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Relativistic plane-wave impulse approximation for nuclear inelastic scattering of protons and electrons

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We establish the general form of inelastic scattering amplitudes based on invariance principles and derive simple relations between these amplitudes and spin-transfer observables. We then examine a plane-wave relativistic impulse approximation for (p,p′) which is a simple extension of the relativistic impulse approximation for elastic scattering. We show under what approximations this amplitude reduces to the standard (nonrelativistic) impulse approximation and then examine the novel features of the free relativistic impulse approximation which results from less restrictive assumptions. We also derive free relativistic impulse approximation inelastic electron scattering form factors and then use our (p,p′) and (e,e′) formulations to extract nuclear structure information for the first two 1^+ levels in ^{12}C. We consider possible experimental signatures of the strong scalar and timelike vector potentials of modern relativistic theories.

I. INTRODUCTION

Following the successful application to intermediate energy, proton-nucleus elastic scattering, the relativistic impulse approximation is currently being extended to inelastic scattering. The basic elements of the inelastic approach have already been established.[1] A computer program DRIA (Ref. 2) has been written to calculate inelastic observables in the relativistic impulse approximation and a considerable amount of experience in the application of the model to experimental data is currently being amassed. The process of distilling understanding from experience of this kind is greatly facilitated by an approximate version of the full theoretical treatment, which is simple enough that the dynamical dependences on the various input quantities are obvious, but at the same time is comprehensive enough so that these dynamical dependences are at least qualitatively correct. The plane-wave approximation to the standard distorted wave impulse approximation has repeatedly been shown to be useful in this regard. Moss,[3] for example, has used the plane-wave approximation to demonstrate in a pedagogically appealing way the relationship between inelastic scattering spin observables and the nucleon-nucleon amplitude. In many cases the simple relations which emerge in the plane-wave limit persist when distortion effects are included.

In a very similar spirit, we develop here a plane-wave version of the relativistic impulse approximation which reveals the new elements of the overall approach. We find, in particular, that the structure of the formulation immediately suggests a generalization to the standard nonrelativistic approaches to proton-nucleus inelastic scattering. This generalization consists of including the effects of nuclear *currents* on the transition amplitude in just the way such current contributions are usually included in standard microscopic treatments of electron-nucleus inelastic scattering. We also show that these current contributions *should* be included whether or not we assume that the nuclear dynamics are governed by strong scalar and timelike vector potentials which characterize modern relativistic approaches. However, we also find that the magnitudes of the current terms are sensitive to the presence of such potentials (as is also the case in elastic electron scattering[4]) and that observables sensitive to the currents will in turn reflect relativistic nuclear dynamics to some degree. The present plane-wave formulation permits the straightforward identification of such observables. We also treat inelastic electron scattering in the same relativistic formulation which allows us to make a systematic comparison between proton- and electron-induced inelastic processes.

II. GENERAL STRUCTURE OF THE INELASTIC AMPLITUDES

We begin by examining the general structure of the nucleon-nucleus inelastic scattering amplitude as determined by the requirements of parity, rotational, and time-reversal invariance.[5] For a transition from a 0^+ initial state to a final state of spin parity J^π, we may write the amplitude as

$$T_M(m',m)=\langle J^\pi M;m'\,|\,\mathcal{T}\,|\,0^+;m\rangle\ , \tag{1}$$

where $\mathcal{T}$ is the projectile-target interaction, m (m') is the initial (final) projectile spin projection, and M is the spin projection of the final nuclear state. We now define the following right-handed coordinate system:

$$\begin{aligned}\hat{\mathbf{q}}&=(\mathbf{k}_f-\mathbf{k}_i)/\,|\,\mathbf{k}_f-\mathbf{k}_i\,|\ ,\\ \hat{\mathbf{p}}&=\tfrac{1}{2}(\mathbf{k}_f+\mathbf{k}_i)/\,|\,\mathbf{k}_f+\mathbf{k}_i\,|\ ,\\ \hat{\mathbf{n}}&=\hat{\mathbf{p}}\times\hat{\mathbf{q}}=\mathbf{k}_i\times\mathbf{k}_f/\,|\,\mathbf{k}_i\times\mathbf{k}_f\,|\ ,\end{aligned} \tag{2}$$

where $\mathbf{k}_i$ ($\mathbf{k}_f$) is the initial (final) projectile momentum. (We have ignored the nuclear excitation energy so that $k_i = k_f$.) As shown in Appendix A, when we choose $\hat{\mathbf{q}}$ as the quantization axis, the general structure of the transition amplitude depends on whether the final state has natural $[\pi = (-1)^J]$ or unnatural $[\pi = -(-1)^J]$ parity. It is convenient to express the transition amplitude of Eq. (1) as an operator in the projectile spin space. We then write $T_M = \sum_i t_i^M \theta_i$, where the t_i^M are not active in the projectile spin space and the θ_i are. We choose

$$\{\theta_i\} = \{\mathbf{1}, \sigma_n, \sigma_p, \sigma_q\}$$
$$\equiv \{\mathbf{1}, \sigma\cdot\hat{\mathbf{n}}, \sigma\cdot\hat{\mathbf{p}}, \sigma\cdot\hat{\mathbf{q}}\}$$

and find for natural parity transitions

$$T_0 = A_0 \mathbf{1} + B_0 \sigma_n ,$$
$$T_M^{(+)} = A_M^{(+)} \mathbf{1} + B_M^{(+)} \sigma_n ,$$
$$T_M^{(-)} = C_M^{(-)} \sigma_p + D_M^{(-)} \sigma_q ,$$

while for unnatural parity transitions (3)

$$T_0 = C_0 \sigma_p + D_0 \sigma_q ,$$
$$T_M^{(+)} = C_M^{(+)} \sigma_p + D_M^{(+)} \sigma_q ,$$
$$T_M^{(-)} = A_M^{(-)} \mathbf{1} + B_M^{(-)} \sigma_n ,$$

where we have defined

$$T_M^{(+)} \equiv (i/\sqrt{2})(T_M + T_{-M})$$

and

$$T_M^{(-)} \equiv (-1/\sqrt{2})(T_M - T_{-M})$$

for $M > 0$.

The time-reversal properties of the inelastic scattering amplitudes are also derived in Appendix A. These can be summarized as follows: for A_M, B_M, and C_M;

$$\eta_i + M = \text{even} ,$$

while for D_M; (4)

$$\eta_i + M = \text{odd} ,$$

where $\eta_i = 0$ (1) implies that the target-space operator giving rise to the nuclear matrix element implicit in the A, B, C, or D is time-reversal even (odd). (See Appendix A.) For a given value of $M > 0$, these relations apply to both $T_M^{(+)}$ and $T_M^{(-)}$.

We may now express all inelastic observables in a simple form using the results of Eq. (3). We define the spin transfer observables

$$ID_{ij} = \tfrac{1}{2}\,\mathrm{tr} \sum_{\text{all } M} \theta_i T_M \theta_j T_M^\dagger$$
$$= \tfrac{1}{2}\,\mathrm{tr}\theta_i T_0 \theta_j T_0^\dagger$$
$$+ \tfrac{1}{2}\,\mathrm{tr} \sum_{M>0} \left(\theta_i T_M^{(+)} \theta_j T_M^{(+)\dagger} + \theta_i T_M^{(-)} \theta_j T_M^{(-)\dagger}\right) , \quad (5)$$

where

$$I = \tfrac{1}{2}\,\mathrm{tr} \sum_{\text{all } M} T_M T_M^\dagger .$$

It is useful to recall the relation between some familiar observables and those defined in Eq. (5). For instance, the polarization is given by $P = D_{n0}$ while the analyzing power is given by $A_y = D_{0n}$. Similarly the spin-rotation function[6] is given by $Q = D_{qp}$; we also define a complementary quantity[7] $B \equiv -D_{pq}$. Certain combinations of these particular observables are very simple functions of the amplitudes presented in Eq. (3). For example,

$$I\Sigma_S = I[i(Q+B) + (P + A_y)] = 4\sum_M A_M B_M^* , \quad (6a)$$

$$I\Delta_S = I[(Q-B) + i(P - A_y)] = 4\sum_M C_M^* D_M , \quad (6b)$$

which defines the spin *sum* function, Σ_S, and the spin *difference* function, Δ_S. We also have

$$I(1 + D_{nn} + D_{pp} + D_{qq}) = 4\sum_M |A_M|^2 , \quad (6c)$$

$$I(1 + D_{nn} - D_{pp} - D_{qq}) = 4\sum_M |B_M|^2 , \quad (6d)$$

$$I(1 - D_{nn} + D_{pp} - D_{qq}) = 4\sum_M |C_M|^2 , \quad (6e)$$

$$I(1 - D_{nn} - D_{pp} + D_{qq}) = 4\sum_M |D_M|^2 . \quad (6f)$$

We observe that no other independent combinations of observables (with the final nuclear polarization being unobserved) can be constructed since combinations involving A_M or B_M and C_M or D_M (e.g., $A_M C_M^*$) never occur in Eq. (3) due to parity and rotational invariance. It is interesting to note that the spin difference function defined in Eq. (6b) involves the interference of amplitudes with *opposite* time-reversal properties. [See Eq. (4).]

III. THE (RELATIVISTIC) PLANE-WAVE IMPULSE APPROXIMATION

We now formulate a relativistic plane-wave impulse approximation for nucleon-nucleus inelastic scattering which is a simple extension of the relativistic impulse approximation for *elastic* scattering. We work in the Breit frame with the three-momenta defined in Fig. 1. Assuming that the bound state wave functions have a simple harmonic time dependence, the explicit treatment of the time integrations is trivial[1] and the inelastic amplitude can be written as

$$T_{fi} = \int \frac{d\mathbf{P}}{(2\pi)^3} u_p^\dagger(\mathbf{K}+\mathbf{q}/2)\Psi_f^\dagger(\mathbf{P}-\mathbf{K}/A-\mathbf{q}/2)\hat{t}_{\mathrm{NN}}(s,t)$$
$$\times u_p(\mathbf{K}-\mathbf{q}/2)\Psi_i(\mathbf{P}-\mathbf{K}/A+\mathbf{q}/2) , \quad (7)$$

where the u_p's are the usual Dirac free spinors for the projectile, the Ψ's are four-component spinors for the nucleus, and the NN t matrix is given by

$$\hat{t}_{\mathrm{NN}}(s,t) = (-8\pi i p_{\mathrm{c.m.}}/E_{\mathrm{c.m.}})\gamma^0(1)\gamma^0(2)\hat{F}_{\mathrm{NN}}(s,t) . \quad (8)$$

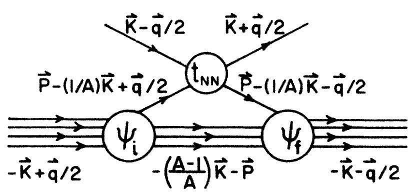

FIG. 1. Kinematical variables used to describe the (p,p′) reaction.

In Eq. (8), $p_{\text{c.m.}}$ and $E_{\text{c.m.}}$ are the nucleon momentum and energy in the nucleon-nucleon center of momentum frame, γ^0 is the usual Dirac γ matrix and, F_{NN} is the relativistic invariant representation of the free NN amplitude which depends on the kinematical invariants s and t. In the present work, we take F_{NN} to have the following form:[8,9]

$$F_{\text{NN}}=F_S+\gamma^{\mu}(1)\gamma_{\mu}(2)F_V+\gamma^5(1)\gamma^5(2)F_P +\gamma^5(1)\gamma^5(2)\gamma^{\mu}(1)\gamma_{\mu}(2)F_A+\sigma^{\mu\nu}(1)\sigma_{\mu\nu}(2)F_T \ . \quad (9)$$

The carets over F_{NN} and t_{NN} in Eqs. (7) and (8) indicate that they are taken to be operators active in the space of the nuclear bound states. This is the full relativistic plane-wave impulse approximation. With relatively minor approximations (to be discussed in Sec. V) it can be evaluated as it stands given some model for the four-component bound state spinors.

IV. THE STANDARD IMPULSE APPROXIMATION

We now show how the standard nonrelativistic plane-wave impulse approximation can be recovered from the relativistic formulation. For on-shell, positive-energy nucleons, we have the following identity[9]

$$M_{\text{NN}}(1,2)=u_1'^{\dagger}u_2'^{\dagger}t_{\text{NN}}u_1u_2 \ , \quad (10)$$

where M_{NN} is the representation of the NN t matrix *in the space of Pauli spinors* (in contrast to t_{NN} which is the representation in the space of Dirac spinors). The quantity M_{NN} is frequently expressed in terms of the Wolfenstein amplitudes; in the NN center of momentum frame, we may write

$$M_{\text{NN}}(1,2)=a+\sigma_1\cdot\sigma_2 b+iq(\sigma_1+\sigma_2)\cdot\hat{\mathbf{n}}c +\sigma_1\cdot\hat{\mathbf{q}}\sigma_2\hat{\mathbf{q}}q^2d+\sigma_1\cdot\hat{\mathbf{p}}\sigma_2\hat{\mathbf{p}}e \ . \quad (11)$$

As will be discussed in Sec. VI, we decompose the full nuclear spinors in terms of four-component, single-particle, bound state wave functions which for the moment we construct according to the following prescription:

$$\psi(\mathbf{p})=N\begin{bmatrix}1\\ \dfrac{\sigma\cdot\mathbf{p}}{E+m}\end{bmatrix}\phi(\mathbf{p})=Nu(\mathbf{p})\phi(\mathbf{p}) \ , \quad (12)$$

where $\phi(\mathbf{p})$ is the usual two-component (Schrödinger) wave function and N is a normalization constant. The relation between upper and lower components in Eq. (12) is clearly that prescribed by the free Dirac equation,

$$(\gamma^0E-\gamma\cdot\mathbf{p}-m)\Psi=0 \ .$$

With the preceding definitions, we may rewrite Eq. (7) as

$$T_{fi}=N_fN_i\int\frac{d\mathbf{P}}{(2\pi)^3}\Phi_f^{\dagger}(\mathbf{P}-\mathbf{K}/A-\mathbf{q}/2)[u_p^{\dagger}(\mathbf{K}+\mathbf{q}/2)u_t^{\dagger}(\mathbf{P}-\mathbf{K}/A-\mathbf{q}/2)\hat{t}_{\text{NN}}(s,t) \times u_t(\mathbf{P}-\mathbf{K}/A+\mathbf{q}/2)u_p(\mathbf{K}-\mathbf{q}/2)]\Phi_i(\mathbf{P}-\mathbf{K}/A+\mathbf{q}/2) \ , \quad (13)$$

where the Φ's are the usual two-component nuclear wave functions.

