PHYSICAL REVIEW C VOLUME 34, NUMBER 3 SEPTEMBER 1986

Internal conversion in an intense radiation field

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(Received 22 January 1986)

The transition probability per unit time of a K shell internal conversion process is calculated in an intense laser field where the interaction energy of electrons with the laser field is comparable to the binding energy of the electrons in the atomic potential. The intense field K shell internal conversion coefficient is deduced and some numerical examples are given.

I. INTRODUCTION

Rapidly growing interest is being shown in studying nuclear processes in the presence of intense laser or other radiation fields. An intense radiation field can reduce the multipolarity of gamma transitions,[1] modify β decay,[2] and, through changing the atomic surroundings, affect internal conversion.[3] The influence of a laser field on internal conversion has, until now, been investigated only in the following special case.[3] If (such as in the 75 eV isomeric state of ^{235}U) the binding energy of the electrons participating in the conversion process is very small, and with the help of a laser we remove one of the electrons giving a significant contribution to internal conversion, then the coefficient of internal conversion will be diminished.

In the present work we deal with the direct modification of internal conversion caused by a laser field. With intense lasers the interaction energy of the electrons with the laser field is comparable to the binding energy of the electrons in the atomic potential. For internal electrons (K, L, etc. shells) it is still the Coulomb potential of the nucleus that dominates and the modification of the electron states due to the laser can be treated by perturbation theory. For free electrons, however, we can use the Volkov solutions which are determined by the radiation field. Decay of the excited nucleus may take place via generating transitions between such (i.e., laser perturbed Coulomb and Volkov) electron states instead of by multipole radiation.

Exact treatment of the original internal conversion process is very complicated. Based as it is, among other things, on solutions of the Dirac equation, one has to take into account the finite size of the nucleus as well as the shielding effect of charge clouds.[4,5] This is the reason why, in the present calculation, instead of the precise treatment of internal conversion, we shall use a simplified but highly intuitive model[6] to describe the laser induced modifications of the process.

II. DESCRIPTION OF THE MODEL

The Hamiltonian of the system "nucleus + electron + radiation field" is $H=H_0+H_I$, where $H_0=H_{0N}+H_{0e}$ is the sum of the Hamiltonians of the unperturbed nucleus H_{0N} (its explicit form is not necessary for the following treatment) and the single electron H_{0e} participating in internal conversion,

$$H_{0e}=-\frac{Ze^2}{R}+\frac{1}{2m}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}\right]^2 . \tag{1}$$

Here, $\mathbf{A}$ is the vector potential of the external radiation field

$$\mathbf{A}=a[\hat{\mathbf{e}}_1\cos(\omega t-\mathbf{k}\cdot\mathbf{R})+\hat{\mathbf{e}}_2\sin(\omega t-\mathbf{k}\cdot\mathbf{R})] \tag{2}$$

corresponding to a circularly polarized plane wave.

We assume that the interaction between the nucleus and the electron is of Coulomb type, arising between the individual protons of the nucleus and the electron,

$$H_I=-\sum_{j=1}^{Z}\frac{e^2}{|\mathbf{R}-\mathbf{x}_j|}+\frac{Ze^2}{R} , \tag{3}$$

where $\mathbf{x}_j$ denotes proton and $\mathbf{R}$ denotes electron coordinates.

As in and out electron states for the process we take two different approximate solutions of H_{0e} since no exact solution is available. The bound states are taken as hydrogen-like states including modifications due to the laser field.[7] These states are considered initial states of the above process and can be written in the form

$$v_i=\sum_{L=-\infty}^{\infty}v_i^{(L)} , \tag{4a}$$

with

$$v_i^{(L)}=\phi_0(\xi)J_L(b\xi\sin\vartheta)e^{iL\varphi}e^{-i[(E_0+\Delta E_0)/\hbar-\omega L]t} . \tag{4b}$$

