Simultaneous electromagnetic enhancement of nuclear beta decay and internal conversion

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A nuclear system is treated in which a forbidden beta decay leads to a state metastable against gamma-ray emission. Considering both the beta decay and the internal conversion component of the isomeric decay to be accelerated by an applied electromagnetic field, the rate equation for emission from the isomeric state is solved for the case where the nucleus is subjected to alternating episodes of field on and field off. There are important qualitative differences in the solution between the cases where enhancement of the beta transition rate is greater than enhancement of the internal conversion rate, and vice versa. Application is made to the example of ^{137}Cs.

Beta decays are forbidden because the angular momentum change exceeds the zero or one unit involved in an allowed decay (or because there is a change in nuclear parity). In a similar fashion, an excited nuclear state will be metastable against gamma emission if there is a large angular momentum change involved in the transition. There is a further correspondence between the two processes if one considers the internal conversion pathway to deexcitation of an isomeric state. In that case, the beta decay and the internal conversion both involve an unbound electron as a final-state particle. The particular relevance of this is that a free electron in an intense plane-wave electromagnetic field can have significant angular momentum components induced by the field in addition to its normal spin-$\frac{1}{2}$ angular momentum. This can then make important alterations in the transition rate both for forbidden beta decay[1–3] and for internal conversion in a transition of high multipolarity.[3]

Experiments designed to test the effects of an external electromagnetic field on the first-forbidden beta decay in ^{137}Cs were carried out at the Amoco Research Center[4] and at the University of Arizona.[5] These experiments measured the 662 keV gamma rays that are emitted from ^{137}Ba^m after the beta decay of ^{137}Cs. To interpret the results of these experiments, a study was done[6] of the solution of the rate equation for population of ^{137}Ba^m by the beta decay of ^{137}Cs and depopulation of this state by the isomeric decay channels when there were alternating episodes with an electromagnetic field present and not present. The rate equation study was done allowing for possible field effects on the beta decay and/or on the gamma emission from the isomeric state. It was concluded that the experimental results showed a dominance of beta decay effects over gamma decay effects, and any influence of the field on the gamma process had to be extremely small. That work, however, did not include effects of the field on the internal conversion channel in the isomeric transition.

The earlier work is now altered to consider field effects on the beta decay and on a succeeding internal conversion. The rate equation solutions already found for the population of the isomeric state are again applicable. What is changed is the way in which these solutions are applied to the prediction of the measured rates of gamma emission. This is done below. The consequences of these results are examined later. It is concluded that the experiments can establish that enhancement of beta emission exceeded enhancement of internal conversion, but they cannot establish the magnitude of the acceleration of the beta decay. The difficulty is that any internal conversion enhancement which occurs directly subtracts from the apparent beta decay enhancement. The experiments that were performed can then place only a lower limit on field-induced increases in forbidden beta decay. Absolute measurements would require new experiments in which internal conversion rates were detected along with the gamma emission rates.

As in Ref. 6, we consider three nuclear levels labeled a, b, and c. Level a decays by beta emission to level b in the daughter nucleus. Level b is metastable against gamma emission to a lower-lying level c. The population of level b, $N_b(t)$, is found when the system is exposed to an electromagnetic field that follows a periodic pattern. The field is turned on at a constant amplitude for a time T. This is followed by a time interval of length T where there is no field, after which the entire cycle of length $2T$ is repeated numerous times. The experiments are conducted so that during each field-on or field-off episode, the emission of gamma rays from level b is detected, and these counts are accumulated and recorded at time intervals of $T/4$. This is referred to as a "block" of data. That is, there are four blocks of data collected with the field on, followed by four more with the field off, for a total of eight blocks in each complete cycle. (Field-off blocks were regarded in the experiment as occurring before field-on blocks in each cycle, so the results are presented in that order.) In the experiments, T was selected so that $T/4$ is approximately equal to a natural half-life of level b. This was done so that any field-induced changes would cause an easily identified pattern to appear in the results. Such a pattern is seen in the experimental results shown in Fig. 1.

The natural transition rate for the decay of state a is W_a, and this rate may be accelerated by the presence of a field to Ω_a, represented as

$$W_a \rightarrow \Omega_a = W_a + \Delta_a \quad . \tag{1}$$

A similar statement can be made for the transition rate for decay of state b. However, this rate consists of γ and internal conversion portions, so

$$W_b = W_{b\gamma} + W_{bic} \quad , \tag{2}$$

$$W_b \rightarrow \Omega_b = W_b + \Delta_b \quad . \tag{3}$$

Equations (1) and (3) are identical to Eq. (4) of Ref. 6. However, the Δ_b quantity was assumed in Ref. 6 to come about entirely through a change in $W_{b\gamma}$. It is now assumed