If we ignore the $\mathbf{P}$ dependence inside the square brackets, we can use the identity of Eq. (10) to obtain the standard impulse approximation (SIA):

$$T_{fi}=\int\frac{d\mathbf{p}'}{(2\pi)^3}\Phi_f^{\dagger}(\mathbf{p}'-\mathbf{q}/2)M_{\text{NN}}\Phi_i(\mathbf{p}'+\mathbf{q}/2) \ , \quad (14a)$$

where M_{NN} is the Pauli representation of the NN amplitude in the appropriate frame of reference.[9] In configuration space, we have

$$T_{fi}=\langle f\,|\,M_{\text{NN}}(q)e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,i\rangle \ . \quad (14b)$$

In evaluating the amplitude of Eq. (14b), it is convenient to write the Pauli representation of M_{NN} in Eq. (11) somewhat differently. We have

$$M_{\text{NN}}(1,2)=\mathcal{A}(q,\sigma_1)+\sigma_2\cdot\mathcal{B}(q,\sigma_1) \ , \quad (15a)$$

where

$$\mathcal{A}=a+i\sigma_1\cdot\hat{\mathbf{n}}qc \ ,$$
$$\mathcal{B}=\sigma_1 b+i\hat{\mathbf{n}}qc+\hat{\mathbf{q}}\sigma_1\cdot\hat{\mathbf{q}}q^2d+\hat{\mathbf{p}}\sigma_1\cdot\hat{\mathbf{p}}e \ . \quad (15b)$$

Then, remembering that we are considering transitions from 0^+ initial states, we have

$$T_{fi}=\mathcal{A}(q,\sigma_p)\langle JM\,|\,e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle +\mathcal{B}(q,\sigma_p)\cdot\langle JM\,|\,\sigma e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle \ , \quad (16)$$

where $|\,JM\rangle$ is the final nuclear state and σ_p is the Pauli spin operator for the projectile. Since we have chosen the quantization axis to lie along $\hat{\mathbf{q}}$, the possible spin projections for the final state are $M=0$ for the first term on the right-hand side of Eq. (16) while $M=0, \pm 1$ is allowed for the second term. We then have

$$T_0=T_q=\mathscr{A}\langle J0\,|\,e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle+\mathscr{B}\cdot\hat{\mathbf{q}}\hat{\mathbf{q}}\langle J0\,|\,\sigma e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle\ , \tag{17a}$$

$$T_1^{(+)}=\frac{i}{\sqrt{2}}(\mathscr{B}\cdot\hat{\mathbf{e}}^{\dagger}_{+1}\langle J,+1\,|\,\sigma_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle+\mathscr{B}\cdot\hat{\mathbf{e}}^{\dagger}_{-1}\langle J,-1\,|\,\sigma_{-1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle)\ , \tag{17b}$$

$$T_1^{(-)}=-\frac{1}{\sqrt{2}}(\mathscr{B}\cdot\hat{\mathbf{e}}^{\dagger}_{+1}\langle J,+1\,|\,\sigma_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle-\mathscr{B}\cdot\hat{\mathbf{e}}^{\dagger}_{-1}\langle J,-1\,|\,\sigma_{-1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle)\ , \tag{17c}$$

where $\hat{\mathbf{e}}_0=\hat{\mathbf{q}}$, $\hat{\mathbf{e}}_{\pm1}=\mp(\hat{\mathbf{n}}+i\hat{\mathbf{p}})/\sqrt{2}$ are the usual spherical unit vectors, and $\sigma_M=\sigma\cdot\hat{\mathbf{e}}_M$. For M greater than one, $T_M^{(+)}=T_M^{(-)}=0$ in this approximation where all transverse operators are at most rank one.

In proceeding further, it is useful to define the following nuclear transition densities (see Appendix B for details): "Coulomb" form factor

$$\rho_J\equiv\langle J,0\,|\,e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,0,0\rangle\ , \tag{18a}$$

longitudinal form factor

$$\Sigma_J^L\equiv\langle J,0\,|\,\sigma_0e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,0,0\rangle\ , \tag{18b}$$

transverse form factors

$$\Sigma_{J,0}^T\equiv\langle J,+1\,|\,\sigma_{+1}(e^{-i\mathbf{q}\cdot\mathbf{r}})_{L=J}\,|\,0,0\rangle\quad(\text{natural }\pi)\ ,$$
$$\Sigma_{J,1}^T\equiv\langle J,+1\,|\,\sigma_{+1}(e^{-i\mathbf{q}\cdot\mathbf{r}})_{L=J\pm1}\,|\,0,0\rangle\quad(\text{unnatural }\pi)\ . \tag{18c}$$

For *unnatural* parity transitions, $\rho_J=\Sigma_{J,0}^T=0$ and we can write

$$T_0=T_q=\mathscr{B}\cdot\hat{\mathbf{q}}\Sigma_J^L=(b+q^2d)\Sigma_J^L\sigma_q\ , \tag{19a}$$

$$T_{M=1}^{(+)}=T_p=\mathscr{B}\cdot\hat{\mathbf{p}}\Sigma_{J,1}^T=(b+e)\Sigma_{J,1}^T\sigma_p\ , \tag{19b}$$

$$T_{M=1}^{(-)}=T_n=\mathscr{B}\cdot\hat{\mathbf{n}}\Sigma_{J,1}^T=(iqc+b\sigma_n)\Sigma_{J,1}^T\ . \tag{19c}$$

In deriving these expressions, we have made use of the fact that

$$\langle J,\pm1\,|\,\sigma_{\pm1}e^{i\mathbf{q}\cdot\mathbf{r}}\,|\,0,0\rangle=\Sigma_{J,1}^T$$

for unnatural parity transitions and the relations $i/\sqrt{2}(\hat{\mathbf{e}}_{+1}+\hat{\mathbf{e}}_{-1})=\hat{\mathbf{p}}$ and $-1/\sqrt{2}(\hat{\mathbf{e}}_{+1}-\hat{\mathbf{e}}_{-1})=\hat{\mathbf{n}}$.

By comparing with Eq. (3), we can establish the following correspondence:

$$C_q=0,\ \ D_q=(b+q^2d)\Sigma_J^L\ ,$$
$$C_p=(b+e)\Sigma_{J,1}^T,\ \ D_p=0\ , \tag{20}$$
$$A_n=iqc\Sigma_{J,1}^T,\ \ B_n=b\Sigma_{J,1}^T\ ,$$

where we have defined $C_q\equiv C_0$, $C_p\equiv C_1^{(+)}$, $A_n\equiv A_1^{(-)}$, etc., as suggested by Eq. (19). We note that two of the amplitudes allowed by parity and rotational invariance, namely C_0 and D_p, are zero in the SIA.

For natural parity transitions, $\Sigma_J^L=\Sigma_{J,1}^T=0$, and we can write

$$T_q=\mathscr{A}\rho_J=(a+iqc\sigma_n)\rho_J\ , \tag{21a}$$

$$T_p=-i\mathscr{B}\cdot\hat{\mathbf{n}}\Sigma_{J,0}^T=(-i\hat{\mathbf{q}}\times\mathscr{B})\cdot\hat{\mathbf{p}}\Sigma_{J,0}^T$$
$$=-i(iqc+b\sigma_n)\Sigma_{J,0}^T\ , \tag{21b}$$

$$T_n=+i\mathscr{B}\cdot\hat{\mathbf{p}}\Sigma_{J,0}^T=(-i\hat{\mathbf{q}}\times\mathscr{B})\cdot\hat{\mathbf{n}}\Sigma_{J,0}^T$$
$$=+i(b+e)\Sigma_{J,0}^T\sigma_p\ , \tag{21c}$$

where, in this instance, we have used the following identity:

$$\langle J,\pm1\,|\,\sigma_{\pm1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,0,0\rangle=\pm\Sigma_{J,0}^T$$

which is valid for natural parity transitions. The correspondence of the natural parity transition amplitude with Eq. (3) is

$$A_q=a\rho_J,\ \ B_q=iqc\rho_J\ ,$$
$$A_p=qc\Sigma_{J,0}^T,\ \ B_p=-ib\Sigma_{J,0}^T\ , \tag{22}$$
$$C_n=i(b+e)\Sigma_{J,0}^T,\ \ D_n=0\ ,$$

where, again, one of the allowed amplitudes, D_n, is found to be zero in the SIA.

We may summarize these results by writing the following expression which is valid for both natural and unnatural parity transitions:

$$T_{fi,M}=(\mathscr{A}\rho_J+\mathscr{B}\cdot\hat{\mathbf{q}}\Sigma_J^L)\delta_{M,q}+\sum_{j=n,p}(-i\hat{\mathbf{q}}\times\mathscr{B}\Sigma_{J,0}^T+\mathscr{B}\Sigma_{J,1}^T)\cdot\hat{\mathbf{e}}_j\delta_{M,j}\ , \tag{23}$$

where we understand that Σ_J^L and $\Sigma_{J,1}^T$ (ρ_J and $\Sigma_{J,0}^T$) are zero for natural (unnatural) parity transitions. If all form factors in Eq. (23) were unity, then the observables calculated using Eq. (5) would be just those for free NN scattering. This fact lies at the heart of the close relationship between the inelastic scattering and NN observables.

We note that the spin-difference function, Δ_S defined in Eq. (6b), is identically zero in the SIA due to the correspondences appearing in Eqs. (20) and (22), even though symmetry considerations do *not* require this result. This is interesting in light of the fact that measurements show that $P-A_y$ is large for some inelastic transitions,[10] suggesting that some important physics has been dropped in the series of approximations leading to the SIA. In what follows, we demonstrate that this is indeed the case and that a straightforward extension of the SIA results in a new class of terms which (among other things) *can* yield a nonzero spin-difference function.

V. THE FREE RELATIVISTIC IMPULSE APPROXIMATION

The proposed extension consists of rewriting Eq. (13) by moving the square brackets in as follows:

$$T_{fi}=N_fN_i\int\frac{d\mathbf{P}}{(2\pi)^3}\Phi_f^\dagger(\mathbf{P}-\mathbf{K}/A-\mathbf{q}/2)u_p^\dagger(\mathbf{K}+\mathbf{q}/2)u_t^\dagger(\mathbf{P}-\mathbf{K}/A-\mathbf{q}/2)$$
$$\times[\hat{t}_{\rm NN}(s,t)]u_t(\mathbf{P}-\mathbf{K}/A+\mathbf{q}/2)u_p(\mathbf{K}-\mathbf{q}/2)\Phi_i(\mathbf{P}-\mathbf{K}/A+\mathbf{q}/2) \quad (24)$$

and again dropping the **P** dependences inside the square brackets. In making this approximation, we have retained the dependence of the lower components of the target nucleon on the nuclear dynamics which bind it. In contrast, the SIA of Eq. (14) implicitly includes only the lower component dependences arising from kinematic effects such as recoil or boosting to an appropriate reference frame. This is equivalent to setting the lower components of the bound state wave function to *zero* in the nuclear rest frame. We refer to the expression in Eq. (24) as the free relativistic impulse approximation (FRIA) since the free space relation between the upper and lower components of the target nucleon is assumed [Eq. (12)]. We note that such an approach is implicit in standard microscopic treatments of electron-nucleus inelastic scattering.[11]

We now wish to rewrite Eq. (24) so as to facilitate comparison with the SIA amplitude appearing in Eq. (23). For notational convenience we assume an infinitely massive target ($1/A\to 0$) in which case the Breit frame and the nuclear rest frame coincide.

We may now express Eq. (24) as

$$T_{fi}=u_p^\dagger(\mathbf{K}+\mathbf{q}/2)\langle\tilde{J}\tilde{M}\,|\,\hat{t}_{\rm NN}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,\tilde{0}\tilde{0}\rangle u_p(\mathbf{K}-\mathbf{q}/2)\,, \quad (25)$$

where the tildes over the bra and ket indicate that they are four-component target wave functions related to the usual two-component bras and kets by Eq. (12) which, in configuration space, becomes

$$|\,\tilde{J}\tilde{M}\rangle=N\begin{bmatrix}1\\ \dfrac{-i\sigma\cdot\nabla}{E+m}\end{bmatrix}|\,JM\rangle\,. \quad (26)$$

It is convenient, when evaluating nuclear matrix elements, to cast the Dirac matrix content of $\hat{t}_{\rm NN}$ in a form which is different from the manifestly covariant expression in Eqs. (8) and (9). Specifically we have

$$\hat{t}_{\rm NN}(q)=\sum_i(f_i+\sigma_1\cdot\sigma_2 g_i)\Gamma_i(1)\Gamma_i(2)\,, \quad (27)$$

where the 2×2 matrices, $\{\Gamma_i\}$, act only in component space and are defined by

$$\Gamma_1=\begin{bmatrix}1&0\\0&1\end{bmatrix},\quad \Gamma_2=\begin{bmatrix}1&0\\0&-1\end{bmatrix},$$
$$\Gamma_3=\begin{bmatrix}0&1\\1&0\end{bmatrix},\quad \Gamma_4=\begin{bmatrix}0&1\\-1&0\end{bmatrix}. \quad (28)$$

The f_i and g_i are determined by the NN invariants appearing in Eqs. (8) and (9).

$$f_1=t_V,\quad f_2=t_S,\quad f_3=t_A,\quad f_4=t_P\,,$$
$$g_1=-t_A,\quad g_2=2t_T,\quad g_3=-t_V,\quad g_4=2t_T\,. \quad (29)$$

The FRIA amplitudes may then be written as

$$T_{fi}=\sum_{i=1}^{4}(f_iu_p^\dagger\Gamma_iu_p\langle\tilde{J}\tilde{M}\,|\,\Gamma_ie^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,\tilde{0}\tilde{0}\rangle$$
$$+g_iu_p^\dagger\sigma_p\Gamma_iu_p\cdot\langle\tilde{J}\tilde{M}\,|\,\sigma\Gamma_ie^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,\tilde{0}\tilde{0}\rangle) \quad (30a)$$

or

$$T_{fi}=\sum_{i=1}^{4}[\mathscr{A}^i(q,\sigma_p)\langle\tilde{J}\tilde{M}\,|\,\Gamma_ie^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,\tilde{0}\tilde{0}\rangle$$
$$+\mathscr{B}^i(q,\sigma_p)\cdot\langle\tilde{J}\tilde{M}\,|\,\sigma\Gamma_ie^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,\tilde{0}\tilde{0}\rangle] \quad (30b)$$

which defines the quantities $\mathscr{A}^i$ and $\mathscr{B}^i$. The expression in Eq. (30b) has been written so as to facilitate comparison with the SIA amplitude appearing in Eq. (16). The momentum dependence of the NN amplitude as well as the projectile spin dependence are carried in the $\mathscr{A}$ and $\mathscr{B}$ quantities in Eq. (16) and in their relativistic generalizations in Eq. (30b). The nuclear transition densities appearing in the SIA are likewise generalized in the FRIA and in analogy with Eq. (18) we define the following quantities:

$$\rho_J^i\equiv\langle\tilde{J},\tilde{0}\,|\,\Gamma_ie^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,\tilde{0},\tilde{0}\rangle\,, \quad (31a)$$

$$\Sigma_J^{i,L}\equiv\langle\tilde{J},\tilde{0}\,|\,\sigma_0\Gamma_ie^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,\tilde{0},\tilde{0}\rangle\,, \quad (31b)$$

$$\Sigma_{J,0}^{i,T}\equiv\langle\tilde{J},+\tilde{1}\,|\,\sigma_{+1}\Gamma_i(e^{-i\mathbf{q}\cdot\mathbf{r}})_{L=J}\,|\,\tilde{0},\tilde{0}\rangle\,,$$
$$\Sigma_{J,1}^{i,T}\equiv\langle\tilde{J},+\tilde{1}\,|\,\sigma_{+1}\Gamma_i(e^{-i\mathbf{q}\cdot\mathbf{r}})_{L=J\pm1}\,|\,\tilde{0},\tilde{0}\rangle\,. \quad (31c)$$

The selection rules for these quantities are trivially obtained from those of their SIA counterparts [Eq. (18)] by noting that upper and lower components of Dirac spinors have opposite parities. Therefore, matrix elements containing Γ_1 or Γ_2, which are diagonal in component space, have the same selection rules as the SIA matrix elements, but those containing Γ_3 or Γ_4, which are off diagonal, have the opposite parity selection rules. Thus, only the following matrix elements can be nonzero:

	$i=1,2$	$i=3,4$
Natural parity	$\rho_J^i,\Sigma_{J,0}^{i,T}$	$\Sigma_J^{i,L},\Sigma_{J,1}^{i,T}$
Unnatural parity	$\Sigma_J^{i,L},\Sigma_{J,1}^{i,T}$	$\rho_J^i,\Sigma_{J,0}^{i,T}$

(32)

Following the procedures used to obtain Eq. (23), we may write

$$T_{fi,M}(\text{FRIA})=\sum_{i=1}^{4}\left[(\mathscr{A}^i\rho_J^i+\mathscr{B}^i\cdot\hat{\mathbf{q}}\Sigma_J^{i,L})\delta_{M,q}+\sum_{j=n,p}(-i\hat{\mathbf{q}}x\mathscr{B}^i\Sigma_{J,0}^{i,T}+\mathscr{B}^i\Sigma_{J,1}^{i,T})\cdot\hat{\mathbf{e}}_j\delta_{M,j}\right], \tag{33}$$

where again we understand that half of the nuclear structure matrix elements appearing in Eq. (33) are zero for a given spin and parity of the final state according to the relation in Eq. (32).

