Four-body calculation of the breakup reaction ^{3}He(p,pd)^{1}H

T. E. Mdlalose and H. Fiedeldey
Department of Physics, University of South Africa, Pretoria, 0001, South Africa

W. Sandhas
Physikalisches Institut der Universität Bonn, D-5300 Bonn 1, Federal Republic of Germany
(Received 4 November 1985)

The Alt, Grassberger, and Sandhas four-body theory has been used to calculate the differential cross sections for the kinematically complete breakup reaction ^{3}He(p,pd)^{1}H at various proton energies and angles. The two-body input consists of Yamaguchi potentials without Coulomb corrections and the (3 + 1) and (2 + 2) amplitudes were expanded using the generalized unitary pole expansion. The resulting effective two-body equations have been solved in the first-order K-matrix approximation. Reasonable agreement with experiment was obtained.

I. INTRODUCTION

Practical integral equations were formulated by Grassberger and Sandhas[1] (GS) for four-body scattering and breakup. In this formalism the original operator equations are reduced to effective two-body equations in two steps by employing separable expansions for the two- and three-body subamplitudes. After partial wave decomposition, the GS equations become manageable one-variable integral equations.

The first calculations based on this method were performed by Alt, Grassberger, and Sandhas (AGS),[2] for $d + d \rightarrow p + t$ ($n + {}^3He$) rearrangement collisions and for elastic $p + {}^3He$ scattering. In first-order K-matrix approximation, at energies both below and above the breakup threshold. Similar results have been obtained by Becker[3] after replacing the Yamaguchi potentials used in Ref. 2 by separable interactions with Gaussian form factors. These results were encouraging but failed to reproduce the second maximum in the differential cross section of the $2+2 \rightarrow 3+1$ rearrangement processes and were quite unable to reproduce the data in the forward direction for $3+1 \rightarrow 3+1$ elastic scattering. Recently Sofianos *et al.*[4] employed improved expansion methods, the energy-dependent pole expansion (EDPE),[5] and the generalized unitary pole expansion (GUPE),[6] for the (3 + 1) and (2 + 2) subamplitudes, and used local and separable two-body forces (with and without a tensor component). In all these calculations it was found that the agreement with the data improved with increasing energy. Going beyond the first-order K-matrix approximation, Sofianos *et al.*[4] included the principal value part of the (2 + 2) propagators essentially exactly by means of a converged EDPE/GUPE expansion. This improved the agreement with experiment also at lower energies in the $d + d \rightarrow n + {}^3He$ reaction considerably and led in the case of $p + {}^3He$ elastic scattering even to a spectacular improvement in the forward direction. However, the lack of structure in the differential cross section (the absence of a second maximum in the $d + d \rightarrow n + {}^3He$ section) persisted. Solutions of the four-body integral equations below the breakup threshold by Tjon[7] and Kröger and Sandhas[8] indicate that this is probably mainly due to the omission of the p-wave contributions to the three-body subamplitudes.

In what follows we study the four-body breakup reaction ^{3}He(p,pd)^{1}H within the framework of this formalism. In this case, as an intermediate step, the rearrangement amplitudes enter again. However, being half off the energy shell, a much larger number than in Ref. 4 is now needed. Apart from many spin-isospin combinations, up to eight GUPE terms had to be included to reach convergence. This extreme complexity forced us to confine ourselves, in contrast to Ref. 4, to the first-order K-matrix approximation.

An earlier attempt to calculate rearrangement processes in second order, and breakup reactions in first-order K-matrix approximation, has been made by Sawicki and Namyslowski[9] and by Sawicki,[10] respectively, who employed a two-term Bateman expansion. This approach, however, not only failed to reproduce the correct triton and ^{4}He binding energies for the Yamaguchi potential used, but their first-order rearrangement results completely disagreed with experiment and with the corresponding results of Refs. 2–4, inexplicably tending to get worse with increasing energy. Moreover, also their second-order results are in disagreement with those of Ref. 4. In our present investigation no agreement could be found with the breakup calculations of Sawicki[10] either.