Here, $\phi_0(\xi)$ is an unperturbed H-like wave function with energy E_0, and ΔE_0 is the power shift due to the laser field (in the present problem one can neglect this contribution). Furthermore, we use the following notation: $|E_0+\Delta E_0|=K$; J_L is the Bessel function of order L,

$$b=\frac{ea}{\hbar c}a_0,\quad a_0=\frac{a_B}{Z} \tag{4c}$$

where a_B is Bohr's radius, and ϑ and φ are polar angles of the vector $\boldsymbol{\xi}=\mathbf{R}/a_0$ in the coordinate frame defined by $\hat{\mathbf{e}}_1,\hat{\mathbf{e}}_2,\hat{\mathbf{k}}$. For the sake of simplicity, we deal with s-states only. In this case ϕ_0 depends only on ξ. (The above is true if shielding of the radiation field is negligible.[8])

As final states of the process, we take nonrelativistic Volkov solutions given as[9]

$$v_f=\sum_{N=-\infty}^{\infty} v_f^{(N)} , \tag{5a}$$

with

$$v_f^{(N)}=\frac{1}{\sqrt{V}}e^{i((a_0/\hbar)\mathbf{p}\cdot\boldsymbol{\xi})}J_N(\beta\sin\theta)e^{i[(\chi-a_0\mathbf{k}\cdot\boldsymbol{\xi})N+(N\omega-E/\hbar)t]} \tag{5b}$$

and

$$\beta=\frac{eap}{mc\hbar\omega} . \tag{5c}$$

Here, V is the normalization volume, $\mathbf{p}$ is the electron momentum, m is the rest mass of the electron $(E=p^2/2m)$, and χ and θ are polar angles of $\mathbf{p}$ in the coordinate system defined previously. The above states are in Coulomb gauge.

For the sake of further simplification, we deal with the case when $pa_0/\hbar \ll 1$ and $ka_0 \ll 1$: that is, we can replace $\exp[i(a_0\mathbf{p}\cdot\boldsymbol{\xi}/\hbar)]$ and $\exp[i(a_0\mathbf{k}\cdot\boldsymbol{\xi}N)]$ by unity.

After these simplifications, (5b) reduces to

$$v_f^{(N)}=\frac{1}{\sqrt{V}}J_N(\beta\sin\theta)e^{i\chi N}e^{i(N\omega-E/\hbar)t} . \tag{5d}$$

In other words, the wave function corresponding to the outgoing electron does not depend on electron coordinates within the interaction volume relevant for the process.

In the subsequent sections we shall determine the transition probability per unit time of internal conversion and the internal conversion coefficient in the presence of an intense laser field with the help of the states given in this section.

III. CALCULATION OF THE TRANSITION PROBABILITY PER UNIT TIME

For the derivation of the transition probability we use the scattering matrix formalism. The matrix element to be determined is, in first order,

$$S_{fi}=\frac{1}{i\hbar}\int v_f^*\psi_f^{b*}H_I\psi_i^a v_i d^3R\, d\tau\, dt . \tag{6}$$

Here, ψ_i^a and ψ_f^b are nuclear wave functions for the initial (a) and final (b) states, respectively.

$$d\tau=\sum_{\text{proton spin}} d^3x_1\cdots d^3x_z ,$$

and $\mathbf{x}_1,\ldots,\mathbf{x}_z$ are proton coordinates. Using expressions (4a) and (5a) for v_i and v_f, we can write S_{fi} as

$$S_{fi}=\sum_{N,L}S_{fi}^{(N,L)} , \tag{7}$$

where

$$S_{fi}^{(N,L)}=\frac{1}{i\hbar}\int v_f^{(N)*}\psi_f^{b*}H_I\psi_i^a v_i^{(L)}d^3R\, d\tau\, dt . \tag{8}$$

After carrying out time integration, this takes the form

$$S_{fi}^{(N,L)}=-iH_{I,ab}^{(L)}\hat{v}_f^{(N)}2\pi\delta(K+E-\hbar\omega_{ab}+(L-N)\hbar\omega) , \tag{9}$$

where we introduced the notation

$$H_{I,ab}^{(L)}=\int\hat{\psi}_f^{b*}H_I\hat{\psi}_i^a\hat{v}_i^{(L)}d^3R\, d\tau , \tag{10}$$

and $\hbar\omega_{ab}=E_a-E_b$ (E_a and E_b are the energies of the initial and final nuclear states) and the caret denotes the space dependent part of the wave functions.

