Two-component exciton model: Basic formalism away from shell closures

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(Received 30 August 1985)

A closed form version of the two-component exciton model is derived. Discussions of corrections for the Pauli exclusion principle, of the expected behavior of the residual two-body matrix elements, and of the relative merits of different types of reaction calculations within the exciton model are presented. Calculations verify the validity of the equal *a priori* probabilities assumption underlying exciton model calculations and demonstrate that the one- and two-component models produce very similar results when similar assumptions are made about the relative probabilities of exciting proton and neutron degrees of freedom.

I. INTRODUCTION

In the first formulation of the Griffin or exciton model for preequilibrium nuclear reactions,[1] all particle and hole degrees of freedom were grouped together as excitons and treated as indistinguishable from one another. Fairly early on, particle degrees of freedom were distinguished from their hole counterparts.[2] Yet the obvious, natural, and straightforward extension of distinguishing between proton and neutron particle degrees of freedom and between proton and neutron hole degrees of freedom has only recently been carried out.[3,4]

The reason for this ten year delay is twofold. The first reason is the extra computational effort required, while the second relates to need. In describing the preequilibrium phase of a nuclear reaction in either the exciton or particle-hole formalism, there is a unique hierarchy of states through which the system passes. Letting n denote the total number of excitons composed of p particle and h hole degrees of freedom so that $n=p+h$, this hierarchy may be denoted in two equivalent forms:

$$n_0 \rightarrow n_0+2 \rightarrow n_0+4 \rightarrow n_0+6 \cdots ,$$

$$p_0,h_0 \rightarrow p_0+1,h_0+1 \rightarrow p_0+2,h_0+2 \rightarrow p_0+3,h_0+3 \cdots .$$

Each stage has one more particle-hole pair (or two more excitons) than the preceding stage. Describing the equilibration process is then, in some sense, a one-dimensional problem. When proton and neutron degrees of freedom are distinguished from one another, each class of states in the above hierarchy is divided into $h=p-A_a$ subclasses (where A_a is the mass number of the projectile) characterized by different divisions of the degrees of freedom into protons and neutrons. The problem is now two dimensional and the effort required to do a calculation is significantly increased. The difference in the two formalisms is shown schematically in Fig. 1.

The increased effort alone would not, however, have been able to stop the early implementation of the two-component exciton model had the need been pressing. The extension is too straightforward. What happened was that for early studies of preequilibrium reactions designed to investigate some of the fundamental aspects of the field, it was adequate to introduce proton-neutron distinguishability in an approximate way where it is most important, namely in the particle emission rates. Only now, as confidence in the model has grown, as better ways are being sought to include pairing and shell effects, and as the need for better proton-neutron yield ratios has grown has there been sufficient motivation to go over to the two-component exciton model.

While many of the equations relevant to the two-component exciton model have been quoted in the literature,[5-7] only two papers[3,4] describing calculations with the model have appeared. In the first,[3] Gupta approximated the two-component master equation solution with a closed form expression and made comparisons with the corresponding one-component formula. Unfortunately he made the serious error[4] of identifying the time integrated occupation probability for a given configuration with the lifetime of that configuration. In fact, the occupation

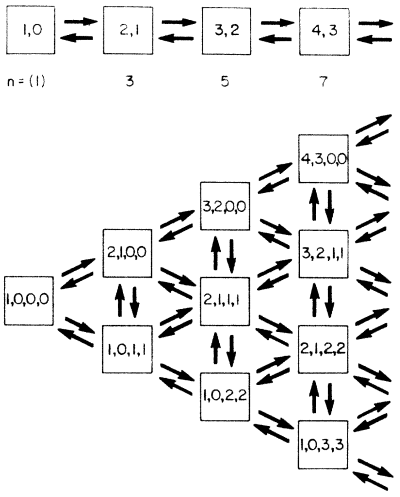

FIG. 1. Schematic diagrams of proton induced reactions in both the one- and two-component exciton models. In the one-component model, the different classes of states are designated by p,h whereas in the two-component model the labels are p_π, h_π, p_ν, and h_ν. The arrows show the allowed residual two-body interactions.

probabilities are also weighted by the fraction of the total strength passing through the configurations of interest. Other shortcomings of this work are the neglect of the Pauli exclusion principle and of the energy and exciton number dependence of the mean square matrix element for the residual two-body interactions.

Dobeš and Běták[4] have remedied all three of these difficulties and have used an approximate solution to the time integrated master equations. They also use different residual two-body matrix elements for proton-proton, neutron-neutron, and proton-neutron interactions. Finally, they compare their results with data and with their own one-component results for a few sample reactions.

The present work is similar to that of Dobeš and Běták[4] but differs in several important regards. First, an alternate set of transition rates for the residual two-body interactions is derived and is shown to be consistent with the full particle-hole state density in the limit of an equilibrated system. Second, a closed form approximation to the two-component exciton model is presented along with a discussion of the advantages and disadvantages of this approach relative to using the time-integrated master equations as in Ref. 4. The residual two-body matrix elements and, in particular, their relationship to free nucleon-nucleon scattering cross sections and to full two-body matrix elements in nuclear structure calculations are discussed in some detail. The assumption of equal mean square matrix elements for the residual proton-neutron, neutron-neutron, and proton-proton interactions is provisionally implemented.

The results of reaction calculations are presented and compared with the results of the corresponding one-component model. Results from the present work are also used to investigate the validity of the assumption that at each stage in a reaction all states of a given exciton number are equally likely to be populated. This assumption underlies the one-component model. Its analog for the various subclasses of states is inherent in the two-component model.

Finally, one of the main purposes for developing the present two-component version of the exciton model was to be able to study corrections to preequilibrium reaction calculations due to pairing and shell structure effects. Since these effects for a given exciton number will obviously depend on the way in which the excitons are divided into proton and neutron degrees of freedom, they cannot be adequately treated in a one-component model. A discussion of various approaches to preequilibrium pairing corrections[8] as well as other factors influencing the relative yields of protons and neutrons for nonclosed shell systems will be presented in paper II in this series along with comparisons with data. Paper III is planned to contain a study of preequilibrium shell corrections in the context of the shell-shifted equispacing model (S^2-ESM).[9]

II. STATE DENSITIES AND TRANSITION RATES

In the exciton or Griffin model of nuclear reactions there are basically three types of quantities which are needed in either the master equations or their closed form counterparts. These are (i) the density of states for the system with a fixed combination of excitons and at a given excitation energy, (ii) the rates for the residual interactions which carry the system from one class of states to another (the arrows in Fig. 1), and (iii) the rates for emitting various types and energies of particles from a given class of states.

A. Two component particle-hole state densities

The state density for a nucleus at excitation energy E containing $p=p_\pi+p_\nu$ particle degrees of freedom and $h=h_\pi+h_\nu$ hole degrees of freedom was given by Williams[5] to be

$$\omega(p_\pi,h_\pi,p_\nu,h_\nu,E)=\frac{g_\pi^{n_\pi}g_\nu^{n_\nu}[E-A(p_\pi,h_\pi,p_\nu,h_\nu)]^{n-1}}{p_\pi!h_\pi!p_\nu!h_\nu!(n-1)!} \tag{1}$$

with

$$A(p_\pi,h_\pi,p_\nu,h_\nu)=\frac{p_\pi^2+h_\pi^2+p_\pi-3h_\pi}{4g_\pi}+\frac{p_\nu^2+h_\nu^2+p_\nu-3h_\nu}{4g_\nu}\ . \tag{2}$$

Here the subscripts π and ν refer to protons and neutrons, respectively; g is the density of single particle states and is evaluated separately for the two components; $n_\pi=p_\pi+h_\pi$, $n_\nu=p_\nu+h_\nu$, and $n=p+h=n_\pi+n_\nu$. These are the formulae employed by Dobeš and Běták.[4] There are, however, two fundamental difficulties with Eq. (2), which contains the effects of the Pauli exclusion principle. Neither makes a large difference in reaction calculations, but it is worth correcting them at this stage since in the presence of shell corrections the value of this function (and thus the impact of the errors) can become much larger.

The first difficulty is that Eq. (2) is not symmetric in particles and holes. This results from the fact that Williams places the Fermi level *at* the last occupied single particle state in the ground state of the nucleus rather than halfway between it and the first vacant single particle state. Fortunately, Eq. (2) can be rewritten in terms of the Pauli energy, the minimum energy required for a given configuration by the exclusion principle. This gives[9,10]

$$A(p_\pi,h_\pi,p_\nu,h_\nu)=E_{\text{Pauli}}^{(\pi)}(p_\pi,h_\pi)+E_{\text{Pauli}}^{(\nu)}(p_\nu,h_\nu)-\frac{p_\pi^2+h_\pi^2+n_\pi}{4g_\pi}-\frac{p_\nu^2+h_\nu^2+n_\nu}{4g_\nu}\ . \tag{3}$$

Thus all of the particle-hole asymmetry is in the Pauli energy which is easily reevaluated.

The second difficulty[9,10] can likewise be corrected by reevaluating the Pauli energy. It is the neglect in Eq. (2) of the presence of passive particles and/or holes. The Pauli energy is determined by the total numbers of particles and holes; both those that are degrees of freedom and those which are passive or adjacent to the Fermi level.