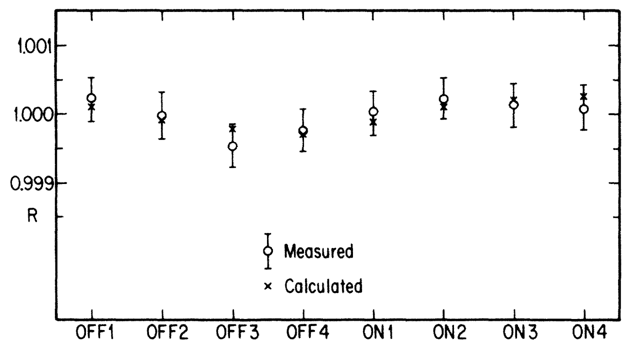

FIG. 1. Experimental results for ^{137}Cs from Ref. 5. The first four points in the figure are integrated gamma-ray counts for field-off conditions, and the last four points are when the field is on. The ordinate measures the integrated gamma count normalized with respect to an average for all eight points shown in the figure.

to come about entirely through a change in $W_{b\kappa}$. This difference in interpretation matters not at all for the basic solution of the rate equation as given in Ref. 6 in general in Eqs. (18) and (19), and for large times in Eqs. (20) and (21). The "large times" asymptotic result is reached within only about two complete off-and-on cycles of the field, and so these solutions are appropriate for our purposes. The results are

$$\downarrow N_b(t) = N_a^0 w e^{-W_a t} + N_a^0 (1-e^{-(\Omega_b+W_b)T})^{-1} \times [(\omega e^{-\Omega_a T} - w) - (\omega - w e^{-W_a T}) e^{-\Omega_b T}] e^{-W_b t} , \quad (4)$$

$$\uparrow N_b(t) = N_a^0 \omega e^{-\Omega_a t} + N_a^0 (1-e^{-(\Omega_b+W_b)T})^{-1} \times [(\omega e^{-\Omega_a T} - w) e^{-W_b T} - (\omega - w e^{-W_a T})] e^{-\Omega_b t} , \quad (5)$$

where $\downarrow N_b$ is the population of state b while the field is off, $\uparrow N_b$ is the population while the field is on, N_a^0 is the population of state a at the start of the experiment, and w, ω are defined by

$$w \equiv W_a (W_b - W_a)^{-1}, \quad \omega \equiv \Omega_a (\Omega_b - \Omega_a)^{-1} . \quad (6)$$

The rate equation results will now be related to measured quantities in the experiments that have been done.[4,5] Those experiments involved integrated gamma-ray counts recorded at intervals of $T/4$ during each field-off and field-on cycle. These integrated counts (after normalization with respect to a comparison source) were then referred to as R_i, with $i=1,2,3,4$ for the field-off blocks of data, and $i=5,6,7,8$ for the field-on blocks. The rate at which gamma rays are emitted is just the product of the gamma transition probability per unit time and the population of level b. Hence the integrated gamma-ray counts in each block of data are

$$R_i = W_{b\gamma} \int_{(i-1)T/4}^{iT/4} dt \downarrow N_b(t), \quad i=1,2,3,4 , \quad (7)$$

$$R_i = W_{b\gamma} \int_{(i-5)T/4}^{(i-4)T/4} dt \uparrow N_b(t), \quad i=5,6,7,8 , \quad (8)$$

since it is assumed that the field does not influence $W_{b\gamma}$. With a slight difference in notation, Eq. (7) is identical to Eq. (22) of Ref. 6. However, Eq. (8) differs from Eq. (23) of Ref. 6 by the factor $(1+\beta)^{-1}$. The integrations involved in Eqs. (7) and (8) when Eqs. (4) and (5) are substituted are elementary. After the integrations are done, some physical approximations can be made. Since the half-life of the isomeric state in the daughter nucleus of a forbidden beta decay is, in all practical cases, far less than the half-life of the beta decay, then[7]

$$W_b >> W_a, \quad \Omega_b >> \Omega_a . \quad (9)$$

Also, since $T/4$ is selected to be approximately $\ln 2/W_b$, then Eq. (9) implies

$$1-e^{-W_a T/4} \approx W_a T/4, \quad 1-e^{-\Omega_a T/4} \approx \Omega_a T/4 ,$$
$$w \approx W_a/W_b, \quad \omega \approx \Omega_a/\Omega_b , \quad (10)$$
$$(\omega e^{-\Omega_a T} - w) \approx \omega - w, \quad (\omega - w e^{-W_a T}) \approx \omega - w .$$