An older purely phenomenological approach to breakup reactions describes them in terms of quasi-two-body processes, such as quasi-free scattering (QFS) and final state interactions (FSI). The plane-wave impulse approximation (PWIA) (Ref. 11) was widely used for QFS processes, while the Watson-Migdal (WM) theory[12] has been employed for the latter. These approximations owe their popularity to their numerical simplicity and their roots in our physical intuition. In addition, most of the structure of the four-nucleon breakup scattering data occurs in the regions where quasi-two-body processes are important. Their disadvantages originate from the absence of a theoretical justification based on a rigorous theory. This

led to serious shortcomings, e.g., where several quasi-two-body processes compete. Even in those regions where these theories are expected to be valid, they only yield the approximate shapes of the cross sections which have to be renormalized to the experimental data. However, they still may serve to interpret certain dominant structures found in our more detailed approach in terms of simple physical mechanisms.

In Sec. II we outline the GS formalism and express the breakup amplitudes in terms of the rearrangement amplitudes and the (3 + 1) and (2 + 2) propagators. The simplification which results from resorting to the first-order K-matrix approximation and our use of a converged expansion of the (3 + 1) and (2 + 2) subamplitudes is also discussed. Although the singularities that are involved in the evaluation of the propagators have to be treated quite carefully, they only involve integrals that already occur in the exact solution of the three-body problem.[13] The numerical results and their analysis are presented in Sec. III.

II. THEORY

The AGS transition operators $U^{\sigma\rho}_{\beta\alpha}$ for a four-body system are given by the integral equation:[14]

$$U^{\sigma\rho}_{\beta\alpha}=\bar{\delta}_{\sigma\rho}\delta_{\beta\alpha}G_0^{-1}t_\alpha^{-1}G_0^{-1}+\sum_{\tau,\gamma}\bar{\delta}_{\sigma\tau}T^{\tau}_{\beta\alpha}G_0t_\gamma G_0U^{\tau\rho}_{\gamma\alpha}\ , \tag{1}$$

where t_α are the two-body amplitudes, $T^\tau_{\beta\alpha}$ the (3 + 1) or (2 + 2) subsystem amplitudes, and G_0 is the two-body free resolvent and $\bar{\delta}_{\sigma\tau}=(1-\delta_{\sigma\tau})$. All the subsystem operators are embedded in four-body space. By inserting the ansatz;

$$t_\gamma=\sum_{n=\varphi,\mathrm{d}}|\,\gamma n\,\rangle\tilde{g}_{0;n\gamma,n\gamma}\langle\,\gamma n\,|\tag{2}$$

in Eq. (1), we obtain effective three-body equations which can be expressed in matrix notation as follows:

$$\tilde{U}^{\sigma\rho}=\bar{\delta}_{\sigma\rho}\tilde{g}_0^{-1}+\sum_\tau\bar{\delta}_{\sigma\tau}\tilde{T}^\tau\tilde{g}_0\tilde{U}^{\tau\rho}\ . \tag{3}$$

The reaction we wish to describe involves two cluster and three clusters in the entrance and exit channels, respectively. Hence we must reduce the entrance channel in Eq. (3) to an effective two-body form. We, moreover, wish to approximate the intermediate states by a finite number of quasiparticles for computational reasons. To this end we insert a separable expansion for the effective three-body operators:

$$\tilde{T}^\tau=\sum_{\nu\mu t}|\,^{\tau t\nu}\,\rangle\mathcal{G}^{\tau t\mu}_{0;\nu\mu}\langle\,^{\tau t\mu}\,|\ , \tag{4}$$

where t represents the spin-isospin state of the subsystem under the partition $\tau=(i,jkl)$ or (ij,kl). We then obtain

$$\tilde{U}^{\sigma\rho}\tilde{g}_0\,|\,^{\rho x\mu}\,\rangle=\sum_{\tau t\nu'\mu'}|\,^{\tau t\nu'}\,\rangle\mathcal{G}^{\tau t}_{0;\nu'\mu'}\times\langle\,^{\tau t\mu'}\,|\,\tilde{g}_0\tilde{U}^{\tau\rho}\tilde{g}_0\,|\,^{\rho x\mu}\,\rangle\ , \tag{5}$$

where we have omitted a term on the right-hand side which vanishes on shell due to energy conservation. In a more concise notation we have