We may now ascertain directly the values of the quantities A_M, B_M, C_M, and D_M of Eq. (3) implied by the FRIA amplitude by Eq. (33). We can then convert these quantities into observables using Eqs. (6a)–(6f). The details of this procedure are given in Appendix C and the results are presented in Table I.

Examination of Table I shows that *none* of the amplitudes allowed by parity and rotational invariance are identically zero in the FRIA. This means, among other things, that the spin-difference function, Δ_S in Eq. (6b), can be nonzero in this approximation. Clearly, nuclear structure terms not present in the SIA appear in the FRIA. In order to identify these new terms—and to facilitate comparison between the two approaches generally—we must establish the relations between the two-component nuclear transition densities of the SIA [appearing in Eq. (23)] and the four-component densities of the FRIA [appearing in Eq. (33)].

We begin by noting that

$$\rho_J^1=\langle\tilde{J},\tilde{0}\,|\,\Gamma_1e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,\tilde{0},\tilde{0}\rangle=N_fN_i\left\langle J,0\left|\left[1+\frac{\sigma\cdot\overleftarrow{\nabla}\sigma\cdot\overrightarrow{\nabla}}{(E+m)^2}\right]e^{-i\mathbf{q}\cdot\mathbf{r}}\right|0,0\right\rangle$$

$$\simeq\langle J,0\,|\,e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,0,0\rangle=\rho_J\ , \tag{34}$$

where we understand here and in the expressions which follow that the gradients in these matrix elements do *not* act on the exponential and where the approximate equality is obtained by assuming that the lower-lower component combination is negligible compared to the upper-upper one. Similar arguments establish additional relations leading to the correspondences presented in Table II for nuclear structure amplitudes containing Γ_1 or Γ_2.

Interpretation of the remaining FRIA nuclear transition densities is somewhat more involved. We proceed by considering ρ_J^4. We have

$$\rho_J^4=\left\langle\tilde{J},\tilde{0}\left|\begin{bmatrix}0&1\\-1&0\end{bmatrix}e^{-i\mathbf{q}\cdot\mathbf{r}}\right|\tilde{0},\tilde{0}\right\rangle$$

$$=\frac{1}{E+m}\langle J,0\,|\,[-i\sigma\cdot(\overleftarrow{\nabla}+\overrightarrow{\nabla})e^{-i\mathbf{q}\cdot\mathbf{r}}]\,|\,0,0\rangle\ .$$

TABLE I. Plane wave proton-nucleus inelastic scattering amplitudes.

$\rho_J^{\pm}\equiv V\rho_J^1\pm S\rho_J^{\mp}$	$\Sigma_J^{\pm}\equiv A\Sigma_J^1\pm 2T\Sigma_J^2$	
$X_1\equiv\dfrac{E+m}{2m}$	$X_2\equiv\dfrac{p^2-q^2/4}{2m(E+m)}$	$X_3\equiv\dfrac{p^2+q^2/4}{2m(E+m)}$
$X_4\equiv\dfrac{p}{m}$	$X_5\equiv\dfrac{q}{2m}$	$X_6\equiv\dfrac{pq}{2m(E+m)}$
	Natural parity	
T_q:	$A_q=X_1\rho_J^{+}+X_2\rho_J^{-}-X_5 2T\Sigma_J^{4L}$ $B_q=-iX_6\rho_J^{-}-iX_4 2T\Sigma_J^{4L}$	
T_p:	$A_p=-X_6\Sigma_{J0}^{+T}-X_4V\Sigma_{J1}^{3T}$ $B_p=iX_1\Sigma_{J0}^{-T}-iX_2\Sigma_{J0}^{+T}+iX_5V\Sigma_{J1}^{3T}$	
T_n:	$C_n=-iX_1\Sigma_{J0}^{-T}+iX_3\Sigma_{J0}^{+T}-iX_5V\Sigma_{J1}^{3T}$ $D_n=iX_4 2T\Sigma_{J1}^{4T}$	
	Unnatural parity	
T_q:	$C_q=X_4A\rho_J^3$ $D_q=-X_5P\rho_J^4-X_1\Sigma_{J1}^{-L}+X_3\Sigma_{J1}^{+L}$	
T_p:	$C_p=-X_1\Sigma_{J1}^{-T}-X_3\Sigma_{J1}^{+T}-X_5V\Sigma_{J0}^{3T}$ $D_p=X_4 2T\Sigma_{J0}^{4T}$	
T_n:	$A_n=-iX_6\Sigma_{J1}^{+T}-iX_4V\Sigma_{J0}^{3T}$ $B_N=-X_1\Sigma_{J1}^{-T}+X_2\Sigma_{J1}^{+T}-X_5V\Sigma_{J0}^{3T}$	

TABLE II. Reduction of four-component nuclear transition densities to two-component form: I.

Four-component	Two-component
ρ_J^1,ρ_J^2	$\longrightarrow\rho_J$
$\Sigma_J^{1L},\Sigma_J^{2L}$	$\longrightarrow\Sigma_J^L$
$\Sigma_J^{1T},\Sigma_J^{2T}$	$\longrightarrow\Sigma_J^T$

Integration by parts yields

$$\rho_J^4 = \frac{1}{E+m}\langle J,0\,|\,\sigma\cdot\mathbf{q}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,0,0\rangle \simeq \frac{q}{2m}\Sigma_J^L\ , \qquad (35)$$

where we have assumed that $E+m \simeq 2m$ for the target nucleon. Equation (35) demonstrates that ρ_J^4 does *not* represent a new transition density appearing in the FRIA but not the SIA.

We now consider ρ_J^3. We have

$$\rho_J^3 = \left\langle \widetilde{J},\widetilde{0}\,\middle|\,\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} e^{-i\mathbf{q}\cdot\mathbf{r}}\,\middle|\,\widetilde{0},\widetilde{0}\right\rangle$$

$$= \frac{1}{E+m}\langle J,0\,|\,i\sigma\cdot(\overleftarrow{\nabla}-\overrightarrow{\nabla})e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,0,0\rangle\ .$$

Manipulation such as integration by parts does not allow reduction of this term and we conclude that it *does* give rise to a new transition density not present in SIA. Its nature is made more apparent by rewriting it as:

$$\rho_J^3 \simeq \langle J,0\,|\,\sigma\cdot\mathbf{j}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,0,0\rangle\ , \qquad (36)$$

where $\mathbf{j} \equiv i/2m(\overleftarrow{\nabla}-\overrightarrow{\nabla})$ is the familiar[12] probability (or convection) current operator. Following Love and Comfort,[13] we call this a composite-current transition density since $\mathbf{j}$ is combined with σ. Referring to Table I, we see that, for unnatural parity transitions, the C_q amplitude, which was identically zero in the SIA, is linear in ρ_J^3. Therefore, the $M=q$ contribution to the spin-difference function, Δ_S, in Eq. (6b), is also linear in ρ_J^3. We also observe that the composite-current operator appearing in the expression for ρ_J^3, in Eq. (36), $\sigma\cdot\mathbf{j}$, is time-reversal even, while that appearing for ρ_J^4 in Eq. (35), $\sigma\cdot\mathbf{q}$, is time-reversal odd. This is consistent with the general time-reversal requirements presented in Eq. (4).

The remaining nuclear transition densities can be interpreted in a like manner. The resulting correspondences are summarized in Table III. In obtaining these expressions, we have eliminated the longitudinal convection current contribution using current conservation and our assumption of zero Q value. We note that, for unnatural parity transitions, the D_p amplitude, which was zero in the SIA, is linear in $\Sigma_{J,0}^{4,T}$ and that the $M=p$ contribution to the spin difference function is therefore linear in this quantity. Similarly, for natural parity transitions, the D_n amplitude is proportional to $\Sigma_{J,1}^{4,T}$ (but is zero in the SIA) implying that the sole contribution to the spin-difference function for natural parity is proportional to this quantity. Examination of Table III shows that $\Sigma_{J,0}^{4,T}$ and $\Sigma_{J,1}^{4,T}$ can be expressed as two-component transition matrix elements of the operator $\sigma\times\mathbf{j}$.

TABLE III. Reduction of four-component nuclear transition densities to two-component form: II. An asterisk (*) denotes transition densities not present in the SIA.

Four-component		Two-component
Σ_J^{3L}	$\longrightarrow$	0
Σ_J^{4L}	$\longrightarrow$	$\frac{q}{2m}\rho_J - \langle J0\,|\,[\sigma\times\mathbf{j}]_0 e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle^*$
Σ_J^{3T}	$\longrightarrow$	$\frac{q}{2m}\Sigma_J^T + \langle J,+1\,|\,j_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle^*$
Σ_J^{4T}	$\longrightarrow$	$-i\langle J,+1\,|\,[\sigma\times\mathbf{j}]_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle^*$

We have identified a class of nuclear transition densities, absent in SIA, but arising naturally in FRIA, involving the convection current operator, $\mathbf{j}$, by itself and in combination with the spin operator, σ, of the target nucleon, namely the composite current operators:

$$\sigma\cdot\mathbf{j} \text{ and } \sigma\times\mathbf{j}\ . \qquad (37)$$

The composite-current transition densities make unique contributions to the spin difference function which would be identically zero without them. The time-reversal properties of these operators ($\mathbf{j}$ is time-reversal odd while $\sigma\cdot\mathbf{j}$ and $\sigma\times\mathbf{j}$ are time-reversal even) are consistent with the general time-reversal constraints presented in Eq. (4).

VI. NUCLEAR STRUCTURE DEPENDENCE IN THE FRIA

It is useful to establish general relationships between the nuclear structure properties of the initial and final target states and the nuclear transition densities appearing in Eqs. (33)–(36) and in Tables I–III. This is most readily accomplished by introducing the concept of nuclear structure amplitudes or, as they are sometimes referred to, one-body density matrix elements.[17] We may consider the following general nuclear matrix element of a one-body operator, θ_{JM}:

$$\langle J_f M_f\,|\,\theta_{JM}\,|\,J_i M_i\rangle\ .$$

We now introduce a complete set of single-particle states $|\,nljm\rangle$. These may be *either* two-component or four-component wave functions. In the latter case, n and l refer to the number of radial nodes and the orbital angular momentum of the *upper* component. [See Eqs. (12) and (26).] Using standard techniques we may write

$$\langle J_f M_f\,|\,\theta_{JM}\,|\,J_i M_i\rangle = -(J_i J M_i M\,|\,J_f M_f)\sum_{\alpha_f,\alpha_i}\mathscr{A}_{J(\alpha_f,\alpha_i)}^{J_f,J_i}\hat{j}_f\hat{J}^{-1}\langle\psi_{\alpha_f}||\theta_J||\psi_{\alpha_i}\rangle\ , \qquad (38a)$$

where

$$\alpha = (nlj) \qquad (38b)$$

and

$$\mathscr{A}_{J(\alpha_f,\alpha_i)}^{J_f,J_i} \equiv \langle J_f||[a_{\alpha_f}^\dagger b_{\alpha_i}^\dagger]_J||J_i\rangle\ , \qquad (38c)$$

where $a_{nljm}^\dagger$ and $b_{nljm}^\dagger$ are particle and hole creation operators, respectively, and where $\hat{j} \equiv \sqrt{2j+1}$. We have defined the reduced matrix element according to

$$\langle J_f M_f \mid T_{kq} \mid J_i M_i \rangle = (J_i k M_i q \mid J_f M_f)\langle J_f || T_k || J_i \rangle \ . \tag{38d}$$

The operators giving rise to *two-component* transition densities can usually be characterized by the orbital and spin angular momentum transfer they imply. Such a characterization is *not* straightforward for *four-component* transition densities which are of necessity formulated in a j-j coupling scheme as in Eq. (38). For two-component transition densities, we may write, omitting the principle quantum number index for the single particle states,

$$\langle J_f M_f \mid [\theta_L \theta_S]_{JM} \mid J_i M_i \rangle = -(J_i J M_i M \mid J_f M_f) \sum_{l_f l_i} \mathscr{A}^{J_f J_i}_{J(l_f l_i, L; 1/2\, 1/2, S)} \frac{\sqrt{2}\hat{l}_f}{\hat{L}\hat{S}} \langle l_f || \theta_L || l_i \rangle \langle 1/2 || \theta_S || 1/2 \rangle \ , \tag{39}$$

where the LS representation of the nuclear structure amplitude is related to its j-j counterpart, [Eq. (38c)], by

$$\mathscr{A}^{J_f J_i}_{J(l_f l_i, L; 1/2\, 1/2, S)} = \sum_{j_f j_i} \hat{L}\,\hat{S}\,\hat{j}_f \hat{j}_i \begin{Bmatrix} l_f & l_i & L \\ \frac{1}{2} & \frac{1}{2} & S \\ j_f & j_i & J \end{Bmatrix} \mathscr{A}^{J_f J_i}_{J(j_f, j_i)} \tag{40}$$

and its obvious inverse.

Equation (39) implies that, if l_f and l_i are restricted to the same fixed value (as in the $1p$ shell, for example), the magnitude of a transition density characterized by given values of L and S is proportional to the nuclear structure amplitude $\mathscr{A}^{J_f J_i}_{J(LS)}$. The identification of the L-S structure amplitudes governing the SIA transition densities is quite straightforward. This procedure is less obvious in the case of the composite-current transition densities but is dealt with in detail in Appendix D. All results (for $l_f = l_i$) are summarized in Table IV. Referring to this table, we note that the composite-current transition densities are governed by so-called "abnormal parity" amplitudes. We may then conclude that the magnitude of the spin-difference function, for example, is closely related to the spin-transfer abnormal parity amplitudes and, in fact, is proportional to $\mathscr{A}^{J0}_{J(J1)}$ for unnatural parity transitions such as the $0^+ \to 1^+$ excitation in ^{12}C. These arguments also apply to transitions in the s-d shell since only the $l=2$ single particle wave functions can participate in transitions with odd L.