When both of these difficulties are corrected, the Pauli energies in Eq. (3) become[9]

$$E^{(\pi)}_{\text{Pauli}}(p_\pi,h_\pi)=p^2_{\pi m}/g_\pi \ , \tag{4}$$

$$p_{\pi m}=\text{maximum}(p_\pi,h_\pi) \ , \tag{5}$$

and similarly for $E^{(\nu)}_{\text{Pauli}}$. These results agree with Williams's original formula in the limit $p_\pi=h_\pi$ and $p_\nu=h_\nu$. Their relative accuracies can be tested in a few numerical examples for cases where $p_\pi\neq h_\pi$. For simplicity, the results for only one active component are considered. These may be obtained, for instance, by taking $p=p_\pi$, $h=h_\pi$, and $p_\nu=h_\nu=0$. The results of Eq. (1) using both Eq. (2) (Williams's formula) and Eqs. (3)–(5) are compared in Table I with the numbers of states obtained by direct counting. Here unusually low values of $g_\pi E$ have been chosen so that differences in the correction function $A(p_\pi,h_\pi,p_\nu,h_\pi)$ would have a larger effect on the calculated state densities. Clearly the problems in Williams's formula become more significant at larger values of p-h and for low excitation energies or high exciton numbers. The same is true of the results of Eqs. (3)–(5), but the errors are very much smaller. Equation (1) was derived in the limit of an infinite set of single particle states and is adequate so long as the excitation energy does not exceed the depth V of the potential well. For higher excitation energies it must be corrected by subtracting those configurations which have one or more holes below the bottom of the well. Thus one gets

$$\omega(p_\pi,h_\pi,p_\nu,h_\nu,E) = \frac{g_\pi^{n_\pi} g_\nu^{n_\nu}[E-A(p_\pi,h_\pi,p_\nu,h_\nu)]^{n-1}}{p_\pi!h_\pi!p_\nu!h_\nu!(n-1)!} f(h) \ , \tag{6}$$

$$f(h)=\sum_{i=0}^{h}(-1)^i \begin{bmatrix} h \\ i \end{bmatrix} \left[\frac{E-iV(h)}{E}\right]^{n-1} \Theta[E-iV(h)] \ . \tag{7}$$

Here the function $f(h)$ is the same as that used in the one-component model[11] except that the well depth is allowed to vary with h to take into account that states populated in the first few interactions might be limited to the surface region of the nucleus and thus have a lower effective well depth. In practice this correction is only applied for states with $h \lesssim 2$. The quantity $\Theta(E)$ is the Heaviside function which is zero for a negative argument and unity for a positive one.

B. Transition rates and residual two-body matrix elements

The residual two-body interactions in the exciton model are the following: $\Delta p=\Delta h=+1$ or the creation of a particle-hole pair; $\Delta p=\Delta h=-1$ or the destruction of a particle-hole pair; and $\Delta p=\Delta h=0$ which involves the exchange of energy between degrees of freedom. In the two-component exciton model, the transition rates which need to be considered are $\lambda_{\pi+}$ and $\lambda_{\pi-}$ for the creation and destruction of proton particle-hole pairs; $\lambda_{\nu+}$ and $\lambda_{\nu-}$ for neutron pairs; and $\lambda_{\pi\nu}$ and $\lambda_{\nu\pi}$ for the conversion of a proton pair into a neutron pair and *vice versa*. Each of these can be written in terms of a corresponding transition state density using time dependent perturbation theory so that, for example,

$$\lambda_{\pi+}(p_\pi,h_\pi,p_\nu,h_\nu)=\frac{2\pi}{\hbar}M^2\omega_{\pi+}(p_\pi,h_\pi,p_\nu,h_\nu,E) \ , \tag{8}$$

where M^2 is some appropriate effective residual mean square matrix element for the interaction. If, as assumed by Dobeš and Běták,[4] M^2 is different for interactions between different kinds of nucleons so that $M^2_{\pi\pi}\neq M^2_{\nu\nu}\neq M^2_{\pi\nu}$ then Eq. (8) becomes

$$\lambda_{\pi+}(p_\pi,h_\pi,p_\nu,h_\nu)=\frac{2\pi}{\hbar}[M^2_{\pi\pi}\omega_{\pi\pi+}(p_\pi,h_\pi,p_\nu,h_\nu,E) + M^2_{\pi\nu}\omega_{\nu\pi+}(p_\pi,h_\pi,p_\nu,h_\nu,E)] \tag{9}$$

with the first term corresponding to excitation of a proton particle-hole pair by a proton degree of freedom (either a particle or a hole) and the second to excitation by a neutron degree of freedom.

The question of whether M^2 should be assumed to have a constant average value, as assumed by Gupta,[3] or to have different values for p-p, n-n, and p-n interactions is a complex one and is discussed further in Sec. III of this pa-

TABLE I. Calculated and exact numbers of states.[a]

n	p_π,h_π	$g_\pi E$	Exact	$(1/g_\pi)\omega(p_\pi,h_\pi,0,0,E)$[b] Williams[c]	This work[d]
2	1,1	21	21	21	21
	2,0	21	9	10	9
3	2,1	13	30	36	30
	2,1	17	56	64	56
	2,1	21	90	100	90
4	2,2	21	330	333	333
	3,1	21	102	176	103
5	3,2	21	249	406	257

[a]Calculated numbers have been rounded to the nearest integer.
[b]Given by Eq. (1).
[c]$A(p_\pi,h_\pi,0,0)$ from Eq. (2).
[d]$A(p_\pi,h_\pi,0,0)$ from Eqs. (3)–(5).

per. Results for different $M^2_{\pi\pi}$, $M^2_{\nu\nu}$, and $M^2_{\pi\nu}$ are presented here and are easily converted to the results for a single M^2.

The method used here to derive the transition state densities is basically that used in Ref. 12 where bound and unbound states were distinguished from one another. The average final state densities in this work are each found from a one-dimensional integral and are solved analytically whereas Dobeš and Běták[4] use a multidimensional integral which was treated with the Cauchy residue theorem. They derive the pair annihilation rates and $\lambda_{\pi\nu}$ and then use the steady state equilibrium condition to obtain the results for pair creation and $\lambda_{\nu\pi}$. Unfortunately, it is the pair creation rates which are most important in preequilibrium calculations, while in Ref. 4 these are obtained only indirectly, and the Pauli-corrected results are never quoted. Here all of the needed transition state densities are derived directly and with the Pauli corrections. The steady state equilibrium criterion can then be used as an internal consistency check. The derivations are done in the limit of an infinitely deep well, and finite well depth corrections are applied to the answers in an approximate way.

For the case of a proton particle-hole pair being excited by a neutron degree of freedom, the transition state density is given by

$$\omega_{\nu\pi+}(p_\pi,h_\pi,p_\nu,h_\nu,E)=\int_{L_1}^{L_2}\frac{\omega'(0,0,1,0,e)\omega(p_\pi,h_\pi,p_\nu-1,h_\nu,E-e)}{\omega(p_\pi,h_\pi,p_\nu,h_\nu,E)}\omega'(1,1,1,0,e)de + \int_{L_3}^{L_4}\frac{\omega'(0,0,0,1,e)\omega(p_\pi,h_\pi,p_\nu,h_\nu-1,E-e)}{\omega(p_\pi,h_\pi,p_\nu,h_\nu,E)}\omega'(1,1,0,1,e)de \ . \tag{10}$$

The first term represents excitation by a neutron particle degree of freedom, the second by a neutron hole degree of freedom. In each term the degrees of freedom are divided into those which are involved in the transition and those which are spectators. The ratio in each integral represents the fraction of the initial states which have the active degree of freedom with energy e. The additional factor is the state density for the active degrees of freedom in the final state. The primes in the state densities for the active parts of the system indicate that the Pauli correction function A is to be evaluated bearing in mind that there are other degrees of freedom in the system and that the exclusion principle operates simultaneously on all of them. Likewise, the limits of integration reflect the requirements of the exclusion principle. Thus, for instance, the maximum energy of the exciting particle is not $L_2=E$ but rather

$$L_2=E-A(p_\pi,h_\pi,p_\nu-1,h_\nu)$$

and the minimum energy is not $L_1=0$ but

$$L_1=A(p_\pi+1,h_\pi+1,p_\nu,h_\nu)-A(p_\pi,h_\pi,p_\nu-1,h_\nu) \ .$$

Evaluating the integral gives the result

$$\omega_{\nu\pi+}(p_\pi,h_\pi,p_\nu,h_\nu,E)=\frac{n_\nu g_\nu g_\pi^2}{n(n+1)}\frac{[E-A(p_\pi+1,h_\pi+1,p_\nu,h_\nu)]^{n+1}}{[E-A(p_\pi,h_\pi,p_\nu,h_\nu)]^{n-1}} \ . \tag{11}$$

The same method is then applied to excitation of a proton pair by a proton degree of freedom. The two answers are then combined and the finite well depth correction function for the final states is applied to give

$$\lambda_{\pi+}(p_\pi,h_\pi,p_\nu,h_\nu)=\frac{2\pi}{\hbar}\frac{g_\pi^2}{2n(n+1)}\frac{[E-A(p_\pi+1,h_\pi+1,p_\nu,h)]^{n+1}}{[E-A(p_\pi,h_\pi,p_\nu,h_\nu)]^{n-1}}(n_\pi g_\pi M^2_{\pi\pi}+2n_\nu g_\nu M^2_{\pi\nu})f(h+1) \ . \tag{12}$$

In a similar way, the other transition rates are given by

$$\lambda_{\pi-}(p_\pi,h_\pi,p_\nu,h_\nu)=\frac{2\pi}{\hbar}\frac{p_\pi h_\pi}{2}[(n_\pi-2)g_\pi M^2_{\pi\pi}+2n_\nu g_\nu M^2_{\pi\nu}]f(h-1) \ , \tag{13}$$

$$\lambda_{\pi\nu}(p_\pi,h_\pi,p_\nu,h_\nu)=\frac{2\pi}{\hbar}M^2_{\pi\nu}\frac{p_\pi h_\pi}{n}g_\nu^2 f(h)\left[\frac{E-B(p_\pi,h_\pi,p_\nu,h_\nu)}{E-A(p_\pi,h_\pi,p_\nu,h_\nu)}\right]^{n-1} \times\{2[E-B(p_\pi,h_\pi,p_\nu,h_\nu)]+n\mid A(p_\pi,h_\pi,p_\nu,h_\nu)-A(p_\pi-1,h_\pi-1,p_\nu+1,h_\nu+1)\mid\} \ , \tag{14}$$

where

$$B(p_\pi,h_\pi,p_\nu,h_\nu)=\text{maximum}[A(p_\pi,h_\pi,p_\nu,h_\nu),A(p_\pi-1,h_\pi-1,p_\nu+1,h_\nu+1)] \ . \tag{15}$$

There are analogous expressions for $\lambda_{\nu+}$, $\lambda_{\nu-}$, and $\lambda_{\nu\pi}$ in which the roles of protons and neutrons are interchanged.