$$X^{\cdot,\rho x}_{\beta n}=\sum_{\tau t\nu'\mu'}|\,^{\tau t\nu'}_{\beta n}\,\rangle\mathcal{G}^{\tau t}_{0;\nu'\mu'}\mathbf{T}^{\tau t,\rho x}_{\mu'\mu}\ , \tag{6}$$

where T is the four-body effective two-body rearrangement amplitude and β identifies the pair that is bound in the final state, its quantum numbers being collectively denoted by n. The operator breakup amplitudes can be shown to be given by,

$$X^{\sigma\nu,\rho x}_{\beta n}=\langle\,^{\sigma\nu}_{\beta n}\,|\,X^{\cdot,\rho x}_{\beta n} \tag{7}$$

where $|\,^{\sigma\nu}_{\beta n}\,\rangle$ is the spin-isospin factor in $|\,^{\sigma\nu\nu}_{\beta n}\,\rangle$ such that,

$$|\,^{\sigma\nu\nu}_{\beta n}\,\rangle=|\,\Gamma^{\sigma\nu\nu}_{\beta n}\,\rangle\,|\,^{\sigma\nu}_{\beta n}\,\rangle \tag{8}$$

and $|\,\Gamma\,\rangle$ is the form factor of our separable expansion of the (2 + 2) and (3 + 1) subamplitudes with $\sigma=(i,j,kl)$, $\beta=(kl)$. The equation for the breakup amplitude can therefore be written as follows:

$${}^{IS}X^{\sigma\nu,\rho x}_{\beta n}=\sum_{\tau t\nu'\mu'}{}^{IS}\langle\,^{\sigma\nu}_{\beta n}\,|\,^{\tau t}_{\beta n}\,\rangle^{IS}\,|\,\Gamma^{\tau t\nu'}_{\beta n}\,\rangle\mathcal{G}^{\tau t}_{0;\nu'\mu'}{}^{IS}\mathbf{T}^{\tau t,\rho x}_{\mu',\mu}\ , \tag{9}$$

where ${}^{IS}\langle\,^{\sigma\nu}_{\beta n}\,|\,^{\tau t}_{\beta n}\,\rangle^{IS}$ is the spin-isospin recoupling coefficient for total S and isospin I.

After symmetrization the breakup amplitude (9) takes the form

$${}^{IS}X^{\nu,x}_n(\mathbf{q}_{1,1}\mathbf{q}_2;\mathbf{p})=\sum_t{}^{IS}X^{\nu,t,x}_n(\mathbf{q}_{1,1}\mathbf{q}_2;\mathbf{p})\ , \tag{10}$$

where the terms on the right-hand side are given by

$${}^{IS}X^{\nu,t,x}_n(\mathbf{q}_{1,1}\mathbf{q}_2;\mathbf{p})=\sum_{\nu\mu}{}^{IS}_1\Lambda_{\nu,t}\Gamma^{t\nu}_n({}_1\mathbf{q}_2)\mathcal{G}^t_{0;\nu\mu}(E-\tfrac{2}{3}q_1^2)\,{}^{IS}\mathbf{T}^{t,x}_{\mu,1}(\mathbf{q}_1,\mathbf{p})+\sum_{\nu\mu}{}^{IS}_2\Lambda_{\nu,t}\Gamma^{t\nu}_n({}_1\mathbf{Q}_2)\mathcal{G}^t_{0;\nu\mu}(E-\tfrac{2}{3}Q_1^2)\,{}^{IS}\mathbf{T}^{t,x}_{\mu,1}(\mathbf{Q}_1,\mathbf{p})\ , \tag{11a}$$

if t is a state corresponding to a (3 + 1) partition, and

$${}^{IS}X^{\nu,t,x}_n(\mathbf{q}_{1,1}\mathbf{q}_2;\mathbf{p})=\tfrac{1}{2}({}^{IS}_3\Lambda_{\nu,t}+{}^{IS}_4\Lambda_{\nu,t})\sum_{\nu\mu}\Gamma^{t\nu}_{nm}(q_{12})\mathcal{G}^t_{0;\nu\mu}\left[E-\frac{q^2_{(12)}}{2}\right]{}^{IS}\mathbf{T}^{t,x}_{\mu,1}(\mathbf{q}_{(12)}\mathbf{p}) \tag{11b}$$

if it corresponds to a (2 + 2) channel.

Our convention is that when a pole exists in a subsystem amplitude it is contained in the first term of its expansion.

