Nuclear friction and lifetime of induced fission

K. H. Bhatt* and P. Grangé
Centre de Recherches Nucléaires et Université Louis Pasteur, 67037 Strasbourg Cedex, France

B. Hiller
Centre d'Etudes Nucléaires, Département de Recherche Fondamentale-Physique Nucléaires, 38041 Grenoble Cedex, France

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Induced nuclear fission is described as a transport process of the fission degree of freedom over the fission barrier. The lifetime of the event is defined in terms of the probability of finding the nuclear system in the potential well corresponding to the ground-state deformation. This definition appears as a natural generalization to nonstationary transport processes of the usual expression for the lifetime. Using the conservation law for the current we relate the lifetime to the time-integrated escape rate across the collective potential barrier. We envisage a schematic model in which the escape rate attains a constant value only after a certain transient time τ. In this case we show that the lifetime evaluated at the saddle point of the collective potential is made up of two contributions: one—$\hbar/\Gamma_f^{\rm stat}$—identified as the quasistationary transition-state expression of the statistical model and one proportional to τ, hence directly related to the transient behavior of the transport process. As long as $\hbar/\Gamma_f^{\rm stat} \gg \tau$, fission can well be described as a quasistationary transport phenomenon. For $\hbar/\Gamma_f^{\rm stat} \ll \tau$, which occurs for excitation energies of a few hundreds of MeV and small fission barriers, the fission process becomes a transient phenomenon of duration of the order of τ. For a single collective variable and its canonically conjugate momentum, we study the transient time τ as a function of the nuclear friction constant β. Thereby we extend and complete the findings of earlier studies. For a specific system of mass $A=248$, we calculate the lifetime at the saddle point of the collective potential and find results in keeping with our schematic model. We assume further that the collective potential beyond the saddle point can be correctly represented by an inverted parabola and we obtain an analytical expression for the current evaluated at the scission point. We find that this current can be expressed reliably as the current evaluated at the saddle point but delayed by a constant time $\bar{\tau}_1$ which we obtain and interpret. Thereby we bring support to the conjectures made in several studies of fissioning systems within the same framework. As a result we also extend trivially the schematic model to the escape rate evaluated at the scission point and obtain the lifetime evaluated at scission as the sum of the lifetime evaluated at the saddle point and of the time delay $\bar{\tau}_1$.

I. INTRODUCTION

Dissipative processes in atomic nuclei have been observed for a long time, primarily in nuclear fission.[1] The deep inelastic heavy-ion reactions provide a large variety of nuclear dissipative systems often analyzed within the framework of transport theories.[2] In such descriptions dissipation, i.e., the irreversible flow of energy between various degrees of freedom of the system, is modeled in terms of a nuclear friction tensor which may be determined either from microscopic considerations[2] or entirely phenomenologically.[3] However, in deep inelastic collisions the nuclear matter overlap amounts to 10—20 % of the total volume of the system and the information obtained here from about the nuclear friction tensor concerns only a limited surface region of the nuclei. Yet the need for understanding the mechanism of nuclear friction deep in the nuclear interior has grown since the recent production[4,5] of highly excited composite nuclear systems. At such high excitation energies the standard statistical model[1] predicts lifetimes seemingly inconsistent with observations. However, it is recognized[6,7] that the fission widths derived in this approach are obtained from phase-space arguments only with no consideration for the effects of nuclear friction. Hence the important question arises: how precisely are the lifetimes at different excitation energies affected by nuclear friction? To study this problem a transport description of fission is certainly useful as it includes dynamical features not contained in the statistical model.

The domain of applicability of transport theories has been extensively discussed in the case of deep inelastic heavy-ion reactions.[8] More recently it was found that transport theories were also legitimate[9] for describing the competitive decay of composite nuclear systems. Here we take for granted the applicability of a transport approach to the fission process and we want to focus on the general definition and properties of the lifetime in this framework. In Sec. II we define this lifetime independently of any specific form of the transport equation and in terms of the probability of finding the composite system in a certain initial state. The conservation law for the current density enables one to relate this lifetime to the escape rate. We show further how this relation eventually leads

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to the usual expression of the lifetime in terms of the statistical model decay width. As the probability current over the fission barrier rises smoothly from zero at time $t=0$ to a quasistationary value, the escape rate accordingly reaches a constant value over a characteristic time, the transient time τ. Using a schematic model for the time dependence of the escape rate, it is shown simply how the lifetime evaluated at the saddle point of the collective potential depends upon the transient time τ and upon the quasistationary value reached by the escape rate. Thereby we exhibit clearly the dominant role of transients at high excitation energies (≥ 100 MeV) and accordingly why in such cases the lifetime of fission is determined by τ. The precise value of τ depends strongly upon the dissipation tensor[10] and one may thus expect novel information about nuclear friction from the studies of lifetimes and competitive decays of systems produced in this domain of excitation energy.

The essential quantity in the present analysis of the lifetime is the time dependent escape rate. In Sec. III we consider first its evaluation at the saddle point of the collective potential. We use the very same simple one-dimensional model of an earlier study[10] which we complete and extend. The novel features concern the derivation of an approximate analytical expression for the escape rate which takes into account the existing anharmonicities of the collective potential as well as the overall underdamped or overdamped motion of the fissioning system along the collective coordinate. These analytic expressions are found to reproduce very reliably the time dependent escape rates and the transient times τ obtained directly from a numerical solution of the transport equation. From these expressions we calculate the lifetime τ_f^{sad} at the saddle point and study the ratio τ/τ_f^{sad} as a function of the strength β of the coupling of the collective degrees of freedom to a heat bath of given temperature. The schematic model of Sec. II enables one to interpret simply the variations of this ratio with β.

In Sec. IV we turn to the study of the modifications of the escape rates and lifetimes in going from the saddle point to the scission point. We assume that the collective potential beyond the saddle point can be correctly modeled as an inverted parabola. We derive the probability of finding the fissioning system at any point beyond the saddle point in terms of the equivalent probability at the saddle point. The time-dependent flux at scission is obtained and we show that it can be expressed simply as the time dependent flux at the saddle point delayed by a constant time which we derive and interpret. Finally the value of the lifetime evaluated at the scission point is discussed. For a quasistationary situation of probability flow across the potential barrier it appears essentially as the sum of three contributions: one which relates to the decay width of the statistical model and two directly connected to the dynamics of the system on its way to the saddle point and beyond. Finally Sec. V contains the conclusions.

We stress that our findings related to the lifetime of the fission process are generic of transport approaches to this phenomenon and mostly independent of the simplifications we have introduced. They serve only to give quantitative estimates of the various contributions to this lifetime within a simple one-dimensional model often used in the analysis of experimental data.

II. GENERAL DEFINITION OF THE LIFETIME

Transport theories assume the existence of a heat bath formed by a certain collection of microscopic variables that couple to the collective degrees of freedom of the system. In our case the collective variables are the fission modes. The domain of applicability of a transport description in terms of time scales and excitation energy of the fissioning system has been previously discussed.[9,10] We assume in the present paper that the necessary conditions for the separation between intrinsic and collective coordinates are met.

We consider the fission variables $\{X_i\}$, $i=1-n$, as classical variables with associated momenta $\{p_i\}$ and a time variable t. The time evolution of the decaying composite system is then described through the propagation in phase space $\{X_i,p_i\}$ of the distribution function $P(X_i,p_i;t)$ which describes the probability of finding the system in the domain of the phase space limited by the hypershells $\{X_i+\Delta X_i,p_i+\Delta p_i\}$ and $\{X_i,p_i\}$. In the absence of the coupling to the heat bath the collective variables obey classical equations of motion and the distribution function satisfies the Liouville equation. The driving force is related to the collective potential which presents a barrier that the system must overcome to undergo fission.

Let X_i^0, X_i^{sc}, and X_i^1 be, respectively, the positions of the saddle point, of the scission point, and of the minimum in the potential pocket corresponding to ground state deformation.

The probability that the system is to the left of the scission point is given by

$$\Pi(X_i^{\text{sc}};t)=\int_{-\infty}^{X_i^{\text{sc}}} dX_i \int_{-\infty}^{\infty} dp_i P(X_i,p_i;t) \ . \tag{2.1}$$

At time $t=0$ we assume that the composite system is formed in the potential pocket at $X=X_i^1$ and $\Pi(X_i^{\text{sc}};t=0)$ may be normalized to one.

We now define the lifetime of the system as the time $\tau=\tau_f$ at which $\Pi(X_i^{\text{sc}};t)$ has been reduced to e^{-1} of its initial value at $t=0$:

$$\Pi(X_i^{\text{sc}};t=\tau_f)=e^{-1}\Pi(X_i^{\text{sc}};t=0) \ . \tag{2.2}$$

The value of τ_f is thus determined by the time evolution of the probability $\Pi(X_i^{\text{sc}};t)$ defined in Eq. (2.1). Quite generally and independently of the specific form of the transport equation obeyed by $P(X_i,p_i;t)$ the conservation of probability implies the following continuity equation:

$$\frac{d}{dt}\Pi(X_i^{\text{sc}};t)=-\int_S \mathbf{dX}\cdot\mathbf{J}(X_i^{\text{sc}};t) \ , \tag{2.3}$$

$$=-I(X_i^{\text{sc}};t) \ . \tag{2.3$'$}$$

Here $\mathbf{J}(X_i^{\text{sc}};t)$ is the current density operator at the scission point and S is the $(n-1)$-dimensional hyperplane through the point X_i^{sc} with a normal vector in the direction of $\mathbf{J}$. The leakage of probability through the hyperplane S is measured by the escape rate $\lambda_f^{\text{sc}}(t)$ obtained by dividing the total current I by the probability $\Pi(X_i^{sc};t)$:

$$\lambda_f^{sc}(t) = -\Pi^{-1}(X_i^{sc};t)d\Pi(X_i^{sc};t)/dt \ , \quad (2.4)$$

$$= \Pi^{-1}(X_i^{sc};t)I(X_i^{sc};t) \ . \quad (2.4')$$

Relating (2.2) and (2.4) the lifetime of the process is such that

$$\int_0^{\tau_f} \lambda_f^{sc}(s)ds = 1 \ . \quad (2.5)$$

We shall qualify the process as quasistationary if after a certain time τ (the transient time), $\lambda_f^{sc}(t)$ becomes independent of time. In the statistical model one generally considers the extreme case of a transient time which is so small that the rate $\lambda_f^{sc}(t)$ is virtually a constant λ_f^{sc} over the whole time interval $(0,\infty)$ and one obtains,

$$\Pi(X_i^{sc};t) = \Pi(X_i;t=0)e^{-\lambda_f^{sc}t} \ , \quad (2.6)$$

and we express τ_f in terms of a decay width Γ_f^{sc} as

$$\tau_f^{stat} = (\lambda_f^{sc})^{-1} \equiv \hbar/\Gamma_f^{sc} \ . \quad (2.7)$$

Relation (2.2) appears therefore as the natural generalization to nonquasistationary processes of the usual definition of the lifetime. We note that for a quasistationary process as defined above neither the probability (2.1) nor the current density $\mathbf{J}(X_i;t)$ are individually time independent.