TABLE IV. Nuclear structure amplitudes $\mathscr{A}^{J0}_{J(LS)}$ governing various transition densities. (For $0^+ \to J^\pi$ and $l_f = l_i$.)

Transition density	Natural parity L	Natural parity S	Unnatural parity L	Unnatural parity S
ρ_J	J	0		
Σ_J	J	1	$J\pm1$	1
$\langle J0 \mid \sigma\cdot \mathbf{j} e^{-i\mathbf{q}\cdot\mathbf{r}} \mid 00\rangle$			J	1
$\langle J0 \mid [\mathbf{j}]_{+1} e^{-i\mathbf{q}\cdot\mathbf{r}} \mid 00\rangle$	J	0	J	0
$\langle JM \mid [\sigma\times\mathbf{j}]_M e^{-i\mathbf{q}\cdot\mathbf{r}} \mid 00\rangle$	$J\pm1$	1	J	1

VII. APPLICATION OF FRIA FORMULATION

Because the FRIA formulation discussed above works directly with the relativistic invariant form of the fundamental NN interaction, it is especially easy to generalize to the inelastic scattering of other fundamental probes for which the relativistic invariant form of their interaction with nucleons is known. The electron is such a probe and the FRIA electron-nucleus inelastic scattering amplitude is derived in Appendix E. We now wish to demonstrate how the similarity between the FRIA amplitudes for (p,p′) and (e,e′) can be exploited to extract nuclear structure information. This is most readily accomplished by rewriting the FRIA (p,p′) amplitudes and (e,e′) form factors making use of the FRIA-SIA correspondences of Tables II and III. The resulting expressions appear in Tables V and VI for (p,p′) and (e,e′), respectively. In the (p,p′) expressions, we have used a hybrid notation for the NN interaction, mixing the relativistic invariant form of Eq. (8) with the Wolfenstein representation of Eq. (11). Those quantities common to the FRIA and SIA are written in the SIA form consistent with Eq. (23) while those unique to the FRIA retain the form of Table I. We note that the (e,e′) form factors of Table VI are the same as those obtained in standard nonrelativistic microscopic formulations.

We may now express the (p,p′) *observables* in a very compact form using Eqs. (6a)–(6f). The resulting (p,p′) expressions appear in Table VII. The expressions of Tables VI and VII explicitly demonstrate the relatively simple relations between (p,p′) observables and the (e,e′) form factors in the FRIA limit. We may now demonstrate the utility of these relationships by using them to analyze and interpret (p,p′) spin transfer data[14] and to relate these data to measured (e,e′) form factors[15] for the transitions to the 12.71 MeV 1^+ $T=0$ and 15.11 MeV 1^+ $T=1$ levels in ^{12}C. Specific relations are implied by the FRIA amplitude and the full calculations can be used to determine if the FRIA relations persist when additional effects arising from, e.g., distortion, nonzero Q-value, off-shell extrapolation of t_{NN}, etc., are included. The data can also be examined to see if the implied relations are present. Finally, as will be discussed more fully below, we can use the FRIA relations to identify observables which

TABLE V. FRIA inelastic scattering amplitudes for (p,p′).

	Natural parity
$T_0=T_q$:	$A_q=a\rho_J+i2Tq/(2m)\langle J0\,|\,[\sigma\times\mathbf{j}]_0 e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle$ $B_q=iqc\rho_J-2Tp/m\,\langle J0\,|\,[\sigma\times\mathbf{j}]_0 e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle$ $C_q=0\;\;D_q=0$
$T_M^{(+)}=T_p$:	$A_p=qc\Sigma_{J0}^T-Vp/m\,\langle J,+1\,|\,[\mathbf{j}]_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle$ $B_p=-ib\Sigma_{J0}^T+iVq/(2m)\langle J,+1\,|\,[\mathbf{j}]_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle$ $C_p=0\;\;D_p=0$
$T_M^{(-)}=T_n$:	$A_n=0\;\;B_n=0$ $C_n=i(b+e)\Sigma_{J0}^T-iVq/(2m)\langle j,+1\,|\,[\mathbf{j}]_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle$ $D_n=2Tp/m\,\langle J,+1\,|\,[\sigma\times\mathbf{j}]_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle$
	Unnatural parity
T_q:	$A_q=0\;\;B_q=0$ $C_q=Ap/m\,\langle J0\,|\,\sigma\cdot\mathbf{j}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle$ $D_q=(b+q^2d)\Sigma_J^L$
T_p:	$A_p=0\;\;B_p=0$ $C_p=(b+e)\Sigma_{J1}^T-Vq/(2m)\langle J,+1\,|\,[\mathbf{j}]_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle$ $D_p=-i2Tp/m\,\langle J,+1\,|\,[\sigma\times\mathbf{j}]_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle$
T_n:	$A_n=iqc\Sigma_{J1}^T-iVp/m\,\langle J,+1\,|\,[\mathbf{j}]_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle$ $B_n=b\Sigma_{J1}^T-Vq/(2m)\langle J,+1\,|\,[\mathbf{j}]_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle$ $C_n=0\;\;D_n=0$

are sensitive to differences between the FRIA and the full DRIA calculations. These differences are due to the presence of the strong scalar and timelike vector potentials which characterize modern relativistic models of the nucleon-nucleus interaction and which are included in the dynamical RIA (or DRIA). We may then be able to identify experimental signatures of strong relativistic dynamics in (p,p′) just as a possible signature has already been identified in (e,e′).[4]

To begin, let us consider how we might extract the transverse spin transition density, $\Sigma^{\rm trans}$, for the $T=1$ transition. This extraction is the most straightforward if the transverse convection current transition density vanishes, i.e., $j^{\rm trans}\rightarrow 0$. In this case, we have, from Tables VI and VII:

$$\frac{1}{2}\frac{2mZ}{qg}\,|F_{\rm mag}|^2=|\Sigma^{\rm trans}|^2\,, \tag{41a}$$

$$\frac{A^2}{\kappa\,|qc|^2}=|\Sigma^{\rm trans}|^2\,, \tag{41b}$$

$$\frac{B^2}{\kappa\,|b|^2}=|\Sigma^{\rm trans}|^2\,, \tag{41c}$$

where $g=e_{\rm N}+\kappa_{\rm N}$ and the other symbols are defined in the tables. If in addition, we may ignore composite current transition densities, we also have

$$\frac{C^2}{\kappa\,|b+c|^2}=|\Sigma^{\rm trans}|^2\,. \tag{41d}$$

TABLE VI. FRIA form factors for (e,e′).

Natural parity

$$|F_{\rm long}|^2=Z^{-2}\left[\left|\left[e_{\rm N}-\frac{q^2}{4m^2}\kappa_{\rm N}\right]\rho_J\right|^2+\left|\frac{q}{2m}\kappa_{\rm N}\langle J0\,|\,[\sigma\times\mathbf{j}]_0e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle\right|^2\right]$$

$$|F_{\rm elec}|^2=2Z^{-2}\left|-\frac{q}{2m}(e_{\rm N}+\kappa_{\rm N})\Sigma_{J0}^T+e_{\rm N}\langle J,+1\,|\,j_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle\right|^2$$

Unnatural parity

$$|F_{\rm mag}|^2=2Z^{-2}\left|-\frac{q}{2m}(e_{\rm N}+\kappa_{\rm N})\Sigma_{J1}^T+e_{\rm N}\langle J,+1\,|\,j_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle\right|^2$$

TABLE VII. Observables for unnatural parity transitions in plane wave FRIA.

$$A^2=\frac{1}{4}\frac{d\sigma}{d\Omega}(1+D_{nn}+D_{pp}+D_{qq})=\kappa\left|qc\Sigma_{J1}^{T}-\frac{p}{m}V\langle \mathbf{j}\rangle^{T}\right|^2,$$

$$B^2=\frac{1}{4}\frac{d\sigma}{d\Omega}(1+D_{nn}-D_{pp}-D_{qq})=\kappa\left|b\Sigma_{J1}^{T}-\frac{q}{2m}V\langle \mathbf{j}\rangle^{T}\right|^2,$$

$$C^2=\frac{1}{4}\frac{d\sigma}{d\Omega}(1-D_{nn}+D_{pp}-D_{qq})=\kappa\left[\left|(b+e)\Sigma_{J1}^{T}-\frac{q}{2m}V\langle \mathbf{j}\rangle^{T}\right|^2+|p/mA\langle\sigma\cdot\mathbf{j}\rangle|^2\right],$$

$$D^2=\frac{1}{4}\frac{d\sigma}{d\Omega}(1-D_{nn}-D_{pp}+D_{qq})=\kappa\left[|(b+q^2d)\Sigma_{J}^{L}|^2+\left|\frac{2p}{m}T\langle\sigma\times\mathbf{j}\rangle^{T}\right|^2\right],$$

where

$$\langle j\rangle^{T}\equiv\langle J,+1\,|\,[\mathbf{j}]_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle,$$

$$\langle\sigma\cdot\mathbf{j}\rangle\equiv\langle J0\,|\,\sigma\cdot\mathbf{j}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle,$$

$$\langle\sigma\times\mathbf{j}\rangle^{T}\equiv\langle J,+1\,|\,[\sigma\times\mathbf{j}]_{+1}e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle,$$

$$\kappa\equiv\left[\frac{E_{\rm NN}P_{{\rm N}A}}{4\pi p_{\rm NN}}\right]^2,$$

where NN and NA refer to the nucleon-nucleon and nucleon-nucleus center-of-momentum frames, respectively.

To proceed further, we must somehow determine the validity of the preceding assumptions.

The structure of the eN and NN amplitudes for $\Delta T=1$ imply that the convection current receives little weight compared to the spin amplitude. Explicit (e,e′) and (p,p′) calculations also show this to be true. Furthermore, the observable combination A^2 appearing in Table VII is *maximally* sensitive to $j^{\rm trans}$ at small momentum transfers and experimentally is found to be small there. All this leads to the conclusion that the convention current can be ignored in this case.

The (p,p′) data show that

$$\frac{B^2}{\kappa|b|^2}\simeq\frac{C^2}{\kappa|b+e|^2},$$

which can only happen if $|\langle\sigma\cdot\mathbf{j}\rangle|^2$ is small. DRIA calculations also indicate that the composite current transition density makes a negligible contribution to C^2 and we conclude that it also can be ignored for this combination of observables. We therefore conclude that our assumptions are justified and that Eq. (45) provides a valid means of extracting $|\Sigma^{\rm trans}|^2$. Note that, when $j^{\rm trans}$ and $\langle\sigma\cdot\mathbf{j}\rangle$ are dropped in Table VII, we obtain the same relations between the (p,p′) spin transfer observables and the nuclear spin transition densities as those derived by Moss[3] and by Bleszynski *et al.*[16]

In Fig. 2 we compare $|\Sigma^{\rm trans}|^2$ as extracted[15] from $|F_{\rm mag}|^2$ according to Eq. (41a) with the same quantity as extracted from (p,p′) spin transfer data[14] using Eqs. (41c) and (41d). Also shown is $|\Sigma^{\rm trans}|^2$ as calculated using the Cohen and Kurath nuclear structure amplitudes;[17] by construction this quantity is obtained from both (e,e′) *and*

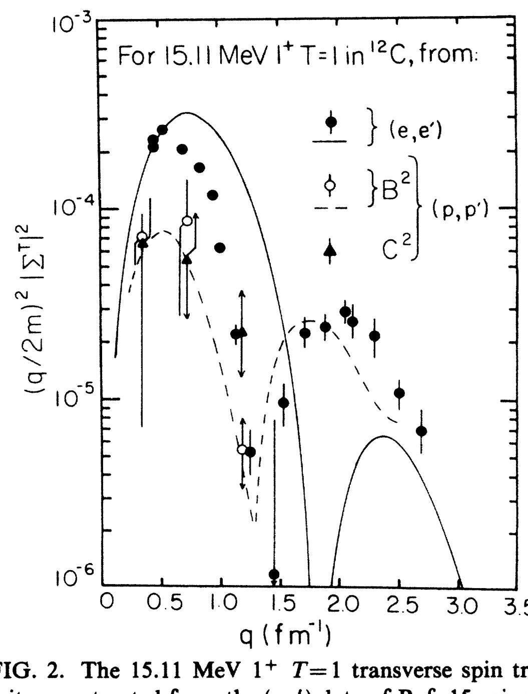

FIG. 2. The 15.11 MeV 1^+ $T=1$ transverse spin transition density as extracted from the (e,e′) data of Ref. 15 using the expression for $|F_{\rm mag}|^2$ in Table VI and assuming $j^{\rm trans}=0$. This density is compared with the same quantity as extracted from the (p,p′) data of Ref. 14. The points labeled B^2 and C^2 were extracted using corresponding expressions in Table VII and assumed no convection or composite current contribution. The solid curve is the transverse spin transition density as given by the Cohen and Kurath (Ref. 17) wave function. The dashed curve is the (p,p′) quantity C^2 as calculated with the code DRIA (Ref. 2) including distortion.

(p,p′) calculations in the plane-wave approximation. Figure 2 shows that the extracted values of $|\Sigma^{\rm trans}|^2$ are very similar for the two (p,p′) observable combinations. In turn, these quantities have a similar momentum transfer dependence to that extracted from the (e,e′) data, but are systematically lower by $\sim\frac{1}{3}$. This renormalization is due to distortion (attentuation) effects as is demonstrated by the full distorted wave calculation of the quantity of Eq. (41c) which is shown as the dashed line in Fig. 2. (Recall that in the plane-wave approximation, this same quantity would be identical to the solid line.) The good agreement between the dashed line and the (p,p′) data suggests that distortion effects can be accounted for quantitatively and that, under the assumptions outlined above, information about $|\Sigma^{\rm trans}|^2$ can be extracted in a consistent fashion from both (e,e′) *and* (p,p′) data.

Let us now consider extraction of $|\Sigma^{\rm trans}|^2$ for the 1^+ $T=0$ level. Here the situation is more complicated than for the 1^+ $T=1$ level because both the convection current and isospin mixing produce large effects in $|F_{\rm mag}|^2$. The sensitivity to the convection current is demonstrated in Fig. 3 which shows the (e,e′) observable given by Eq. (8). The data are compared with calculations using Cohen and Kurath amplitudes[17] with (solid line) and without (dashed line) convection-current contribution which is quite large. Consequently $|\Sigma^{\rm trans}|^2$ is *not* directly accessible in (e,e′) for the 1^+ $T=0$ transition as it was for the 1^+ $T=1$ one. In order to extract $|\Sigma^{\rm trans}|^2$ from (p,p′) we must also address the roles of the convection current and isospin mixing as well as the contributions of the composite currents. Because of the empirical strengths of the NN amplitudes, isospin mixing is a negligible effect for (p,p′). In some observable combinations, as will be discussed below, currents give rise to large effects. However, in the observable combination B^2, composite currents do not enter and the convection current is minimized by virtue of the factor of q multiplying it (see Table VII). Full DRIA calculations also show that B^2 is insensitive to currents and therefore directly reflects $|\Sigma^{\rm trans}|^2$ in a way that the (e,e′) data do not. Unfortunately, the existing (p,p′) spin transfer data are not of sufficient quality to permit a meaningful comparison with either the (e,e′) data or distorted wave (p,p′) calculations.