The physical difference between the first and second term in (11a) is that the free nucleons in the final state are interchanged.

The right-hand sides of both Eqs. (11) are essentially determined by the symmetrized rearrangement amplitudes ${}^{IS}\mathbf{T}^{t,x}_{\mu,1}$.

The Jacobi momentum variables q_1 and ${}_1q_2$ are chosen as in Ref. 2. In addition we use the notation

$$\begin{aligned}
\mathbf{Q}_1 &= {}_1\mathbf{q}_2 - \tfrac{1}{3}\mathbf{q}\,, \\
{}_1\mathbf{Q}_2 &= \tfrac{1}{3}\,{}_1\mathbf{q}_2 + \tfrac{8}{9}\mathbf{q}_1\,, \\
\mathbf{q}_{(12)} &= -{}_1\mathbf{q}_2 - \tfrac{2}{3}\mathbf{q}_1\,, \\
\mathbf{q}_{12} &= -\tfrac{1}{2}\,{}_1\mathbf{q}_2 + \tfrac{2}{3}\mathbf{q}_1\,,
\end{aligned} \tag{12}$$

The half on-shell rearrangment amplitudes in Eq. (11) satisfy the integral equation

$${}^{IS}\mathbf{T} = {}^{IS}\mathbf{V} + {}^{IS}\mathbf{V}\mathcal{G}_0^{IS}\mathbf{T}\,, \tag{13}$$

which is extremely difficult to solve exactly at energies above the breakup threshold, because of its complex cut structure. Consequently, we employ as in Refs. 2–4 the K-matrix approach, confining ourselves to its first-order approximation, for reasons already pointed out in the Introduction. In essence, this involves the replacement of the effective free Green's function by its delta function part which reduces Eq. (13) to

$${}^{IS}\mathbf{T}(z) = {}^{IS}\mathbf{V}(z) + {}^{IS}\mathbf{V}(z)\mathcal{G}_0^{\delta}(z,z^*)\,{}^{IS}\mathbf{T}(z)\,, \tag{14}$$

where

$$\mathcal{G}_0^{\delta}(z,z^*) = [\mathcal{G}_0(z) - \mathcal{G}_0(z^*)]\,. \tag{15}$$

It is evident that in those spin-isospin channels, where no bound states exist, the corresponding contribution is just the Born approximation. Furthermore, unlike the rearrangement scattering calculations, where only the pole terms are required in the expansion of the $(3+1)$ and $(2+2)$ subsystem amplitudes, we have to make the full expansion since the rearrangement amplitudes are required half on shell.

After eliminating the delta function integration in Eq. (14), we have

$${}^{IS}\mathbf{T}^{x,y}_{\mu,\nu} = {}^{IS}\mathbf{V}^{xy}_{\mu,\nu} + \sum_t {}^{IS}\mathbf{V}^{x,t}_{\mu,1} A_t\, {}^{IS}\mathbf{T}^{t,y}_{1,\nu}\,, \tag{16}$$

where

$$\begin{aligned}
A_t &= (-3\pi^2 q_{\rm tr} R_{\rm tr})i, \text{ for the triton pole} \\
&= (-4\pi^2 q_{\rm dd} R_{\rm dd})i, \text{ for the deuteron-pole} \\
&= 0, \text{ otherwise}
\end{aligned}$$

and q_t, R_t are the corresponding on-shell momenta and residues, respectively.

The allowed spin-isospin channels for the reaction ^{3}He(p,pd)^{1}H, contributing as intermediate states in the summation (10), are given in Table I, where the spin-isospin state of an n-body subsystem is denoted by (S_n, I_n).

The spin-isospin recoupling coefficients are given in Table II.

Finally, we obtain the differential cross section for a complete experiment from the relation:

$$\frac{d^3\sigma}{d\Omega_4 d\Omega_3 dE_3} = \frac{1}{16}\left[3\frac{d^3\sigma^{10}}{d\Omega_4 d\Omega_3 dE_3} + 9\frac{d^3\sigma^{11}}{d\Omega_4 d\Omega_3 dE_3}\right], \tag{17}$$

TABLE I. Allowed spin-isospin channels.