To determine transition probability per unit time, we have to calculate the quantity

$$|S_{fi}|^2\frac{Vp^2dp\, d\Omega_p}{(2\pi\hbar)^3}=\sum_{N,L,N',L'}(2\pi)^2H_{I,ab}^{(L)}H_{I,ab}^{(L')}\delta(N,L)\delta(N',L')\times\hat{v}_f^{(N)*}\hat{v}_f^{(N')}V\frac{mp}{(2\pi\hbar)^3}dE\, d\Omega_p , \tag{11}$$

where $\delta(N,L)$ denotes the Dirac delta function in (9) and $\delta(N',L')$ has the same meaning, but N' and L' are used in the argument instead of N and L.

As $\hat{v}_f^{(N)}$ and $\hat{v}_f^{(N')}$ have no ξ dependence and are of the form given in (5d), furthermore, $d\Omega_p=d\chi\, d(\cos\theta)$, the integration by χ can be carried out, giving $2\pi\delta_{L,L'}$. Using this and the identity

$$\delta(N,L)\delta(N',L')=\frac{1}{\hbar}\delta(\omega(N-N'))\delta(N,L) ,$$

the sums over N' and L' can be executed and we can obtain the following result in the usual way:[10]

$$dw_{fi}(m_a\to m_b)=\sum_{N,L}n_0\frac{4\pi^2}{\hbar}|H_{I,ab}^{(L)}|^2\frac{mp(N,L)}{(2\pi\hbar)^3}\times J_N^2(\beta\sin\theta)d(\cos\theta) , \tag{12}$$

where the symbol $p(N,L)$ denotes that p has to take a value determined by the argument of $\delta(N,L)$; n_0 is the density of the initial state, which is 2 in our case as only S states are investigated, and m_a,m_b are the magnetic quantum numbers of the nucleus in states a and b, respectively. From formula (12) it can be seen that the outgoing electrons have cylindrically symmetric distribution around the laser beam as the symmetry axis.

The main contribution to $H_{I,ab}^{(L)}$ comes from values of $\mathbf{R}$ larger than the nuclear coordinates $\mathbf{x}_j$; thus, we use the approximate form of the expansion of $1/|\mathbf{R}-\mathbf{x}_j|$ in terms of spherical harmonics (A1) (see Appendix) valid if $x_j<R$, which is true for all the nuclei except the heavy ones.[6] In this way

$$H_{I,ab}^{(L)}=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{4\pi}{2l+1}Q_{lm}(a,b)I_{lm}^{(L)} , \tag{13}$$

where

$$Q_{lm}(a,b)=e\sum_{j=1}^{Z}\int r_j^l Y_{lm}^*(\vartheta_j,\varphi_j)\hat{\psi}_f^{b*}\hat{\psi}_i^a d\tau \quad (14)$$

is the matrix element of the electric multipole moment of order l,m between nuclear states a and b, and

$$I_{lm}^{(L)}=\int \hat{v}_i^{(L)}(\xi)Y_{lm}(\vartheta,\varphi)\xi^{1-l}d\xi\, d\varphi\, d(\cos\vartheta)\ . \quad (15)$$

IV. K SHELL CASE

If $\hat{v}_i$ is the wave function of the innermost electron shell, i.e., it is a K shell wave function which is given here as a laser modified, $1S$ hydrogen-type one, then with the aid of (4b), (15) has the form

$$I_{lm}^{(L)}=Ci_{lm}^{(L)}\ , \quad (16a)$$

where

$$i_{lm}^{(L)}=\int e^{-\xi}J_L(b\xi\sin\vartheta)e^{iL\varphi}\times Y_{lm}(\vartheta,\varphi)\xi^{1-l}d\xi\, d\varphi\, d(\cos\vartheta) \quad (16b)$$

and

$$C=\frac{e}{(\pi a_0^3)^{1/2}}a_0^{2-l}\ . \quad (17)$$

The integral over φ gives $2\pi\delta_{-L,m}$, which means that $|L|\leq l$ can occur in (13). By summing over m in (13), we get terms containing $I_{lm}^{(L)}$ of type $I_{l,-L}^{(L)}$ only. Because of the identities $J_{-L}=(-1)^L J_L$ and $Y_{l-|m|}=(-1)^{|m|}Y_{l\,|m|}$, the relation

$$I_{l,-L}^{(L)}=I_{l,L}^{(-L)},\ \ L>0 \quad (18)$$

holds, which means that it is sufficient to deal with $I_{l,-L}^{(L)}$, $L\geq 0$, and it is denoted I_{lL} (and the corresponding $i_{l,-L}^{(L)}$ as i_{lL}) in the following.