We now turn our attention to the extraction of information about $j^{\rm trans}$ which is very important in (e,e′) for the 1^+ $T=0$ transition. Examination of Table VII shows that the observable combination A^2 is maximally sensitive to $j^{\rm trans}$ since it is multiplied by a factor of p here, but by a factor of q in all other observable combinations where it appears. DRIA calculations with and without the convection current contribution are compared with the (p,p′) data in Fig. 4. A very strong signature of the convection current is indeed observed in A^2, but the data clearly indicate that convection current contribution should be *much smaller* than that implied by the Cohen and Kurath wave functions.[17] [It is interesting to note that the Cohen and Kurath amplitude governing the convection current $(L,S=1,0)$ is relatively small for both the 1^+ $T=0$ and $T=1$ transitions, but that the $T=0$ amplitude is nearly *four* times the $T=1$ amplitude.] It may therefore be that the convection current nuclear structure amplitude is poorly determined in the Cohen and Kurath wave functions and that the (p,p′) data demonstrate that this particular amplitude is incorrect. Such a conclusion would have major implications for the interpretation of the (e,e′) data for the 1^+ $T=0$ level.

The preceding discussion suggests that $\Sigma^{\rm trans}$ and $j^{\rm trans}$

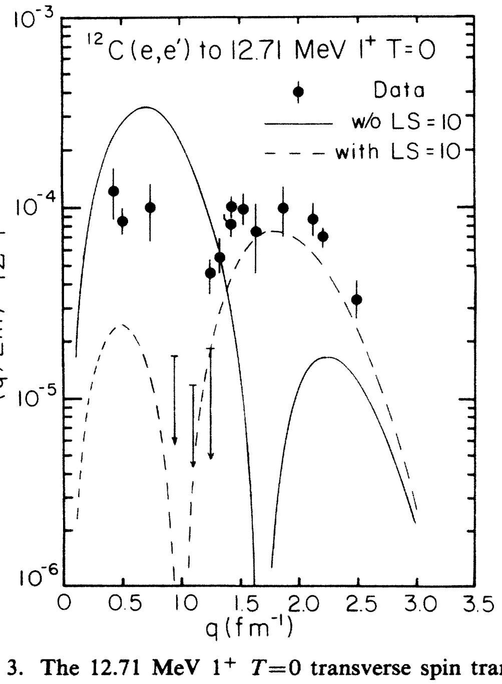

FIG. 3. The 12.71 MeV 1^+ $T=0$ transverse spin transition density as extracted from the (e,e′) data of Ref. 15 using the expression for $|F_{\rm mag}|^2$ in Table VI and assuming $j^{\rm trans}=0$. The solid (dashed) curve is the same quantity as computed using the Cohen and Kurath (Ref. 13) wave functions without (with) the $(L,S)=(1,0)$ amplitude which governs the convection current.

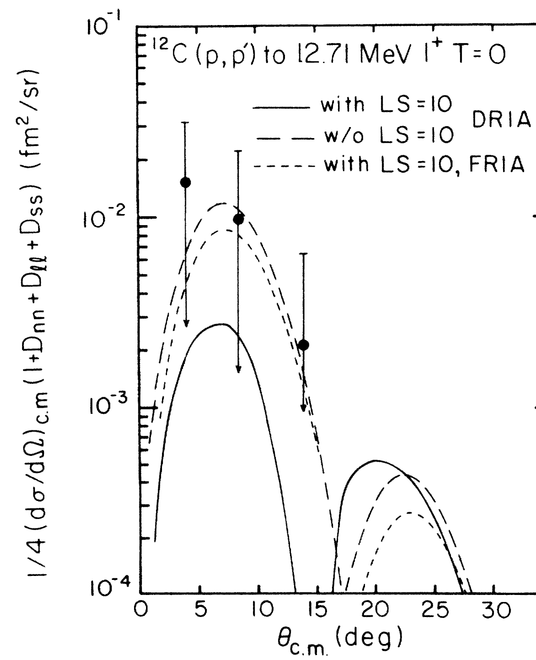

FIG. 4. The 150 MeV (p,p′) data of Ref. 14 are compared with DRIA (Ref. 2) calculations. See the text.

can be separated in (p,p′) in a manner which is not possible using (e,e′) data alone. We now consider how the composite currents might manifest themselves in the (p,p′) observables. [They do not enter in (e,e′), but the $\langle \sigma\cdot \mathbf{j}\rangle$ transition density *would* appear in reactions involving weak probes, e.g., (ν,ν').] We note that $|\langle \sigma\cdot \mathbf{j}\rangle|^2$ appears in the observable combination C^2. As noted above, this composite current does *not* make a significant contribution to the relevant (p,p′) observable for the 1^+ $T=1$ transition. This is due to the detailed nature of the isovector NN amplitude which strongly emphasizes the $|\Sigma^{\rm trans}|$ contribution to C^2 (see Table VII). In contrast $|\langle \sigma\cdot \mathbf{j}\rangle|^2$ has a very large effect on C^2 for the 1^+ $T=0$ transition. This is demonstrated in Fig. 5 where the (p,p′) data are compared to DRIA calculations with (solid line) and without (dashed line) the composite current amplitudes (i.e., $\mathscr{A}^{10}_{1(L=1,S=1)}\to 0$). At $\theta=15°$, the calculated composite current contribution is more than an order of magnitude greater than all the others. Unfortunately, the existing data are too poor to allow for a meaningful comparison with the calculations even when the latter contain such a strong signature of the composite currents. We also observe that the composite currents play an essential role in determining the spin difference function $\Delta_S \equiv (P-A_y)+i(Q-B)$ and can be constrained by this observable combination, too. (See Ref. 7.)

It is interesting to note that the work of Moss[3] and of Bleszysnki *et al.*,[16] where current contributions are ignored, advocates the use of the observable combination C^2 for the extraction of $|\Sigma^{\rm trans}|^2$. As we have seen, this happens to be an acceptable procedure for the 1^+ $T=1$ transition but would be disastrous for the 1^+ $T=0$ transition due to the overwhelming contribution of the composite current. In general, the contribution B^2 is to be preferred for examining $|\Sigma^{\rm trans}|^2$.

We now turn to the extraction of $|\Sigma^{\rm long}|^2$ from the (p,p′) data. Again, as is well known, there is no contribution to (e,e′) from this quantity. Table VII shows that $|\Sigma^{\rm long}|^2$ is directly accessible from D^2 *if* the $\langle \sigma\times \mathbf{j}\rangle^{\rm trans}$ contribution is small. DRIA calculations show that this is indeed the case for the 1^+ $T=1$ transition and the relevant observable combination is compared with data in Fig. 6. Although the error bars are quite large, there does appear to be agreement, suggesting that the Cohen and Kurath wave functions give a reasonable description of $|\Sigma^{\rm long}|^2$.

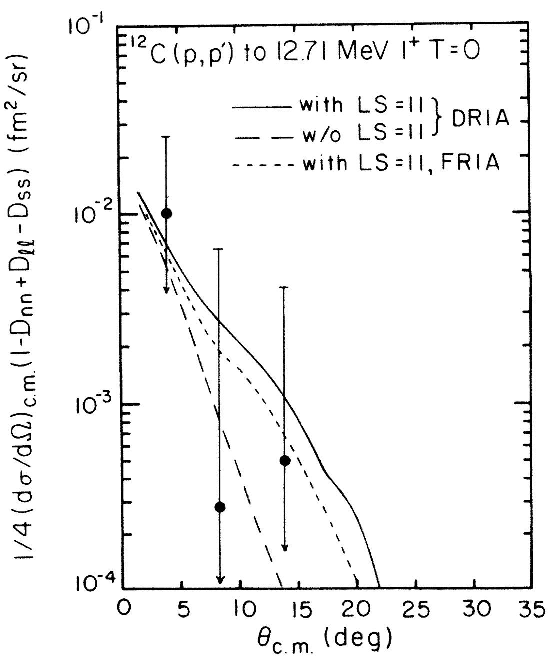

FIG. 5. The 150 MeV (p,p′) data of Ref. 14 are compared with DRIA (Ref. 2) calculations. See the text.

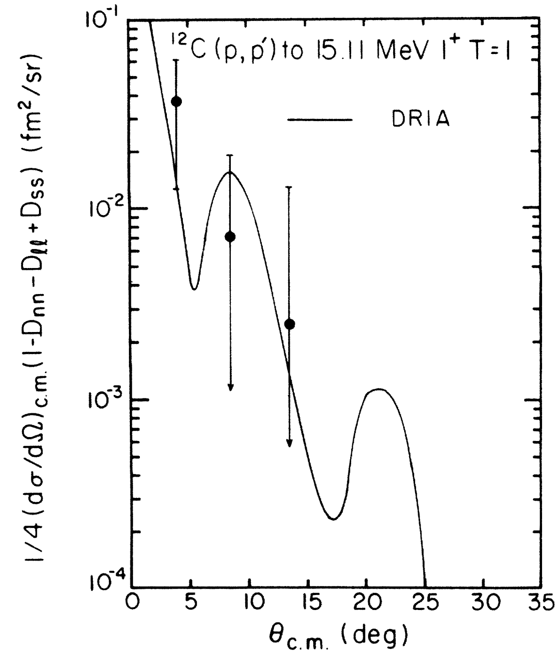

FIG. 6. The (p,p′) data of Ref. 14 are compared with DRIA (Ref. 2) calculations. See the text.

VIII. BEYOND FRIA

Let us now consider possible signatures of the strong scalar and timelike vector potentials of relativistic models of the nucleon-nucleus interaction. Implicit in the FRIA is the free-space relation between upper and lower components appearing in Eqs. (12) and (26). In many modern relativistic models, single particle wave functions satisfy

$$[\not p - m - S(r) - \gamma^0 V(r)]\psi = 0 \ , \qquad (42)$$

where S and V are the scalar and vector potentials, respectively. We may then write the *DRIA* wave function as

$$\psi(r) \approx N \begin{pmatrix} s^{-1/2} \\ \dfrac{-is\sigma\cdot\nabla s^{-1/2}}{E+m} \end{pmatrix} \phi(r) \ , \qquad (43)$$

where

$$s(r) \equiv \frac{E+m}{E+m-V(r)+S(r)} \ . \qquad (44)$$

Since $S \simeq -450$ MeV and $V \simeq +350$ MeV, $s \simeq 1.75$ in the nuclear interior, implying that the upper/lower component ratio can be quite different for FRIA and DRIA. This difference will manifest itself most strongly for the inelastic transition driven by matrix elements quadratic in lower components such as those which would be identified with the natural parity spin-orbit interaction in nonrelativistic treatments. We do not consider such matrix elements here, but instead examine the *current* transition densities which are linear in the lower components and hence will also be sensitive—although to a lesser degree—to the differences between FRIA and DRIA. In Figs. 4 and 5, FRIA calculations of the observable combinations sensitive to convection and composite currents, respectively, are compared with DRIA results discussed earlier. The differences are not large—generally less than a factor of 2—but some sensitivity is apparent.

We finally consider the sensitivity of (p,p′) observables to ambiguities in the form of NN invariants appearing in Eq. (9). In particular, we examine the pseudoscalar invariant which can be written in two different forms which are totally equivalent on shell by using the following relation:

$$\frac{1}{2m}\bar{u}\not{q}\gamma^5 u=\bar{u}\gamma^5 u \ , \tag{45}$$

which is proved using the on-shell condition

$$(\not{p}-m)u=\bar{u}(\not{p}-m)=0 \ . \tag{46}$$

These two forms are *not* equivalent off shell and if we consider wave functions satisfying Eq. (46), we may write

$$\frac{1}{2m}\bar{\psi}\not{q}\gamma^5\psi=\bar{\psi}\left[1+\frac{S_f+S_i}{2m}+\gamma^0\frac{V_f-V_i}{2m}\right]\gamma^5\psi$$
$$\neq\bar{\psi}\gamma^5\psi \ . \tag{47}$$

The third term on the right-hand side is present only for the projectile wave function since we assume $V_f=V_i$ for the bound state. As shown above, the spin-difference function, Δ_S, arises from the interference of a pseudoscalar amplitude (containing γ^5) with a timelike axial vector amplitude (containing $\gamma^0\gamma^5$). Clearly the third term of Eq. (47) can interfere with the first two to make a contribution to Δ_S. This happens *only if* the $\not{q}\gamma^5$ form of the pseudoscalar invariant is used. Hence Δ_S is particularly sensitive, in DRIA, to the form of the pseudoscalar invariant which *cannot* be determined by the free NN data. Preliminary calculations of $P-A_y$ for the ^{12}C(p,p′)^{12}C* (15.11 MeV 1^+ $T=1$) transition at $T_p=150$ MeV show that the data are much better described when $\not{q}\gamma^5$ is used. This is consistent with the finding that the energy dependence of the impulse approximation scalar and timelike vector potentials for elastic scattering is correct only when the pseudoscalar invariant, which contributes through exchange processes, is taken to have the $\not{q}\gamma^5$ form.[18]

IX. SUMMARY AND CONCLUSIONS

We have established the general properties of inelastic scattering amplitudes assuming parity, rotational, and time-reversal invariance and have then shown simple relations between these amplitudes and spin-transfer observables. We have also examined a relativistic plane-wave impulse approximation for proton-nucleus inelastic scattering which is a simple generalization of the Born amplitude for elastic scattering in the relativistic impulse approximation. We have shown that this amplitude corresponds to the standard impulse approximation (SIA) if we drop the dynamical dependence of the lower components of the target nucleons. Retaining this dependence leads to the free relativistic impulse approximation (FRIA) which differs from the SIA by the appearance, in lowest order, of convection and composite current amplitudes. These latter amplitudes make unique contributions to the spin-difference function which is identically zero in the SIA.

We have also examined the nuclear structure dependence of the various nuclear transition densities appearing in the FRIA. We have shown, for example, that under a restriction to a basis of single-particle wave functions with a single orbital angular momentum, the composite current transition densities depend only on abnormal parity spin transfer amplitudes.

We have also used the plane-wave relativistic impulse approximation to establish a simple relation between (p,p′) observables and various nuclear transition densities. The formulation has been shown to be particularly useful because it facilitates the comparison of (p,p′) and (e,e′) observables. These relations can be exploited to extract $|\Sigma^{\rm trans}|^2$ for the transition to the 15.11 MeV 1^+ $T=1$ level in ^{12}C from both (p,p′) and (e,e′) data in a consistent manner. We have found that, in general, spin and convection current contributions can be examined separately in (p,p′) while this is often not possible (e.g., for the 12.71 MeV 1^+ $T=0$ level in ^{12}C) in (e,e′). We have also identified (p,p′) quantities which are maximally sensitive to convection and composite currents and have found large signatures of the currents in the 1^+ $T=0$ transition. The existing data suggest that the Cohen and Kurath amplitude governing the convection current for this transition is too large. We have also shown that $|\Sigma^{\rm long}|^2$ can be straightforwardly extracted from the (p,p′) data for the 1^+ $T=1$ transition, but probably not for the 1^+ $T=0$ transition. Moderately strong signatures of relativistic dynamics of the DRIA have been identified for the 1^+ $T=0$ transition. We have also demonstrated that the spin-difference function is potentially sensitive to the off-shell extrapolation of the pseudoscalar invariant implied by the operator γ^5 and $\not{q}\gamma^5/2m$.