Equations (12)—(14) and their analogs agree with the results of Běták *et al.*[6] and of Dobeš and Běták[4] when the Pauli corrections are ignored, but they differ from those of the latter reference in the way that the effects of the Pauli principle are included. It should also be noted that in Ref. 6 it was assumed that $g_\pi = g_\nu$. Neither of these references use finite well depth corrections. The corrections used here on the transition rates are admittedly somewhat *ad hoc*, but they are the ones proposed in Ref. 11 and shown there to agree with the more complicated exact results to within 5% for excitation energies below 2 V.

C. Consistency of transition rates and state densities

At statistical equilibrium all states of the system should be equally likely to be populated so that the probability of finding the nucleus in any one of its states specified by p_π, h_π, p_ν, and h_ν should be proportional to the density of such states. An important check for consistency between the state densities and the transition rates is that the transitions not disturb the equilibrium; that forward and backward transitions occur with equal rates. The three consistency conditions in the exciton model are

$$\omega(p_\pi,h_\pi,p_\nu,h_\nu)\lambda_{\pi+}(p_\pi,h_\pi,p_\nu,h_\nu)=\omega(p_\pi+1,h_\pi+1,p_\nu,h_\nu)\lambda_{\pi-}(p_\pi+1,h_\pi+1,p_\nu,h_\nu)\,, \tag{16a}$$

$$\omega(p_\pi,h_\pi,p_\nu,h_\nu)\lambda_{\nu+}(p_\pi,h_\pi,p_\nu,h_\nu)=\omega(p_\pi,h_\pi,p_\nu+1,h_\nu+1)\lambda_{\nu-}(p_\pi,h_\pi,p_\nu+1,h_\nu+1)\,, \tag{16b}$$

$$\omega(p_\pi,h_\pi,p_\nu,h_\nu)\lambda_{\pi\nu}(p_\pi,h_\pi,p_\nu,h_\nu)=\omega(p_\pi-1,h_\pi-1,p_\nu+1,h_\nu+1)\lambda_{\nu\pi}(p_\pi-1,h_\pi-1,p_\nu+1,h_\nu+1)\,. \tag{16c}$$

All three of these conditions can be shown to be exactly satisfied by state densities from Eqs. (3)—(7) and transition rates given by Eqs. (12)—(15).

It should be pointed out that this consistency is a necessary but not a sufficient condition for the accuracy of the transition rates based on the accuracy of the total state densities. In particular the present results cannot be exact because the pair annihilation rates contain no corrections for the effects of the exclusion principle. In this respect the results of Ref. 4 have a slight advantage over the present transition rates.

D. Particle emission rates

Particle emission rates in the two-component exciton model are, like those in the one-component model, derived from microscopic reversibility. The result has exactly the same form except that there is now one emission rate for each proton-neutron configuration at a given exciton number and there are no longer any factors of the type[13] $Q_b(p)$ to approximate the effects of proton-neutron distinguishability.

The two-component emission rate for protons of energy ϵ from a specific group of configurations is

$$W_\pi(p_\pi,h_\pi,p_\nu,h_\nu,\epsilon)=\frac{(2s_\pi+1)}{\pi^2\hbar^3}\mu_\pi\epsilon\sigma_\pi(\epsilon)\frac{\omega(p_\pi-1,h_\pi,p_\nu,h_\nu,E-B_\pi-\epsilon)}{\omega(p_\pi,h_\pi,p_\nu,h_\nu,E)} \tag{17}$$

with an analogous expression for neutron emission. In general, for a particle of type b with proton number Z_b and neutron number N_b the emission rate becomes

$$W_b(p_\pi,h_\pi,p_\nu,h_\nu,\epsilon)=\frac{(2s_b+1)}{\pi^2\hbar^3}\mu_b\epsilon\sigma_b(\epsilon)\frac{\omega(p_\pi-Z_b,h_\pi,p_\nu-N_b,h_\nu,E-B_b-\epsilon)}{\omega(p_\pi,h_\pi,p_\nu,h_\nu,E)}\,, \tag{18}$$

where s_b, B_b, and $\sigma_b(\epsilon)$ are the spin, binding energy in the composite system, and inverse reaction cross section, respectively, for the emitted particle. Equation (18) is the result also given by Gupta,[3] and Eq. (17) agrees with the result of Dobeš and Běták[4] for nucleon emission.

III. RESIDUAL TWO-BODY MATRIX ELEMENTS

As was mentioned in the previous section, Gupta[3] assumes a constant residual two-body matrix element while Dobeš and Běták[4] have used different matrix elements depending on the nature of the interacting particles. In addition to relatively small differences which disappear in the $N=Z$ limit, their value of $M^2_{\pi\nu}$ is nearly three times their values of $M^2_{\pi\pi}$ or $M^2_{\nu\nu}$. The reason for this factor is that the free nucleon-nucleon scattering cross section is about a factor of 3 larger between unlike nucleons than between like ones. On the other hand, it does not follow automatically that this factor should apply inside a nucleus, and in particular there is no *a priori* reason why it should apply to *residual* interactions inside the nucleus. These questions are considered below.

A. Full two-body interactions in nuclei

With regard to full two-body interactions in real nuclei, the nuclear matter calculations of Kikuchi and Kawai,[14] which include the effects of the Pauli exclusion principle, indicate that the difference in the free scattering cross sections should appear also in the cross sections for scattering inside the nucleus.

A more detailed look in which angular momentum and isospin coupling restrictions are considered can be obtained by examining the centroid shifts of particle-particle, particle-hole, and hole-hole multiplets of states

outside of a closed shell configuration. These multiplets result from the coupling of two particles with single particle angular momenta j_1 and j_2. Because the terminology used by spectroscopists in discussing the multiplets has led to some confusion as to their nature, it is worth examining them in some detail.

As explained in Refs. 15 and 16 the centroid shift is the average energy of the multiplet relative to the energy it would have in the absence of any interaction between the two particles. For example, the nucleus ^{210}Bi has one proton and one neutron outside of a closed ^{208}Pb core. From the energies of the single proton states in ^{209}Bi and of the single neutron states in ^{209}Pb, it is possible to calculate the unperturbed energy of the configurations in ^{210}Bi having a proton in the j_1 single particle state and a neutron in the j_2 single particle state. This is the energy in the absence of any two-body interaction between the proton and neutron. But the interaction between the two particles exists and furthermore depends on the relative orientations of their angular momentum vectors so that the states with different spin J are no longer degenerate in energy and, in general, the average energy for the resulting multiplet of states is shifted from its unperturbed value.[15] This shift represents an estimate of the matrix elements for the *full* interaction between the two particles averaged over all possible orientations of their angular momentum vectors.

The confusion has arisen because this interaction is sometimes termed "residual" by nuclear spectroscopists. It is, however, residual only in the sense that it represents a small part of the total Hamiltonian of the system. The presence of the core generates the zero-body part while the interactions of individual valence particles with the core give the one-body part. The full interaction between the two valence particles gives the two-body part of the Hamiltonian. This division of the Hamiltonian is possible in shell model calculations because a relatively inert core of nucleons exists and the active particles can be identified. In statistical model calculations, on the other hand, such a division is not possible. Any of the particles may be active.

The usual approach in the exciton model is to leave the Hamiltonian suitably undefined and to assume that the main part of the two-body interaction between all $A(A-1)/2$ pairs of nucleons (where A is the mass number of the system) has gone into the potential well in which the assumed set of single particle states exists. The resulting configurations with varying numbers of particle and hole degrees of freedom are not true eigenstates of the system, and transitions between configurations are caused by residual *parts* of the two-body interactions. Thus it is not clear that one should ever expect the average effective residual matrix element used in exciton model calculations to be the same as that extracted from multiplet centroid analyses (as Dobeš and Běták[4] imply that one should) even if all of the important parameters were included in the statistical bookkeeping. Nor should their systematics necessarily be the same. The systematics of the residual matrix elements will depend on which parts of the full interaction are producing them.

Another way to look at things is to realize that the energy levels used in the multiplet centroid analysis are related to the diagonal matrix elements and the eigenvalues of the system, while the residual matrix elements occurring in exciton model transition rates are more like off-diagonal matrix elements. If the particle-hole states were really eigenstates of the system, then these matrix elements would be zero and there would be no transitions.

In spite of these differences and because so many people start with free nucleon-nucleon interactions for preequilibrium calculations, it may still be instructive to look at estimates of the full two-body interaction in nuclei.

To apply the results of multiplet centroid analysis to the problem of the relative matrix elements for proton-proton and proton-neutron interactions, it is necessary to take the isospin of the multiplets into consideration. Here the treatment used by Schiffer and True[17] in looking at a large body of data is most useful. To simplify matters, only data for multiplets for nonidentical orbits will be considered. In general it is assumed that (neglecting the Coulomb force) the same basic interaction is applicable for proton-proton, neutron-neutron, and proton-neutron interactions. The difference is that proton-neutron multiplets have mixed isospin (the T of the pair is 0 or 1) while multiplets involving like particles have only $T=1$ so that only the $T=1$ component of the interaction is operative. Schiffer and True[17] actually use this fact to separate the $T=0$ and $T=1$ components of the two-body interaction.