With the use of (A2) (see Appendix), $J_L(b\xi\sin\vartheta)$ can be factored into $b\xi$ and ϑ dependent terms. The integral over $\cos\vartheta$ in (16b) has nonzero value if $l-L=$even, which can be seen from the forms of the spherical harmonics (A7a) and (A7b), i.e., the Y_{lm} has $P_l^{|m|}(\cos\vartheta)$-type $\cos\vartheta$ dependence. Thus with the use of (A3), (16b) has the form

$$i_{lL}=N_{lL}\int_0^\infty e^{-\xi}2\pi\sum_{k\geq(l-L)/2}f_{lLk}\frac{b^k}{k!2^k}\times J_{L+k}(b\xi)\xi^{k+1-l}d\xi\ ,\quad l-L=\text{even},\ \ L\geq 0 \quad (19)$$

$$i_{lL}=0,\ \ l-L=\text{odd}$$

where

$$N_{lL}=\left[\frac{2l+1}{4\pi}\frac{(l-L)!}{(l+L)!}\right]^{1/2} \quad (20a)$$

and

$$f_{lLk}=\frac{2^{-L}\Gamma(k+\frac{1}{2})k!(l+L)!}{\Gamma((L-l)/2+k+1)\Gamma((L+l)/2+k+\frac{3}{2})(l-L)!}\ . \quad (20b)$$

With the aid of a further integral formula, (A4), the integral over ξ can be carried out in (19) and we obtain, as a result,

$$i_{lL}=S_{lL}(y)(1+b^2)^{(l-2)/2}y^{L/2},\ \ l-L=\text{even}$$
$$i_{lL}=0, l-L=\text{odd} \quad (21)$$

where

$$y=b^2/(1+b^2)\ , \quad (22a)$$

$$S_{lL}(y)=\sum_{k\geq(l-L)/2}^{\infty}F_{lLk}y^k\ , \quad (22b)$$

with

$$F_{lLk}=2\pi N_{lL}f_{lLk}\frac{(2k+L-l+1)!}{(L+k)!k!2^{2k+L}}\times {}_2F_1\left[\frac{L-l}{2}+k+1,\frac{l+L-1}{2};1+L+k;y\right]\ . \quad (22c)$$

For $L<0$, I_{lL} (and also i_{lL}) can be obtained from (18).

From the nuclear electric multipole matrix elements $Q_{lm}(a,b)$, we can take those nonvanishing ones which have the smallest l and we can neglect all the others in (13). Thus, l is determined by the nuclear states a and b. Averaging over the magnetic quantum number of the initial nuclear state (a) and summing over the magnetic quantum number of the final state (b) as usual, we obtain

$$\frac{1}{2J_a+1}\sum_{m_a,m_b}|H_{I,ab}|^2=\left[\frac{4\pi}{2l+1}\right]^2 I_{lL}^2\frac{|\langle a||Q_l||b\rangle|^2}{(2J_a+1)(2l+1)}\ , \quad (23)$$

where $\langle a||Q_l||b\rangle$ is the reduced matrix element[11] of the electric multipole momentum of order l. With the aid of the above and (12) we obtain

$$dw_{fi}=\frac{32\alpha}{\hbar\lambda_C(2l+1)^3}\sum_{N,L}{}'\frac{p(N,L)}{\hbar}i_{l|L|}^2\frac{|\langle a||Q_l||b\rangle|^2}{a_0^{2l-1}(2J_a+1)}\times J_N^2(\beta\sin\theta)d(\cos\theta)\ , \quad (24)$$

where λ_C is the Compton wavelength of the electron, α is the fine structure constant, and the prime over the sum denotes that summation over l must be carried out for $l-L=$even in the $-l\leq L\leq l$ interval and the sum over N must be carried out from that minimum (generally negative) N value which is determined by energy conservation, i.e., by the argument of δ in (9).

