets. The method that was developed, even if very involved, as the problem of handling four particle excitations is complicated, has a rather simple conceptual structure.

The treatment presented is also a natural extension of some of the ideas developed by Flowers and Vujicic.[24] The main difference is that we have shown that it is necessary to take into account in a self-consistent way that the quartet must also be formed by two pairs.

Another characteristic of the present description is that it is *essentially* number conserving, as the main assumptions and approximations that we have done are the following.

(i) We assumed that the ground state of the system can be described as a condensate of quartets.

(ii) We only considered a certain subset of diagrams, those of the principal series. These are the only types of diagrams contributing for large values of K and N. To determine which diagrams belong to the PS and which do not belong to them, we assume that some amplitudes (shown in Fig. 1) are of the same order of magnitude.

(iii) We use of a factor $\sqrt{K}$ each time that an external boson is destroyed and created [one must in fact use a factor $K!/(K-n)! \approx K^n$ if $K \gg n$ where n is the number of connected initial or final external bosons].

In the limit of K and N very large, the PS set of diagrams is the only one contributing to the (exact) description of the system and it will be therefore number conserving. In a sense the present description has many similarities with Nambu's gauge invariant description[32] of superconductivity.

The treatment was developed having in mind its application to well deformed heavy nuclei. The main characteristics of these nuclei can be understood in terms of a Hartree-Fock-Bogoliubov (HFB) description, or, to make a simpler description that retains the most striking physical characteristics of these systems, one can say that they can be understood in terms of the Nilsson model.[33]

As was already said in Sec. III, the equation related to Fig. 10(a) guarantees that in all the odd systems (one particle or one hole states) one will have excitations that have the same structure as the one given in the Nilsson (or HFB) model.

The main difference between the present description and the standard model of deformed nuclei is the emergence of a particular set of three-particle excitations that have a *collective character.* The number of these three-particle states will be small, compared, for example, with the number of usual three-quasiparticle states. One must then expect a special selectivity in three particle transfer reactions. Another striking feature of the model is the existence of sets of *three-hole* states in the one-particle part of the spectra. We may also comment, that the obtaining of the present solution through an expansion around the normal (particle-conserving) initial state is possible because the expansion parameters are the degeneracy and the number of particles and not the interaction strength.

One may ask which was the role that the main features of the four body correlations play in simplifying the diagram selection. We have assumed that the ground state of the system is a condensate of quartets. In order that this assumption make any sense, it is necessary that the interaction between the quartets be not of the same order of magnitude (in terms of K and N) as the processes incorporated in their definition. The particular property that the Hamiltonian must have is to have as ground state a quartet condensate. This property reduces the sums appearing in the diagrams to the small number of collective states, and therefore the requirement is not on the structure of the Hamiltonian but on its ground state.

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