The data from their paper for all p-n multiplets and for nonidentical orbit $T=1$ (n-n or p-p) multiplets have been analyzed to extract centroid shifts. These have been supplemented in Fig. 2 by centroid shifts (or average two-

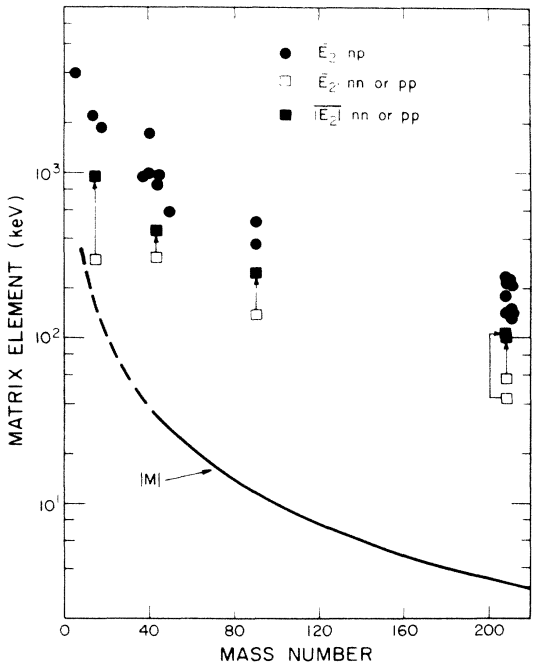

FIG. 2. Full two-body matrix elements derived by multipole centroid analysis of multiplets of states. The symbols indicate whether the matrix element is for like or unlike nucleons. The curve shows the effective residual rms matrix elements provisionally adopted in this work.

body matrix elements) for p-n multiplets from Refs. 15 and 18. Indeed, while there is significant scatter, the matrix elements for the like particle, $T=1$ case, shown as open squares, tend to fall well below the values for the corresponding p-n multiplets. The factors range from 2.7 to 7.6 corresponding to factors of 7.3 to 58 in M^2, much larger than the factors found in the free nucleon-nucleon scattering cross sections.

This may, however, be somewhat misleading since the $T=0$ matrix elements for individual spin members of a given multiplet always have the same sign while the $T=1$ matrix elements will typically have both signs and will largely cancel out in computing the centroid shift. Using the absolute value of the matrix elements for the individual J to compute the average for the $T=1$ multiplets yields M values that are factors of 1.5 to 2.4 below the values for the p-n multiplets corresponding to factors of 2.25 to 5.8 in M^2, much closer to the factor of 3 seen in the free nucleon-nucleon scattering cross section.

Thus there seems to be some evidence that the full two-body matrix elements for interactions inside the nucleus should be smaller between like nucleons (two protons or two neutrons) than between a proton and a neutron, but exactly how much smaller is hard to estimate. A factor of 3 is probably not a bad estimate.

B. Residual two-body interactions in nuclei

What about the case of the *residual* two-body interactions used in the exciton model? For formation and annihilation of a particle-hole pair, distinguishability alone reduces the interaction rate between like particles by a factor of about 2, quite apart from any difference in the residual matrix element. The main question with regard to M^2 is to what extent the parts of the full two-body interaction which are isospin dependent have been included in the potential well which generates the single particle states. This question cannot be answered *a priori*.

It is possible that some guidance can be obtained from the imaginary part of the optical potential, which determines the damping width of the initial $p,h=1,0$ (or target plus projectile) configuration. On the other hand, these widths include the creation of both neutron and proton particle-hole pairs and thus the effects of both n-p and p-p (or n-n) interactions. This fact combined with a fair amount of diversity in the form of the imaginary optical potential makes the use of this potential for unraveling information on the relative matrix elements for residual interactions among different kinds of nucleons somewhat difficult.

In view of this uncertainty it seems reasonable to assume, at least initially, that all of the mean squared matrix elements are equal for the purposes of model calculations. In the analysis of data, the possibility that $M^2_{\pi\nu}$ may be larger than the other two should be kept in mind and the factor of 3 introduced if and only if it is required by a wide variety of data. This question will be dealt with in the second paper in this series.

C. Numerical value of M^2

Assuming, as mentioned above, that

$$M^2=M^2_{\pi\pi}=M^2_{\pi\nu}=M^2_{\nu\nu}\ , \tag{19}$$

it is still necessary to assign a numerical value to the mean square residual matrix element.

The functional dependence of M^2 used here is a simplified reparametrization of the results of Ref. 19 and has the form

$$M^2(n,E)=K_A A^{-3}(e+20.9\ \text{MeV})^{-3}\ , \tag{20}$$

where

$$e=E/n \tag{21}$$

is the average excitation energy per degree of freedom. The functional form used in Ref. 19 was slightly different in each of four domains of the variable e. Converting to the present form requires that the normalization constant K from that work be multiplied by 3×10^4 to get K_A. Equation (20) then agrees with the earlier results to within 7% for 1 MeV $\leq e \leq$ 32 MeV and can probably be better extrapolated to higher e values. (The results of Ref. 19 would very likely require a fifth domain of e to be defined.) For the e values important in the early stages of most preequilibrium reactions (5—30 MeV) the agreement is no worse than 4%.

In the one-component exciton model a value of $K=135$ MeV^3 was used. However, $\omega_{\pi+}+\omega_{\nu+}$ in the present two-component model is roughly $\frac{3}{8}$ of ω_+ in the one-component model if $M^2_{\pi\pi}=M^2_{\pi\nu}$. This suggests that to retain the same normalization of the interaction rates relative to the particle emission rates (and thus retain the same intensity for the calculated preequilibrium emission spectra), K should be multiplied by $\frac{8}{3}$ and take on the value $K=360$ MeV^3. In the simplified reparametrization of M^2 used here corresponds to

$$K_A=1.08\times10^6\ \text{MeV}^5\ . \tag{22}$$

This is adopted as a provisional value only, because the division of strength between the proton and neutron emission channels in the present work is different from that in the one-component model used to set the value of K. Possible adjustments to K_A based on comparisons with data will be discussed in paper II. The matrix elements of Eqs. (20)—(22) for $E/n=1.5$ MeV are also shown in Fig. 2.

IV. REACTION CALCULATIONS

The primary quantity calculated in the exciton model is the angle integrated energy spectrum of particles emitted from the initial composite nucleus. The question of angular distributions (or double differential cross sections) is discussed in Sec. V, and extensions to multiple particle emission have been made even in the two-component model,[4] but this paper focuses on the energy spectrum for the first emitted particle. The preequilibrium component to this spectrum for a particle of type b is given by

$$\left[\frac{d\sigma_b(\epsilon)}{d\epsilon}\right]_{\text{pre}} = \sigma_a(\epsilon_a)\sum_p \sum_{p_\pi} S_{\text{pre}}(p_\pi,h_\pi,p_\nu,h_\nu) \times W_b(p_\pi,h_\pi,p_\nu,h_\nu,\epsilon) , \quad (23)$$

where σ_a is the cross section for forming the composite nucleus when a particle of type a is incident at energy ϵ_a. The quantity S_{pre} is the time-integrated strength passing through the specified set of configurations during the preequilibrium phase of the reaction. The equilibrium component can then be evaluated using the traditional compound nucleus model or from the equilibrium occupation probabilities in the exciton model. Alternatively, if S is evaluated over the entire course of the reaction (i.e., for both the preequilibrium and equilibrium phases) the result is S_{tot}, and an equation analogous to Eq. (23) gives the total reaction cross section.

There are three basic ways in which S has been evaluated in the one-component exciton model. Their advantages and disadvantages, especially with regard to the two-component model, are discussed below.

A. Numerical solutions to the master equations

The master or flow equations describing the equilibration of the composite nucleus are a straightforward extension of their one-component counterparts. They are[7]

$$\begin{aligned}\frac{dP(p_\pi,h_\pi,p_\nu,h_\nu,t)}{dt} &= P(p_\pi+1,h_\pi+1,p_\nu,h_\nu,t)\lambda_{\pi-}(p_\pi+1,h_\pi+1,p_\nu,h_\nu) + P(p_\pi,h_\pi,p_\nu+1,h_\nu+1,t)\lambda_{\nu-}(p_\pi,h_\pi,p_\nu+1,h_\nu+1) \\ &+ P(p_\pi-1,h_\pi-1,p_\nu,h_\nu,t)\lambda_{\pi+}(p_\pi-1,h_\pi-1,p_\nu,h_\nu) + P(p_\pi,h_\pi,p_\nu-1,h_\nu-1,t)\lambda_{\nu+}(p_\pi,h_\pi,p_\nu-1,h_\nu-1) \\ &+ P(p_\pi+1,h_\pi+1,p_\nu-1,h_\nu-1,t)\lambda_{\pi\nu}(p_\pi+1,h_\pi+1,p_\nu-1,h_\nu-1) \\ &+ P(p_\pi-1,h_\pi-1,p_\nu+1,h_\nu+1,t)\lambda_{\nu\pi}(p_\pi-1,h_\pi-1,p_\nu+1,h_\nu+1) - P(p_\pi,h_\pi,p_\nu,h_\nu,t)/\tau(p_\pi,h_\pi,p_\nu,h_\nu) , \end{aligned} \quad (24)$$

where τ is the lifetime of the specified class of states for all six types of residual interaction plus particle emission. The occupation probabilities P give the probability of finding the system in the specified configuration at time t.

Once the master equations have been solved numerically starting from some specified set of initial conditions and continuing to time t_{eq} at which equilibrium can be said to have been reached, S_{pre} is evaluated as

$$S_{\text{pre}}(p_\pi,h_\pi,p_\nu,h_\nu) = \int_0^{t_{\text{eq}}} P(p_\pi,h_\pi,p_\nu,h_\nu,t)dt . \quad (25)$$

One advantage of numerical integration of the master equations is that the time evolution of the system can be studied. This is primarily useful for pedagogical studies but is not an advantage in routine calculations for comparisons with data or for practical applications of the model. The main advantage in these cases is that all of the residual interactions have been fully taken into account.