The integral over $\cos\theta$ can be carried out with the aid of formulae (A5) and (A6) as

$$\int J_N^2(\beta\sin\theta)d(\cos\theta)=T_{|N|}(\beta)\ , \quad (25)$$

with

$$T_{|N|}(\beta)=\frac{1}{2\beta}\int_0^{2\beta}J_{2|N|}(x)dx\ . \quad (26)$$

Thus the total transition probability per unit time is

$$w_{fi}=\frac{32\alpha}{\hbar\lambda_C(2L+1)^3}\sum_{N,L}{}'\frac{p(N,L)}{\hbar}i_{l|L|}^2\frac{|\langle a||Q_l||b\rangle|^2}{a_0^{2l-1}(2J_a+1)}\times T_{|N|}(\beta)\ . \tag{27}$$

V. INTERNAL CONVERSION COEFFICIENT IN INTENSE RADIATION FIELD

The intense field internal conversion coefficient is defined as the ordinary one, viz.,

$$\alpha=\frac{w_{fi}(\text{las})}{w_{fi}(\gamma)}\ , \tag{28}$$

where $w_{fi}(\text{las})$ is the total transition probability per unit time of the internal conversion process in an intense radiation (laser) field [for the $1S$ state given by (27)] and $w_{fi}(\gamma)$ is the same for ordinary γ emission and it is given[12] as

$$w_{fi}=\frac{8\pi(l+1)}{l[(2l+1)!!]^2\hbar}\left[\frac{\omega_{ab}}{c}\right]^{2l+1}\frac{|\langle a||Q_l||b\rangle|^2}{(2J_a+1)}\ . \tag{29}$$

Thus, α_{1S} can be obtained from (27)–(29) as

$$\alpha_{1S}=\frac{4l[(2l-1)!!]^2}{\pi(l+1)(2l+1)}\left[\frac{\alpha E_e}{E_\gamma}\right]^{2l+1/2}\frac{Z_{\text{eff}}^{2l-1}}{\alpha^{1/2}}\times\sum_{N,L}{}'\left[\frac{2E(N,L)}{E_\gamma}\right]^{1/2}i_{l|L|}^2T_{|N|}(\beta)\ , \tag{30}$$

where $E_e=mc^2$, $E_\gamma=E_a-E_b$, and Z_{eff} is the effective nuclear charge,[13] which is introduced in order to take into account the shielding of the nucleus to some extent. We took $a_0=a_B/Z_{\text{eff}}$. The effective charge eZ_{eff} corresponds to the charge of the nucleus in a hypothetical hydrogen-like ion in which the electron has the same binding energy as in the real atom while the electron has the same quantum numbers in both systems.

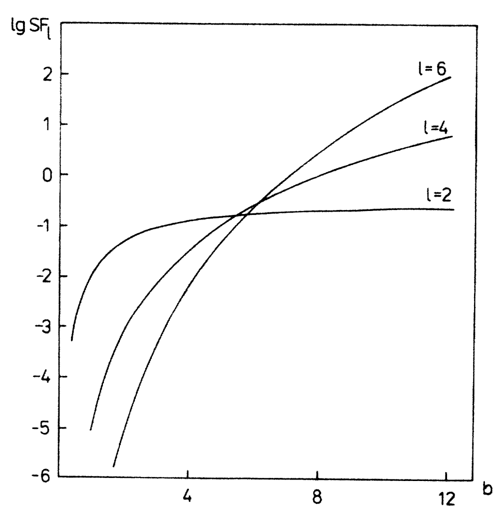

FIG. 1. The quantity $lgSF_l$ vs b for $l=2,4,6$.

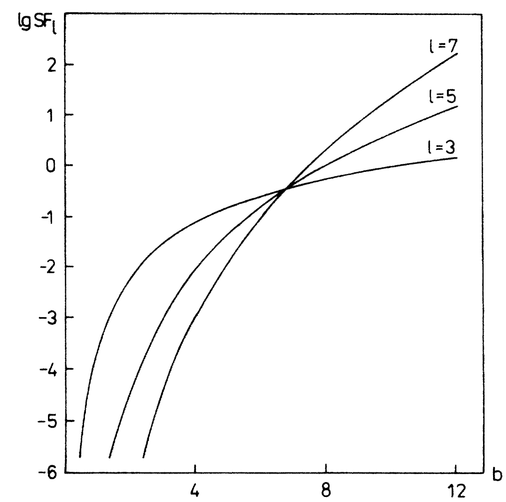

FIG. 2. The quantity $lgSF_l$ vs b for $l=3,5,7$.