The crucial quantities which govern the time evolution of the current across the hyperplane S are the excitation energy of the system, the height of the barrier, and the dissipation and inertia tensors. Let us assume that the conditions in terms of these quantities are such that a quasistationary regime is established after a finite time τ. Schematically the situation may be represented as in Fig. 1. It corresponds to

$$\lambda_f^{sc}(t) = \frac{\lambda_f^{sc}}{\tau} t\theta(\tau - t) + \theta(t-\tau)\lambda_f^{sc} \ . \quad (2.8)$$

Using (2.5) one finds

$$\tau_f = (\lambda_f^{sc})^{-1} + \tfrac{1}{2}\tau$$
$$\equiv \tau_f^{stat} + \tfrac{1}{2}\tau \ . \quad (2.9)$$

Although oversimplified this schematic model is useful to show that the transient behavior may completely modify the usual expression of the lifetime in cases where $\tau/2 \gtrsim \tau_f^{stat}$. In the statistical model[1] Γ_f^{sc} increases very rapidly with excitation energy. It appears that at excitation energies above a few hundreds of MeV the quasistationary value Γ_f^{sc} loses its usual physical significance[9,10,12] as fission becomes altogether a transient phenomenon of duration of the order of τ.

According to our definition Eq. (2.1) of the lifetime it is clear that a transport approach of induced nuclear fission incorporates transient features not contained in the statistical model description. Thus we may expect novel information about nuclear friction to emerge from the study of the nonquasistationary behavior of the competitive decay of highly excited composite nuclei. Such a possibility has been recently shown[9] and quantitatively implemented[11] in the specific case of the decay of the nucleus ^{158}Er formed in the reaction $^{16}O + ^{142}Nd$ at an incident energy of 207 MeV.

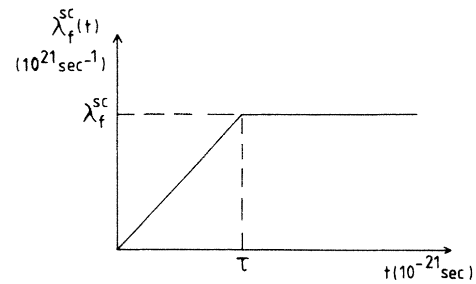

FIG. 1. Schematic sketch of the time dependence of the escape rate as defined in Eq. (2.4). τ is the transient time.

III. THE LIFETIME EVALUATED AT THE SADDLE POINT IN A ONE-DIMENSIONAL DIFFUSION MODEL

We consider the most simple case where the transport equation obeyed by the distribution function reduces to a Fokker-Planck equation (FPE) in a single variable X and its associated momentum $\tilde{p}$. The strength of the coupling to the heat bath is thus measured in terms of the ratio β of the friction coefficient to the inertia along the fission coordinate. It is measured in units of time^{-1} and considered to be a parameter independent of the collective coordinate. It is probably an oversimplification which has to be tested in specific cases like in Ref. 11.

The quantitative numerical studies of the time dependence of the escape rate at the saddle point of the collective potential can be found in Refs. 9, 10, 12, and 13. We extend here the derivation of the approximate analytical expressions for $\lambda_f^{sad}(t)$ presented in Ref. 10. The new developments concern the effect of the anharmonicities of the collective potential on the escape rate and the derivation of a general expression for this rate valid in the regime of overdamping. This regime is characterized by $\beta \gg 2\omega_1$ where ω_1 is the frequency of the harmonic oscillator osculating the collective potential at its first minimum. The overdamped case has been already studied in Ref. 12 with, however, two important restrictions. On the one hand the fission potential used did not exactly correspond to a realistic physical situation and on the other hand there is no simple way of telling how the results of a calculation will change with the nuclear temperature T if the potential is kept fixed. The approximate analytic expression we derive for the time dependent rate in the overdamped situation is valid for any potential shape presenting a saddle point and thereby allows us to complete the investigations of Ref. 12.

A. Classical consideration for the existence of a quasistationary regime

Even for temperatures of the heat bath which are small in comparison to the height E_f of the fission barrier the physical effect of the diffusion process is to lead the composite nucleus to overcome the potential barrier and then to undergo fission. However, qualitatively we expect the time scales for escape over the barrier to be very different according to whether or not there is classically enough kinetic energy E_K in the collective motion along the fission coordinate to overcome the effect of friction and of the potential barrier of height E_f.

Let $E_{\rm loss}$ be the energy lost due to the action of the friction force up to the time t_0 of closest approach to the saddle point. Thus the composite nucleus is classically trapped in the potential pocket to the left of the barrier if the following inequality holds:

$$E_k < E_f + |E_{\rm loss}| \ . \tag{3.1}$$

The distribution function $P(X,p;t)$ is centered around the mean value $\bar{X}$ of X and $\bar{X}$ obeys essentially a classical equation of motion in the collective potential. If condition (3.1) is fulfilled we expect that the passage over the barrier will take place by diffusion only and thereby be dominantly a quasistationary process. If in relation (3.1) the converse inequality holds the process of diffusion accelerates an inevitable passage over the barrier and the transient behavior may now dominate.

Let $\bar{X}$ be the main fission coordinate in the sense of Ref. 14. For a simple estimate of relation (3.1) in terms of the nuclear temperature and reduced dissipation β we may assume that the initial kinetic energy at time $t=0$ is given by T. Thus

$$\left.\frac{d\bar{X}}{dt}\right|_{t=0} = \left[\frac{2T}{\mu}\right]^{1/2} , \tag{3.2}$$

where μ is the reduced mass of the composite system. For later times, the solution of the classical equation of motion taking into account the presence of the friction force $F=-\mu\beta(d\bar{X}/dt)$ gives in general,[15]

$$\frac{d\bar{X}}{dt} = \left[\frac{2T}{\mu}\right]^{1/2} e^{-\beta t/2} f(t) \ . \tag{3.3}$$

Here $f(t)$ is oscillatory or not according to whether the motion in the collective potential is underdamped or overdamped. The escape over the potential barrier takes place only if the coordinate $\bar{X}(t)$ reaches the saddle point. This point must be reached the latest at the first maximum of $\bar{X}(t)$, for if it does not the saddle point will never be reached again due to the damping of the motion. In the case of underdamped motion the time t_0 to reach the first maximum is about $\frac{1}{4}$ of the pseudoperiod $2\pi\omega_r^{-1}$ with $\omega_r^2=\omega_1^2-\beta^2/4$ where ω_1 is the frequency of the harmonic oscillator osculating the collective potential at its minimum. In the converse situation of overdamped motion it is given by

$$t_0 \simeq \omega_r'^{\,-1} \tanh^{-1}(2\omega_r'/\beta)$$

with $\omega_r'^{\,2}=-\omega_r^2$. The total energy lost up to time t_0 reads

$$E_{\rm loss} = -\mu\beta \int_0^{t_0} \left[\frac{d\bar{X}}{dt}\right]^2 dt$$

$$= -2\beta T \int_0^{t_0} e^{-\beta t} f^2(t) dt \ . \tag{3.4}$$

Taking for $f(t)$ the expression valid for underdamped or overdamped motion in the harmonic oscillator of frequency ω_1 we express in both cases the energy lost as

$$E_{\rm loss} = -T(1-e^{-\beta t_0}) \ . \tag{3.5}$$

The condition (3.1) now reads $\beta < 2\omega_1$ (underdamping):

$$T < E_f[1+\beta\pi/(2\omega_1)] \ , \tag{3.6}$$

$\beta > 2\omega_1$ (overdamping):

$$T < E_f[1+4\ln(\beta/\omega_1)] \ . \tag{3.6$'$}$$

These qualitative considerations indicate that a quasistationary situation of probability flow may be expected even for temperatures higher than the height of the barrier. This is due to the presence of the nuclear dissipation and borne out by the numerical calculations of Sec. III.

For a given fissioning system, E_f itself may decrease[16] rapidly with increasing temperature T. The precise dependence of E_f upon T thus determines the domain in temperature that leads to a quasistationary diffusion process. In the sequel and due to the lack of experimental information we do not choose any specific model for the temperature dependence of E_f and restrict our numerical study to quasistationary situations which, for a given E_f, occur with changing values of T and β compatible with the condition (3.6) or (3.6′). However, the analytical expressions we derive below for the escape rate contains E_f explicitly and are therefore valid for any dependence of the barrier height upon the temperature. We shall comment on the qualitative changes one may expect in our numerical results when a variation of E_f with temperature is considered.

B. Approximate analytic expressions for the time-dependent escape rate evaluated at the saddle point

Let $U(X)$ be the one-dimensional collective potential with a minimum at $X=X_1$ and a saddle point at $X=X_0$ $(X_1<X_0)$. We define $\langle X\rangle_-$ by

$$\langle X\rangle_- = \int_{-\infty}^{X_0} X\,dX \int_{-\infty}^{\infty} dp\, P(X,p;t) \ . \tag{3.7}$$

At time $t=0$, $\langle X\rangle_- = X_1$. If the condition (3.6) or (3.6′) is met, $\langle X\rangle_-$ will remain located for later times to the left of the saddle point although possibly moving towards the top of the barrier. Hence the escape of probability over the barrier can only occur through diffusion and a time-dependent approximate solution of the diffusion equation may be constructed following a procedure discussed in Ref. 10. It is modeled after Kramers's derivation[6] of the stationary decay rate under the same circumstances. We show below how to extend the approach of Ref. 10 to include the effects of the anharmonicities of the potential as well as those of the overall motion of the distribution towards the top of the barrier. This particular motion develops with either increasing temperatures or shallower potential pockets. It leads to an enhancement of the escape rate above the final quasistationary value during an important fraction of the total decay time. The analytical developments of Ref. 10 did not incorporate these effects as they assumed $\langle X\rangle_-$ to be frozen at its initial value X_1. This is a severe limitation which the present derivation overcomes.

We begin with the same FPE as in Ref. 10. It reads

$$\frac{\partial}{\partial t}P(X,p;t)+p\frac{\partial}{\partial X}P(X,p;t)-K(X)\frac{\partial}{\partial p}P(X,p;t) =\beta\frac{\partial}{\partial p}[pP(X,p;t)]+\epsilon\frac{\partial^2}{\partial p^2}P(X,p;t) . \quad (3.8)$$

Here X is the fission variable, $\tilde{p}$ its conjugate momentum with $p=\tilde{p}/\mu$ the velocity, μ the reduced mass; $K(X)=\mu(dU/dX)$, where $U(X)$ is the collective potential with a minimum $U(X=X_1)=0$ and a local maximum (saddle point) at $X=X_0=0$ with $U(0)=E_f$; $\epsilon=\beta T/\mu$. We distinguish between two limits, the underdamped and overdamped cases.