(IS)	Intermediate states	Final states
(1,0)	$(I_3,S_3) = (\frac{1}{2},\frac{1}{2})\to$(tr)	$(I_3,S_3) = (\frac{1}{2},\frac{1}{2})\to$(tr)
(1,1)	$(I_3,S_3) = (\frac{1}{2},\frac{1}{2})\to$(tr)	$(I_3,S_3) = (\frac{1}{2},\frac{1}{2})$
	$= (\frac{1}{2},\frac{3}{2})\to$(qu)	$= (\frac{3}{2},\frac{1}{2})$
	$(I_2,S_2) = (1,0)\to\varphi$	

where

$$\frac{d^3\sigma^{IS}}{d\Omega_4 d\Omega_3 dE_3} = \frac{(2\pi)^4 2k_3 k_4}{p_{\rm lab}} \frac{|\,{}^{IS}X_{fi}\,|^2}{3 - 2\hat{\mathbf{k}}_4(\mathbf{p}_{\rm lab} - \mathbf{k}_3)/k_4}\,, \tag{18}$$

$\mathbf{k}_i$, $i=3,4,5$ are the laboratory momenta of the three clusters in the final state and $\mathbf{p}_{\rm lab}$ is the initial laboratory momentum. The breakup amplitudes are given by

$$\begin{aligned}
|\,{}^{10}X_{fi}\,|^2 &= |\,{}^{10}X^{\rm tr,tr,tr}\,|^2\,, \\
|\,{}^{11}X_{fi}\,|^2 &= \Big|\sum_{y={\rm tr,qu},\varphi{\rm d},{\rm d}\varphi} {}^{11}X^{{\rm tr},y,{\rm tr}}\Big|^2 \\
&\quad + \Big|\sum_{y={\rm tr,qu},\varphi{\rm d}} {}^{11}X^{{\rm qu},y,{\rm tr}}\Big|^2\,,
\end{aligned} \tag{19}$$

the abbreviations tr, qu, φ and d for the channel indices being listed in Table I (compare also Ref. 2).

III. RESULTS

In all our calculations we used the following low energy input parameters for the Yamaguchi form factors: singlet scattering length, $a_\varphi = -23.68$ fm; triplet scattering length, $a_{\rm d} = 5.416$ fm; singlet effective range, $r_\varphi = 2.67$ fm; deuteron binding energy, $E_{\rm d} = 2.224$ MeV, yielding the triton binding energy $E_{\rm t} = 10.98$ MeV.

As already mentioned the GUPE was used to expand

TABLE II. Spin-isospin recoupling coefficients.

	${}^{IS}_1\Lambda_{x,y}$	${}^{IS}_2\Lambda_{x,y}$	${}^{IS}_3\Lambda_{x,y}$	${}^{IS}_4\Lambda_{x,y}$
$(I,S) = (1,0)$				
$(x,y) = $ (tr,tr)	1	-1		
$(I,S) = (1,1)$				
$(x,y) = $ (tr,tr)	1	$-\frac{1}{3}$		
= (tr,qu)	0	$-\frac{2}{3}\sqrt{2}$		
= (qu,tr)	0	$-\frac{2}{3}\sqrt{2}$		
= (qu,qu)	1	$\frac{1}{3}$		
= (tr,φd)			$-\sqrt{1/3}$	$-\sqrt{1/3}$
= (qu,φd)			$\sqrt{2/3}$	$\sqrt{2/3}$

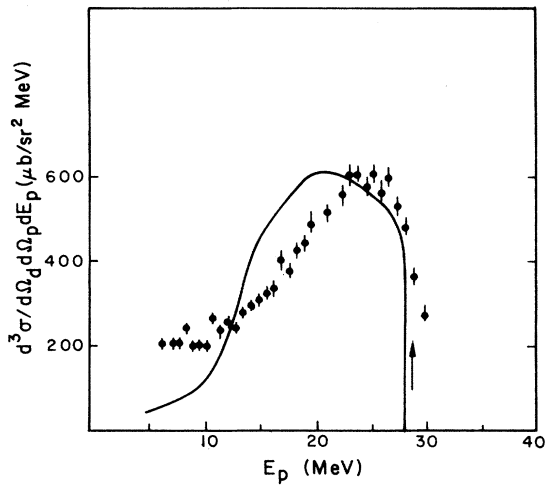

FIG. 1. ^{3}He(p,pd)^{1}H cross section at fixed proton-deuteron angles $\theta_p=\theta_d=35°$, in coplanar geometry for incident proton laboratory energy, $E_{lab}=45$ MeV, as a function of outgoing proton energy E_p. All allowed spin-isospin channels are included. Experimental results are from Ref. 19.