Using formulae (21), (22a), and (22b) and $E(N,L)=E_\gamma-K+(N-L)\hbar\omega$, we can get

$$\alpha_{1S}=\frac{Z_{\text{eff}}^{2l-1}}{\alpha^{1/2}}\left[\frac{\alpha E_e}{E_\gamma}\right]^{2l+1/2}\left[\frac{8\Delta}{E_\gamma}\right]^{1/2}\times\sum_{N,L}{}'\left[1+(N-L)\frac{\hbar\omega}{\Delta}\right]^{1/2}SF_{l|L|}(b)T_{|N|}(\beta)\ , \tag{31}$$

where $\Delta=E_\gamma-K$ and

$$SF_{l|L|}=(1+b^2)^{l-2}y^{|L|}S_{l|L|}^2\frac{2l[(2l-1)!!]^2}{\pi(l+1)(2l+1)}\ . \tag{32}$$

VI. RESULTS AND DISCUSSION

The quantities in formula (31) are grouped so that before the sum the laser independent, i.e., nuclear (E_γ,l) and atomic (Z_{eff},Δ), parameters occur, and the part after it depends on the multipolarity (l), on the two laser intensity dependent parameters b and β, and on the ratio of the laser photon energy and the outgoing electron energy in the laser free case $(\hbar\omega/\Delta)$. Our result is valid only in the so called optical frequency range because of the assumptions and computation methods employed.

Generally, $\hbar\omega/\Delta \ll 1$ and $|L|\le l \ll N$ for most

TABLE I. The values of the coefficients A_l in formula (35) (l is the order of multipolarity).

l	A_l
2	0.249
3	1.087×10^{-2}
4	3.110×10^{-4}
5	5.320×10^{-6}
6	2.299×10^{-7}
7	2.650×10^{-9}

TABLE II. Intense field K shell internal conversion coefficients computed with laser intensity $I=10^{18}$ W/cm^2 and laser photon energy $E=1.16$ eV. Nuclear and atomic data necessary to the calculation are also given. (For the notation, see the text).

	E_γ (keV)[a]	El	K (keV)[a]	Δ (keV)	τ[a]	Z_{eff}[b]	α_{las}
$^{135}_{56}$Ba	47.3	$E5$	37.44	9.86	28.7h ($M4$)	51.49	1×10^{-1}
$^{148}_{61}$Pm	61.5	$E4$	45.19	16.22	41.3d (95% β)	56.34	3×10^{-2}
$^{160}_{67}$Ho	60	$E3$	55.62	4.38	5.02h	62.18	2×10^{-1}

[a]Reference 15.
[b]Reference 16.

values of N; therefore the L dependence in the square root can be neglected and the sums over L and N can be executed independently. This leads to the result

$$\alpha_{1S}=BSF_l T\ , \tag{33}$$

where

$$B=Z_{eff}^{2l-1}\alpha^{-1/2}\left[\frac{\alpha E_e}{E_\gamma}\right]^{2l+1/2}\left[\frac{8\Delta}{E_\gamma}\right]^{1/2}, \tag{34a}$$

$$SF_l=\sum_{L=-l}^{l}SF_{l\,|L|},\quad l-L=\text{even}\ . \tag{34b}$$

and

$$T=\sum_{N>-\Delta/\hbar\omega}\frac{1}{2\beta_0}\int_0^{2\beta(N)}J_{2|N|}(x)dx\ , \tag{34c}$$

with

$$\beta_0=\frac{ea}{\hbar\omega}\left[\frac{2\Delta}{mc^2}\right]^{1/2}$$

and

$$\beta(N)=\beta_0\left[1+(N-L)\frac{\hbar\omega}{\Delta}\right]^{1/2}.$$

Here, with the notation $\beta=\beta(N)$, it is shown that β may have an N dependence because of $\beta\sim p(N,L)$ [see (5c)] which will be neglected later on. The L dependence of β is neglected too.