1. The underdamped case

We seek an approximate solution of Eq. (3.8) in the spirit of Ref. 10 and only emphasize here the point related to the new developments we introduce. We write the full solution as

$$P(X,p;t)=C_0P_0(X,p;t)F(X,p;t) , \quad (3.9)$$

and take for $P_0(X,p;t)$ an ansatz of the form

$$P_0(X,p;t)=C_0'\exp\left[-\frac{1}{2(1-\rho^2)}\left\{\frac{(p-\langle p\rangle)^2}{2\sigma_p^2}+\frac{2U(X)}{\mu\omega_X^2\sigma_X^2}-2\rho\frac{(p-\langle p\rangle)[dU(X)/dX]}{\mu\omega_X^2\sigma_X\sigma_p}\right\}\right] . \quad (3.10)$$

Here $\sigma_p(t)$, $\sigma_X(t)$ are time-dependent variances, $\rho(t)$ a correlation function, and $\omega_X(t)$ an effective oscillator frequency all determined as explained below. We note that if the total potential $U(X)$ were the osculating harmonic potential of the first well with $\omega_X=\omega_1$, $P_0(X,p;t)$ would then be the exact solution of Eq. (3.8) with the corresponding variances and correlation functions and thus $F(X,p;t)=1$. This observation, supported by numerical findings, has led the authors of Ref. 10 to retain these variances, the correlation function, and the oscillator frequency ω_1 in the ansatz (3.10). However, these quantities may be obtained in a more stringent and realistic way.

The function $F(X,p;t)$ in Eq. (3.9) is determined by inserting $P(X,p;t)$ into the FPE (3.8) and assuming that $P_0(X,p;t)$ *satisfies* this equation. Due to the departure of $U(X)$ from an harmonic oscillator this requirement cannot obviously be met in the whole phase space but it requires that the first and second moments of the distribution $P_0(X,p;t)$ satisfy the following set of coupled equations:

$$\begin{aligned}
&\frac{d\langle X\rangle}{dt}=\langle p\rangle ,\\
&\frac{d\langle p\rangle}{dt}=-\beta\langle p\rangle-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}K(X)P_0(X,p;t)dp\,dX ,\\
&\frac{d\langle X^2\rangle}{dt}=2\langle Xp\rangle , \qquad (3.11)\\
&\frac{d}{dt}\langle Xp\rangle=\langle p^2\rangle-\beta\langle Xp\rangle\\
&\qquad -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}XK(X)P_0(X,p;t)dp\,dX ,\\
&\frac{d}{dt}\langle p^2\rangle=-2\beta\langle p^2\rangle+2\epsilon\\
&\qquad -2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}K(X)pP_0(X,p;t)dp\,dX .
\end{aligned}$$

Here we have denoted the mean value of the quantity z over the distribution $P_0(X,p;t)$ by $\langle z\rangle$ Since we have assumed that the classical requirements of Eq. (3.6) are met we are justified in solving the set of Eq. (3.11) in the Gaussian approximation with the initial conditions[10] $\langle X\rangle_{t=0}=X_1$, $\langle p\rangle_{t=0}=0$, $\sigma_X^2(t=0)=\sigma_{X_0}^2$, $\sigma_p^2(t=0)=\sigma_{p_0}^2$, $\omega_X=\omega_1$, and $\rho(t=0)=0$. The variances $\sigma_X^2(t)$, $\sigma_p^2(t)$ and the correlation function $\rho(t)$ are then completely determined. The frequency ω_X present in the ansatz (3.10) appears thus as the frequency of the harmonic oscillator centered at $\langle X\rangle$ which at time t gives the same variance σ_X as the one emerging from the numerical solution of the set of Eq. (3.11). It is clear now that this procedure incorporates in the variances all the complex dynamics that the anharmonicities of the collective potential may generate.

Inserting the ansatz (3.9) in the FPE Eq. (3.8) and using the fact that $P_0(X,p;t)$ solves this FPE in the Gaussian approximation we obtain for $F(X,p;t)$ an equation which is valid around the saddle point $X_0=0$ where the locally osculating harmonic oscillator frequency is ω_0. It reads

$$\frac{\partial F}{\partial t}+p\frac{\partial F}{\partial X}+\omega_0^2X\frac{\partial F}{\partial p} =\beta p\frac{\partial F}{\partial p}-\frac{2\epsilon}{1-\rho^2}\left[\frac{p-\langle p\rangle}{\sigma_p^2}+\frac{\rho\omega_0^2X}{\omega_X^2\sigma_p\sigma_X}\right]\frac{\partial F}{\partial p}+\epsilon\frac{\partial^2F}{\partial p^2} . \quad (3.12)$$

The solution of this equation is constructed by writing F as a function of the variable $\xi=p-\bar{p}(t)-a(t)X$. This ansatz is compatible with Eq. (3.12) if $a(t)$ and $\bar{p}(t)$ fulfill specific differential equations. The equation satisfied by the function $a(t)$ has been discussed in Ref. 10 and the solution obtained therein remains unchanged.

The equation that $\bar{p}(t)$ must satisfy reads

$$\frac{d\bar{p}}{dt}+\bar{p}\left[\beta+a-\frac{2\epsilon}{(1-\rho^2)\sigma_p^2}\right]+\frac{2\epsilon\langle p\rangle}{(1-\rho^2)\sigma_p^2}=0 . \quad (3.13)$$

From the set of Eq. (3.11) it is easy to show that as $t\to\infty$ $\sigma_p^2(t)$ has the asymptotic value $\epsilon/\beta=T/\mu$. Using the known asymptotic value of $a(t)$ we obtain

$$\bar{p}(t\to\infty)=-\frac{2\beta\langle p\rangle_{t\to\infty}}{(\beta^2/4+\omega_0^2)^{1/2}-\beta/2} . \qquad (3.14)$$

If in Eq. (3.11) $K(X)$ derives only from the harmonic oscillator osculating the full potential in the first pocket then $\langle p\rangle_{t\to\infty}=\langle p\rangle_{t=0}=0$. The results and conclusions of Ref. 10 are retrieved. All the time-dependent coefficients present in Eq. (3.13) are determined by solving the set of Eqs. (3.11). Then Eq. (3.13) can be solved in a standard way with the initial condition $\bar{p}(t=0)=0$. In general we find that if the condition (3.6) is fulfilled $\langle p\rangle_{t\to\infty}$ remains small.

The formal analytical solution of Eq. (3.12) permits one to write the total approximate distribution $P(X,p;t)$ which is valid up to the saddle point $X_0=0$ where it is written as

$$P(0,p;t)=C_0\exp\left[-\frac{(p-\langle p\rangle)^2}{2(1-\rho^2)\sigma_p^2}-\frac{E_f}{(1-\rho^2)\mu\omega_X^2\sigma_X^2}\right]\times\left[\frac{C}{2\pi}\right]^{1/2}\int_{-\infty}^{p-\bar{p}(t)}e^{-\xi^2C/2}d\xi . \qquad (3.15)$$

Here C_0 is a normalization constant and the positive[10] quantity C is defined as

$$C=\{(a+\beta)/\epsilon-2/[(1-\rho^2)\sigma_p^2]\} . \qquad (3.16)$$

From expression (3.15) the flux and the escape rate at the saddle point are derived in a straightforward fashion. We obtain

$$\lambda_f^{\rm sad}(t)=\Pi^{-1}(X_0=0;t)\left\{\exp\left[-\frac{(\bar{p}-\langle p\rangle)^2}{2(1-\rho^2)\sigma_p^2}\frac{C}{C+\epsilon}\right](1-\rho^2)\sigma_p^2\left[\frac{C}{C+\epsilon}\right]^{1/2}+\langle p\rangle[\pi(1-\rho^2)\sigma_p^2/2]^{1/2}[1+{\rm erf}(Z)]\right\}\exp\{-E_f/[(1-\rho^2)\mu\omega_X^2\sigma_X^2]\} . \qquad (3.17)$$

In this relation $\Pi(X_0=0;t)$ is the probability defined in Eq. (2.1) where we use for $P(X,p;t)$ the approximate distribution just built up. The error function

$${\rm erf}(Z)=(2/\sqrt{\pi})\int_0^Z e^{-u^2}du$$

has the argument

$$Z=\{C/[2(1-\rho^2)\sigma_p^2(C+\epsilon)]\}^{1/2}(\langle p\rangle-\bar{p}) . \qquad (3.18)$$

It is important to notice that expression (3.17) is limited to the underdamped situation $\beta<2\omega_1$. We next obtain an expression for the overdamped situation.

2. *The overdamped case*

In the overdamped situation where $\beta>2\omega_1$ the FPE (3.8) implies that the variance $\sigma_p(t)$ equilibrates on a time scale of the order of β^{-1} while $\sigma_X(t)$ equilibrates on a much longer time scale. Thus during the equilibration process in the momentum variable the distribution in the coordinate variable remains unchanged and localized around its initial mean position at $X=X_1$. This is expressed in the following ansatz for the full distribution $P(X,p;t)$ if $t\lesssim\beta^{-1}$:

$$P(X,p;t)=C\tilde{P}_0(p;t)E(X) , \qquad (3.19)$$

where $E(X)$ is the distribution in the X variable at time $t=0$. For $t>\beta^{-1}$ the evolution concerns only the distribution in the X variable. Writing the full solution under the form (3.9) with the ansatz (3.10) for $P_0(X,p;t)$ appears therefore legitimate only for the time evolution of the underdamped case when both $\sigma_p(t)$ and $\sigma_X(t)$ vary on the same time scales or for the asymptotic quasistationary regime as shown by Kramers.[6]

The observation that in the overdamped situation the equilibrium in momentum is very rapid is the basic assumption used to derive[6,15] the Smoluchowski equation from the full FPE (3.8). It reads

$$\frac{\partial}{\partial t}Q(X;t)=\beta^{-1}\frac{\partial}{\partial X}[K(X)Q(X;t)]+\epsilon\beta^{-2}\frac{\partial^2}{\partial X^2}Q(X;t) . \qquad (3.20)$$

The quantities present in Eq. (3.20) are defined after Eq. (3.8) and $Q(X;t)$ is obtained from $P(X,p;t)$ by integrating over all p space. The conditions of validity of Eq. (3.20) in the context of nuclear fission are discussed in Ref. 12 where it is also shown that the domain of values of the reduced friction parameter β covered in using (3.20) joins in practice the domain covered in using relation (3.17).

