Resonant $E2$ capture into the ^{8}Be ground state

K. Langanke* and C. Rolfs†
W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125
(Received 15 October 1985)

We have calculated the γ-transition strength from the broad 2^+ resonance in $\alpha+\alpha$ scattering at $E=2.9$ MeV into the ^{8}Be (0^+) ground state in a bremsstrahlung model and find a γ width of $\Gamma_\gamma=8.3$ meV.

We have studied the electromagnetic transition strength between the first excited 2^+ state in ^{8}Be at $E_x=2.94$ MeV and the ^{8}Be ground state (0^+), which is experimentally as well as theoretically unknown.[1] Since both states are well described as resonances in the elastic $\alpha+\alpha$ channel and can be reproduced within microscopically founded $\alpha+\alpha$ potential models[2] we have calculated the γ transition between these states on the basis of an $\alpha+\alpha$ potential model. However, to account for the resonant character of both states we have evaluated the transition via a bremsstrahlung formalism introduced by Tanimura and Mosel to study electromagnetic transitions between ^{12}C+^{12}C resonances.[3] As a first step our study will treat the two α particles as structureless fragments. We will then take into account the internal degrees of freedom of the α particles.

The γ transition under consideration is of the $E2$ type. Hence the total bremsstrahlung cross section from an $\alpha+\alpha$ scattering state at energy E in the partial wave l, denoted by $u_l(E)$, into a resonance at energy E' with angular momentum l' [wave function $u_{l'}(E')$] is given by[3]

$$\left[\frac{d\sigma}{dE_\gamma}\right](E)=\frac{4\pi^2e^2}{15k^2}(2l+1)^2(2l'+1)^2(l020\,|\,l'0)^2\left[\frac{E_\gamma}{\hbar c}\right]^5\left|W(2l'l0;ll')\int u_l(E)r^2u_{l'}(E')dr\right|^2. \tag{1}$$

Here $E_\gamma=E-E'$ is the energy of the emitted photon, while k is the wave number in the entrance channel. In (1) we have adopted the notation as well as the asymptotic normalization of the scattering states from Tanimura and Mosel:[3,4]

$$u_l(E)\underset{r\to\infty}{\longrightarrow}\left[\frac{2\mu}{\pi\hbar^2k}\right]^{1/2}e^{i\delta_l}[\cos\delta_lF_l(kr)+\sin\delta_lG_l(kr)]\ , \tag{2}$$

where F_l (G_l) is the regular (irregular) Coulomb function and δ_l is the nuclear phase shift.

We have derived the scattering wave functions $u_l(E)$ and $u_{l'}(E')$ needed in (1) by solving the Schrödinger equation of relative motion for the two α particles

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{l(l+1)\hbar^2}{2\mu r^2}+V(r)-E\right]u_l(r)=0 \tag{3}$$

in which we adopted the microscopically founded $\alpha+\alpha$ potential of Buck *et al.* as the nucleus-nucleus potential.[5] This potential fits the experimental $\alpha+\alpha$ scattering phase shifts nearly perfectly for $E\lesssim20$ MeV. For our study it is important to note that the model exhibits: (i) a very narrow resonant ^{8}Be ground state at $E=92.12$ keV with a width of $\Gamma=6$ eV which is in good agreement with the experimental values[1] of $E=92.12\pm0.05$ keV, and $\Gamma=6.8\pm1.7$ eV and (ii) a broad 2^+ resonance at $E=2.9$ MeV with a width of $\Gamma=1.4$ MeV (experimental values:[1] $E=2.94\pm0.03$ MeV, $\Gamma=1.56\pm0.03$ MeV).

The radial integral as defined in (1) will, of course, diverge for scattering states if evaluated numerically. To obtain a physically obvious convergence of the integral we introduced a radial shielding factor as it is used for the evaluation of dipole radiation.[6] Our numerical analysis of the integral followed closely the method discussed in Ref. 3 integrating the radial integral by parts and substituting the derivative of the scattering states by using the Schrödinger equation (3) and repeating this procedure.

We have calculated the bremsstrahlung cross section (1) as a function of final energy covering the energy range of the d wave $\alpha+\alpha$ resonance in the entrance channel. Typical results are plotted in Fig. 1 for $E=2.7$–3.5 MeV in the initial channel. The cross sections clearly exhibit the resonant behavior of the final states having a maximum at $E'=92.128$ keV which is the resonance energy of the ^{8}Be ground state as calculated with the potential of Ref. 5.