In order to obtain numerical values for the laser modified internal conversion coefficient for some nuclei, the sum over L was executed and the SF_l values were numerically computed for the values of $b\leq12$. The results (for example, SF_l versus b) are depicted in Figs. 1 and 2 for $l=2,3,\ldots,7$. For $b>12$ the following approximative formula is used:

$$SF_l=A_l(1+b^2)^{l-2}\ , \tag{35}$$

where the numerical values of the coefficients A_l are given in Table I.

Instead of carrying out the sum over N in (34c), we give an order of magnitude estimation[14] leading approximately to $T\simeq1$.

Our numerical results are given in Tables II and III for those nuclei where $\Delta\ll K$ and our simplifications made at the beginning (see Sec. II) are valid. Table II gives the energy of the nuclear transition, i.e., the outgoing γ (E_γ) and its multipolarity (El), the K shell binding energy (K), the energy of the outgoing electron ($\Delta=E_\gamma-K$), the half-life of the nuclear state (τ) and its decay mode,[15] the effective charge (Z_{eff}) belonging to the K shell,[16] and the intense field internal conversion coefficient (α_{las}) obtained from formulae (33) and (34). For the computation we used $b=1.45\times10^{-7}I^{1/2}Z_{eff}^{-1}E_{las}^{-1}$ and $\beta_0=3.39\times10^{-5}I^{1/2}\Delta^{1/2}E_{las}^{-2}$ obtained from (4c) and (5c), respectively, where I is the laser intensity in W/cm^2, Δ is in keV, and E_{las} is the laser photon energy in eV. The α_{las} values in Table II are computed with

$$I=10^{18}\ \text{W/cm}^2,\quad E_{las}=1.16\ \text{eV}\quad(\lambda=1.06\ \mu\text{m})\ .$$

The same α_{las} values can be obtained in the case of

$$I=10^{16}\ \text{W/cm}^2,\quad E_{las}=0.116\ \text{eV}\quad(\lambda=10.6\ \mu\text{m})\ .$$

In Table III our results and the ordinary laser free K shell internal conversion coefficients[17] are given.

Thus, our results indicate that in the range of the available optical frequencies laser assisted internal conversion coefficients are less than the usual internal conversion ones.

Finally, we investigate the following special case where the influence of an external radiation field may significantly modify the process. In some nuclei (e.g., $^{105}_{47}$Ag) the energy of the outgoing gamma ($E_\gamma=25.47$ keV, $E3$ for $^{105}_{47}$Ag) is less compared to the K shell binding energy ($K=25.514$ keV, $Z_{eff}=42.77$ for $^{105}_{47}$Ag) and thus it is not enough to produce K shell internal conversion. In our notation it means that $\Delta<0$. However, if $|\Delta|$ is in the order of magnitude of the laser photon energy (in the above mentioned case it is $|\Delta|\sim44$ eV), the originally energeti-

TABLE III. Comparison of the intense field K shell internal conversion coefficients (given also in Table II) and the ordinary ones.

	α_K[a]	α_{las}
$^{135}_{56}$Ba	172 (<1000)	1×10^{-1}
$^{148}_{61}$Pm	33.4	3×10^{-2}
$^{160}_{67}$Ho	4.41	2×10^{-1}

[a]Reference 17.

cally forbidden ($\alpha_{1S}=0$) K shell internal conversion process may pass off via absorption of the necessary amount of laser photons. With a similar train of thought as above, we can obtain the laser induced internal conversion coefficient of this process in the same form as in (33), but with

$$\beta_0=\frac{ea}{\hbar\omega}\left[\frac{2\hbar\omega}{mc^2}\right]^{1/2},$$

$$\beta=\beta_0(N-L+\Delta/\hbar\omega)^{1/2},$$

$$B=Z_{\rm eff}^{2l-1}\alpha^{-1/2}\left[\frac{\alpha E_e}{E_\gamma}\right]^{2l+1/2}\left[\frac{8\hbar\omega}{E_\gamma}\right]^{1/2}. \quad (36)$$

We can conclude that the K shell internal conversion coefficient, which is originally zero, can have a nonzero value in this case.