We remark first that if a quasistationary situation is established for a particular value of $\beta>2\omega_1$, holding the collective potential and the temperature fixed, a quasistationary situation is *a fortiori* established if the motion is largely overdamped, i.e., $\beta\gg2\omega_1$. This is due to a simple scaling property—as β is changed—of Eq. (3.20) as discussed in Ref. 12. For two solutions of Eq. (3.20) associated with different values β_0 and β of the friction constant, but subject to identical initial conditions (independent of β) at time $t=0$, one has[12]

$$\lambda_f^{\rm sad}(t;\beta)=(\beta_0/\beta)\lambda_f^{\rm sad}(\beta t/\beta_0;\beta_0) . \qquad (3.21)$$

Thus, if after some transient time τ, $\lambda_f^{\rm sad}(\beta t/\beta_0;\beta_0)$ reaches a constant value, so does $\lambda_f^{\rm sad}(t;\beta)$. The dependence of τ on β is then known from the study of the rate

λ at just one value β_0 of β, as

$$\tau(\beta)=(\beta/\beta_0)\tau(\beta_0)\ . \qquad (3.22)$$

Using the property that we have fixed the saddle point at $X_0=0$ with $U(X_0)=E_f$,

$$K(X_0)=\mu\frac{dU}{dX}\bigg|_{X=X_0}=0\ ,$$

the time dependent rate at this point is obtained from Eq. (3.20) as

$$\lambda_f^{\rm sad}(t)=-\Pi^{-1}(X_0=0;t)\epsilon\beta^{-2}\frac{\partial}{\partial X}Q(X;t)\mid_{X=X_0=0}\ . \qquad (3.23)$$

To derive an approximate expression for (3.23) we follow the same approach as before and write the solution $Q(X;t)$ of Eq. (3.20) under the form

$$Q(X;t)=C_0Q_0(X;t)F(X;t)\ . \qquad (3.24)$$

For $Q_0(X;t)$ we take the ansatz

$$Q_0(X;t)=C_0'\exp[-U(X)/(\mu\omega_X^2\sigma_X^2)]\ , \qquad (3.25)$$

and determine $\sigma_X^2(t)$ and ω_X through the method of moments. $Q_0(X;t)$ thus solves Eq. (3.20) in the Gaussian approximation and $F(X;t)$ is found to obey a partial differential equation which is solved analytically. This procedure determines completely the distributions $Q(X;t)$ in the vicinity of the saddle point and an expression for $\lambda_f^{\rm sad}(t)$ valid for $\beta>2\omega_1$ is obtained in a straightforward fashion. It reads

$$\lambda_f^{\rm sad}(t)=\Pi^{-1}(X_0=0;t)[b(t)/\beta](T/2\pi\mu)^{1/2} \times\exp[-E_f/(\mu\omega_X^2\sigma_X^2)]\ . \qquad (3.26)$$

Here $b(t)$ obeys the differential equation

$$\frac{db}{dt}+(b^2-\omega_0'^2)(b/\beta)=0\ , \qquad (3.27)$$

with

$$\omega_0'^2=\omega_0^2[2T/(\mu\omega_X^2\sigma_X^2)-1]$$

and ω_0 the frequency of the inverted harmonic oscillator osculating the potential at the saddle point. We recall that the frequency ω_X is obtained as the frequency of the harmonic oscillator centered at $\langle X\rangle$ which gives the same variance at each time as the one obtained in the numerical solution of the equations for the first and second moments. Thus $\mu\omega_X^2\sigma_X^2$, $\omega_0'^2$, and b have the asymptotic ($t\to\infty$) values T, ω_0^2, and ω_0, respectively. If we evaluate the probability $\Pi(X_0=0;t\to\infty)$ following Kramers, i.e., by assuming the mean value $\langle X\rangle$ to be frozen at its initial position $X=X_1$ and thereby $\omega_X=\omega_1$, we find

$$\Pi(X_0=0,t\to\infty)=(2\pi T/\mu)^{1/2}\omega_1^{-1}$$

and $\lambda_f^{\rm sad}$ Eq. (3.26) reduces to Kramers's expression.[6]

To discuss the function $b(t)$ we observe that with the change of variable $b^2=v^{-1}$ the solution of Eq. (3.27) is easily found. It reads

$$b^2(t)=\exp\left[\int_0^{(t/\beta)}\omega_0'^2(s)ds\right] \times\left\{C+2\int_0^{(t/\beta)}ds\exp\left[2\int_0^{s}dr\,\omega_0'^2(r)\right]\right\}^{-1}, \qquad (3.28)$$

with C an arbitrary constant to obey the initial condition. Let us consider that $\sigma_X^2(t)$ reaches its asymptotic value $T/(\mu\omega_X^2)$ after a time scale given by t_0 with $b^2=b^2(t_0)$. If the total potential were the osculating harmonic oscillator of the first well with $\omega_X=\omega_1$ we would have $t_0=\beta/(2\omega_1^2)$. Using Eq. (3.28) we find for $t>t_0$,

$$b^2(t)=b^2(t_0)\omega_0^2\{b^2(t_0)+[\omega_0^2-b^2(t_0)] \times\exp[-2\omega_0^2(t-t_0)/\beta]\}^{-1}\ . \qquad (3.28')$$

This shows that $b^2(t)$ reaches its asymptotic value ω_0^2 after a time scale given by $t_0+\beta/(2\omega_0^2)$. Hence the important effect of the function $b(t)$ is to increase the transient time τ needed for the attainment of the quasistationary value of $\lambda_f^{\rm sad}(t)$ with respect to its value[9,10] obtained only from the leading exponential present in Eq. (3.26). Finally it is important to notice that our expression (3.26) for $\lambda_f(t)$ satisfies the scaling property (3.21) since all the time-dependent quantities present in (3.26) obey differential equations with the same scaling property of Eq. (3.20).

C. Numerical results

The initial conditions and the numerical procedure used to solve the FPE (3.8) are described in Ref. 10. We apply them to study the fission of the nucleus $A=248$. The collective potential is taken the same as in Ref. 10 (Sec. II B) with a barrier height E_f of 4 MeV. We use Eq. (2.4) to calculate the escape rate $\lambda_f^{\rm sad}(t)$ in units of 10^{21} sec^{-1} as a function of time in units of 10^{-21} sec. On the left-hand side of Fig. 2 we display λ_f for values of β characteristic of the underdamped case and increasing nuclear temperatures. The right-hand side corresponds to the overdamped situation. In both cases the dashed horizontal lines show the prediction of Kramers's formula[6] and the dashed curves result from the direct numerical calculations. The dash-dotted curves originate from the approximate expression (3.17) and (3.26) shown, respectively, on the left-hand and on the right-hand side of the figure.

We first discuss the underdamped case. For $\beta=0.5$ and $T=1$ MeV a comparison with Fig. 4 of Ref. 10 shows that our new expression for $\lambda_f^{\rm sad}(t)$ Eq. (3.17) and the scheme we have proposed to determine the variances result in a marked improvement over our earlier treatment. The time scales for the rise of $\lambda_f(t)$ are reproduced with very good accuracy in all the cases we have studied and not displayed here. The present derivation is of particular interest for temperatures $T\gtrsim E_f$ where according to classical arguments a quasistationary situation may develop although with considerable overshooting with respect to Kramers's rate. A typical example is shown for $\beta=1.5$ and $T=5$ MeV. The analysis of Ref. 10 has shown that for such a value of β the mobility of the system in the

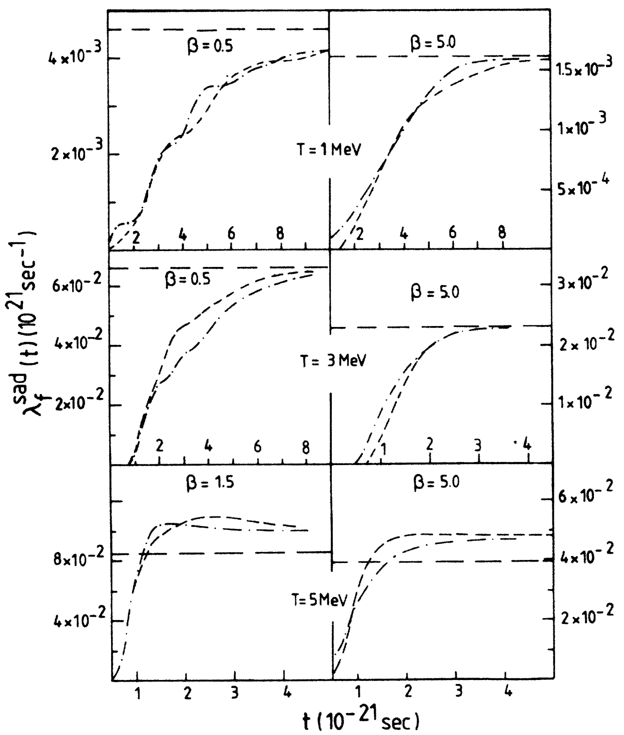

FIG. 2. The fission rate $\lambda_f(t)$ (in units of 10^{21} $\sec^{-1}$) evaluated at the saddle point. The left-hand side of the figure corresponds to underdamped motion of the collective variable in the potential pocket. The right-hand side to overdamped motion. The dashed curves are the results of the numerical calculations and the dash-dotted curves the results of the analytical approximations Eqs. (3.17) and (3.26) for the left- and right-hand sides, respectively. The dashed straight line gives the quasistationary value of Kramers.

direction of the fission coordinate attains a maximum and results in the enhancement of the rate with respect to Kramers's value. We see that the dynamical treatment of the mean values in position and momentum and of the variances takes this mobility into account in a very satisfactory way. The domain of excitation energy explorable with expression (3.17) is thus considerably enhanced with respect to the derivation of Ref. 10 and the upper bound (3.6) seems a reasonable estimate.

Turning to the overdamped case we recall that the approximate analytical expression of Ref. 10 for the escape rate was completely unsatisfactory in this case for reasons we have analyzed above. Figure 2 shows good overall agreement between the results of the direct numerical calculations and of relation (3.26) for the escape rate. Although the numerical solution of Eq. (3.20) can be obtained with fast and accurate standard methods, the solution of the first-order differential equations for the mean values, variance, and $b(t)$ as defined in Eq. (3.27) is very simple. Our results justify the use of expression (3.26) to investigate for large viscosity the competitive decay of composite systems created at high excitation energies.

In this study we keep the barrier height E_f fixed irrespective of the nuclear temperature T considered. In a realistic situation E_f may be[16] a rapidly decreasing function of T. However, the classical considerations of Sec. III A indicate that it is only the ratio E_f/T which determines the existence of the quasistationary regime. Thus we expect that the patterns of the escape rates shown in Fig. 2 will stay nearly unchanged with increasing temperatures T' such that the ratios $E_f(T')/T'$ are the same as those in Fig. 2. As a consequence the onset of the overshooting of the escape rate over Kramers's quasistationary value may now show up at relatively lower temperatures than those indicated from our model calculation with a temperature-independent barrier. In any case the analytic approximate expressions (3.17) and (3.26) can be used unchanged since the barrier height E_f may assume any temperature dependence.

Following Ref. 10 we define the transient time τ as the time needed for $\lambda_f(t)$ to reach 90% of its final quasistationary value. In our one-dimensional model τ is a function of the reduced dissipation parameter β and of the nuclear temperature T. Figure 3 shows the transient time τ in units of 10^{-21} sec as a function of β in units of 10^{21} $\sec^{-1}$ for temperatures $T=1$, 3, and 5 MeV. The dashed curves are obtained from the direct numerical calculation of $\lambda_f(t)$. The dash-dotted curves result from the calculation of $\lambda_f(t)$ with formula (3.17) up to the value $\beta=2.5\times10^{21}$ $\sec^{-1}$ for which the results of formula (3.26) start to join smoothly those of formula (3.17) for all three temperatures. Despite the restrictive conditions imposed in the derivations of our approximate analytic expressions it is plain to see that they cover in a continuous way the whole domain of β values ranging from $\beta \gtrsim 0.2\times10^{21}$ $\sec^{-1}$. For smaller values of β which are probably unrealistic in the nuclear context the fission rate varies linearly with β and a specific derivation is necessary. Semiquantitative estimates[9,10] of τ based on the behavior of the leading exponential $\exp(-E_f/\mu\omega_X^2\sigma_X^2)$ lead to

$$\begin{aligned} &\beta<2\omega_X:\ \ \tau=\beta^{-1}\ln(10E_f/T)\ , \\ &\beta>2\omega_X:\ \ \tau=\frac{\beta}{2\omega_X^2}\ln(10E_f/T)\ . \end{aligned} \qquad (3.29)$$

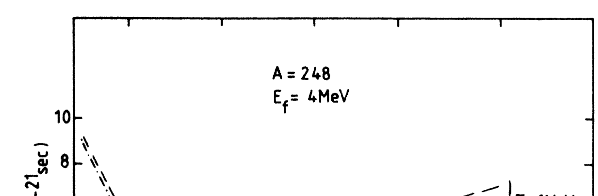

FIG. 3. The transient time τ (in units of 10^{-21} sec) evaluated at the saddle point as a function of the reduced dissipation parameter β (in units of 10^{21} $\sec^{-1}$) for various nuclear temperatures. The dashed curves result from the numerical calculation of $\lambda_f(t)$ and the dashed dotted curves from formula (3.17) for $\beta<2.5\times10^{21}$ $\sec^{-1}$ and from formula (3.26) beyond.