the (3 + 1) and (2 + 1) subsystem amplitudes in separable terms. Our choice was motivated solely by the fact that this is the simplest expansion. Its numerical advantage lies in the fact that the form factors are not energy dependent and therefore they are evaluated once and for all through an eigenfunction analysis. This advantage in our case outweighs the slower convergence of the GUPE compared to those expansions which have energy-dependent form factors, like the EDPE,[5] SE1, and SE2.[15] An idea of the numerical effort required to solve the problem is given

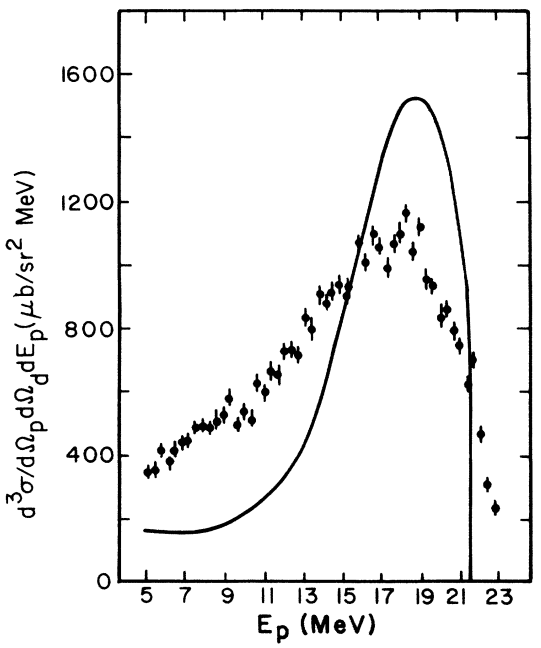

FIG. 2. Same as for Fig. 1 but with angles $\theta_p=\theta_p=35°$ and incident energy, $E_{lab}=35$ MeV. Experimental results are from Ref. 18.

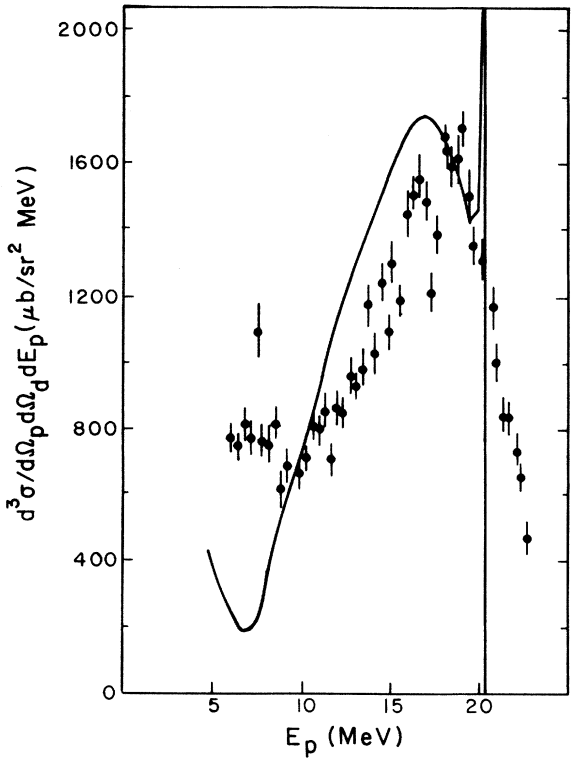

FIG. 3. Same as for Fig. 2 but with angles $\theta_p=\theta_d=45°$ and incident energy $E_{lab}=35$ MeV. Experimental results are from Ref. 17.

