Microscopic calculations in the $A=6$ system

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The $A=6$ structure problem is solved within the framework of the nuclear shell model. Model states are expanded upon a basis of properly symmetrized, translationally invariant harmonic oscillator eigenstates including states of up to $6\hbar\omega$ of excitation. The model interaction is based upon a modification of the two-body Sussex interaction. Eigenvalues and (J^π,T) values are predicted for the ground states of the following systems: ^{6}n, ^{6}H, ^{6}He, ^{6}Li, ^{6}Be, ^{6}B, and ^{6}C. The ground state binding energies are within 4 percent of experiment for ^{6}He, ^{6}Li, and ^{6}Be. Excited states for the ^{6}He, ^{6}Li, and ^{6}Be systems are determined and compared to experiment and other calculations. The model spectra for the ^{6}Li system is similar to that proposed by Ajzenberg-Selove. However, the ^{6}He and ^{6}Be spectra contain levels in addition to those suggested by existing compilations. In particular, the ^{6}He and ^{6}Be level spectra have the following level ordering: $(0^+,1)$, $(2^+,1)$, $(1^+,0)$, $(0^+,1)$, $(0^+,1)$, $(3^+,0)$ $(2^+,1)$, $(4^-,1)$, $(2^-,1)$, $(3^-,1)$, and $(4^+,1)$.

I. INTRODUCTION

A recent data compilation by Ajzenberg-Selove[1] summarizes the available $A=6$ data. Although there are considerable data which describe the ground and excited states of the $A=6$ systems (^{6}n, ^{6}H, ^{6}He, ^{6}Li, ^{6}Be, ^{6}B, and ^{6}C), no single theoretical calculation has been performed which addresses the ground and excited states of these seven nuclei. Calculations have addressed ground state properties or specific energy levels. For example, early calculations by Cohen and Kurath[2] and Irvine *et al.*[3,4] addressed levels in the ^{6}Li and the ^{6}He, ^{6}Li, and ^{6}Be nuclei, respectively. More recently, Hofmann and Zahn[5] addressed the ^{6}Be system in considerable detail. Binding energy summaries have been presented by Nguyen and Úlehla[6] and Beiner *et al.*[7] Other work is summarized in Refs. 1, 8, and 9.

Although these calculations have advanced our knowledge of the $A=6$ system, they do not yield a consistent picture of the ground and excited states of this system or analyze the $A=6$ system in a consistent fashion with neighboring $A=2$–5 nuclei. This paper will attempt such a study by utilizing a model interaction which yielded reasonable results in $A=2$–5 systems.[10–16] Specifically, this work will calculate the binding energies of ^{6}n, ^{6}H, ^{6}He, ^{6}Li, ^{6}Be, ^{6}B, and ^{6}C. In addition, excited states of ^{6}He, ^{6}Li, and ^{6}Be will be presented.

II. BASIS STATES AND FORMALISM

In general, the nuclear wave function Ψ can be written as a sum over a set of basis or expansion states $|\lambda\rangle$

$$\Psi=\sum_\lambda A_\lambda |\lambda\rangle \ . \tag{1}$$

Following previous $A=2$–4 (Refs. 10 and 11) and $A=5$ (Ref. 16) studies, the $A=6$ expansion states are a set of properly symmetrized, translationally invariant harmonic oscillator eigenstates which permit recoil to be treated correctly. These states are defined in terms of five internal coordinates and associated quantum numbers. The definition of the basis state is completed once a coupling scheme is chosen. The coupling scheme specifies how the individual angular momenta are coupled to form the various J^π states in the system of interest.

The generalized R-matrix methodology of Lane and Robson[17] and the methods of Philpott and George[18] are most easily applied to the $A=6$ system if the basis states are limited to two separate clusters. The reader should note that the use of the terminology "clusters" does not imply the inclusion or omission of binary reaction channels. The term clusters is merely a convenient way of explaining the coupling structure of the basis state. For example, the choice of ^{6}Li basis states is defined in terms of ^{4}He + d clusters.