We see therefore that τ may be written as

$$\tau(\beta,T,E_f)=\alpha(T,E_f)\{\beta^{-1}+\beta/[2\omega_X^2(T)]\} . \qquad (3.30)$$

Thus the value $\beta_m=\sqrt{2}\omega_X$ at which τ attains a minimum measures the overall effective harmonic oscillator frequency which results from the deviation of the true collective potential from its harmonic approximation around the minimum $X=X_1$. This deviation becomes more and more important with increasing temperature, hence the dependence of ω_X upon T. For this value β_m we have found that the rate $\lambda_f(t)$ overshoots Kramers's quasistationary value for nuclear temperatures of the order $0.5E_f$. Figure 3 also shows that the coefficient $\alpha(T,E_f)$ in Eq. (3.30) has a temperature dependence which deviates markedly from $\ln(10E_f/T)$. This feature is well reproduced by our approximate analytic expressions since at $T=5$ MeV for $\beta<3.10^{21}$ sec^{-1} the dashed and dash-dotted curves are not distinguishable.

The transient time τ is a central quantity in the competitive decay of the compound nucleus[9,10] since its value determines the importance of decay channels other than fission in the early stage of the time evolution. Therefore multiplicities of particles emitted prior to fission are related to transients and provide insight into the role of β. The specific study of the decay of the compound nucleus ^{158}Er is devoted to this aspect and reported elsewhere.[11] Here we focus only on the total lifetime evaluated at the saddle point. In Fig. 4 we show the ratio $\tau/\tau_f^{\rm sad}$ as a function of the reduced dissipation parameter β for the same three temperatures as before. The lifetime $\tau_f^{\rm sad}$ evaluated at the saddle point was obtained from relation (2.5). To discuss the trends shown in Fig. 4 we recall that in the schematic model of Sec. II, the lifetime evaluated at the saddle point is the sum of two contributions $(\hbar/\Gamma_f)_{\rm stat}$ and $\frac{1}{2}\tau$ which both vary with β and T. At $T=1$ MeV for the nucleus studied the inverse of the quasistationary rate varies from 21×10^{-21} sec for $\beta=0.5\times10^{21}$ sec^{-1} to 62×10^{-21} sec for $\beta=5.0\times10^{21}$ sec^{-1} whereas the corresponding values of τ are 9.2×10^{-21} sec and 7.0×10^{-21} sec. Thus in this case the influence of τ on the total lifetime is very pronounced only for small values of β. At $T=3$ MeV the inverse of the quasistationary rate varies from 15×10^{-21} sec for $\beta=0.5\times10^{21}$ sec^{-1} to 40×10^{-21} sec for $\beta=5.0\times10^{21}$ sec^{-1}. The corresponding values of τ are 4.5×10^{-21} sec and 2.2×10^{-21} sec. Thus in going from $T=1$ MeV to $T=3$ MeV the transient time diminishes faster than the inverse of the quasistationary rates which explain the trends in Fig. 4. However, at $T=5$ MeV the inverse of the quasistationary rate is of the order of 9×10^{-21} sec over the whole range of β values and $\tau\simeq2.0\times10^{-21}$ sec. The influence of the transient time on the total lifetime increases again with respect to $T=3$ MeV. We see therefore that the trends observed in Fig. 4 are in keeping with the qualitative analysis based on the schematic model and Fig. 3.

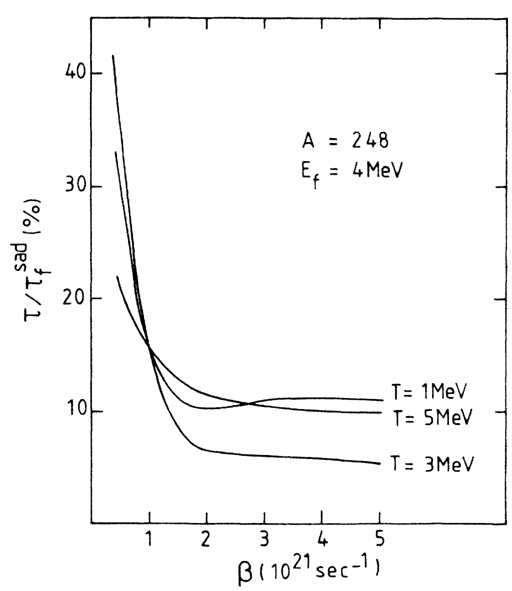

FIG. 4. Ratio in (%) of the transient time τ over the total lifetime τ_f, evaluated at the saddle point from Eq. (2.5), as a function of the reduced dissipation parameter β and for various nuclear temperatures.

IV. TRANSIENT BEHAVIOR AND LIFETIME AT THE SCISSION POINT

For the dynamics of the fission process and competition against particle emission the quantity of interest is the lifetime at scission. The approximate solutions of the FPE we have built up are valid only in a restricted range of X values up to and including the saddle point. To evaluate physical quantities at the scission point we need to build up explicitly the solution of the FPE for values of the collective coordinate bigger than X_0 (the position of the saddle point). In this section we derive first this solution under the assumption that the collective potential beyond the saddle point can be correctly modeled by an inverted harmonic oscillator of frequency ω_0. We obtain the flux at the scission point $X_{\rm sc}$ in terms of the flux at the saddle point X_0. We close this study with the analysis of the lifetime evaluated at the scission point.

A. Solution of the FPE for $X>0$ and expression for the current at the scission point

The potential of the fissioning nucleus tends to zero for $X\to\infty$ and we should set $\beta=0$ beyond the scission point. However, we use the FPE with a constant value of the nuclear friction β and the inverted harmonic oscillator of frequency ω_0 for all values of $X>X_0$. Although only semirealistic, this choice—discussed in Refs. 9, 10 and 17—allows analytic derivations with clear physical interpretations.

We define the Fourier-Laplace transform $g(u,v;t)$ of $P(X,p;t)$ as

$$g(u,v;t)=(2\pi)^{-1}\int_{-\infty}^{\infty}e^{-ipv}dp\int_0^{\infty}e^{-Xu}P(X,p;t)dX . \qquad (4.1)$$

Equation (3.8) is transformed to a first-order partial differential equation for $g(u,v;t)$ which, under the change of variables $\xi=iav+u$, $\eta=ibv-u$ with $a=\beta/2+\omega_r$, $b=\beta/2-\omega_r$, and $\omega_r^2=\omega_0^2+\beta^2/4$, takes the symmetric form

$$\frac{\partial}{\partial t}g(\xi,\eta;t)=f(\xi,\eta;t)+B\xi\frac{\partial}{\partial\xi}g(\xi,\eta;t)-A\eta\frac{\partial}{\partial\eta}g(\xi,\eta;t)+[\epsilon/(4\omega_r^2)](\xi+\eta)^2g(\xi,\eta;t)\ ; \qquad (4.2)$$

here

$$B=(2\omega_0^2+\beta b)/(2\omega_r)\ ,$$
$$A=(2\omega_0^2+\beta a)/(2\omega_r)\ ,$$

and $f(\xi,\eta;t)$ is a source term which originates from the boundary condition that at $X=0$ the solution is the known function $P(0,p;t)$. We have

$$f(\xi,\eta;t)\equiv f[(\xi+\eta)/(2i\omega_r);t]=f(v,t)=(2\pi)^{-1}\int_{-\infty}^{\infty}e^{-ipv}p\,P(0,p;t)dp\ . \qquad (4.3)$$

The solution of (4.2) is easily found.[18] Transforming back to the original variables (X,p) we obtain after some calculations the solution $P(X,p;t)$ valid for $X>0$. It reads

$$P(X,p;t)=Q_0(X,p;t)+\int_0^t dt'\int_{-\infty}^{\infty}p'dp'P(0,p';t')\times G_0(Xp;0p';t-t')\ . \qquad (4.4)$$

In this equation $G_0(Xp;X'p';t-t')$ is the Gaussian propagator which solves the FPE with $K(X)=-\omega_0^2X$ in the full (X,p) space with the initial condition at time $t=t'$,

$$G_0(Xp;X'p';0)=\delta(X-X')\delta(p-p')\ . \qquad (4.5)$$

The initial conditions in terms of the distribution $P(X,p;t=0)$ are contained in the term $Q_0(Xp;t)$ which we discuss below. The exact form of G_0 appears as a double Gaussian in the variable $z_1=p+aX$, $z_2=p+bX$ with a and b defined after Eq. (4.1),

$$G_0(Xp;X'p';\tau)=N^{-1}\exp\left\{-\frac{1}{2(1-\rho_0^2)}[(z_1-z_1'e^{-b\tau})^2/\sigma_0^2+(z_2-z_2'e^{-a\tau})^2/\delta_0^2-2\rho_0(z_1-z_1'e^{-b\tau})(z_2-z_2'e^{-a\tau})/(\sigma_0\delta_0)]\right\}\ . \qquad (4.6)$$

Here $z_1'=p'+aX'$, $z_2'=p'+bX'$, $N=2\pi(1-\rho_0^2)^{1/2}\sigma_0\delta_0$, and

$$\sigma_0^2=\epsilon(1-e^{-2b\tau})/b\ ,$$
$$\delta_0^2=\epsilon(1-e^{-2a\tau})/a\ , \qquad (4.7)$$
$$\rho_0\sigma_0\delta_0=2\epsilon(1-e^{-\beta\tau})/\beta\ ;$$

the diffusion constant is $\epsilon=\beta T/\mu$. Finally the term $Q_0(X,p;t)$ in Eq. (4.4) originates from the initial distribution at time $t=0$ propagated through $G_0(Xp;X'p';t)$,

$$Q_0(X,p;t)=\int_{-\infty}^{\infty}dp'\int_0^{\infty}dX'P(X',p';t=0)G_0(Xp;X'p';t)\ . \qquad (4.8)$$

For the physical situation of a composite system formed initially with unit probability in the collective potential pocket, $P(X,p;t=0)$ is entirely localized in the region $X<X_0=0$ and $Q_0(X,p;t)$ in Eq. (4.4) does not contribute to $P(X,p;t)$ for $X>0$. Henceforth we omit $Q_0(X,p;t)$ and obtain the flux $J(X,t)$ at any point X beyond the saddle point. It reads

$$J(X,t)=\int_0^t d\tau\int_{-\infty}^{\infty}p\,dp\,P(0,p;t-\tau)g_0(Xp;\tau)\ , \qquad (4.9)$$

with $g_0(Xp;\tau)$ given as

$$g_0(Xp;\tau)=(2\pi\sigma_X^2)^{-1/2}[(X-pr)\rho\delta_p/\sigma_X+p\dot{u}]\times\exp[-(X-pr)^2/(2\sigma_X^2)]\ . \qquad (4.10)$$