The principal disadvantage is the computational time required. For the simple one-component model where each equation is coupled to only two others and the total number of equations is of the order $p_{\max}=(2gE)^{1/2}$, the problem is quite tractable and has been frequently used beginning in 1971.[2] Indeed, in this case much of the computation time is used in calculating the two-body transition rates and particle emission rates. The method has, however, not been applied in the two-component model where there are of order $p_{\max}^2/2=gE$ equations each coupled to six others. Because of this increased complexity, the numerical solution of the master equations is not used in this work either.

B. Solutions of the time-integrated master equations

Since the quantities of interest for practical reaction calculations are not the occupation probabilities themselves but rather their time integral, a number of approaches in the one-component model involve solving the time integrated master equations directly. Both exact[20,21] and approximate[22] solutions have been given in the literature. The latter approach is the one carried over by Dobeš and Běták[4] into the two-component model.

The advantage to these methods as opposed to the numerical integration of the original master equations is that they are computationally somewhat faster, for which one pays the price of not following the time evolution of the system. Of course, as with the numerical solution, all of the transition types are considered exactly (at least for the exact solutions).

The feature of this approach that has been proposed as an advantage over the closed form results discussed below is that the preequilibrium and equilibrium parts of the reaction are treated together and the time integral is typically taken from zero (the time the projectile strikes the target) to infinity. (In the numerical integration discussed above, the equilibrium component may be taken either from the equilibrium limit of the exciton model or from evaporation models.) Pedagogically it is an advantage to consider all the phases of the reaction with a single formalism. But greater consistency in the calculations does not necessarily mean better physics. There are features and approximations in the exciton model which make it less suitable in the equilibrium limit than the more conventional Weisskopf-Ewing evaporation model.

One of the most important of these relates to the state densities used to calculate particle emission rates. For the

exciton model emission rates to be consistent with the evaporation model in the equilibrium limit, the particle-hole state densities summed over all accessible configurations should equal (or at least be proportional to) the corresponding total state densities.[2] As has been pointed out,[13] one of the difficulties is that in the exciton model it is assumed that particle and hole degrees of freedom are always created in pairs so that the quantity p-h is a constant of the nucleus considered. (In the two-component model p_π-h_π and p_ν-h_ν are both constant.) In reality, however, passive particles and holes can be either created or destroyed in the two-body interactions so that p-h is not a constant of the system. For example, in a pair annihilation interaction a particle degree of freedom may "annihilate" a passive hole, giving up its energy to another degree of freedom so that p is decreased by one unit in the interaction but h is not. Thus in the equilibrium limit all states of the composite nucleus should be populated, regardless of their value of p-h.

For the case where $p=h$ there is no problem. The state density $\omega(p,h=p,E)$ contains all states at excitation energy E which have p particle-hole pairs regardless of how many of the particles and holes are degrees of freedom and how many are passive. Thus the sum of $\omega(p,h=p,E)$ over p includes all states of the system and, as Williams demonstrated,[5] is, to a good approximation, consistent with the usual one-Fermi-gas state density frequently used in evaporation calculations. Thus

$$\sum_p \omega(p,h=p,E) \approx \frac{\exp[2(\pi^2 gE/6)^{1/2}]}{(48)^{1/2}E} . \tag{26}$$

On the other hand, when $p \neq h$, the near equivalence of Eq. (26) disappears because not all of the particle-hole states have been counted. Consider the case where $p > h$. The quantity $\omega(p,h,E)$ now includes only those states at excitation energy E and with p particle-hole pairs which have at least p-h passive holes. Thus in general it is and should be true that

$$\sum_p \omega(p,h\neq p,E) < \frac{\exp[2(\pi^2 gE/6)^{1/2}]}{(48)^{1/2}E} . \tag{27}$$

The size of this effect is illustrated for an extreme case in Table II. The summed state densities for both the (incorrect) Williams expression for the Pauli correction function and the particle-hole symmetric expression are given. These corrections are the one-component analogs of Eqs. (2) and (3)–(5), respectively.

These results imply that if the state densities in the exciton model always had $p=h$ then the equilibrium emission rates would be roughly consistent with what is used in the evaporation model. But this condition is not fulfilled. In general, the composite nucleus starts out with p greater than h by the number of nucleons in the projectile so that $p=h+A_a$ where A_a is the mass number of the projectile. In the residual nuclei typically $p=h+A_a-A_b$ where A_b is the mass number of the emitted particle. Thus in the early stages of a reaction, the composite nucleus and the residual nuclei will generally have different p-h values and residual nuclei populated by nucleon emission will have a different value than those populated by complex particle emission. Because it is assumed in the exciton model that p-h is constant with time, the relative equilibrium emission rates for different mass particles in the exciton model are not consistent with those from the Weisskopf-Ewing evaporation model.[13] (Considering cluster emission for complex particles in the exciton model so that only one degree of freedom is lost during emission does not solve the problem because the "single particle" state density used for the emitted particle must now be that for the cluster states, thus introducing a different inconsistency with the evaporation model.)

The difficulty is that the assumption of a constant p-h is probably a reasonable one in the early stages of the equilibration process, but as more interactions occur, the probability that passive particles or holes can become degrees of freedom or that newly created particles and holes may be passive increases. Until and unless this sort of effect is taken into account in the exciton model, it seems preferable to restrict that model to the preequilibrium phase of the reaction and to use a compound nucleus model to treat the equilibrium phase. For this reason it was decided not to pursue the approach of the time integrated master equations in the two-component exciton model. Efforts are currently underway,[23] however, to unify the preequilibrium and equilibrium models.

It should be pointed out that there has also been proposed[24,25] a related matrix method for solving the master equations themselves. This method relies[24] on the specific tridiagonal nature of the matrix relating the set of time derivatives of the occupation probabilities to the occupation probabilities themselves. It, in principle, can yield occupation probabilities at any arbitrary time following the onset of the reaction process but in practice has generally been used to get the time integrated spectra for the entire reaction including the equilibrium phase. It is more awkward to use if only the preequilibrium component is to be obtained because the time at which equilibrium can

TABLE II. Comparison of summed particle-hole state densities with total state densities for $gE=20$.

	$\sum_p \omega(p,h,E)/g$			
	Williams		Kalbach	
$\lvert p-h \rvert$	$p \geq h$	$p \leq h$		$\omega(E)/g$[a]
0	671	671	671	692
1	529	674	311	
2	327	539	116	

[a] $\omega(E)=\frac{\exp[2(\pi^2 gE/6)^{1/2}]}{(48)^{1/2}gE}$.

be said to have been achieved must be defined. Further its application in the two-component model is hindered by the increased dimensionality of the problem.

C. Closed form reaction equations

The third and original[1] method for reaction calculations is what is known as the closed form method. It is used exclusively for the preequilibrium part of the reaction and utilizes the fact that in the early stages (where most of the preequilibrium emission occurs) pair creation rates are much larger than pair annihilation rates. It is less precise than using the master equations, but the level of approximation is consistent with other approximations in the model (e.g., using an equispacing model or neglecting angular momentum) and the calculations are quite simple.

In the one-component exciton model, it is assumed that the strength of the reaction passes through a single hierarchy of states (as shown in Fig. 1) in the "never come back" approximation in which pair annihilation interactions are ignored. In this case $S_{\rm pre}(p,h)$ can be approximated by the average lifetime $\tau(p,h)$ of the configuration with an extra factor to reduce the strength according to the amount which has already been lost to particle emission from simpler states. Either the full lifetime (considering pair creation, pair annihilation, and particle emission) or a partial lifetime which neglects pair annihilation can be used.

In the present two-component model, exciton scattering interactions must be considered. The reaction is then envisioned to occur as a series of pair creation interactions with the possibility of converting proton particle-hole pairs to neutron pairs or *vice versa* after each pair creation. Pair annihilation is neglected.

A quantity similar to the occupation probabilities is defined. It is denoted $P_1(p,p_\pi)$ and represents the strength populating the specified set of states by pair creation from states with $p-1$ particle degrees of freedom. To simplify the notation, configurations are here denoted by the quantities p and p_π. The total strength which passes through these configurations is greater than P_1, however, because the same states can also be populated by exchange interactions. The total strength passing through a given set of configurations is thus denoted by $P_2(p,p_\pi)$.