The model $A=6$ basis state clusters (a and b) are defined in terms of an internal coupling scheme which leads to states of total angular momentum J_a^π and J_b^π for the a and b clusters, respectively. A coordinate ($\mathbf{r}_C$) connects the centers of mass of the a and b clusters, and is characterized by the oscillator quantum numbers N_C and L_C, which describe radial and orbital excitation. Given these definitions, the basis states are defined as

$$|\lambda_j\rangle = |((J_a^\pi N_C L_C)X J_b^\pi)J_j^\pi\rangle \ , \tag{2}$$

where X is the result of coupling J_a and L_C, and J_j^π is the total angular momentum for the jth $A=6$ nucleus. The parity of the basis state is defined in terms of the parities of the individual clusters

$$\pi=\pi_a\pi_b(-)^{L_C} \ . \tag{3}$$

Examples of basis states used in the $A=3$ (Ref. 13), $A=4$ (Ref. 10), and $A=5$ (Ref. 16) systems illustrate the coupling scheme and basis state methodology.

The number of basis states required to describe the various J^π configurations rapidly increases as the allowable oscillator excitation increases. The total oscillator excita-

tion p can be expressed in terms of the properties of the a and b clusters, and the oscillator quanta in the $\mathbf{r}_C$ coordinate,

$$p = p_a + p_b + 2N_C + L_C \ , \tag{4}$$

where

$$p_a = \sum_{i=1}^{A_a - 1} 2N_i + L_i \ , \tag{5}$$

$$p_b = \sum_{j=1}^{5-A_a} 2N_j + L_j \ . \tag{6}$$

An example of basis size increase with increasing p is illustrated in Ref. 10 for the ^{4}He system. A consideration of previous $A=4$ (Ref. 10) and $A=5$ (Ref. 16) model space restrictions and increased $A=6$ degrees of freedom leads to a practical size limit of $6\hbar\omega$ ($p_{\max}$) for our $A=6$ basis states.

In addition to a choice for $p_{\max}$, the internal coupling scheme for the a and b clusters must be chosen. The coupling scheme is chosen to be that used in the solution of previous few-nucleon problems, ^{2}H, ^{3}H, ^{3}He, ^{4}He (Refs. 10–15), ^{5}H, ^{5}He, ^{5}Li, and ^{5}Be (Ref. 16).

The choice of specific a and b clusters is arbitrary because the solution of the Schrödinger equation is in principle not dependent on the wave function coupling scheme. For simplicity, we choose the cluster configurations for the $A=6$ basis states to be those of the lowest lying binary reaction channels in each nucleus.

The generalized R-matrix equation can be written in the form[17,18]

$$\sum_{\lambda'} \left[\langle \lambda | H - E | \lambda' \rangle + \sum_c \gamma_{\lambda c} (b_{\lambda' c} - b_c) \gamma_{\lambda' c} \right] A_{\lambda'} = 0 \ , \tag{7}$$

where H is the Hamiltonian describing the system of interest and $\gamma_{\lambda c}$ and $b_{\lambda c}$ are the reduced widths[19] and logarithmic derivatives associated with the expansion states $|\lambda\rangle$. The expansion states are introduced in order to describe the nuclear wave function within the interaction region, $r_c \leq a_c$ in all channels (c). The other quantities appearing in Eq. (7) are defined in Ref. 10. If the channel radii are chosen such that $a_c \to \infty$, the generalized R-matrix solution becomes the shell-model solution. Herein, we choose $a_c = \infty$, and use the shell model solution

$$\sum_{\lambda'} \langle \lambda | H - E | \lambda' \rangle A_{\lambda'} = 0 \tag{8}$$

to determine the ground state binding energies and excited states in the $A=6$ system.