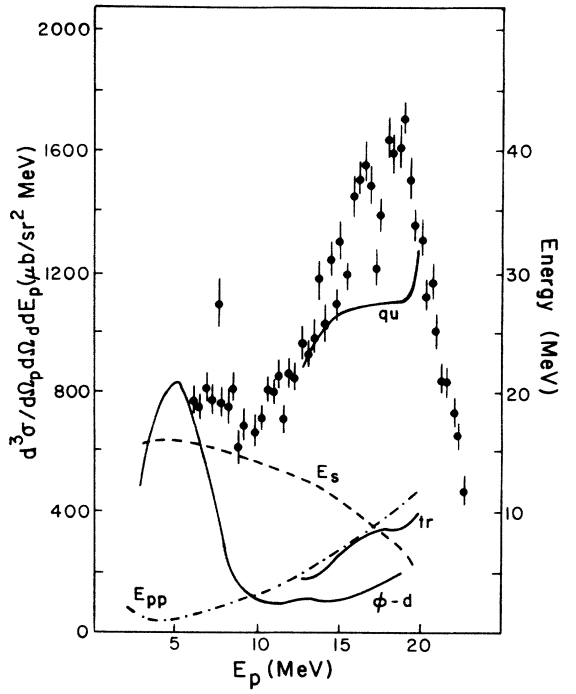

FIG. 4. Partial ^{3}He(p,pd)^{1}H cross section at proton-deuteron angles $\theta_p=\theta_d=45°$ and proton incident energy $E_{lab}=35$ MeV is given by the continuous curves. These curves represent the contribution of the channels qu, tr, and φd, respectively. The dashed curve represents the laboratory energy of the spectator E_s, while the dash-dot curve gives the proton-proton relative energy E_{pp}.

by the fact that a single point on the cross-section curve requires about one hour CPU time on a Burroughs 7900.

We denote the number of expansion terms for the subsystem amplitudes in the qu, tr, and φd-dφ states by N31, N33, and N22. Up to an accuracy of $\lesssim 7\%$, it was sufficient to go up to N31 = 4 and N33 = N22 = 6.

The process ^{3}H(p,pd)^{1}H is well known to be predominantly coplanar,[16] indicating that the reaction proceeds preferentially as a quasi-two-body reaction which can result from either proton-proton (proton-deuteron) quasi-free scattering or proton-proton final state interaction (FSI) in the $^1S_0$ state. These quasi-two-body phenomena result in the enhancement of the cross section when appropriate kinematical conditions are satisfied. At a QFS peak the recoil momentum of the deuteron (p-p QFS) or proton (p-d) QFS is a minimum, while the FSI peaks occur at minima of the energy of relative motion between the protons in the final state. In our formalism the ^{3}He nucleus is a mixture of d + nucleon and φ + nucleon. Consequently the p-p, $^1S_0$ state FSI can be viewed as resulting from a neutron pickup reaction.[16] The greatest contribution to the cross section for this mechanism can therefore be expected to come from the amplitude $^{SI}X^{x,\varphi\text{d-d}\varphi,\text{tr}}$. The rest of the amplitudes would then describe QFS processes.

In Figs. 1–3 we have plotted the differential cross section for a selection of kinematical conditions. Our results are in reasonable agreement with experiment. In this context it should be noted that the overbinding of the triton in our model, based on Yamaguchi potentials, results in a shortening of the kinematically allowed region for the measured energy of the outgoing proton. Hence, there are experimental values beyond our theoretical limits. A typical feature of breakup spectra at both three-body (Ref. 20) and four-body levels is that there exist proton energies and angles (of reaction products) for which the pole of the phase space factor, Eq. (18), moves sufficiently near to the kinematically allowed region to cause a sharp increase in the cross section near the maximum proton energy; hence the spike in Fig. 3. In this case, it is aggravated by our distorted kinematics. In Fig. 4 we have selected one set of kinematical conditions in order to illustrate the contributions of various channels and the kinematical variables that govern various quasi-two-body processes. It is clearly seen that for each quasi-two-body process there are intermediate channels in Eq. (19) which dominate the cross section. In particular, we find that the amplitude $^{SI}X^{x,\varphi\text{d}-\text{d}\varphi,\text{tr}}$ essentially determines the FSI while the p-d QFS is dominated by the quartet channel.

In Ref. 17 "anomalies in QFS from p + ^{3}He reactions" were reported which consist of shifts of the experimental peaks away from the minimum of the spectator energy. This shift was partly explained[17] by starting from the *ad hoc* assumption that a pickup amplitude calculated in the diffraction model can be added coherently to the QFS amplitude. According to Fig. 4, the combination of amplitudes for various processes is seen to arise automatically in the AGS approach with the result that the peak is quite naturally shifted towards its experimental position.