This work is intended to serve as a "handbook" which can be used to understand (p,p′) in a relativistic formulation containing all of the features implicit in standard microscopic formulations of (e,e′). The vast majority of the material presented is "relativistic" in form only and does not make reference to the dynamics of modern relativistic theories of nuclear structure and scattering any more than do the (e,e′) formulations referred to above. However, one of the important advantages of the present treatment is that it can readily incorporate relativistic nuclear dynamics. Furthermore, as stressed earlier, any inelastic scattering process can be treated using the present formulation given a relativistic invariant representation of the interaction between the probe and a nucleon.

ACKNOWLEDGMENTS

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APPENDIX A: SYMMETRIES OF TRANSITION AMPLITUDES

This development follows closely that found in Satchler.[5] We begin by examining the properties of the transition amplitude under reflection in the scattering plane. Such a transformation implies that the initial and final projectile momenta are unchanged. We use the coordinate system defined in Eq. (2) and choose the quantization axis along $\hat{\mathbf{q}}=\hat{\mathbf{z}}$. Reflection in the scattering plane (the y-z plane) can be described by the following sequence of transformations:

reflection in the y-z plane

$$=R(-\pi/2,z)R(\pi,y)R(\pi/2,z)P\ , \tag{A1}$$

where P is the parity or spatial inversion operator and $R(\theta,i)$ represents a rotation of the coordinate system through an angle θ about the $\hat{\imath}$ axis. In general, wave functions transform as follows under these transformations:

$$\begin{aligned} &P\,|\,\mathbf{K}_a;I_aM_a\rangle=\pi_a\,|\,-\mathbf{K}_a;I_aM_a\rangle\ ,\\ &R(\theta,z)\,|\,I_aM_a\rangle=(-1)^{-iM_a\theta}\,|\,I_aM_a\rangle\ ,\\ &R(\pi,y)\,|\,I_aM_a\rangle=(-1)^{I_a-M_a}\,|\,I_a,-M_a\rangle\ , \end{aligned} \tag{A2}$$

where π_a is the parity of a. With this information and the assumption that the projectile-target interaction, $\mathcal{T}$, is invariant with respect to parity and rotational transformations, we may write

$$\begin{aligned} T_{fi}&=\langle\mathbf{K}_b;I_BM_B,I_bm_b\,|\,\mathcal{T}\,|\,\mathbf{K}_a;I_AM_A,I_am_a\rangle\\ &=\pi_B\pi_b\pi_A\pi_a(-1)^{I_B+I_A+I_b+I_a}(-1)^{-2(M_A+m_a)}\\ &\quad\times\langle\mathbf{K}_b;I_B-M_B,I_b-m_b\,|\,\mathcal{T}\,|\,\mathbf{K}_a;I_A-M_A,I_a-m_a\rangle. \end{aligned} \tag{A3}$$

For $I_A=0$, $I_a=I_b=\frac{1}{2}$, $\pi_A=\pi_a=\pi_b=+1$, this expression becomes

$$\begin{aligned} T_{fi}=T_{M_B}(m_b,m_a)&\equiv\langle\mathbf{K}_b;I_BM_B,m_b\,|\,\mathcal{T}\,|\,\mathbf{K}_a;m_a\rangle\\ &=\pi_B(-1)^{I_B}\langle\mathbf{K}_b;I_B-M_B,-m_B\,|\,\mathcal{T}\,|\,\mathbf{K}_a;-m_a\rangle\\ &=\pi_B(-1)^{I_B}T_{-M_B}(-m_b,-m_a)\ . \end{aligned} \tag{A4}$$

We observe that the quantity $\pi_B(-1)^{I_B}=+1\ (-1)$ for so-called natural (unnatural) parity transitions.

We now consider the case of $M_B=0$. Then we have, for *natural* parity,

$$T_0(m_b,m_a)=T_0(-m_b,-m_a)\ . \tag{A5}$$

Expressed in terms of operators in the projectile spin space (i.e., in terms of Pauli spin matrices), this equation implies

$$T_0(m_b,m_a)\rightarrow A_0\mathbf{1}+B_0\sigma_x=A_0\mathbf{1}+B_0\sigma_n\ . \tag{A6}$$

For *unnatural* parity transitions, we have

$$T_0(m_b,m_a)=-T_0(-m_b,-m_a) \tag{A7}$$

or

$$T_0(m_b,m_a)\rightarrow C_0\sigma_y+D_0\sigma_z=C_0\sigma_p+D_0\sigma_q\ . \tag{A8}$$

We now consider $M_B\neq 0$. We define the following quantities:

$$T_{M_B}^{(+)}(m_b,m_a)\equiv\frac{i}{\sqrt{2}}[T_{M_B}(m_b,m_a)+T_{-M_B}(m_b,m_a)]\ , \tag{A9a}$$

$$T_{M_B}^{(-)}(m_b,m_a)\equiv-\frac{1}{\sqrt{2}}[T_{M_B}(m_b,m_a)-T_{-M_B}(m_b,m_a)]\ , \tag{A9b}$$

for $M_B>0$ where the coefficients have been chosen for later convenience. For natural parity transitions, we find

$$T_{M_B}^{(+)}(m_b,m_a)=T_{M_B}^{(+)}(-m_b,-m_a)$$

or

$$T_{M_B}^{(+)}(m_b,m_a)\rightarrow A_{M_B}^{(+)}\mathbf{1}+B_{M_B}^{(+)}\sigma_n\ . \tag{A10a}$$

Likewise

$$T_{M_B}^{(-)}(m_b,m_a)\rightarrow C_{M_B}^{(-)}\sigma_p+D_{M_B}^{(-)}\sigma_q\ . \tag{A10b}$$

For *unnatural* parity transitions, these relationships are just reversed

$$T_{M_B}^{(+)}(m_b,m_a)\rightarrow C_{M_B}^{(+)}\sigma_p+D_{M_B}^{(+)}\sigma_q\ , \tag{A11a}$$

$$T_{M_B}^{(-)}(m_b,m_a)\rightarrow A_{M_B}^{(-)}\mathbf{1}+B_{M_B}^{(-)}\sigma_q\ . \tag{A11b}$$

We now examine the time-reversal properties of the scattering amplitudes. For microscopic treatments of nucleon-nucleus inelastic scattering, the projectile-nucleus interaction is taken to be some nucleon-nucleon effective interaction, $t_{\rm NN}$, which we assume to be invariant under time reversal, i.e., $\theta t_{\rm NN}\theta^{-1}=t_{\rm NN}$, where θ is the time-reversal operator.[5] We may express $t_{\rm NN}$ as a sum of symmetric combinations of operators acting individually on the interacting nucleons:

$$t_{\rm NN}=\sum_i t_i(1)t_i(2)\ .$$

The individual operators, t_i, may be even *or* odd under time reversal:

$$\theta t_i\theta^{-1}=(-1)^{\eta_i}t_i,\quad \eta_i=\begin{cases}0 \text{ for even,}\\ 1 \text{ for odd}\ .\end{cases} \tag{A12}$$

We can write the inelastic amplitude as

$$\langle \mathbf{K}_b;I_BM_B,\tfrac{1}{2}^+m_b\,|\,t_{\rm NN}\,|\,\mathbf{K}_a;I_AM_A,\tfrac{1}{2}^+m_a\rangle=\sum_i T(i)=\sum_i \langle I_BM_B\,|\,t_i(2)\langle \mathbf{K}_b;m_b\,|\,t_i(1)\,|\,\mathbf{K}_a;m_a\rangle\,|\,I_AM_A\rangle\ , \tag{A13}$$

where 1 (2) refers to the projectile (target) nucleon.

We now apply the time-reversal transformation to the partial matrix element involving the projectile. We have, for the ith term,

$$\langle \mathbf{K}_b;m_b\,|\,\theta_1^{-1}\theta_1 t_i(1)\theta_1^{-1}\theta_1\,|\,\mathbf{K}_a;m_a\rangle=(-1)^{1/2+1/2-m_b-m_a}(-1)^{\eta_i}\langle -\mathbf{K}_a;-m_a\,|\,t_i\,|\,-\mathbf{K}_b;-m_b\rangle \tag{A14}$$

or

$$T(i)=\langle I_BM_B\,|\,t_i(2)\langle \mathbf{K}_b;m_b\,|\,t_i(1)\,|\,\mathbf{K}_a;m_a\rangle\,|\,I_AM_A\rangle$$
$$=(-1)^{\eta_i}(-1)^{1-(m_b+m_a)}\langle I_BM_B\,|\,t_i(2)\langle \mathbf{K}_b;-m_a\,|\,t_i(1)\,|\,\mathbf{K}_a;-m_b\rangle\,|\,I_AM_A\rangle\ .$$

We now apply $R(\pi,z)$ and obtain

$$T(i)=(-1)^{\eta_i}(-1)^{M_B-M_A}\langle I_BM_B\,|\,t_i(2)\langle \mathbf{K}_b;-m_a\,|\,t_i(1)\,|\,\mathbf{K}_a;-m_b\rangle\,|\,I_AM_A\rangle\ , \tag{A15}$$

where we have used the fact that, under $R(\pi,z)$, $-\mathbf{K}_a\rightarrow\mathbf{K}_b$ and $-\mathbf{K}_b\rightarrow\mathbf{K}_a$ if $|\mathbf{K}_a|=|\mathbf{K}_b|$ as is the case for $Q=0$ which we assume throughout.

For $I_A=0$ and $\pi_A=+1$, we have

$$T_{M_B}(i;m_b,m_a)=\langle I_BM_B\,|\,t_i(2)\langle \mathbf{K}_b;m_b\,|\,t_i(1)\,|\,\mathbf{K}_a;m_a\rangle\,|\,M_A\rangle$$
$$=(-1)^{\eta_i+M_B}\langle I_BM_B\,|\,t_i(2)\langle \mathbf{K}_b;-m_a\,|\,t_i(1)\,|\,\mathbf{K}_a;-m_b\rangle\,|\,M_A\rangle$$
$$=(-1)^{\eta_i+M_B}T_{M_B}(i;-m_a,-m_b)\ . \tag{A16}$$

We now consider $M_B=0$ and $m_a=m_b=m$. We then have

$$T_0(i;m,m)=(-1)^{\eta_i}T_0(i;-m,-m)\ .$$

For time-reversal even operators ($\eta_i=0$), we have

$$T_0(i;m,m)=T_0(i;-m,-m)\text{ or }T_0(i)\rightarrow A_0\mathbf{1}\ . \tag{A17a}$$

For time-reversal odd, we have

$$T_0(i;m,m)=-T_0(i;-m,-m)\ \text{ or }T_0(i)\rightarrow D_0\sigma_q\ . \tag{A17b}$$

If we now assume $m_a=-m_b=m$, we have

$$T_0(i;-m,m)=(-1)^{\eta_i}T_0(i;,-m,m)\ ,$$

which *requires* $\eta_i=0$ or time-reversal even. We then conclude that A_0, B_0, and C_0 arise from time-reversal even operators while D_0 originates with time-reversal odd operators.

If we allow $M_B\neq0$, we follow the above procedure and find that, in general, for

$$T_{M_B}(i;m_b,m_a)\rightarrow A_{M_B}\mathbf{1}+B_{M_B}\sigma_n+C_{M_B}\sigma_p+D_{M_B}\sigma_q$$

we require

$$\eta_i+M_B=\text{even}\Longrightarrow A_{M_B},B_{M_B},C_{M_B}\text{ terms}\ , \tag{A18a}$$

$$\eta_i+M_B=\text{odd}\Longrightarrow D_{M_B}\text{ term}\ . \tag{A18b}$$

These findings are consistent with the time-reversal relations

$$\sigma\rightarrow-\sigma,\ \ \hat{\mathbf{n}}\rightarrow-\hat{\mathbf{n}},\ \ \hat{\mathbf{p}}\rightarrow-\hat{\mathbf{p}},\ \ \hat{\mathbf{q}}\rightarrow\hat{\mathbf{q}}\ .$$

Hence $\mathbf{1}$, σ_n, and σ_p are time-reversal even operators in the projectile space while σ_q is time-reversal odd.

APPENDIX B: INELASTIC SCATTERING NUCLEAR STRUCTURE AMPLITUDES

We wish to evaluate explicitly the nuclear structure matrix elements appearing in the standard impulse approximation of Eq. (18). Similar developments have appeared elsewhere but we wish to formulate the problem in a specific notation which we then generalize for use in the relativistic impulse approximation (either FRIA or DRIA).

Initially we will consider the "Coulomb" or non-spin-transfer matrix element:

$$\rho_J(q)\equiv\langle J,0\,|\,e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,0,0\rangle$$
$$=\sqrt{4\pi(2J+1)}(-i)^J\langle J||j_J(qr)Y_J(\hat{\mathbf{r}})||0\rangle\ , \tag{B1}$$

where the reduced matrix element is defined by Eq. (38d).