III. EFFECTIVE INTERACTION

In Ref. 10, an effective interaction for oscillator basis states was determined for the two-, three-, and four-nucleon systems. This interaction also leads to reasonably accurate $A=5$ ground state and excited state properties.[16] The interaction was derived from the Sussex matrix elements,[20] and is of the form

$$V^{\text{mod Suss}} = C V^{\text{Suss}} \ , \tag{9}$$

where C is a strength parameter of order unity. The changes from the original Sussex matrix elements implied by our choice of C are typically of the same order of magnitude as the expected uncertainties in the matrix elements themselves.[20] The modified Sussex matrix elements are used to formulate the model Hamiltonian.[10,16]

In particular, the model interaction predicts a ^{4}He binding energy result which is in agreement with experiment.[10] The ^{3}H and ^{3}He binding energies are within 5% of experiment and the ^{2}H eigenvalue is within 20% of experiment. In addition, the modified Sussex interaction yields a good representation of the ^{4}He (Ref. 10), ^{4}H, and ^{4}Li (Ref. 11) spectra and the binding and excited state properties in ^{5}H, ^{5}He, ^{5}Li, and ^{5}Be.[16]

IV. $A=6$ GROUND STATE CALCULATIONS

An initial test of the $A=6$ model is provided by calculating the ground state binding energies for the ^{6}n, ^{6}H, ^{6}He, ^{6}Li, ^{6}Be, ^{6}B, and ^{6}C systems. The model calculations are compared with the data in Table I. Table I also summarizes shell model calculations of Nguyen and Úlehla[6] which use a Skyrme-type force (SK) and the energy density formulation calculations of Beiner, Lombard, and Mas (BLM).[7] The experimental binding energies are based on the mass excess values of Ref. 1.

Model results can be compared directly with data for the ^{6}He, ^{6}Li, and ^{6}Be systems. For these systems, model results are within 4 percent of available data and are somewhat closer to the experimental values than the Skyrme[6] or energy density formulations.[7] Comparisons of model and Skyrme results show considerable differences in the calculated ^{6}B binding energies. The Skyrme interaction yields a binding energy of -6.6 MeV, while our model results suggest a value of $+5.4$ MeV. The source of this difference is not clear because both calculations are near the experimental values in other $A=6$ nuclei. However, the Skyrme results overbind ^{6}Li by 3.3 MeV, but underbind ^{6}He and ^{6}Be by 1.4 MeV and 2.6 MeV, respectively. If the trend of underbinding higher isospin ground states would continue in ^{6}B, one might expect the Skyrme interaction to underbind this system. The model predicts binding energies for the ^{6}n, ^{6}H, and ^{6}C systems which have the values -6.499 MeV, $+7.144$ MeV, and -16.405 MeV, respectively. Since no data exist for ^{6}n, ^{6}H, and ^{6}C, it is difficult to judge the adequacy of results for these systems. However, the success of the model in the description of $A=2$–5 systems[10,16] and the agreement of model and experimental results in ^{6}He, ^{6}Li, and ^{6}Be suggests that reasonable results will also be obtained in ^{6}n, ^{6}H, and ^{6}C.

The model also predicts (J^π, T) values which are in agreement with those of Ajzenberg-Selove.[1] The model confirms the tentative ground state assignment of $(0^+,3)$ for ^{6}n and ^{6}C. In addition, $(1^+,2)$ values are suggested for ^{6}H and ^{6}B ground states. These assignments are not provided in Ref. 1.

V. ^{6}He SPECTRUM

Model calculations for the ^{6}He spectrum are summarized and compared with available data in Table II. The

TABLE I. $A=6$ ground state properties.