We define the total bremsstrahlung cross section into the ^{8}Be ground state by integrating (1) over all final energies

$$\sigma(E)=\int\left[\frac{d\sigma}{dE_\gamma}\right](E)dE_\gamma\ . \tag{4}$$

The results are given in Fig. 2. The cross section resembles roughly a resonant structure and has its maximum at $E=2.68$ MeV with a value of 14.3 nb. Approximating the $\sigma(E)$ cross sections near the resonance energy by a

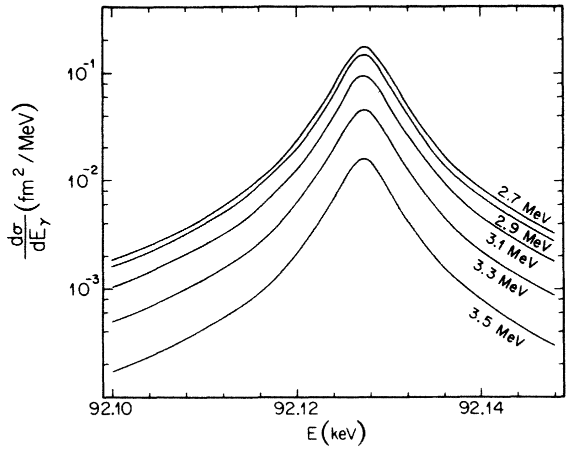

FIG. 1. The $E2$ bremsstrahlung cross section, $d\sigma/dE_\gamma$, to the (unbound) ^{8}Be ground state as a function of the final energy, E', for various energies in the entrance channel. Note that the energy scale E' spans about 45 eV.

Breit-Wigner parametrization we find the quotient $\Gamma_\gamma/\Gamma_{tot} \approx 5.9\times 10^{-8}$ equivalent to a γ width of 8.3 meV when adopting the elastic width $\Gamma = 1.4$ MeV as total width.

To estimate the effect of internal degrees of freedom of the α particles in the results given above we have repeated our study approximating the ^{8}Be states by antisymmetrized product wave functions

$$\Psi_l = \mathcal{A}\{\Phi_\alpha \Phi_\alpha g_l\} \tag{5}$$

which has been shown to be an adequate ansatz for all states relevant to the present investigation.[2] In (5), Φ_α describes the internal degrees of freedom of an α fragment by its harmonic oscillator shell model ground state with oscillator parameter[2] $b = 1.38$ fm. The relative wave functions g_l can be evaluated from a Schrödinger equation of relative motion

$$\Lambda_l \left\{ -\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V(r) - E \right\} h_l(r) = 0 \tag{6}$$

and the folding relation

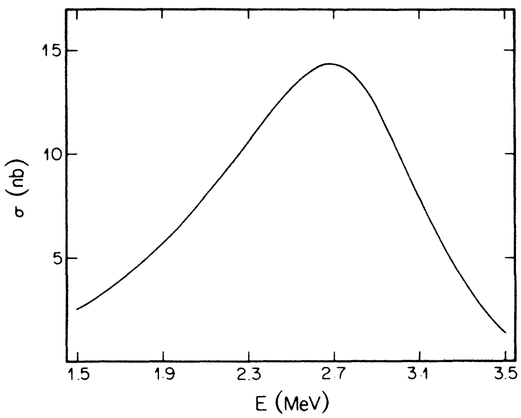

FIG. 2. Bremsstrahlung cross section integrated over the ^{8}Be ground state as a function of energy in the entrance channel, E.

$$g_l(r) = \sum_n \frac{1}{\sqrt{\mu_N}} \langle \phi_{Nl} \mid h_l \rangle \phi_{Nl}(r) \tag{7}$$

where the ϕ_{Nl} are radial harmonic wave functions with $N = 2n + l$ quanta excited and of width $\beta = b/\sqrt{\mu}$. The projector Λ_l in (6) which eliminates the Pauli forbidden states[7] in h_l and the normalization kernels μ_N which are analytically given in Ref. 8 ensure the proper orthonormalization of the many body states (5). Following Buck *et al.*[5] we approximated the nuclear part of the potential in (6) by an l independent local Gaussian form factor

$$V_N(r) = V_0 \exp\{-r^2/a^2\} \tag{8}$$

and adopt the Coulomb potential from Refs. 5 and 9. The parameters V_0 and a were determined by adjusting the phase shifts calculated from (6) to the experimental phase shifts[10] at $E \le 20$ MeV. Using $V_0 = -120.5172$ MeV and $a = 2.132$ fm we obtained a fit of comparable quality to the results of Ref. 5 both in excellent agreement with the experimental phase shifts.