The reaction is assumed to start with the population of states containing Z_a protons, N_a neutrons, and one more particle-hole pair (either a proton or a neutron pair). Here Z_a and N_a are the proton and neutron numbers of the projectile. The relative probabilities of exciting a proton or a neutron pair in the initial target-projectile interaction are determined by the pair creation rates for the target plus projectile configuration. Thus the initial strengths are assumed to be

$$P_1(A_a+1,Z_a) = \frac{\lambda_{\nu+}(Z_a,0,N_a,0,E)}{\lambda_{\nu+}(Z_a,0,N_a,0,E)+\lambda_{\pi+}(Z_a,0,N_a,0,E)} , \tag{28a}$$

$$P_1(A_a+1,Z_a+1)=1-P_1(A_a+1,Z_a) . \tag{28b}$$

In the case where any arbitrary number of exchange interactions can occur before the strength leaves the states with $p=A_a+1$, the total strength passing through the initial configurations is

$$P_2(A_a+1,Z_a)=[P_1(A_a+1,Z_a)+P_1(A_a+1,Z_a+1)\Gamma_{\pi\nu}(A_a+1,Z_a+1)]\sum_{i=0}^{\infty}[\Gamma_{\nu\pi}(A_a+1,Z_a)\Gamma_{\pi\nu}(A_a+1,Z_a+1)]^i , \tag{29}$$

where the terms proportional to $P_1(A_a+1,Z_a)$ have an even number of exchange interactions and those proportional to $P_1(A_a+1,Z_a+1)$ have an odd number. There is an analogous expression for $P_2(A_a+1,Z_a+1)$. The quantities $\Gamma_{\pi\nu}$ and $\Gamma_{\nu\pi}$ are branching ratios for exchange interactions. These and the corresponding quantities for pair creation are defined as

$$\Gamma_{\pi\nu}(p,p_\pi)=\lambda_{\nu\pi}(p,p_\pi)\tau(p,p_\pi) , \tag{30a}$$

$$\Gamma_{\pi\nu}(p,p_\pi)=\lambda_{\pi\nu}(p,p_\pi)\tau(p,p_\pi) , \tag{30b}$$

$$\Gamma_{\nu+}(p,p_\pi)=\lambda_{\nu+}(p,p_\pi)\tau(p,p_\pi) , \tag{30c}$$

$$\Gamma_{\pi+}(p,p_\pi)=\lambda_{\pi+}(p,p_\pi)\tau(p,p_\pi) , \tag{30d}$$

$$\tau(p,p_\pi)=\left[\lambda_{\nu+}(p,p_\pi)+\lambda_{\pi+}(p,p_\pi)+\lambda_{\nu\pi}(p,p_\pi)+\lambda_{\pi\nu}(p,p_\pi)+\sum_b\int W_b(p,p_\pi,\epsilon)d\epsilon\right]^{-1} . \tag{31}$$

For the simple states which are populated early in the reaction and from which most of the preequilibrium particle emission occurs, the branching ratios for the exchange interactions are fairly small, and it is sufficient to retain only the leading $i=0$ term in the sum of Eq. (29). This approximation says that there will be no more than one exchange interaction before particle emission or pair creation occurs. Thus strength that arrives at states with a specified p,p_π through pair creation can undergo exchange, pair creation, or particle emission and is characterized by the branching ratios and lifetime given by Eqs. (30) and (31). On the other hand, strength arriving by exchange from states specified by $p,p_\pi-1$ or $p,p_\pi+1$ only has the choice between pair creation and particle emission in this approximation. The appropriate lifetime to use for this strength is

$$\tau'(p,p_\pi)=\left[\gamma_{\nu+}(p,p_\pi)+\gamma_{\pi+}(p,p_\pi)+\sum_b\int W_b(p,p_\pi,\epsilon)d\epsilon\right]^{-1} \tag{32}$$

and the branching ratios are defined analogously to Eq. (30) but with τ' replacing τ. In order to simplify later equations it is useful to define

$$L(p,p_\pi)=\tau'(p,p_\pi)/\tau(p,p_\pi) . \tag{33}$$

In the one-exchange approximation with the initial con-

ditions of Eq. (28), Eq. (29) becomes approximated as

$$P_2(A_a+1,Z_a)=P_1(A_a+1,Z_a)+P_1(A_a+1,Z_a+1)\times\Gamma_{\pi\nu}(A_a+1,Z_a+1)L(A_a+1,Z_a)\ , \tag{34}$$

where the factor L, which is greater than unity, helps to correct for the effects of multiple exchange processes. In general the strength passing through states specified by p,p_π is

$$P_2(p,p_\pi)=P_1(p,p_\pi)+[P_1(p,p_\pi-1)\Gamma_{\nu\pi}(p,p_\pi-1)+P_1(p,p_\pi+1)\Gamma_{\pi\nu}(p,p_\pi+1)]L(p,p_\pi) \tag{35}$$

and the strength arriving at states specified by $p+1,p_\pi$ by pair creation is

$$P_1(p+1,p_\pi)=P_2(p,p_\pi-1)\Gamma_{\pi+}(p,p_\pi-1)+P_2(p,p_\pi)\Gamma_{\nu+}(p,p_\pi)\ . \tag{36}$$

Finally, the time integrated strengths for Eq. (23) are given by

$$S_{\rm pre}(p,p_\pi)=P_2(p,p_\pi)\tau(p,p_\pi)\ . \tag{37}$$

The equilibrium component needed to supplement Eq. (23) is given by the evaporation model expression

$$\left[\frac{d\sigma_b(\epsilon)}{d\epsilon}\right]_{\rm eq}=\sigma_{a,{\rm eq}}(\epsilon_a)\frac{W_b(\epsilon)}{\sum_b\int W_b(\epsilon)d\epsilon}\ , \tag{38}$$

$$W_b(\epsilon)=\frac{(2s_b+1)}{\pi^2\hbar^3}\mu_b\epsilon\sigma_b(\epsilon)\frac{\omega(E-B_b-\epsilon)}{\omega(E)}\ . \tag{39}$$

In Eq. (38) the quantity $\sigma_{a,{\rm eq}}$ is that part of the initial reaction cross section which did not involve preequilibrium emission of nucleons or alpha particles. The state densities in Eq. (39) can be evaluated either for a single Fermi gas, as in the right-hand side of Eq. (26), for consistency with calculations in the one-component exciton model, or, more appropriately, they can be evaluated using the corresponding two-Fermi-gas expression

$$\omega(E)=\frac{\exp[2(aE)^{1/2}]}{12\pi^{-1/2}a^{1/4}E^{5/4}}\ , \tag{40}$$

where the level density parameter is given by $a=\pi^2(g_\pi+g_\nu)/6$.

V. NUMERICAL RESULTS

A. Model code and input

The formalism outlined in the previous three sections has been coded into the computer program PRECO-E with the provisional assumptions that the matrix elements for proton-proton, proton-neutron, and neutron-neutron interactions are equal and have a normalization constant given by Eq. (22). Particle numbers up to the most probable equilibrium value $\bar{p}$ (or up to 20 if $\bar{p}$ exceeds 20) are considered in the calculations. Additionally, the sum over p in Eq. (23) is terminated if the total strength passing through states with a given number of particle degrees of freedom falls below 0.001. For each value of p, all possible values of p_π from Z_a to $p-N_a$ are considered. The one-Fermi-gas state density given on the right-hand side of Eqs. (26) and (27) is temporarily used in calculating the equilibrium components in order to facilitate comparison with the one-component exciton model. Emission of more than one particle is not considered in this code.

The program PRECO-E also includes two important features of PRECO-D (Ref. 26) (the corresponding one-component exciton model code). One is the calculation of angular distributions using the Kalbach and Mann systematics[27,12] but with the multistep direct (MSD) cross section here approximated by the preequilibrium cross section. The second is the calculation of direct reaction components such as stripping, pickup, and knockout for reactions involving complex particles. Provisions in this code for making pairing and shell corrections will be discussed in papers II and III in this series.

For the purposes of the present work, only (N,N) reactions are considered, but alpha particle emission is automatically calculated in the program along with neutron and proton emission since these are usually the dominant exit channels of a reaction.

Binding energies for the calculations have been taken from Ref. 28. The neutron and proton reaction cross sections used were those of Mani *et al.*[29] while the alpha particle reaction cross sections were those of Huizenga and Igo.[30] These are the choices used in correpsponding one-component exciton model calculations. The question of an optimal choice of optical model reaction cross sections is discussed in paper II in this series. An option in the code allows the reaction and inverse cross sections to be either read in as input or generated internally using the approximation of Chatterjee *et al.*[31] and Murthy *et al.*[32] This approximation is good to about 5% except in the region of the Coulomb barrier for charged particles or below 3 MeV for neutrons. Since the agreement between the approximation and the exact optical model calculations for a given optical potential is very much better than the agreement between reaction cross sections obtained with different optical potentials, the approximation has been used in the calculations presented here. The single particle state densities have been assumed to be $g_\pi=Z/(13\ {\rm MeV})$ and $g_\nu=N/(13\ {\rm MeV})$ corresponding to $g=A/(13\ {\rm MeV})$ in the one-component calculations. A potential well depth of $V(h)=38$ MeV is assumed for states with one- or two-hole degrees of freedom. For more complex states no finite well depth correction is made.

B. State densities and transition rates

The particle-hole state densities given by Eqs. (3)–(7) are shown in Fig. 3 for the composite nucleus formed by 29 MeV protons incident on ^{54}Fe. Here the excitation energy of the composite nucleus is $E=33.5$ MeV. The value of $\bar{p}_\pi$ for a given number p of particle degrees of freedom increases steadily with increasing p and remains close to $p/2$. Further, the summed state density increases most rapidly with p for small numbers of degrees of freedom

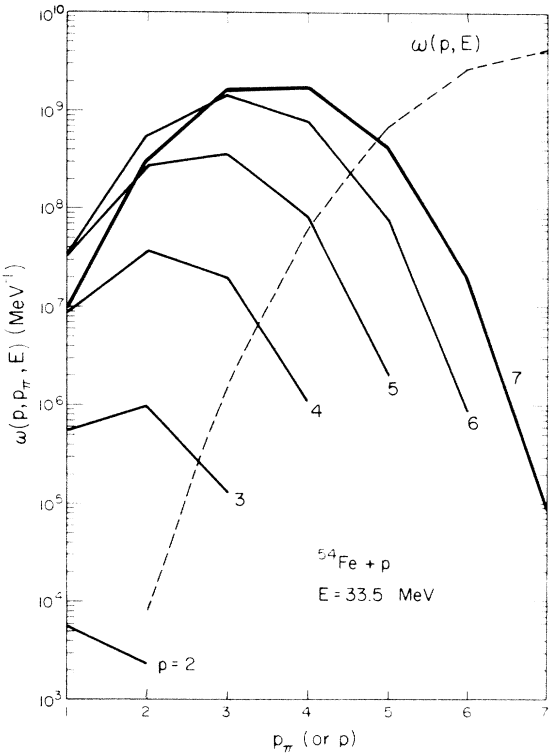

FIG. 3. Particle-hole state densities in the two-component exciton model for a sample system. The solid lines connect values of $\omega(p,p_\pi,E)$ for a given value of p. The results for $p=\bar{p}=7$ are shown with a heavier line. The dotted line shows $\sum_{p_\pi}\omega(p,p_\pi,E)$ as a function of p.

and levels out for higher p, as is expected. In this calculation, $\bar{p}$ has a value of 7 and the value of

$$\omega(p,E)=\sum_{p_\pi}\omega(p,p_\pi,E)$$

decreases for $p>7$.