Next we treat the spin-transfer matrix element.

$$\Sigma_{JM}=\langle JM\,|\,\sigma\cdot\hat{\mathbf{e}}_M e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,00\rangle$$
$$=\sqrt{4\pi(2J+1)}(-i)^J\{i\sqrt{(2J-1)/(2J+1)}(J-1\,1\,0M\,|\,JM)\langle J||j_{J-1}(qr)[Y_{J-1}(\hat{\mathbf{r}})\sigma_1]_J||0\rangle$$
$$+(J\,1\,0M\,|\,JM)\langle J||j_J(qr)[Y_J(\hat{\mathbf{r}})\sigma_1]_J||0\rangle-i\sqrt{(2J+3)/(2J+1)}$$
$$\times[(J+1)\,1\,0M\,|\,JM]\langle J||j_{J+1}(qr)[Y_{J+1}(\hat{\mathbf{r}})\sigma_1]_J||0\rangle\}\ . \quad \text{(B2)}$$

Substituting explicit formulas for the Clebsch-Gordan coefficients enables us to write

$$\Sigma_{JM}=\delta_{M,0}(\sqrt{2J}\,\Sigma_{J,J-1}+\sqrt{2J+2}\Sigma_{J,J+1})+(\delta_{M,+1}+\delta_{M,-1})$$
$$\times(\sqrt{J+1}\Sigma_{J,J-1}-\sqrt{J}\,\Sigma_{J,J+1})+(\delta_{M,+1}-\delta_{M,-1})(-\sqrt{2J+1}\Sigma_{J,J})$$
$$=\delta_{M,0}\Sigma_J^L+(\delta_{M,+1}+\delta_{M,-1})\Sigma_{J,1}^T+(\delta_{M,+1}-\delta_{M,-1})\Sigma_{J,0}^T\ , \quad \text{(B3)}$$

which defines the Σ^L and Σ^T terms and where we have used the following definitions:

$$\Sigma_{J,J\pm1}\equiv\sqrt{2\pi}(-i)^{J-1}\langle J||j_{J\pm1}(qr)[Y_{J\pm1}(\hat{\mathbf{r}})\sigma_1]_J||0\rangle\ , \quad \text{(B4a)}$$

$$\Sigma_{J,J}\equiv\sqrt{2\pi}(-i)^J\langle J||j_J(qr)[Y_J(\hat{\mathbf{r}})\sigma_1]_J||0\rangle\ . \quad \text{(B4b)}$$

In these expressions, L and T refer to longitudinal and transverse, respectively. The transition operator carries the parity of the spherical harmonic, i.e., $\Delta\pi=(-1)^L$. Therefore Σ_J^L and $\Sigma_{J,1}^T$ can contribute only to unnatural parity transitions while $\Sigma_{J,1}^T$ can be present only for natural parity transitions. Note that, by construction, we have

$$\Sigma_J^L=\langle J,0\,|\,\sigma_0(e^{-i\mathbf{q}\cdot\mathbf{r}})_{L=J\pm1}\,|\,0,0\rangle\ , \quad \text{(B5a)}$$

$$\Sigma_{J,1}^T=\langle J,+1\,|\,\sigma_{+1}(e^{-i\mathbf{q}\cdot\mathbf{r}})_{L=J\pm1}\,|\,0,0\rangle\ , \quad \text{(B5b)}$$

$$\Sigma_{J,0}^T=\langle J,+1\,|\,\sigma_{+1}(e^{-i\mathbf{q}\cdot\mathbf{r}})_{L=J}\,|\,0,0\rangle\ . \quad \text{(B5c)}$$

APPENDIX C: THE FRIA INELASTIC SCATTERING AMPLITUDE

We wish to obtain the values of the $A_M^{\pm}$, $B_M^{\pm}$, $C_M^{\pm}$, and $D_M^{\pm}$ [see Eq. (3)] implied by the FRIA for proton-nucleus inelastic scattering [Eq. (33)]. To begin, we require the expressions for the quantities $\mathscr{A}^i$ and $\mathscr{B}^i$ appearing in Eqs. (30b) and (33). These are related to the Dirac spinor matrix elements given in Table VIII. According to Eq. (30), we then obtain the projectile spin functions, $\mathscr{A}^i$ and $\mathscr{B}^i$, presented in Table IX.

The FRIA plane-wave, proton-nucleus inelastic scattering amplitudes appearing in Table I are obtained using Eq. (33) in conjunction with the definitions of Eq. (3) and the projectile spin functions appearing in Table IX.

TABLE VIII. Dirac spinor matrix elements.

$u^\dagger\Gamma_1 u=\dfrac{E+m}{2m}\left[1+\dfrac{p^2-q^2/4}{(E+m)^2}-\dfrac{i\sigma\cdot\hat{\mathbf{n}}pq}{(E+m)^2}\right]$	
$u^\dagger\Gamma_2 u=\dfrac{E+m}{2m}\left[1-\dfrac{p^2-q^2/4}{(E+m)^2}+\dfrac{i\sigma\cdot\hat{\mathbf{n}}pq}{(E+m)^2}\right]$	
$u^\dagger\Gamma_3 u=\dfrac{\sigma\cdot\mathbf{p}}{m}$	$u^\dagger\Gamma_4 u=-\dfrac{\sigma\cdot\mathbf{q}}{2m}$
$u^\dagger\sigma\Gamma_1 u=\sigma\dfrac{E+m}{2m}\left[1-\dfrac{p^2-q^2/4}{(E+m)^2}\right]+\dfrac{1}{2m(E+m)}(2\sigma\cdot\mathbf{p}\mathbf{p}-\tfrac{1}{2}\sigma\cdot\mathbf{q}\mathbf{q}+i\hat{\mathbf{n}}pq)$	
$u^\dagger\sigma\Gamma_2 u=\sigma\dfrac{E+m}{2m}\left[1+\dfrac{p^2-q^2/4}{(E+m)^2}\right]-\dfrac{1}{2m(E+m)}(2\sigma\cdot\mathbf{p}\mathbf{p}-\tfrac{1}{2}\sigma\cdot\mathbf{q}\mathbf{q}+i\hat{\mathbf{n}}pq)$	
$u^\dagger\sigma\Gamma_3 u=\dfrac{1}{2m}(2\mathbf{p}+i\sigma\times\mathbf{q})$	$u^\dagger\sigma\Gamma_4 u=-\dfrac{1}{2m}(\mathbf{q}+2i\sigma\times\mathbf{p})$

APPENDIX D: NUCLEAR STRUCTURE DEPENDENCE OF COMPOSITE-CURRENT TRANSITION DENSITIES

We rewrite Eq. (38a), specifying four-component wave functions, $J_i^\pi = 0^+$, and $n_i = n_f = n$, $l_i = l_f = l$:

$$\langle \tilde{J}\tilde{M} \,|\, \theta_{JM} \,|\, \tilde{0}\tilde{0} \rangle = -\sum_{j_i j_f} \mathscr{A}^{J0}_{J(j_f j_f)} \hat{j}_f \hat{J}^{-1} \langle \tilde{j}_f \| \theta_J \| \tilde{j}_i \rangle \,, \tag{D1}$$

where $J = l \pm \frac{1}{2}$. We note that a four-component, bound-state wave function satisfying the time-independent Dirac equation

$$[\not{p} - m - S(r) - \gamma^0 V(r)]\psi_{ljm} = 0$$

can be written in the following general form:

$$\psi_{ljm} = | \widetilde{ljm} \rangle = \begin{bmatrix} u_{ljm} \\ iw_{ljm} \end{bmatrix} = \begin{bmatrix} u_{lj}(r)[Y_l \chi_{1/2}]_{jm} \\ iw_{lj}(r)[Y_{l'} \chi_{1/2}]_{jm} \end{bmatrix} , \tag{D2}$$

where $l' = 2j - l$ and where $u_{lj}(r)$ and $w_{lj}(r)$ are real. This is also, of course, the form of the FRIA wave functions [Eqs. (12) and (26)] where, in effect, $S(r)$ and $V(r)$ are zero.

We now use Eq. (38) to write, for an unnatural parity transition (J odd),

$$\begin{aligned} \rho_J^3(q) &\equiv \langle \tilde{J}\tilde{0} \,|\, \Gamma_3 e^{-i\mathbf{q}\cdot\mathbf{r}} \,|\, \tilde{0}\tilde{0} \rangle \\ &= -(-i)^J \sqrt{4\pi} \hat{J} \sum_{j_f j_i} \mathscr{A}^{J0}_{J(j_f j_i)} \hat{j}_f \hat{J}^{-1} \langle \tilde{j}_f \| \Gamma_3 j_J(qr) Y_J(\hat{\mathbf{r}}) \| \tilde{j}_i \rangle \\ &= (-i)^{J+1} \sqrt{4\pi} [\mathscr{A}_{j_- j_-} \hat{j}_- (\langle u_{j_-} \| j_J Y_J \| w_{j_-} \rangle - \langle w_{j_-} \| j_J Y_J \| u_{j_-} \rangle) \\ &\qquad + \mathscr{A}_{j_+ j_-} \hat{j}_+ (\langle u_{j_+} \| j_J Y_J \| w_{j_-} \rangle - \langle w_{j_+} \| j_J Y_J \| u_{j_-} \rangle) \\ &\qquad + \mathscr{A}_{j_- j_+} \hat{j}_- (\langle u_{j_-} \| j_J Y_J \| w_{j_+} \rangle - \langle w_{j_-} \| j_J Y_J \| u_{j_+} \rangle) \\ &\qquad + \mathscr{A}_{j_+ j_+} \hat{j}_+ (\langle u_{j_+} \| j_J Y_J \| w_{j_+} \rangle - \langle w_{j_+} \| j_J Y_J \| u_{j_+} \rangle)] \,, \end{aligned} \tag{D3}$$

where $j_\pm = l \pm \frac{1}{2}$. This expression can be rearranged with the help of the following identity:

$$\langle j_i \| [Y_L \sigma_S]_J \| j_f \rangle = (-)^{j_i - j_f + L + S + J} \hat{j}_f \hat{j}_i^{-1} \times \langle j_f \| [Y_L \sigma_S] \| j_i \rangle \,. \tag{D4}$$

We then obtain

$$\rho_J^3(q) = (-i)^{J+1} \sqrt{4\pi} (\mathscr{A}_{j_+ j_-} + \mathscr{A}_{j_- j_+}) \hat{j}_+ \times (\langle u_{j_+} \| j_J Y_J \| w_{j_-} \rangle - \langle w_{j_+} \| j_J Y_J \| u_{j_-} \rangle) \,. \tag{D5}$$

We may write

$$\mathscr{A}^{J0}_{J(j_+ j_-)} + \mathscr{A}^{J0}_{J(j_- j_+)} = \sum_{LS} \hat{j}_+ \hat{j}_- \hat{L} \hat{S} [1 + (-)^{\mathscr{S}}] \times \begin{Bmatrix} l & \frac{1}{2} & j_+ \\ l & \frac{1}{2} & j_- \\ L & S & J \end{Bmatrix} \mathscr{A}^{J0}_{J(LS)} \,, \tag{D6}$$

where $\mathscr{S}$ is the sum of all entries in the 9-j symbol and evidently must be even. This amounts to requiring $L+S=$even since $j_+ + j_- = 2l$ and J is odd. This latter fact also requires that $S=1$ and therefore that $L=$odd. Equation (D5) then becomes

$$\rho_J^3 = 2(-i)^{J+1} \sqrt{12\pi} \sum_{L \text{ odd}} \hat{j}_+ \hat{j}_- \hat{L} \begin{Bmatrix} l & \frac{1}{2} & j_+ \\ l & \frac{1}{2} & j_- \\ L & 1 & J \end{Bmatrix} \mathscr{A}^{J0}_{J(L1)} \hat{j}_f (\langle u_{j_+} \| j_J Y_J \| w_{j_-} \rangle - \langle w_{j_+} \| j_J Y_J \| u_{j_-} \rangle) \,. \tag{D7}$$

If we are in the p shell, for example, and consider the excitation of a 1^+ level, then only the $L=S=J=1$ nuclear structure amplitude will contribute to ρ_J^3, or $\langle 1^+,0 \,|\, \sigma\cdot\mathbf{j} e^{-i\mathbf{q}\cdot\mathbf{r}} \,|\, 0^+,0 \rangle$ in the FRIA limit. We now consider

$$\Sigma_J^{4,T}=\langle \tilde{J},+\tilde{1}\,|\,\sigma_{+1}\Gamma_4 e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,\tilde{0},\tilde{0}\rangle$$
$$=-\sum_L(-i)^L\sqrt{4\pi}\hat{L}(L\,101\,|\,J\,1)\sum_{j_fj_i}\mathscr{A}^{J0}_{J(j_fj_i)}\hat{j}_f\hat{J}^{-1}\langle \tilde{j}_f||\Gamma_4 j_L[Y_L\sigma_1]_J||\tilde{j}_i\rangle$$
$$=\sum_L(-i)^{L+1}\sqrt{4\pi}\hat{L}\,\hat{J}^{-1}(L\,101\,|\,J\,1)$$
$$\times[\mathscr{A}_{j_-j_-}\hat{j}_-(\langle u_{j_-}||j_J[Y_L\sigma_1]_J||w_{j_-}\rangle-\langle w_{j_-}||j_J[Y_L\sigma_1]_J||u_{j_-}\rangle)$$
$$+\mathscr{A}_{j_+j_-}\hat{j}_+(\langle u_{j_+}||j_J[Y_L\sigma_1]_J||w_{j_-}\rangle-\langle w_{j_+}||j_J[Y_L\sigma_1]_J||u_{j_-}\rangle)$$
$$+\mathscr{A}_{j_-j_+}\hat{j}_-(\langle u_{j_-}||j_J[Y_L\sigma_1]_J||w_{j_+}\rangle-\langle w_{j_-}||j_J[Y_L\sigma_1]_J||u_{j_+}\rangle)$$
$$+\mathscr{A}_{j_+j_+}\hat{j}_+(\langle u_{j_+}||j_J[Y_L\sigma_1]_J||w_{j_+}\rangle-\langle w_{j_+}||j_J[Y_L\sigma_1]_J||u_{j_+}\rangle)]\,. \quad \text{(D8)}$$

For unnatural parity transitions, we have $L=J$ and, consequently,

$$\Sigma^{4T}_{J0}=-(-i)^{J+1}\sqrt{2\pi}(\mathscr{A}_{j_+j_-}+\mathscr{A}_{j_-j_+})\hat{j}_+(\langle u_{j_+}||j_J[Y_J\sigma_1]_J||w_{j_-}\rangle+\langle w_{j_+}||j_J[Y_J\sigma_1]_J||u_{j_-}\rangle) \quad \text{(D9)}$$

which gives

$$\Sigma^{4T}_{J0}=-2(-i)^{J+1}\sqrt{6\pi}\sum_{L\text{ odd}}\hat{j}_+\hat{j}_-\hat{L}\begin{Bmatrix} l & \frac{1}{2} & j_+\\ l & \frac{1}{2} & j_-\\ L & 1 & J\end{Bmatrix}\mathscr{A}^{J0}_{J(L1)}\hat{j}_+(\langle u_{j_+}||j_J[Y_J\sigma_1]_J||w_{j_-}\rangle+\langle w_{j_+}||j_J[Y_J\sigma_1]_J||u_{j_-}\rangle) \quad \text{(D10)}$$

indicating that this transition density depends on the same nuclear structure amplitudes as $\rho_J^3(q)$.