When calculating the bremsstrahlung cross section for $E2$ radiation using the many-body wave functions (5) and a many-body quadrupole operator it can be shown, using the techniques as discussed in Refs. 11 and 12, that the contribution of the internal coordinates vanishes. However, in the microscopic description the radial integral as given in (1) has to be replaced by

$$\int h_l(E) r^2 h_{l'}(E') dr - \sum_{N,N'} (1 - a_{NN'}) \langle \phi_{Nl} \mid h_l(E) \rangle \langle h_{l'}(E') \mid \phi_{N'l'} \rangle \int \phi_{Nl} r^2 \phi_{N'l'} dr \tag{9}$$

with

$$a_{NN'} = \begin{cases} \left[\dfrac{\mu_N}{\mu_{N'}}\right]^{1/2} & \text{for } N \le N' \\[2ex] \left[\dfrac{\mu_{N'}}{\mu_N}\right]^{1/2} & \text{for } N' \le N \end{cases} \tag{10}$$

as a consequence of the internal structure of the α particles, which is reflected in the occurrence of the second term in (9). Since the normalization kernel μ_N rapidly converge to 1 the sum in (9) is restricted to only a few terms. Furthermore, the ϕ_{Nl} are short-ranged. Consequently a numerical evaluation of the second term in (9) is straightforward. We find that it amounts to approximately 1% of the results found for the radial integral in (1). We conclude therefore that a microscopic description of the ^{8}Be states is not necessary for a reasonable estimation of the $2^+ \rightarrow 0^+$ γ-transition strength provided the orthogonality condition of the Pauli forbidden states is fulfilled by the $\alpha+\alpha$ relative wave functions.

The $2^+ \rightarrow 0^+$ γ width of 8.3 meV corresponds to an $E2$ strength of 75 W.u., which is among the strongest $E2$ transitions[13] in light nuclei ($A \leq 40$). A direct measurement of the predicted capture cross section of 14 nb near $E=2.7$ MeV ($E_{\rm lab}=5.4$ MeV) is hampered by background reactions such as the ^{13}C(α,n)^{16}O reaction. However, the use of high-resolution Ge(Li) detectors in combination with sophisticated differentially pumped ^{4}He gas targets and α-ion beams of good optical quality might allow a measurable counting rate to be achieved. Alternatively, the 2^+ state can be populated by a reaction such as ^{11}B(p,α_1)^{8}Be and the ratio predicted by $\Gamma_\gamma/\Gamma_{\rm tot} = 5.9\times 10^{-9}$ determined via α_1-γ coincidences requiring time resolutions of $\leq$ 1 nsec. Experimental approaches are in progress.

The authors want to thank O. Tanimura for a helpful communication and W. A. Fowler and C. A. Barnes for comments on the manuscript. This work was supported in part by the National Science Foundation grants PHY-8505682 and PHY82-07332.

*Present address: Institut für Theoretische Physik I, Münster, Federal Republic of Germany.

†Permanent address: Institut für Kernphysik, Münster, Federal Republic of Germany.

[1]F. Ajzenberg-Selove, Nucl. Phys. **A320**, 1 (1979).

[2]D. R. Thompson, I. Reichstein, W. McClure, and Y. C. Tang, Phys. Rev. **185**, 1351 (1969).

[3]O. Tanimura and U. Mosel, Nucl. Phys. **A440**, 173 (1985).

[4]O. Tanimura, private communication.

[5]B. Buck, H. Friedrich, and C. Wheatley, Nucl. Phys. **A275**, 246 (1977).

[6]K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. **28**, 432 (1956).

[7]S. Saito, Prog. Theor. Phys. **41**, 705 (1969); H. Friedrich and K. Langanke, Phys. Rev. C **28**, 1385 (1983).

[8]D. A. Zaikin, Nucl. Phys. **A170**, 584 (1971).

[9]H. Friedrich, Phys. Rev. C **30**, 1102 (1984).

[10]N. P. Heydenberg and G. M. Temmer, Phys. Rev. **104**, 123 (1956); T. A. Tombrello and L. S. Senhouse, *ibid.* **129**, 2252 (1963); R. Nilson *et al.*, *ibid.* **109**, 850 (1958).

[11]Y. Suzuki, Prog. Theor. Phys. **56**, 111 (1976); S. Okabe, Nucl. Phys. **A404**, 179 (1983).

[12]K. Langanke and S. E. Koonin, Nucl. Phys. **A410**, 334 (1983); **A439**, 384 (1985).

[13]P. M. Endt, At. Data Nucl. Data Tables **28**, 3 (1979).