System	(J^π,T) Experiment	(J^π,T) Model	Binding energy (MeV)[a] Experiment	Model	SK[b]	BLM[c]
^{6}n	((0$^+$),3)	(0$^+$,3)		−6.499		
^{6}H	(?,2)	(1$^+$,2)		7.144		
^{6}He	(0$^+$,1)	(0$^+$,1)	29.265	30.066	27.9	31.4
^{6}Li	(1$^+$,0)	(1$^+$,0)	31.993	30.973	35.3	31.5
^{6}Be	(0$^+$,1)	(0$^+$,1)	26.923	26.156	24.3	28.9
^{6}B		(1$^+$,2)		5.418	−6.6	
^{6}C	((0$^+$),3)	(0$^+$,3)		−16.405		

[a]A negative value indicates that the system is unbound.
[b]Reference 6.
[c]Reference 7.

compilation of Ref. 1 has assigned J^π values to only the lowest two levels in ^{6}He. Three additional levels are suggested in Ref. 1, but no J^π assignments are provided. The model predicts energy levels in close agreement to those of Ref. 1. However, additional $T=0$ and $T=1$ levels are suggested by the model. The model yields the ground and first excited states as having the same J^π values as suggested in Ref. 1. In addition, the first excited state $J^\pi=2^+$ is predicted to have a $T=1$ assignment. Two additional states are predicted to lie above the (2$^+$,1) 1.8 MeV level and below the 13.6 MeV level—i.e., the (1$^+$,0) 4.4 MeV and (0$^+$,1) 6.5 MeV levels. These levels have also been suggested in ^{6}Be (see Sec. VII).

The model also suggests the following (J^π,T) assignments for the three other levels identified in Ref. 1: (0$^+$,1) 13.6 MeV, (3$^+$,0) 15.5 MeV, and (4$^-$,1) 23.2 MeV. Model predictions for these states are within 1.1 MeV of the experimental positions. In addition, the following new levels are predicted: (2$^+$,1) 23.3 MeV, (2$^-$,1) 25.6 MeV, (3$^-$,1) 27.9 MeV, and (4$^+$,1) 31.5 MeV.

VI. ^{6}Li SPECTRUM

The levels in ^{6}Li are more established than those in other $A=6$ systems. The model results are within 3 MeV of the experimental values and are summarized in Table III. The model predicts the (2$^-$,1) 21.0 MeV and (0$^-$,1) 21.5 MeV levels to lie at 21.4 MeV and 21.1 MeV, respectively. In addition, the (4$^+$,0) 23.0 MeV and (4$^-$,1) 25.0 MeV levels are predicted to lie at 24.6 MeV and 22.3 MeV, respectively. With the exception of these four levels, the model and experimental energy level order agree.

VII. ^{6}Be SPECTRUM

Table IV summarizes the model results for the ^{6}Be system and compares these results to available data and to the calculations of Hofmann and Zahn.[5] The calculations of Hofmann and Zahn (HZ) suggest four levels [1$^+$ (3.3 MeV), 0$^+$ (4.6 MeV), 2$^+$ (19.6 MeV), and 4$^+$ (>20.6 MeV)] which are not included in the compilation of Ref. 1. However, the HZ calculation does not predict the 2$^-$ (26.0 MeV) level.[1] The model calculations presented herein predict all levels of Ref. 1 and also obtain the four new levels suggested by Hofmann and Zahn. In addition, the model predicts two additional levels—i.e., the (0$^+$,1) 12.8 MeV and (3$^+$,0) 14.7 MeV levels.

The HZ levels were derived from microscopic multichannel resonating group calculations. Resonating group results for phase shifts, differential cross sections,

TABLE II. Levels in the ^{6}He system. All energies are in MeV.

(J^π,T)	Experiment	This work
(0$^+$,1)	0.00	0.00
(2$^+$,1)	1.797±0.025	2.0
(1$^+$,0)	a	4.4
(0$^+$,1)	a	6.5
(0$^+$,1)	13.6±0.5	13.9
(3$^+$,0)	15.5±0.4	14.4
(2$^+$,1)	a	23.3
(4$^-$,1)	23.2±0.7	23.7
(2$^-$,1)	a	25.6
(3$^-$,1)	a	27.9
(4$^+$,1)	a	31.5

[a]Not observed or reported in Ref. 1.

TABLE III. Levels in the ^{6}Li system. All energies are in MeV.