Here σ_X, δ_p are the variances and ρ the correlation function appearing in G_0 written explicitly in the variables (Xp); u and r are functions of τ defined as

$$u\equiv u(\tau)=(be^{-b\tau}-ae^{-a\tau})/(b-a)\ ,$$
$$r\equiv r(\tau)=(e^{-b\tau}-e^{-a\tau})/(a-b)\ . \qquad (4.11)$$

B. Analysis of the flux at scission

In Fig. 5 we show the projection of a three-dimensional plot of the function $g_0(Xp;\tau)$ calculated at the scission point located at $X_{sc}=7$ fm for $\beta=0.5\times10^{21}$ sec^{-1} and $T=1$ MeV and for $\beta=1.5\times10^{21}$ sec^{-1} and $T=5$ MeV, for a frequency $\omega_0=1.65\times10^{21}$ sec^{-1} and a range of values of p (in fm 10^{21} sec^{-1}) and τ (in 10^{-21} sec). We note that g_0 is small for all $p<0$ and for each $p>0$ it is a strongly peaked function of τ with a maximum around a value $\bar{\tau}(X_{sc},p)$. This is a characteristic pattern of the function $g_0(X,p;\tau)$ in all physical situations. In Fig. 6 the continuous curves show the values of $\bar{\tau}(X_{sc},p)$ found numerically in units of 10^{-21} sec as a function of p in units of (fm 10^{21} sec^{-1}). The dashed curves correspond to the classical time τ_{cl} to reach the scission point X_{sc} starting at time $t=0$ from the saddle point with an initial velocity p. This time τ_{cl} is the value of τ for which the quantity $(X_{sc}-pr)$ cancels in the argument of the exponential in Eq. (4.10). Obviously τ_{cl} tends to infinity if p tends to zero. However, due to the exponential divergence with τ of the variance $\sigma_X^2(\tau)$, $\delta_p^2(\tau)$ and of the functions $u(\tau)$ and $r(\tau)$ the maximum value of $\bar{\tau}(X_{sc},p)$ is actually finite as we now show.

The divergence with τ of the variances and of $u(\tau)$ and $r(\tau)$ has its origin in the term $\exp(-b\tau)$ in Eq. (4.7) as $b=\beta/2-\omega_r$ is negative. This exponential increase is independent of the value taken by the diffusion coefficient if $\tau\gg|b^{-1}|$. The diffusive aspect of the motion becomes irrelevant[19] for such large values of τ as the driving force of the potential strongly accelerates the motion. In this regime the Gaussian propagator $G_0(X_{sc}p;0p';\tau)$ reduces to its "scaling limit" expression $G_0^{sc}(X_{sc}p;0p';\tau)$ obtained in Ref. 19. It reads

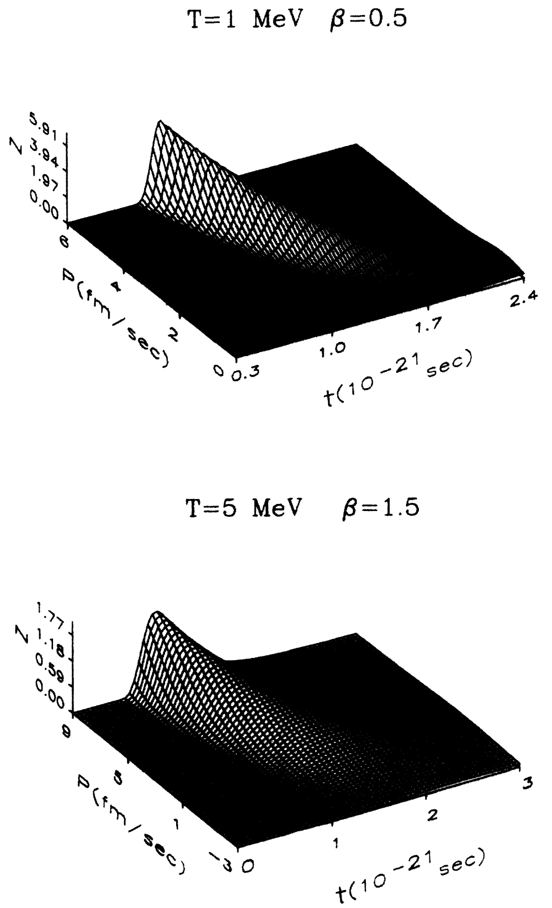

FIG. 5. The function $Z \equiv g_0(Xp;t)$ defined in Eq. (4.10) for a frequency $\omega_0 = 1.65\times10^{21}$ sec^{-1}, $X=7$ fm, $T=1$ MeV, $\beta=0.5\times10^{21}$ sec^{-1} (upper part) and $T=5$ MeV, $\beta=1.5\times10^{21}$ sec^{-1} (lower part). The nucleus is ^{248}Cf.

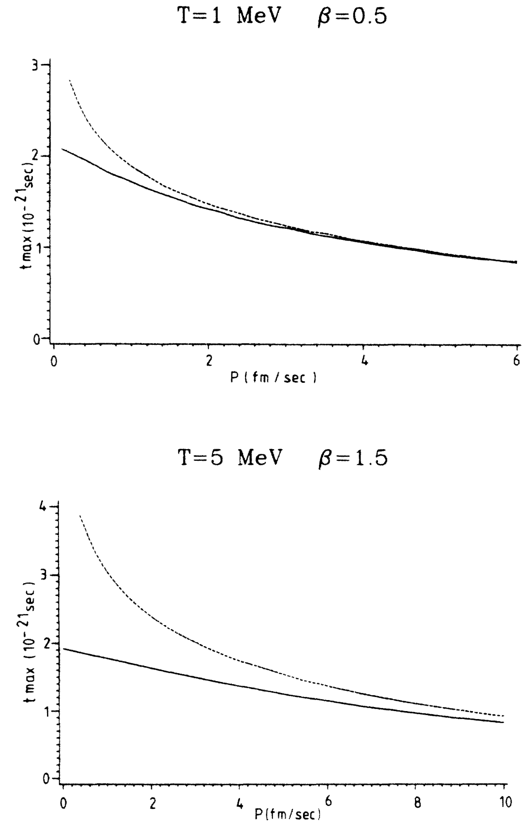

FIG. 6. The value $\overline{\tau}(X,p)$ (continuous curves) where the function $g_0(Xp;\tau)$ defined in Eq. (4.10) is maximum and the classical traveling time $\tau_{\rm cl}$ (dashed curves) to reach the scission point at $X=7$ fm as a function of the initial velocity p. Times are in units of 10^{-21} sec and p in units of fm 10^{21} sec^{-1}. Other parameters are the same as in Fig. 5.

$$G_0^{\rm sc}(X_{\rm sc}p;0p';\tau) = 2\omega_r(2\pi\eta)^{-1/2}\delta(p+bX_{\rm sc}) \times\exp[-2(\omega_r X_{\rm sc})^2/\eta]\ . \qquad (4.12)$$

Here $\eta=(-\epsilon/b)\exp(-2b\tau)$ and $\delta(z)$ is the Dirac delta function. In this limit the memory of the initial condition (4.5) has been lost and $g_0(X_{\rm sc}p';\tau)$ reduces to

$$g_0^{\rm sc}(X_{\rm sc}p';\tau) = \int_{-\infty}^{\infty} p\, dp G_0^{\rm sc}(X_{\rm sc}p;0p';\tau) = -2\omega_r bX_{\rm sc}(2\pi\eta)^{-1/2}\exp[-2(\omega_r X_{\rm sc})^2/\eta]\ . \qquad (4.13)$$

This function $g_0^{\rm sc}$ has a maximum as a function of η for $\eta=(2\omega_r X_{\rm sc})^2$ which corresponds to the value $\overline{\tau}_1(X_{\rm sc})$ of τ,

$$\overline{\tau}_1(X_{\rm sc}) = -(2b)^{-1}\ln|(2\omega_r X_{\rm sc})^2 b/\epsilon|\ . \qquad (4.14)$$

For the parameters of Figs. 5 and 6 we obtain at $X_{\rm sc}=7$ fm the values

$$\overline{\tau}_1(X_{\rm sc}) = 2.4\times10^{-21}\ {\rm sec}$$

using $\beta=0.5\times10^{21}$ sec^{-1} and $T=1$ MeV, and

$$\overline{\tau}_1(X_{\rm sc}) = 1.95\times10^{-21}\ {\rm sec}$$

for $\beta=1.5\times10^{21}$ sec^{-1} an $T=5$ MeV, in good agreement with the numerical findings of Fig. 6.

Due to the characteristic pattern of the function $g_0(X_{\rm sc}p;\tau)$ shown in Fig. 5 only those values of τ close to $\overline{\tau}(X_{\rm sc};p)$ and in the interval $(0,t)$ contribute to the integral over τ present in Eq. (4.9). Thus for asymptotic $(t\rightarrow\infty)$ time and due to the finiteness and independence with respect to the variable p of the maximum value $\overline{\tau}_1(X_{\rm sc})$ of $\overline{\tau}(X_{\rm sc};p)$ we may write,

$$\lim_{t\rightarrow\infty} J(X_{\rm sc},t) = \lim_{t\rightarrow\infty}\int_{-\infty}^{\infty} p\, dp\, P(0,p;t) \times\int_0^t d\tau\, g_0(X_{\rm sc}p;\tau)\ . \qquad (4.15)$$

Using the property that $G_0(Xp;0p';\tau)$ satisfies the FPE with $K(X)=-\omega_0^2X$ and the initial condition (4.5), the last integral in Eq. (4.15) is shown to be equal to unity in the limit $t\rightarrow\infty$. This shows that the asymptotic $(t\rightarrow\infty)$ current is conserved and independent of X as implied by the continuity equation (2.3).

Another important property follows from the finiteness of $\overline{\tau}_1(X_{\rm sc})$. Let us consider the function $pP(0,p;\tau)$ present in Eq. (4.9) with P being the distribution obtained in Eq. (3.15). The maximum of this function in the relevant $p>0$ region is located at $\sigma_p(\tau)$ disregarding small corrections due to $\langle p\rangle$ and the error function present in Eq.

(3.15). In the interval $(0,t)$ this maximum moves in the interval $[\sigma_p(t=0)\simeq 0,\sigma_p(t)]$ of maximum extension $[0,(T/\mu)^{1/2}]$. For the system $A=248$ of reduced mass $\mu=62\ m_p$ where m_p is the mass of the nucleon and for a temperature $T=1$ MeV, one obtains $(T/\mu)^{1/2}=1.2$ fm sec^{-1} whereas at $T=5$ MeV the result is $(T/\mu)^{1/2}=2.7$ fm sec^{-1}. As shown in Fig. 6 $\bar{\tau}(X_{sc},p)$ is a slowly decreasing function of p in these domains and remains close to its value $\bar{\tau}_1(X_{sc})$ at $p=0$. Thus only the region of values of τ around $\bar{\tau}_1(X_{sc})$ contributes effectively to the integral on τ in Eq. (4.9). We conclude that for $t<\bar{\tau}_1(X_{sc})$ the integral over τ is expected to be negligible and $\bar{\tau}_1(X_{sc})$ is therefore the time for the onset of the rise of the flux at the scission point. We surmise that to a good approximation the current at the scission point is given by

$$J(X_{sc},t)\simeq\theta[t-\bar{\tau}_1(X_{sc})]\times\int_{-\infty}^{\infty}p\,dp\,P[0,p;t-\bar{\tau}_1(X_{sc})]\ . \qquad (4.16)$$

Here $\theta(X)$ is the usual step function $\theta(X)=1$ if $X>0$, zero otherwise. This relation expresses the flux at the scission point as the flux at the saddle point delayed by a constant time $\bar{\tau}_1(X_{sc})$.