In order to obtain numerical values, we make estimates for the $^{105}_{47}$Ag case with $E_{\rm las}=5$ eV and $I=10^{16}$ W/cm^2. It means that $N>8$ in (34c), i.e., the process needs the absorption of more than eight laser photons. At this laser intensity and photon energy, $b=6.8\times10^{-2}$, $y=4.3\times10^{-3}$, and therefore $S_{lL}(y)$ can be approximated as

$$S_{lL}(y)=F_{lLk_{\rm min}}y^{k_{\rm min}};$$

furthermore, the hypergeometric function $_2F_1(a,b;c;y)$ in $F_{lLk_{\rm min}}$ can be approximated by unity. These give for SF_l in the case of $l=3$ the following approximation: $SF_3=45\times y^3/50176$. With the values of $E_{\rm las}$ and I mentioned above, we obtain $\alpha_{1S}=1.8\times10^{-8}T$, which gives $\alpha_{1S}=2\times10^{-8}$ with the above $T\sim1$ estimation. Similarly, with $E_{\rm las}=1.16$ eV and $I=10^{18}$ W/cm^2, $\alpha_{1S}=1.1\times10^{-4}$ gives $\alpha_{1S}=1\times10^{-4}T$.

As a consequence, internal conversion of this type seems to be a candidate for a process where the influence of an external radiation field may modify a nuclear process.

ACKNOWLEDGMENT

This work was supported in part by the Air Force Office of Scientific Research.

APPENDIX

The following mathematical formulae are used in the article:

$$\frac{1}{|\mathbf{R}-\mathbf{x}_j|}=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{4\pi}{2l+1}\frac{r_j^l}{R^{l+1}}Y_{lm}(\vartheta,\varphi)Y^*_{lm}(\vartheta_j,\varphi_j), \quad r_j<R \quad (A1)$$

(see Ref. 18),

$$J_\nu(\lambda x)=\lambda^\nu\sum_{k=0}^{\infty}\frac{1}{k!}J_{\nu+k}(x)\left[\frac{1-\lambda^2}{2}x\right]^k \quad (A2)$$

(see Ref. 19),

$$\int_0^1 x^\sigma(1-x^2)^{m/2}P_\nu^m(x)dx=\frac{(-1)^m2^{-m-1}\Gamma\left[\frac{\sigma+1}{2}\right]\Gamma\left[\frac{\sigma+2}{2}\right]\Gamma(1+m+\nu)}{\Gamma(1-m+\nu)\Gamma\left[\frac{\sigma+m-\nu+2}{2}\right]\Gamma\left[\frac{\sigma+m+\nu+3}{2}\right]} \quad (A3)$$

(see Ref. 20),

$$\int_0^\infty e^{-\alpha x}J_\nu(\beta x)x^{\mu-1}dx=\frac{\left[\frac{\beta}{2}\right]^\nu\Gamma(\nu+\mu)\,{}_2F_1\left[\frac{\nu+\mu}{2},\frac{1-\mu+\nu}{2};\nu+1;\frac{\beta^2}{\alpha^2+\beta^2}\right]}{\Gamma(\nu+1)(\alpha^2+\beta^2)^{(\nu+\mu)/2}} \quad (A4)$$

(see Ref. 21),

$$\int_0^{\pi/2}J_\nu^2(\beta\sin\theta)\sin\theta\,d\theta=\frac{1}{\beta}\sum_{k=0}^{\infty}J_{2\nu+2k+1}(2\beta) \quad (A5)$$

(see Ref. 22),

$$2\sum_{k=0}^{\infty}J_{\mu+2k+1}(2\beta)=\int_0^{2\beta}J_\mu(x)dx \quad (A6)$$

(see Ref. 23),

$$Y_{lm}(\vartheta,\varphi)=(-1)^m\left[\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}\right]^{1/2}\times P_l^m(\cos\vartheta)e^{im\varphi},\quad m\ge0, \quad (A7a)$$

$$Y_{l-m}(\vartheta,\varphi)=(-1)^mY_{lm}(\vartheta,\varphi),\quad m\ge0. \quad (A7b)$$

Throughout the paper Y_{lm} denotes the spherical harmonics, J_ν the Bessel function of the first kind, P_l^m the associated Legendre polinom, Γ the gamma function, and $_2F_1$ the hypergeometric function.

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