The transition rates for the same system are shown in Fig. 4. It is seen that for a fixed number of degrees of freedom, the rate of creating a proton particle-hole pair or of converting a neutron pair to a proton pair is greatest for the smallest values of p_π and decreases steadily as p_π increases. The opposite is true for the rates for creating a neutron pair or for transforming a proton pair into a neutron pair. The net effect, then, is for the strength of the reaction to tend to go into the classes of states which have the highest state densities, i.e., those which have $p_\pi \approx p_\nu$.

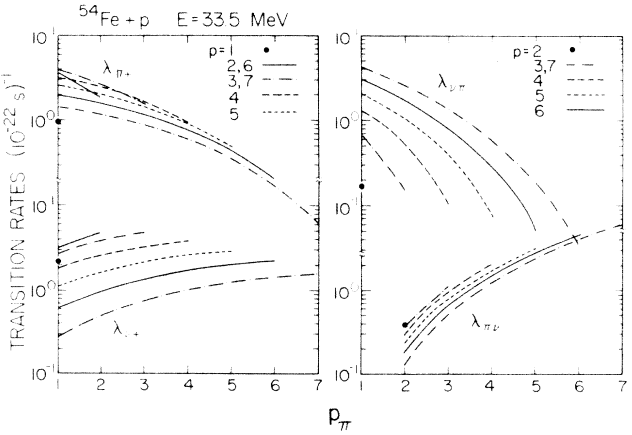

FIG. 4. Transition rates for the residual two-body interactions in a simple system. The curves connect rates for initial states having the same value of p. Pair creation rates for $p=1$ and scattering rates for $p=2$ are shown as points.

As is expected from the one-component exciton model, the average rate for pair creation for states with a given number of degrees of freedom (averaged over all proton-neutron configurations) decreases as p or $n=p+h$ increases. The exchange rates $\lambda_{\nu\pi}$ and $\lambda_{\pi\nu}$ are smaller than the corresponding pair creation rates for the states with low values of p which are populated early in the reaction but become comparable to the pair creation rates for $p\approx\bar{p}$. Once again the effect is to push the system towards those classes of states with the highest state densities and therefore the greatest probability of being populated at equilibrium.

C. Test of the equal *a priori* probabilities assumption

In deriving particle emission rates in the one-component exciton model, it is assumed that all states with the same numbers of particle and hole degrees of freedom are equally likely to be populated at any stage of the reaction. This was verified in a very limited way[33] for the division between closed and open configurations. The

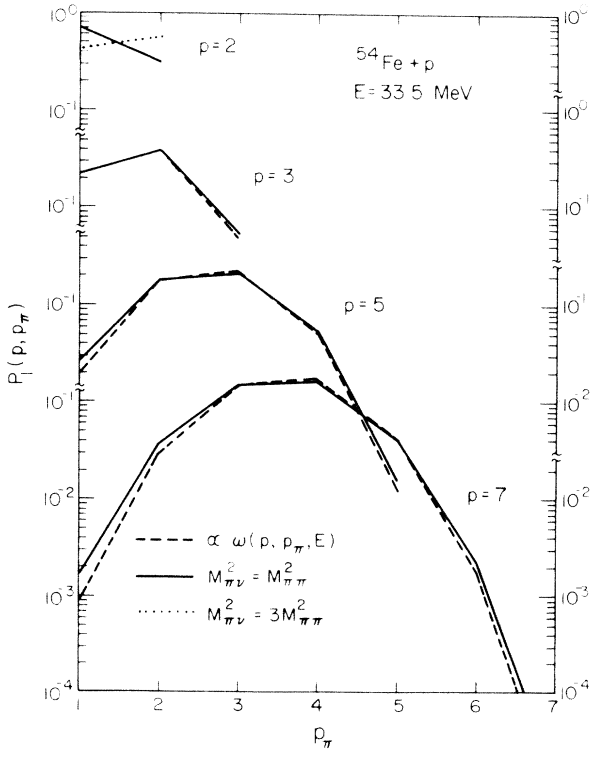

FIG. 5. Test of the equal probabilities assumption in a sample system. The dotted curves are shown only for $p=2$. The solid curves are the results from PRECO-E while the dashed curves are what the results should be if the assumption were exact.

two-component model provides the ability to check the assumption in more detail by investigating the distribution of strength among different proton-neutron configurations having the same number of particle and hole degrees of freedom.

Figure 5 shows the values of $P_1(p,p_\pi)$ for the same reaction system used earlier in this section (^{54}Fe + p at $E=33.5$ MeV). These are compared with the corresponding state densities $\omega(p,p_\pi,E)$ which have been normalized at each value of p to produce the same sum over p_π as $P_1(p,p_\pi)$. It can be seen that the agreement is excellent, particularly near the peaks of the distributions, although at higher p values the P_1 distribution tends to be systematically slightly broader than the ω distribution.

For $p=2$, the dotted line shows the P_1 distribution which would result from assuming that $M^2_{\pi\nu}=3M^2_{\pi\pi}=3M^2_{\nu\nu}$. This is roughly the assumption made by Dobeš and Běták[4] and is clearly in disagreement with the equal *a priori* probabilities assumption as applied to all states of a given p (as is done in the one-component exciton model). This does not necessarily mean that an enhanced rate for proton-neutron interactions as compared to the rates for interactions between like nucleons is wrong, but the inconsistency of such an enhancement with the equal probabilities assumption used in deriving emission rates must be kept in mind.

D. Reaction strengths and the importance of $\Delta n=0$ transitions

The increasing importance of the $\Delta n=0$ transition as p increases has already been noted. Here the size of their effect on the time integrated strength $S_{\rm pre}$ is examined.

For each p in the sample system being studied, Table III compares the sums over p_π of $P_1(p,p_\pi)$ and $P_2(p,p_\pi)$. The difference between them is the amount of strength which has undergone an exchange interaction at that value of p. These quantities are very similar for low values of p indicating that the exchange interactions are relatively unimportant and that the assumption of at most one exchange at each p value is probably a very good one. On the other hand, as p approaches $\bar{p}=7$, the presence of the exchange interactions in the one-exchange approximation doubles, on the average, the amount of strength passing through a given configuration. The one-exchange approximation is thus no longer very good. This situation is mitigated both by the use of the longer lifetimes τ' to treat strength for which no further exchange interactions are allowed in states with a given number of degrees of freedom and also by the fact that most of the preequilibrium particle emission occurs from the very simple states where the approximation is good. Thus the particle emission spectra which are calculated are probably fairly accurate.

The decrease in the sum over $P_1(p,p_\pi)$ with increasing p seen in Table III is due to the loss of strength into particle emission channels.

TABLE III. Reaction strengths as a function of the number of particle degrees of freedom.

p	$\sum_{p_\pi} P_1(p,p_\pi)$	$\sum_{p_\pi} P_2(p,p_\pi)$	Ratio
2	1.00	1.02	1.02
3	0.66	0.71	1.08
4	0.54	0.63	1.17
5	0.47	0.63	1.34
6	0.43	0.70	1.63
7	0.40	0.86	2.17

E. Particle spectra in the one- and two-component exciton models

Figure 6 shows a comparison of preequilibrium components calculated with the present two-component exciton model and with the one-component model formalism of Refs. 12 and 33. Both the multistep direct and the full preequilibrium components are given for the one-component case. As was observed by Dobeš and Běták,[4] the main differences are in the relative intensities of the proton and neutron components. Here the shift occurs primarily in the exchange channel, while the inelastic spectra are virtually unchanged. (This could be altered by shifting the normalization of M^2.)

While Fig. 6 gives a fair comparison of preequilibrium spectral shapes, it is misleading about spectral intensities because the one- and two-component models used in the calculations contain different assumptions about the relative probabilities of exciting proton and neutron degrees of freedom in the reaction. This has been a point of some uncertainty in the exciton model.

In the one-component exciton model the basic particle

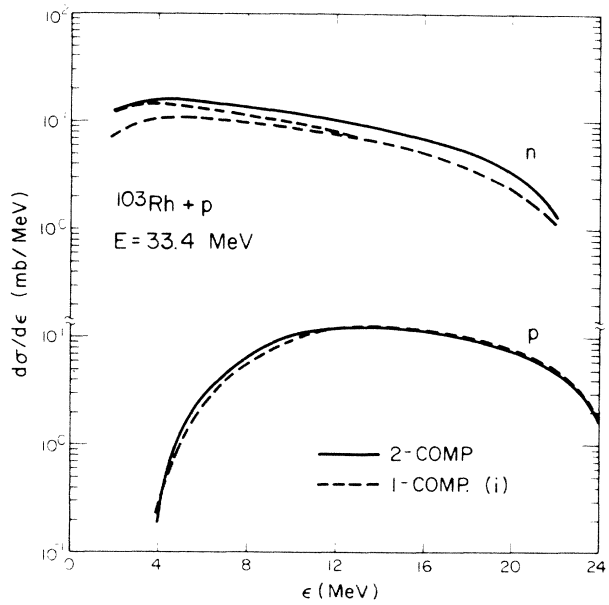

FIG. 6. Comparison of preequilibrium energy spectra in the one- and two-component exciton models. The solid curves are from the present two-component model while the dashed curves correspond to the one-component model where proton and neutron particle-hole pairs are assumed to be excited in proportion to the Z and N of the target [assumption (i) in the text]. The lower dashed curve for neutron emission shows the multistep direct part of the preequilibrium component.

emission rates are derived using the equal probabilities assumption. One of three assumptions is then made in order to modify the emission rates for proton-neutron distinguishability:

(i) The original assumption was that proton and neutron degrees of freedom are excited in proportion to the Z and N of the target.[34,13] This is the approach used in the code PRECO-D to generate Fig. 6. It has the difficulty that it is inconsistent with the equal probabilities assumption used in deriving the basic emission rates.