The current at the scission point is calculated by numerical integration of the exact and approximate expressions (4.9) and (4.16), respectively. The distribution function $P(0,p;t)$ is taken from Eq. (3.15) with the variances, correlation function, and other time-dependent quantities determined as explained in Sec. III B. The two results are shown in Fig. 7 as a function of time in units of 10^{-21} sec. The continuous and the dashed curves correspond to the flux in units of 10^{21} sec^{-1} evaluated according to Eqs. (4.9) and (4.16), respectively. The sets of parameters are $\beta=0.5\times10^{21}$ sec^{-1}, $T=1$ MeV and $\beta=1.5\times10^{21}$ sec^{-1}, $T=5$ MeV. The value of the frequency of the inverted oscillator is $\omega_0=1.65\times10^{21}$ sec^{-1} and the scission point is placed at $X_{sc}=7$ fm. The values of β chosen correspond to an underdamped situation. As we have conjectured, the approximate expression (4.16) is in good overall agreement with the flux obtained from Eq. (4.9). We note, however, that with expression (4.9) the oscillations of the flux present at the saddle point—and characteristic of the underdamped motion in the minimum of the collective potential—are smeared out at scission due to the integration over τ.

In the works of Ref. 11 relation (4.16) was assumed to hold on the basis of qualitative considerations. It is plain to see that our numerical findings support this assumption. However, the time delay $\bar{t}$ used in Ref. 11 is taken, following Ref. 17, as

$$\bar{t}=\frac{\int_0^{X_{sc}}dX\int_{\infty}^{\infty}dp\,P_{Kr}(X,p)}{j}=-(2/b)R[(\tfrac{1}{2}\mu\omega_0^2X_{sc}^2/T)^{1/2}]\ ; \qquad (4.17)$$

here $P_{Kr}(X,p)$ and j are, respectively, Kramers's stationary distribution and position independent current[6] and

$$R(z)=\int_0^z\exp(y^2)dy\int_y^{\infty}\exp(-x^2)dx\ .$$

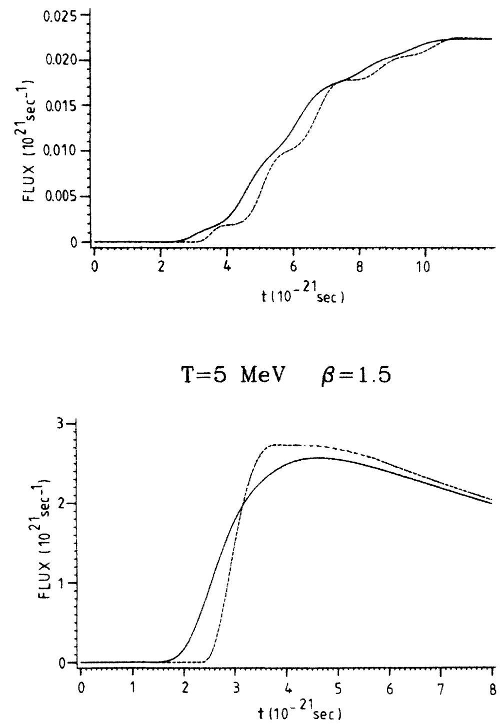

FIG. 7. The flux (in units of 10^{21} sec^{-1}) calculated at the scission point $X_{sc}=7$ fm as a function of time in units of 10^{-21} sec. The continuous curves result from Eq. (4.9) and the dashed curves from Eq. (4.16). The nuclear friction parameter β is in units of 10^{21} sec^{-1}. The nucleus has mass $A=248$ and a fission barrier height $E_f=4$ MeV.

Therefore it is of interest to compare $\bar{t}$ with the time delay $\bar{\tau}_1(X_{sc})$ after which the flux at the scission point starts to rise. In the leading order in the variable

$$Z=4\mu\omega_0^2X_{sc}^2/T=4\omega_0^2X_{sc}^2\beta/\epsilon$$

we have $R(Z)=(1/4)\ln(Z)$ where the first neglected term is of order $O(1/Z)$. Thus

$$\bar{t}=-(2b)^{-1}\ln(4\omega_0^2X_{sc}^2\beta/\epsilon)+O[\epsilon/(4\omega_0^2X_{sc}^2\beta)]\ . \qquad (4.18)$$

Comparing with the expression (4.14) for $\bar{\tau}_1(X_{sc})$ we find that

$$\bar{t}=\bar{\tau}_1(X_{sc})+(2b)^{-1}\ln\{(1+\gamma^2)[(1+\gamma^2)^{1/2}-\gamma]/(2\gamma)\}+O[\epsilon/(4\omega_0^2X_{sc}^2\beta)]\ . \qquad (4.19)$$

Here $\gamma=\beta/(2\omega_0)$ is the reduced dissipation strength. In Eq. (4.19) the argument of the logarithm is unity for $\gamma=\gamma_0=(3+2\sqrt{3})^{-1/2}$ irrespective of the value of the nuclear temperature T and we expect to have $\bar{t}\gtrless\bar{\tau}_1(X_{sc})$ for $\gamma\gtrless\gamma_0$. In Fig. 8 we show the ratio $\bar{t}/\bar{\tau}_1(X_{sc})$ as a function of γ. The continuous curve is for $T=1$ MeV, the dashed curve for $T=3$ MeV, and the dash-dotted curve for $T=5$ MeV. All curves are obtained for the nucleus $A=248$ with $\mu=62m_p$, $X_{sc}=7$ fm, and $\omega_0=1.65\times10^{21}$ sec^{-1}. We see that in the domain of values of β investi-

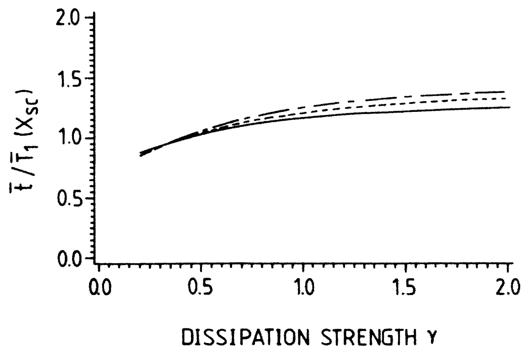

FIG. 8. The ratio of the time $\bar{t}$ defined in Ref. 17 to the time delay $\bar{\tau}_1(X_{sc})$ Eq. (4.14) as a function of the reduced dissipation strength $\gamma=\beta/(2\omega_0)$ with $\omega_0=1.65\times10^{21}$ sec^{-1}. The scission point is at $X_{sc}=7$ fm. Continuous curve $T=1$ MeV, dashed curve $T=3$ MeV, dash-dotted curve $T=5$ MeV. The nucleus has a mass $A=248$ and a fission barrier height $E_f=4$ MeV.

gated in Ref. 11 which corresponds to $\gamma \lesssim 0.8$, $\bar{t}$ does not differ from $\bar{\tau}_1(X_{sc})$ by more than 10% for temperature considered there and by more than 20% in any case. This justifies the use of $\bar{t}$ in Ref. 11 in place of $\bar{\tau}_1(X_{sc})$.

We note finally that the distribution function (4.4) and the properties of the Gaussian propagator $G_0(Xp;X'p';\tau)$ are general and valid for any value of the friction parameter. Therefore the expression of the time delay $\bar{\tau}_1(X_{sc})$ applies to the overdamped as well as the underdamped situations.

C. Lifetime at the scission point

The fulfillment of Eq. (4.16) as we have seen permits one to write the escape rate at the scission point simply in terms of the escape rate at the saddle point,

$$\lambda_f^{sc}(t)=\theta[t-\bar{\tau}_1(X_{sc})]\lambda_f^{sad}[t-\bar{\tau}_1(X_{sc})] \ . \tag{4.20}$$

Let us consider again the schematic model of Sec. II which with the use of Eq. (4.20) now takes the form,

$$\lambda_f^{sc}(t)=\begin{cases}0 \ \text{if} \ t<\bar{\tau}_1(X_{sc}) \\ (\lambda_f^{sad}/\tau)[t-\bar{\tau}_1(X_{sc})] \\ \text{if} \ \bar{\tau}_1(X_{sc})\le t\le\bar{\tau}_1(X_{sc})+\tau \\ \lambda_f^{sad} \ \text{if} \ t>\bar{\tau}_1(X_{sc})+\tau \ .\end{cases} \tag{4.21}$$

The lifetime at the scission point is obtained according to Eq. (2.5). With $\tau_f^{stat}=(\lambda_f^{sad})^{-1}$ we find

$$\tau_f^{sc}=\tau_f^{stat}+\tfrac{1}{2}\tau+\bar{\tau}_1(X_{sc}) \ . \tag{4.22}$$

Thus for a quasistationary situation as described by Eq. (4.21) the total lifetime at scission is the sum of three terms which individually have a clear physical interpretation. The first contribution relates to the asymptotic time behavior of the fissioning system, hence its interpretation as a "decay width" contribution. The second contribution comes from the transient behavior of the composite nucleus in the collective potential pocket and the last term is a saddle-to-scission contribution. In practice as shown in Fig. 2 some departures from our schematic model (4.21) are expected and a clear-cut separation between the first two contributions to the total lifetime is not exact. However, Fig. 7 asserts the occurrence of the saddle-to-scission contribution as a simple additive term to the lifetime evaluated at the saddle point. Combining the results of Figs. 3 and 4 with expression (4.14) for $\bar{\tau}_1(X_{sc})$ one readily obtains the total lifetime τ_f of the fissioning system of mass $A=248$ as a function of the nuclear friction parameter β for various positions of the scission configuration X_{sc} and for the three nuclear temperatures envisaged. The values of τ_f^{sc} are gathered in Table I for three positions of the scission point at 5, 7, and 10 fm, respectively, and a range of values of β in units of 10^{21} sec^{-1}. For each value of X_{sc} the first column corresponds to $T=1$ MeV and the second to $T=3$ MeV. For given temperature T and scission position X_{sc} we note the strong variations of the lifetime with β. For given β and T the lifetime depends very little on the position of the scission point due to the logarithmic dependence of $\bar{\tau}_1(X_{sc})$ on X_{sc}. To discuss further the importance of the time $\bar{\tau}_1(X_{sc})$ in the determination of the lifetime at the scission point we envisage a different and equally plausible physical situation for the nuclear system of reduced mass $\mu=62\ m_p$ at a temperature $T=1$ MeV. For $X>0$ the potential barrier is now modeled by an inverted parabola of much smaller curvature, e.g., $\omega_0=0.5\times10^{21}$ sec^{-1}. Using Kramers's expression for the momentum distribution at the saddle point the mean quasistationary velocity at this point is

$$\langle p\rangle=-b[2T/(\pi\mu)]^{1/2}(2\omega_0)^{-1} \ .$$

It takes the values $\langle p\rangle_0=0.5$ fm sec^{-1} for $\beta=0$ and $\langle p\rangle_\beta=0.31$ fm sec^{-1} for $\beta=0.5\times10^{21}$ sec^{-1}. With a scission point located at $X_{sc}=3.6$ fm the classical traveling times up to X_{sc} are $t_{cl}(\beta=0)=4\times10^{-21}$ sec,

TABLE I. The lifetime (in units of 10^{-21} sec) of the system of mass $A=248$ evaluated at various positions of the scission point X_{sc} and for a series of values of the friction parameter β. For each value of X_{sc} the first column corresponds to $T=1$ MeV and the second to $T=3$ MeV.