Let us finally add some comments on the previous breakup calculations by Sawicki.[10] We have already mentioned the shortcomings of the rearrangement results of Ref. 9 which enter his calculations. Moreover, in the breakup case, as an additional complication not taken care of in Ref. 10, many more expansion terms are needed to reach convergence (compare the above discussion of this point). In fact, only with their full incorporation the magnitude of the experiments is sufficiently well fitted.

In conclusion we have found that, already in first-order K-matrix approximation, the AGS theory enables us to reproduce the main features of those breakup processes considered in this paper. It should be emphasized that our calculations, while being based on a simple two-body interaction, represent a real microscopic approach without any free parameters.

ACKNOWLEDGMENTS

Two of the authors (T.E.M. and H.F.) would like to thank the University of Bonn, and W.S. thanks the University of South Africa for kind hospitality. We thank the Rheinisches Rechenzentrum, Bonn for an allocation of computer time. This work was supported in part by the Deutsche Forschung Gemeinschaft, the University of South Africa, and the Council for Scientific and Industrial Research.

[1] P. Grassberger and W. Sandhas, Nucl. Phys. **B2**, 181 (1967).
[2] E. O. Alt, P. Grassberger, and W. Sandhas, Phys. Rev. C 1, 85 (1970).
[3] K. Becker, Ph.D. thesis, University of Bonn, 1976 (unpublished); see also W. Sandhas, in *Few-Body Dynamics,* edited by A. N. Mitra *et al.* (North-Holland, Amsterdam, 1976), p. 540.
[4] S. A. Sofianos, H. Fiedeldey, and W. Sandhas, Phys. Rev. C **32**, 400 (1985).
[5] S. A. Sofianos, N. J. McGurk, and H. Fiedeldey, Nucl. Phys. **A318**, 295 (1979).
[6] A. Casel, H. Haberzettl, and W. Sandhas, Phys. Rev. C **4**, 359 (1981).
[7] J. A. Tjon, Phys. Lett. **63B**, 391 (1976).
[8] H. Kröger and W. Sandhas, Phys. Rev. Lett. **40**, 834 (1978).
[9] M. Sawicki and J. M. Namyslowski, Phys. Lett. **60B**, 331 (1976).
[10] M. Sawicki, Phys. Lett. **68B**, 43 (1977).
[11] D. R. Lehman, Phys. Rev. C **6**, 2023 (1972).
[12] K. M. Watson, Phys. Rev. **88**, 1163 (1952).
[13] J. H. Stuivenberg, Ph.D. thesis, Vrije Universiteit van Amsterdam, 1976 (unpublished).
[14] E. O. Alt, P. Grassberger, and W. Sandhas, Joint Institute of Nuclear Research Report E4-6688, Dubna, 1972; and in, *Few Particle Problems in the Nuclear Interaction,* edited by I. Slaus *et al.* (North-Holland, Amsterdam, 1972), p. 299; W. Sandhas in, *Progress in Particle Physics,* edited by P. Urban,

Acta Phys. Austriaca, Suppl. XIII, 679, 1974; Czech. J. Phys. **B25**, 251 (1975).

[15]A. C. Fonseca, H. Haberzettl, and E. Cravo, Phys. Rev. C **27**, 939 (1979).

[16]B. J. Wielinga, A. D. Ijpenberg, K. Mulder, R. van Dantzig, and I. Slaus, Phys. Rev. Lett. **27**, 1229 (1971).

[17]I. Slaus, M. B. Epstein, G. Paić, J. R. Richardson, D. L. Shannon, and J. W. Verba, Phys. Rev. Lett. **27**, 751 (1971).

[18]M. B. Epstein, I. Slaus, D. Shannon, J. R. Richardson, J. W. Verba, H. H. Forster, C. Kim, and D. Y. Park, Phys. Lett. **35B**, 305 (1971).

[19]S. N. Bunker, Mahavir Jain, C. A. Miller, J. M. Nelson, and W. T. H. van Oers, Phys. Rev. C **12**, 1396 (1975).

[20]W. Ebenhöh, Nucl. Phys. **A191**, 97 (1972).