Finally, the results of the integrations in Eqs. (7) and (8) in which Eq. (10) is used are

$$R_i = \downarrow K [1 + e^{-(i-1)\tau}(1-e^{-\tau})(1-e^{-4\tau(1+\beta)}) \times (1-e^{-4\tau(2+\beta)})^{-1}(\alpha-\beta)(1+\beta)^{-1}\tau^{-1}] , \quad i=1,2,3,4 , \quad (11)$$

$$R_i = \uparrow K [1 + \alpha - e^{-(i-5)\tau(1+\beta)}(1-e^{-\tau(1+\beta)})(1-e^{-4\tau}) \times (1-e^{-4\tau(2+\beta)})^{-1}(\alpha-\beta)(1+\beta)^{-1}\tau^{-1}] , \quad i=5,6,7,8 , \quad (12)$$

where

$$\alpha \equiv \Delta_a/W_a, \quad \beta \equiv \Delta_b/W_b, \quad \tau \equiv W_b T/4 ,$$
$$\downarrow K \equiv N_a^0 \tau (W_a/W_b)(W_{b\gamma}/W_b), \quad \uparrow K \equiv \downarrow K/(1+\beta) . \quad (13)$$

An examination of the entire pattern of field-off and field-on counts is most conveniently done after normalization so that the average value of R_i is unity. This is done by multiplying Eqs. (11) and (12) by a normalization constant C, and then enforcing the normalization condition

$$\sum_{i=1}^{8} R_i = 8 , \quad (14)$$

in order to evaluate C. The end result of this is

$$C = \frac{1}{\uparrow K} \left[1 + \frac{1}{2}(\alpha+\beta) + \frac{\beta(\alpha-\beta)}{8\tau(1+\beta)} \frac{(1-e^{-4\tau})(1-e^{-4\tau(1+\beta)})}{(1-e^{-4\tau(2+\beta)})} \right] . \quad (15)$$

The normalized versions of Eqs. (11) and (12) are

$$R_i = \frac{1+\beta+e^{-(i-1)\tau}(1-e^{-\tau})(1-e^{-4\tau(1+\beta)})(1-e^{-4\tau(2+\beta)})^{-1}(\alpha-\beta)\tau^{-1}}{1+(\alpha+\beta)/2+(1-e^{-4\tau})(1-e^{-4\tau(1+\beta)})(1-e^{-4\tau(2+\beta)})^{-1}\beta(\alpha-\beta)(1+\beta)^{-1}(8\tau)^{-1}} \quad i=1,2,3,4 , \quad (16)$$

$$R_i = \frac{1+\alpha-e^{-(i-5)\tau(1+\beta)}(1-e^{-\tau(1+\beta)})(1-e^{-4\tau})(1-e^{-4\tau(2+\beta)})^{-1}(\alpha-\beta)(1+\beta)^{-1}\tau^{-1}}{1+(\alpha+\beta)/2+(1-e^{-4\tau})(1-e^{-4\tau(1+\beta)})(1-e^{-4\tau(2+\beta)})^{-1}\beta(\alpha-\beta)(1+\beta)^{-1}(8\tau)^{-1}}, \quad i=5,6,7,8 . \quad (17)$$

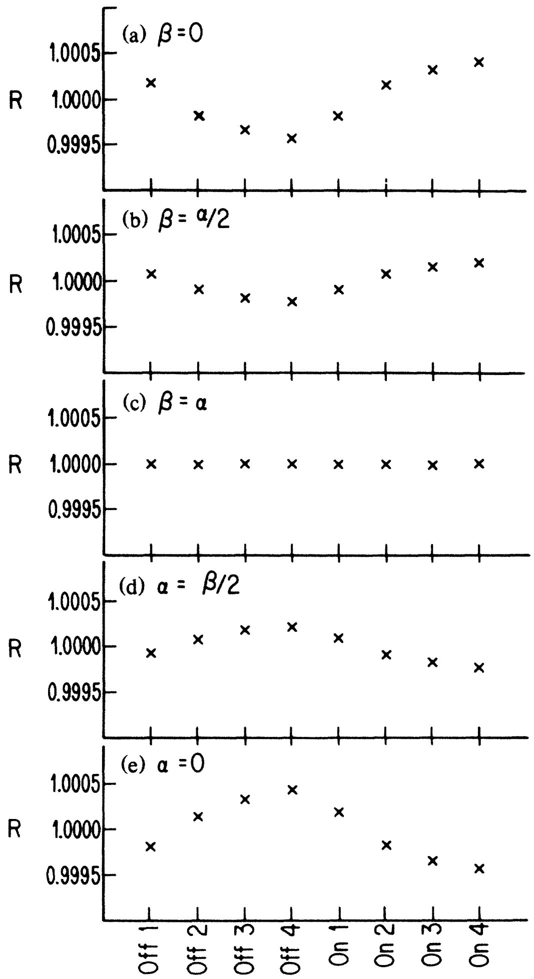

FIG. 2. Integrated gamma-ray counts for several different combinations of field-accelerated beta decay and internal conversion rates. The axes are as in Fig. 1. Parts (a)–(c) are for $\alpha=10^{-3}$, with β values shown in the figures. Parts (c)–(e) are for $\beta=10^{-3}$, with α values shown in the figures. The purpose is to show the qualitative differences between $\alpha>\beta$ and $\beta>\alpha$.