(ii) A second method which *is* consistent with the derivation of the basic emission rates is to assume that all proton-neutron configurations are populated in proportion to their states densities.[35]

(iii) The third method[36,37] has been principally used in the hybrid and geometry dependent hybrid models but is applicable in the exciton model as well. In it an incident nucleon is assumed to excite protons and neutrons in the initial target-projectile interaction in proportion to the corresponding free nucleon-nucleon scattering cross sections. In later pair creation interactions, protons and neutrons are considered to be excited with equal probabilities. Like (i), this assumption is inconsistent with the derivation of the basic emission rates.

Of these three methods, the first gives the highest relative yield in the inelastic channel while the third gives the lowest.

In the two-component exciton model the question of exciting proton versus neutron degrees of freedom is handled more directly. In each case there is a proportionality to the density of *accessible* final states and the appropriate matrix element normalization constant K. Assumptions (i)—(iii) above then correspond to different values

$$R_M = K_{\pi\nu}/K_{\pi\pi} = K_{\nu\pi}/K_{\nu\nu} \ .$$

Assumption (i) would mean in the two-component model that

$$K_{\pi\nu}\omega_{\pi\nu+}/K_{\pi\pi}\omega_{\pi\pi+} = N/Z \cong 1 \ .$$

Since the ratio of the transition state densities is roughly 2, the ratio of the K's is roughly $\frac{1}{2}$, which is also the value of R_M. Assumption (ii) above corresponds to $R_M \cong 1$, as assumed by Gupta[3] and as provisionally adopted here. Assumption (iii) seems to imply that

$$K_{\pi\nu}\omega_{\pi\nu+}/K_{\pi\pi}\omega_{\pi\pi+} \cong 3$$

or $R_M \cong \frac{3}{2}$ whereas the assumption in Ref. 4, also based on free nucleon-nucleon scattering cross sections, is that $R_M \cong 3$.

To best compare the effects of going from a one-component to a two-component exciton model, the assumption relating to proton-neutron distinguishability should be as similar as possible. Thus the fairly new option in PRECO-D of using the assumptions of Gadioli *et al.*[35] from (ii) above was employed. The results of these calculations and calculations done with the present two-component code PRECO-E are shown in Fig. 7. In each case the results from the one- and two-component

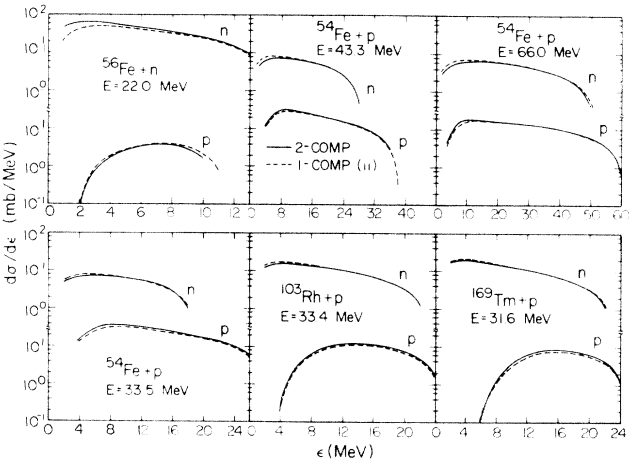

FIG. 7. Comparison of preequilibrium energy spectra in the one- and two-component exciton models. The solid curves are from the present two-component model while the dashed curves come from the one-component model modified to have proton and neutron particle-hole pairs excited in proportion to the state densities of the classes of states populated in the interactions [assumption (ii) in the text].

calculations agree extremely well. There is a tendency for the two-component model to give about 10% more cross section in the inelastic channel than the one-component model with similar assumptions. This difference disappears for excitation energies above about 40 MeV. A similar result can be seen from Ref. 4 where assumption (i) in the one-component model and $R_M = 1/2$ in the two-component model yielded nearly identical spectra.

Thus the one- and two-component exciton models yield essentially the same results if the same basic assumptions about the relative probabilities of exciting proton and neutron degrees of freedom are made. On the other hand, changing the assumptions within either model can have a significant effect on the relative yields. For the one-component model this is illustrated by comparing the results in Figs. 6 and 7 while for the two-component model it can be seen in the results of Ref. 4 where R_M is varied. This illustrates the need to find, in the two-component exciton model, the correct choice for the residual two-body matrix elements. This task is considered further in paper II.

F. Angular distributions in the one- and two-component models

Since the same formalism is used to describe the angular distributions in the one- and two-component versions of the computer code PRECO, the only differences in the shapes of the angular distributions calculated in the two programs is due to the fact that in the two-component model the multistep direct (MSD) cross section is approximated by the full preequilibrium cross section.

In the Kalbach and Mann systematics[27] the MSD component of the cross section has a forward peaked angular distribution while the multistep compound (MSC) component is assumed to have an angular distribution which is symmetric about 90° in the center of mass. The pre-

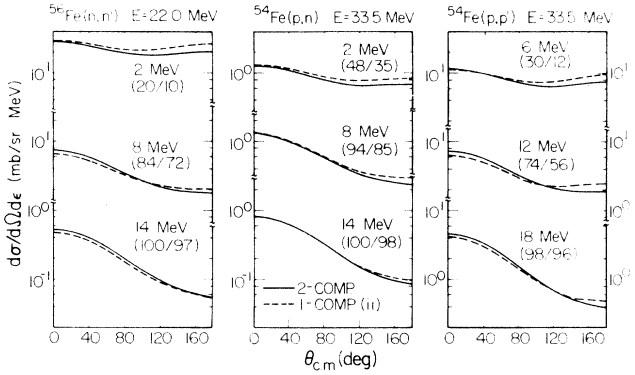

FIG. 8. Comparison of angular distributions in the one- and two-component exciton models. The solid and dashed curves have the same significance as in Fig. 7. The numbers next to each curve are the channel energy for the particle emission, the percent of the cross section which comes from preequilibrium emission, and the percent which is MSD. These curves include both preequilibrium and evaporation components, but multiple particle emission is not considered.

equilibrium component to an energy spectrum consists mainly of the MSD cross section but, particularly at lower emission energies, contains some MSC cross sections as well. Thus it should be expected that the angular distributions calculated in the two-component exciton model described here should be slightly more forward peaked than those from the one-component model, where MSD components are directly calculated.

Figure 8 shows some comparisons of one- and two-component model angular distributions for emission energies at which the percent preequilibrium emission is higher than the percent multistep direct. The differences are noticeable but not large in the context of the approximate nature of the systematics and the typical uncertainties present in the available data. At higher emission energies where the cross section is pure MSD, the differences, of course, disappear.

VI. SUMMARY AND CONCLUSIONS

A closed form version of the two-component exciton model has been developed here. The state densities used employ a more accurate Pauli correction than that used in earlier work, and the transition rates are shown to be consistent with a steady state equilibrium condition. A remaining difficulty is that the pair annihilation rates contain no corrections for the exclusion principle. A discussion of the effective residual two-body matrix elements in the context of full two-body matrix elements found in nuclei near closed shells is presented in an effort to understand the relative strengths for interactions between like and unlike nucleons. The assumption provisionally adopted is that the same residual M^2 should be used for proton-proton, proton-neutron, and neutron-neutron interactions.

A comparison has been made of the various methods which were developed in the one-component exciton model to describe the reaction process itself and to calculate the energy spectra of the emitted particles. Arguments for using the closed form approach in the two-component model have been presented, and the appropriate reaction equations are derived assuming that at most one $\Delta n=0$ transition will occur at each value of p in the course of the reaction.

Calculations with the two-component exciton model code PRECO-E demonstrate the following conclusions:

(1) The equal probabilities assumption used in deriving the one-component model emission rates is verified to a remarkable degree when M^2 is taken to be independent of the isospins of the interacting nucleons.

(2) Excellent agreement with the one-component exciton model results is obtained when similar assumptions are made about the relative probabilities of exciting proton and neutron particle-hole pairs. (Having different assumptions causes the predicted proton-neutron yield ratios to differ while the preequilibrium spectra are still relatively similar in shape.)

(3) The assumption of at most one exchange or $\Delta n=0$ interaction at each p value is a good one for the simple states from which most of the preequilibrium emission occurs. It becomes increasingly worse as p approaches $\bar{p}$, but this should have only a small effect on the calculated energy spectra.

(4) Because the MSD cross section is approximated by the preequilibrium cross section for a given reaction, the angular distributions in the two-component exciton model are generally slightly more forward peaked than those given in the one-component model where the MSD contribution is calculated directly. The differences are, however, not large and disappear at high emission energies where the cross section is pure MSD.

The two-component exciton model thus confirms much of the work with the earlier one-component model, and for simple calculations presents no significant advantage over the latter. It does, however, open new avenues for studying preequilibrium pairing and shell structure effects which should depend not only on the number of degrees of freedom present but on the specific proton-neutron configuration. New possibilities for studying the long standing questions of proton-neutron yield ratios are also made available. These are the subjects of the remaining papers in this series.

ACKNOWLEDGMENTS

This work was supported under the auspices of the U.S. Department of Energy for Los Alamos National Laboratory. The author would like to acknowledge the hospitality of the Duke University Department of Physics and the Triangle Universities Nuclear Laboratory. She is grateful to Dr. E. D. Arthur and Dr. F. M. Mann for their helpful comments on this manuscript.

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