β (10^{21} sec^{-1})	$X_{sc}=5$ fm τ_f^{sc}	$X_{sc}=5$ fm τ_f^{sc}	$X_{sc}=7$ fm τ_f^{sc}	$X_{sc}=7$ fm τ_f^{sc}	$X_{sc}=10$ fm τ_f^{sc}	$X_{sc}=10$ fm τ_f^{sc}
0.5	24.4	20.0	24.6	20.2	24.9	20.5
1.0	27.7	20.8	28.0	21.1	28.3	21.4
1.5	32.0	22.8	32.3	23.1	32.6	23.4
2.0	37.8	26.1	38.1	26.4	38.5	26.8
3.0	45.0	30.2	45.4	30.6	45.9	31.1
5.0	67.0	44.2	67.7	44.8	68.4	45.5

$t_{\rm cl}(\beta=0.5)=8.3\times10^{-21}$ sec, and $t_{\rm cl}(\beta=0.7)=11\times10^{-21}$ sec. For $\beta=0.5\times10^{21}$ sec^{-1} the mean time delay is $\bar{t}=3.5\times10^{-21}$ sec and $\bar{\tau}_1(X_{\rm sc})=3.1\times10^{-21}$ sec. We see therefore that there exist cases in terms of β and T for which $\bar{\tau}_1(X_{\rm sc})<t_{\rm cl}$. This may appear plausible since $\bar{\tau}_1(X_{\rm sc})$ is only the time for *the onset of the rise* of the flux at $X_{\rm sc}$. Thus through considerations based only on classical traveling time one may overestimate the lifetime evaluated at the scission point.

Irrespective of the value ω_0 of the curvature of the inverted parabola the transient time from Fig. 3 is $\tau\simeq9\times10^{-21}$ sec for $T=1$ MeV and β around 0.5×10^{21} sec^{-1}. In this case the transient behavior of the system on its way to the saddle point is most effective in modifying the usual expression of the lifetime in terms of a decay width. The situation changes somewhat for β around 1.5×10^{21} sec since, for the range of temperatures envisaged, the values of τ and $\bar{\tau}_1(X_{\rm sc})$ are now comparable and thus equally important in modifying the usual expression of the lifetime.

V. SUMMARY AND CONCLUSIONS

Nuclear fission is viewed as a transport process over a potential barrier. In this context we define the lifetime of the decaying system independently of the specific form of the transport equation obeyed by the distribution function for the collective variables. We show that if the transport process is stationary from the outset then the lifetime of the system is given in terms of a decay width. Hence our definition appears as a natural generalization to nonstationary processes of the usual expression of the lifetime. Using the conservation law for the current we relate this lifetime to the time integrated escape rate at the saddle point and at the scission point of the collective potential. A schematic model of this time-dependent escape rate permits one to exhibit in a clear way the respective contributions of the quasistationary rate $\hbar/\Gamma_f^{\rm stat}$ and of the transient behavior of the system to its lifetime. This transient behavior is characterized[9,10] by a transient time τ after which a quasistationary probability flow across the barrier is established. As long as $\hbar/\Gamma_f^{\rm stat}\gg\tau$, fission can be well described as a quasistationary phenomenon. For $\hbar/\Gamma_f^{\rm stat}\ll\tau$, which occurs for excitation energies of a few hundreds of MeV and a small fission barrier, the fission process becomes a transient phenomenon of duration of the order of τ.

Since the key ingredient to determine the lifetime of the system is the time-dependent escape rate over the barrier, we study this quantity both at the saddle point and at the scission point of a one-dimensional collective potential. We use the same diffusion model as in an earlier study[10] which we develop and complete. The developments concern the determination of the time-dependent rate at the saddle point which incorporates in a realistic and yet easily calculable manner the anharmonicities of the collective potential and the effects of the overall motion of the system along the main collective coordinate. In the case of large nuclear friction β, i.e., the ratio of the dissipation strength over the reduced mass of the system, we obtain a new approximate analytic expression for the time-dependent escape rate. This expression reproduces very closely the results of a direct numerical calculation and obeys the important scaling properties discussed in Ref. 12. With these new expressions we find that the whole domain of β values ranging from $\beta\gtrsim0.2\times10^{21}$ sec^{-1} is covered in a continuous way for temperatures compatible with the existence of a quasistationary regime. From simple classical considerations these temperatures are bounded through relations which involve the height E_f of the barrier and the value of the nuclear friction β. We perform a direct numerical integration of the Fokker-Planck diffusion equation and thereby verify the existence of a quasistationary regime of probability flow across the barrier for temperatures $T\gtrsim E_f$ as indicated by these inequalities. For such temperatures the mobility of the system towards the top of the barrier is an important feature of the dynamics. The quasistationary regime is finally established when the distribution settles around a position in between the bottom of the first well and the saddle point and the quasistationary rate obtained here differs from Kramers's usual expression. Without solving the full Fokker-Planck equation our method takes into account all the complex dynamics of the problem. Thus we can deal with situations where important overshooting with respect to Kramers's rate occurs. This aspect and the failure to cover the overdamped case had put limitations to the practical use of the approximate analytical rates derived in Ref. 10. It is now possible to investigate simply the fissioning system in the whole range of β values and within a large domain of nuclear temperatures.

We study first the lifetime evaluated at the saddle point. For $T<E_f$ we find that the influence of the transient time τ on the lifetime is significant only for values of β characteristic of the underdamped case. For $T\gtrsim E_f$ the influence of τ becomes more important with increasing temperature over the whole range of β values. For the system of mass $A=248$ with a barrier height $E_f=4$ MeV the transient time contribution to the total lifetime evaluated at the saddle point increases from 10% to 40% at $T=1$ MeV, from 5% to 30% at $T=3$ MeV, and from 10% to 20% at $T=5$ MeV with decreasing values of β in the range $0.2\leq\beta\leq5.0$ in units of 10^{21} sec.$^{-1}$.

We complete this study by evaluating the lifetime of the system at the scission point. Modeling the potential beyond the saddle point by an inverted harmonic oscillator of frequency ω_0 we obtain the time-dependent distribution for the collective variables valid beyond the saddle point in terms of the known time-dependent solution at the saddle point. We derive the flux at the scission point and show that it can be expressed reliably in terms of the flux at the saddle point delayed by a constant time $\bar{\tau}_1(X_{\rm sc})$. This result justifies the conjectures of Refs. 9–11. The schematic model for the time-dependent rate evaluated at the saddle point is trivially extended at the scission point. We obtain the lifetime at scission as a sum of three contributions which individually have a clear physical interpretation. The first two contributions relate to the lifetime at the saddle point and the last one is the time for the onset of the rise of the flux at the scission point.

In heavy-ion-induced reactions, the angular momentum is large and reduces the fission barrier with respect to neutron-induced fission. The nuclear temperatures may also reach a few MeV. In such cases statistical model estimates of the lifetimes are usually inconsistent with physical observations. Our analysis and the results of Refs. 9, 10, and 12 indicate that the main contributions to the total lifetime are the transient time τ and the time delay $\overline{\tau}_1(X_{sc})$ for the onset of the rise of the flux at the scission point. This suggests that systematic studies of the lifetime of such systems may provide useful information on τ and $\overline{\tau}_1(X_{sc})$ and thereby on nuclear friction.

In this study of induced fission we retain a simple one-dimensional diffusion model often used in analyzing experimental results. Some of the simplifications we introduce such as the temperature independence of β and of the fission barrier, may be improved without modification of our analytical derivations. The most important restriction concerns our limitation to a single collective variable and its conjugate momentum. Several such variables can be identified and have in fact been used in the realistic treatment of the fission process. However, to our knowledge only the case of a quasistationary diffusion for n degrees of freedom has been solved analytically.[20] If possible, a generalization of our time-dependent treatment to many degrees of freedom would prove very useful. Nevertheless our conclusions concerning the various contributions to the total lifetime are generic of transport approaches and independent of the number of degrees of freedom considered. It is only the relative interplay of these contributions which may be affected by the dimensionality of the collective system.

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*On leave of absence from Physical Research Laboratory, Ahmedabad 380 009, India.

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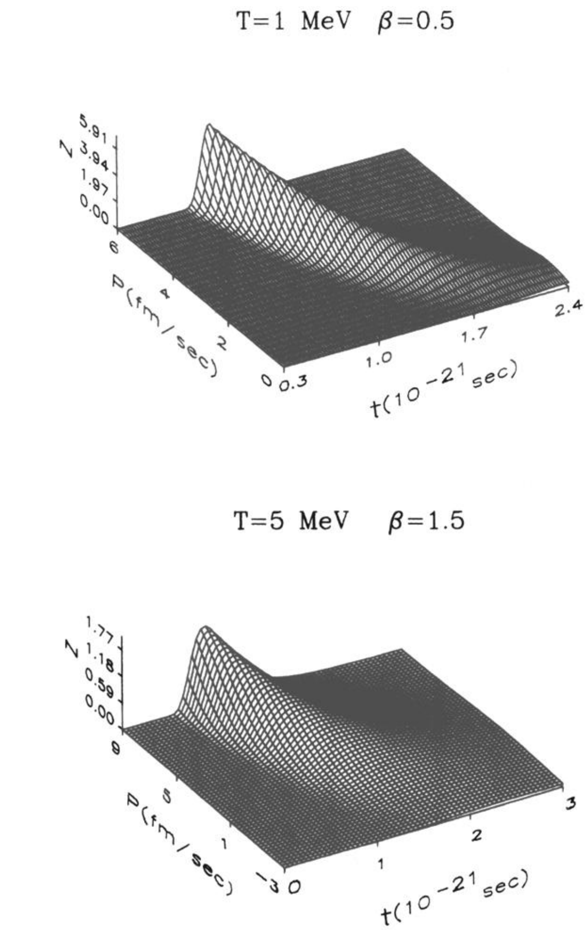

FIG. 5. The function $Z \equiv g_0(Xp;t)$ defined in Eq. (4.10) for a frequency $\omega_0 = 1.65 \times 10^{21}$ sec^{-1}, $X = 7$ fm, $T = 1$ MeV, $\beta = 0.5 \times 10^{21}$ sec^{-1} (upper part) and $T = 5$ MeV, $\beta = 1.5 \times 10^{21}$ sec^{-1} (lower part). The nucleus is ^{248}Cf.