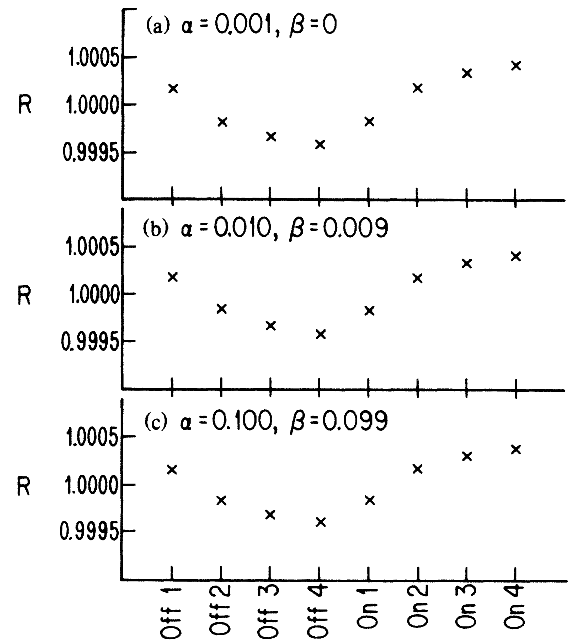

FIG. 3. Integrated gamma-ray counts for several combinations of α and β. The axes are as in Fig. 1. The purpose is to show that the qualitative nature of the results depends primarily on having $\alpha>\beta$, and not on the magnitude of α.

The experiment that gave rise to Fig. 1 gave a value for α of $(6.5\pm2.0)\times10^{-4}$ when the data were analyzed with the presumption that $\beta=0$. To see the qualitative implications of Eqs. (16) and (17), a similar value of α to that found experimentally is substituted. Figure 2(a) is a plot of Eqs. (16) and (17) for the case $\alpha=10^{-3}$ and $\beta=0$, Fig. 2(b) shows the result for $\alpha=10^{-3}$, $\beta=\alpha/2$, and Fig. 2(c) is for $\alpha=\beta=10^{-3}$. Figures 2(d) and 2(c) are for the $\beta>\alpha$ situation, with $\beta=10^{-3}$ in both cases. Figure 2(d) is for $\alpha=\beta/2$, and $\alpha=0$ in Fig. 2(e). A comparison of experimental results in Fig. 1 with the various parts of Fig. 2 shows clearly that the experiment is consistent with $\alpha>\beta$, while $\alpha\lesssim\beta$ gives the wrong qualitative pattern.

While one can conclude that field effects on beta decay exceed field effects on internal conversion, the question remains open about the justification for choosing $\beta=0$ in order to analyze the data. The experiments were done with no measurement of internal conversion rates. Only gamma emission was measured. Qualitatively, it is expected that if the field enhances internal conversion, the effect is to drain population from state b (the isomeric state), without that loss of population being manifested in gamma counts. In fact, if the relative field enhancement of internal conversion is as large as the relative enhancement of beta decay, the increase in beta decay is totally masked, as shown in Fig. 2(c). This raises the possibility that the enhancement of beta decay in the experiments might be substantially larger than the result inferred by assuming $\beta=0$ for purposes of data analysis. Figure 3(a) is a repetition of Fig. 2(a), showing the case $\alpha=10^{-3}$, $\beta=0$. For comparison, Fig. 3(b) gives results for $\alpha=10^{-2}$, $\beta=9\times10^{-3}$. Figures 3(a) and 3(b) are indistinguishable. The same is nearly true for Fig. 3(c), which is for $\alpha=0.1$, $\beta=0.099$.

The conclusion is that field effects on internal conversion subtract almost directly from field effects on forbidden beta decay when those effects are evaluated by examination of the gamma emission from the isomeric state populated by the beta decay. This means that the data analyses of Refs. 4 and 5, done under the presumption that $\beta=0$, give lower limits for the value of α. What is now needed are experiments in which both the gamma emission and internal conversion are measured together. These are being planned.

[1]H. R. Reiss, Phys. Rev. C **27**, 1199 (1983); **28**, 1402 (1983); **29**, 1825 (1984); **29**, 2290 (1984).
[2]H. R. Reiss, Phys. Lett. **103A**, 312 (1984).
[3]H. R. Reiss (unpublished).
[4]H. R. Reiss, E. Watson, and D. J. Vezzetti (unpublished).
[5]H. R. Reiss and D. L. Barker (unpublished).
[6]H. R. Reiss, Phys. Rev. C **31**, 2238 (1985).
[7]There is a typographical error in Eq. (25) of Ref. 6. It should be the same as Eq. (9) here.