J^π,T	Experiment	This work
(1$^+$,0)	0.0	0.0
(3$^+$,0)	2.186±0.002	2.5
(0$^+$,1)	3.56288±0.0001	3.3
(2$^+$,0)	4.31±0.022	4.0
(2$^+$,1)	5.366±0.015	5.4
(1$^+$,0)	5.65±0.05	5.4
(3$^+$,0)	15.8	15.7
(2$^-$,1)	21.0	21.4
(0$^-$,1)	21.5	21.1
(4$^+$,0)	23±2	24.6
(4$^-$,1)	25±1	22.3
(3$^-$,0)	26.6±0.4	27.0
(3$^+$,0)	31.0	30.8

TABLE IV. Levels in the ^{6}Be system. All energies are in MeV.

(J^π, T)	Experiment	This work	Hofmann and Zahn[b]
$(0^+,1)$	0.00	0.0	a
$(2^+,1)$	1.67	1.6	2.3
$(1^+,0)$	a	3.3	3.3
$(0^+,1)$	a	5.4	4.6
$(0^+,1)$	a	12.8	a
$(3^+,0)$	a	14.7	a
$(2^+,1)$	a	23.8	19.6
$(4^-,1)$	23	24.3	18.6–27.6
$(2^-,1)$	26	26.4	a
$(3^-,1)$	27	29.0	18.6–27.6
$(4^+,1)$	a	33.0	>20.6

[a]Not observed or reported in Refs. 1 or 5.
[b]Reference 5 levels have been shifted by 0.59 MeV to account for the shift between the ^{5}Li + p channel and the experimental ground state.

and analyzing power data were used to derive the level sequence of Table IV. The HZ calculations included the ^{5}Li + p, ^{5}Li* (first excited state) + p, ^{3}He + ^{3}He, and ^{5}Li* (second excited state) + p fragmentation channels.

VIII. DISCUSSION OF ^{6}He, ^{6}Li, AND ^{6}Be SPECTRA

The calculated spectra of the ^{6}He, ^{6}Li, and ^{6}Be nuclei exhibit more structure than suggested in the compilation of Ajzenberg-Selove. In addition, the model predicts that the level spectra of these nuclei exhibit considerable similarity. For example, the similarity between the ^{6}He and ^{6}Be spectra is striking, and is reminiscent of the similarity of the ^{4}H and ^{4}Li level spectra.[11] The similarity of model spectra could easily be altered by including channel contributions in the dynamical equations [Eq. (7)].

In the $A=4$ system, the level order and spacing did not change significantly with the inclusion of reaction channels into the dynamical equations.[10] In ^{4}He, for example, the p + ^{3}H, n + ^{3}He, and d + ^{2}H channels were included in the calculations. In ^{6}Li, for example, channel calculations (limited to binary breakup channels) would include ^{4}He + d, ^{5}Li + n, and ^{3}He + ^{3}H channels. The possibility also exists for clusters involving excited states of ^{4}He and ^{5}Li. These possibilities make the ^{6}Li calculations considerably more complicated than those in ^{4}He. Including channels in the dynamical equations may also shift the order and relative position of the predicted levels. However, the work of Hofmann and Zahn[5] included channels, and their level orders and positions are similar to those calculated herein.

The inclusion of binary reaction channels in our model calculations would lead to level width as well as scattering information. This information would lead to an additional assessment of the adequacy of the model calculations. However, the inclusion of channels in our model is beyond the scope of the present calculation.

IX. CONCLUSIONS

The present study of the $A=6$ system yields a solution which is consistent with previous $A=2$–5 calculations. The model results for the ^{6}n, ^{6}H, ^{6}He, ^{6}Li, ^{6}Be, ^{6}B, and ^{6}C systems are in reasonable agreement with available data. Ground state binding energy calculations for ^{6}He, ^{6}Li, and ^{6}Be are within 4 percent of the experimental values. In addition, the ^{6}He, ^{6}Li, and ^{6}Be level sequences are consistent with the compilation of Ajzenberg-Selove. However, additional levels are suggested in both ^{6}He and ^{6}Be.

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