For natural parity transitions, $L=J\pm1$. We may then write

$$\Sigma^{4T}_{J1}=\sum_L(-i)^{L+1}\sqrt{4\pi}\hat{L}\,\hat{J}^{-1}(L\,101\,|\,J\,1)$$
$$\times[2\mathscr{A}_{j_-j_-}\hat{j}_-\langle u_{j_-}||j_L[Y_L\sigma_1]_J||w_{j_-}\rangle+2\mathscr{A}_{j_+j_+}\hat{j}_-\langle u_{j_+}||j_L[Y_L\sigma_1]_J||w_{j_+}\rangle$$
$$+(\mathscr{A}_{j_+j_-}-\mathscr{A}_{j_-j_+})\hat{j}_+(\langle u_{j_+}||j_L[Y_L\sigma_1]_J||w_{j-}\rangle+\langle w_{j_+}||j_J[Y_L\sigma_1]_J||u_{j_-}\rangle)]$$
$$=\sum_L(-i)^{L+1}\sqrt{4\pi}\hat{L}\,\hat{J}^{-1}(L\,101\,|\,J\,1)$$
$$\times\sum_{L'S'}\hat{L}'\hat{S}'\mathscr{A}^{J0}_{J(L'S')}\left\{2(2j_-+1)\begin{Bmatrix} l & \frac{1}{2} & j_-\\ l & \frac{1}{2} & j_-\\ L' & S' & J\end{Bmatrix}\hat{j}_-\langle u_{j_-}||j_L[Y_L\sigma_1]_J||w_{j_-}\rangle+2(2j_++1)\right.$$
$$\times\begin{Bmatrix} l & \frac{1}{2} & j_+\\ l & \frac{1}{2} & j_+\\ L' & S' & J\end{Bmatrix}\hat{j}_+\langle u_{j_+}||j_J[Y_L\sigma_1]_J||w_{j_+}\rangle+\hat{j}_+\hat{j}_-[1+(-)^{1+\mathscr{S}}]\begin{Bmatrix} l & \frac{1}{2} & j_+\\ l & \frac{1}{2} & j_-\\ L' & S' & J\end{Bmatrix}$$
$$\left.\times\hat{j}_+(\langle u_{j_+}||j_L[Y_L\sigma_1]_J||w_{j_-}\rangle+\langle w_{j_+}||j_L[Y_L\sigma_1]_J||u_{j_-}\rangle)\right\}\,. \quad \text{(D11)}$$

The symmetry of the 9-j symbols and the requirement that $\mathscr{S}$ be odd both imply that $L'+S'=$even. In general, $L'=J$, $S'=0$ and $L'=J\pm1$, $S'=1$ will satisfy this condition. However, referring to Table III, we note that, in FRIA,

$$\Sigma^{4T}_{J1}\rightarrow -i\langle J,+1\,|\,[\boldsymbol{\sigma}\times\mathbf{j}]_{+1}(e^{-i\mathbf{q}\cdot\mathbf{r}})_{L=J\pm1}\,|\,00\rangle\,.$$

Comparison of this expression with Eq. (43) tells us that $S'=1$ in the FRIA limit therefore implying $L'=$odd. We may then write, in FRIA,

$$\Sigma_{J1}^{4T} \to 2\sum_{L}(-i)^{L+1}\sqrt{12\pi}\hat{L}\hat{J}^{-1}(L\,101\,|\,J1)$$

$$\times \sum_{L'\text{ odd}} \hat{L}'\mathscr{A}_{J(L'1)}^{J0}\left[(2j_-+1)^{3/2}\begin{Bmatrix} l & \frac{1}{2} & j_- \\ l & \frac{1}{2} & j_- \\ L' & 1 & J\end{Bmatrix}\langle u_{j_-}||j_L[Y_L\sigma_1]_1||w_{j_-}\rangle\right.$$

$$+(2j_++1)^{3/2}\begin{Bmatrix} l & \frac{1}{2} & j_+ \\ l & \frac{1}{2} & j_+ \\ L' & 1 & J\end{Bmatrix}\langle u_{j_+}||j_L[Y_L\sigma_1]_1||w_{j_+}\rangle$$

$$\left.+(2j_++1)\hat{j}_-\begin{Bmatrix} l & \frac{1}{2} & j_+ \\ l & \frac{1}{2} & j_- \\ L' & 1 & J\end{Bmatrix}(\langle u_{j_+}||j_L[Y_L\sigma_1]_1||w_{j_-}\rangle+\langle u_{j_-}||j_L[Y_L\sigma_1]_J||w_{j_+}\rangle)\right]. \tag{D12}$$

Thus, only nuclear structure amplitudes $\mathscr{A}_{J(L'1)}^{J0}$ with $L'=J\pm 1$ contribute to Σ_{J1}^{4T} (in FRIA) for $l_f=l_i$. We note that $\mathscr{A}_{J(J0)}^{J0}$ can contribute to Σ_{J1}^{4L} in FRIA and to Σ_{J1}^{4T} if the FRIA limit is *not* taken.

The amplitudes $\mathscr{A}_{J(LS)}^{J0}$ with $L=$odd are referred to as "abnormal parity" amplitudes for $l_f=l_i$.

TABLE IX. Projectile spin functions ($t_S \to S$, $t_V \to V$, etc.).

$$\mathscr{A}^1 = V\left[\frac{E+m}{2m}+\frac{p^2-q^2/4}{2m(E+m)}-\frac{ipq}{8m(E+m)}\sigma_n\right]$$

$$\mathscr{A}^2 = S\left[\frac{E+m}{2m}+\frac{p^2-q^2/4}{2m(E+m)}+\frac{ipq}{8m(E+m)}\sigma_n\right]$$

$$\mathscr{A}^3 = A\frac{p}{m}\sigma_p \qquad \mathscr{A}^4 = -p\frac{q}{2m}\sigma_q$$

$$\mathscr{B}^1\cdot\hat{\mathbf{q}} = -A\left[\frac{E+m}{2m}-\frac{p^2+q^2/4}{2m(E+m)}\right]\sigma_q$$

$$\mathscr{B}^2\cdot\hat{\mathbf{q}} = 2T\left[\frac{E+m}{2m}+\frac{p^2+q^2/4}{2m(E+m)}\right]\sigma_q$$

$$\mathscr{B}^3\cdot\hat{\mathbf{q}} = 0 \qquad \mathscr{B}^4\cdot\hat{\mathbf{q}} = -2T\left[\frac{q}{2m}+\frac{ip}{m}\sigma_n\right]$$

$$\mathscr{B}^1\cdot\hat{\mathbf{p}} = -A\left[\frac{E+m}{2m}+\frac{p^2+q^2/4}{2m(E+m)}\right]\sigma_p$$

$$\mathscr{B}^2\cdot\hat{\mathbf{p}} = 2T\left[\frac{E+m}{2m}-\frac{p^2+q^2/4}{2m(E+m)}\right]\sigma_p$$

$$\mathscr{B}^3\cdot\hat{\mathbf{p}} = -V\left[\frac{p}{m}-\frac{iq}{2m}\sigma_n\right] \qquad \mathscr{B}^4\cdot\hat{\mathbf{p}} = 0$$

$$\mathscr{B}^1\cdot\hat{\mathbf{n}} = -A\left\{\frac{ipq}{2m(E+m)}+\left[\frac{E+m}{2m}-\frac{p^2-q^2/4}{2m(E+m)}\right]\sigma_n\right\}$$

$$\mathscr{B}^2\cdot\hat{\mathbf{n}} = 2T\left\{\frac{-ipq}{2m(E+m)}+\left[\frac{E+m}{2m}+\frac{p^2-q^2/4}{2m(E+m)}\right]\sigma_n\right\}$$

$$\mathscr{B}^3\cdot\hat{\mathbf{n}} = V\left[\frac{-iq}{2m}\right]\sigma_p \qquad \mathscr{B}^4\cdot\hat{\mathbf{n}} = 2T\left[\frac{ip}{m}\right]\sigma_q$$

APPENDIX E: ELECTRON-NUCLEUS (e,e′) IN FRIA

The electron-nucleus scattering amplitude may be written as

$$M_{eN} = j_e\cdot J_N/q^2 \tag{E1}$$

where the electron and nucleon currents are, respectively,

$$j_e^\mu = \bar{u}'_e\gamma^\mu u_e = u_e'^\dagger\gamma^0\gamma^\mu u_e\ , \tag{E2a}$$

$$J_N^\mu = \bar{u}'_N\tilde{J}^\mu u_N = u_N'^\dagger\gamma^0\tilde{J}^\mu u_N\ , \tag{E2b}$$

and the free nucleon electromagnetic current operator is given by

$$\tilde{J}_N^\mu = e_N F_1(q^2)\gamma^\mu + \frac{i\kappa_N}{2m}F_2(q^2)\sigma^{\mu\nu}(p_f-p_i)_\nu\ , \tag{E3}$$

where $F_i(q^2)$ denotes the nucleon form factors, and e_N and κ_N are the charge and anomalous magnetic moment of the nucleon, respectively. We use p to designate the nucleon momentum, implying $p_f-p_i=k_i-k_f=-q$, where k is the electron momentum.

We may now write the electron-nucleon amplitude in a form analogous to that of the NN t matrix of Eq. (10):

$$M_{eN} = u_e'^\dagger u_N'^\dagger t_{eN} u_e u_N/q^2\ , \tag{E4a}$$

$$t_{eN} = \gamma^0(\mathrm{e})\gamma^0(\mathrm{N})\left[e_N\gamma(\mathrm{e})\cdot\gamma(\mathrm{N})-\frac{i\kappa_N}{2m}\gamma_\mu(\mathrm{e})\sigma^{\mu\nu}(\mathrm{N})q_\nu\right], \tag{E4b}$$

where the form factors are now implicit. The "t matrix" of Eq. (E4) may be rewritten so as to facilitate comparison with the expression for t_{NN} appearing in Eq. (27):

$$t_{eN}=\Gamma_1(\mathrm{e})\left[e_N\Gamma_1(\mathrm{N})-\frac{\kappa_N}{2m}\sigma_N\cdot\mathbf{q}\Gamma_4(\mathrm{N})\right]-\sigma_e\Gamma_3(\mathrm{e})\left[e_N\sigma_N\Gamma_3(\mathrm{N})-\frac{i\kappa_N}{2m}\sigma_N\times\mathbf{q}\Gamma_2(\mathrm{N})-\frac{\kappa_N}{2m}\sigma_N q_0\Gamma_4(\mathrm{N})\right] . \quad \text{(E5)}$$

Note that q_0 is just the overall Q value of the reaction and, since we have assumed $q_0=Q=0$ throughout, we now drop the last term in Eq. (E5).

We may straightforwardly express the electron-nucleus inelastic scattering amplitude in a form analogous to the proton-nucleus amplitude of Eq. (30b):

$$T_{fi}(\mathrm{e,e'})=\mathscr{A}_e\left\langle\tilde{J}\tilde{M}\left|\left[e_N\Gamma_1-\frac{\kappa_N}{2m}\sigma\cdot\mathbf{q}\Gamma_4\right]e^{-i\mathbf{q}\cdot\mathbf{r}}\right|\tilde{0}\tilde{0}\right\rangle-\mathscr{B}_e\left\langle\tilde{J}\tilde{M}\left|\left[e_N\sigma\Gamma_3-\frac{i\kappa_N}{2m}\sigma\times\mathbf{q}\Gamma_2\right]e^{-i\mathbf{q}\cdot\mathbf{r}}\right|\tilde{0}\tilde{0}\right\rangle , \quad \text{(E6a)}$$

where

$$\mathscr{A}_e=\left[\frac{E}{m}-\frac{q^2}{8m(E+m)}\right]-\frac{i\sigma_n pq}{2m(E+m)} , \quad \text{(E6b)}$$

$$\mathscr{B}_e=\frac{\mathbf{p}}{m}+\frac{i}{2m}\sigma\times\mathbf{q} \quad \text{(E6c)}$$

and where $\mathbf{p}\equiv\frac{1}{2}(\mathbf{k}_f+\mathbf{k}_i)$ as in Eq. (2).

Then, in analogy with Eq. (33), we may write

$$T_{fi,M}(\mathrm{e,e'})=\mathscr{A}_e\left[e_N\rho_1-\frac{q}{2m}\kappa_N\Sigma_J^{4L}\right]\delta_{M,q}-\sum_{k=n,p}\left[-i\mathbf{q}\times\mathscr{B}_e\left[e_N\Sigma_{J0}^{3T}-\frac{q}{2m}\kappa_N\Sigma_{J1}^{2T}\right]+\mathscr{B}_e\left[e_N\Sigma_{J1}^{3T}-\frac{q}{2m}\kappa_N\Sigma_{J0}^{2T}\right]\right]\cdot\hat{\mathbf{e}}_k\delta_{M,k} , \quad \text{(E7)}$$

where we have used $\mathscr{B}_e\cdot\hat{\mathbf{q}}=0$ (Table IX).

By referring to Eq. (E7), we may directly obtain the quantities $A_M^{\pm}$, $B_M^{\pm}$, $C_M^{\pm}$, and $D_M^{\pm}$ [see Eq. (3)] for (e,e′). There quantities are presented in Table X which stands as the (e,e′) analog to Table I. It is then a straightforward matter to obtain the (e,e′) differential cross section:

$$\frac{d\sigma}{d\Omega}=\frac{Z^2\sigma_M}{1+2(E/M_T)\sin^2\theta/2}\,|F|^2 , \quad \text{(E8)}$$

TABLE X. Plane-wave inelastic electron scattering amplitudes.

$$\mathscr{F}_L\equiv\left[e_N\rho_J^1-\frac{q}{2m}\kappa_N\Sigma_J^{4L}\right] \qquad \mathscr{F}_T^{N\pi}\equiv\left[e_N\Sigma_{J1}^{3T}-\frac{q}{2m}\kappa_N\Sigma_{J0}^{2T}\right]$$

$$\mathscr{F}_T^{UN\pi}\equiv\left[e_N\Sigma_{J0}^{3T}-\frac{q}{2m}\kappa_N\Sigma_{J1}^{2T}\right]$$

Natural parity

T_q: $A_q=\left[\frac{E+m}{2m}+\frac{p^2-q^2/4}{2m(E+m)}\right]\mathscr{F}_L$, $B_q=\frac{-ipq}{2m(E+m)}\mathscr{F}_L$
$C_q=D_q=0$

T_p: $A_p=-\frac{p}{m}\mathscr{F}_T^{N\pi}$, $B_p=\frac{iq}{2m}\mathscr{F}_T^{N\pi}$, $C_p=D_p=0$

T_n: $A_n=B_n=0$, $C_n=-\frac{iq}{2m}\mathscr{F}_T^{N\pi}$, $D_n=0$

Unnatural parity

T_q: $A_q=B_q=C_q=D_q=0$

T_p: $A_p=B_p=0$, $C_p=-\frac{q}{2m}\mathscr{F}_T^{UN\pi}$, $D_p=0$

T_n: $A_n=-\frac{ip}{m}\mathscr{F}_T^{UN\pi}$, $B_n=-\frac{q}{2m}\mathscr{F}_T^{UN\pi}$, $C_n=D_n=0$

where

$$\sigma_M = \left[\frac{\alpha^2 \cos(\theta/2)}{4E^2 \sin^2\theta/2} \right]^2 ,$$

α is the fine-structure constant, E is the target mass, θ is the scattering angle, and the term in the denominator accounts for the target recoil. The form factor is

$$|F|^2 = |F_{\text{long}}|^2 + (\tfrac{1}{2} + \tan^2\theta/2)(|F_{\text{elec}}|^2 + |F_{\text{mag}}|^2) , \quad \text{(E9a)}$$

where the usual (e,e′) form factors can be expressed as

$$|F_{\text{long}}|^2 = Z^{-2} |\mathcal{F}_L|^2 , \quad \text{(E9b)}$$

$$|F_{\text{elec}}|^2 = 2Z^{-2} |\mathcal{F}_T^{N\pi}|^2 , \quad \text{(E9c)}$$

$$|F_{\text{mag}}|^2 = 2Z^{-2} |\mathcal{F}_T^{UN\pi}|^2 . \quad \text{(E9d)}$$

The expressions of Eq. (E9) summarize the results of Ref. 4.

The amplitudes of Table X reveal the well-known impoverishment of singles electron scattering spin transfer observables. One immediately finds that $P = A_y = 0$ and $Q = B$. Even the nonzero values of Q and B are uninteresting since they depend on the same combinations of nucleon transition densities as the cross section and therefore provide no new information. Finally we observe that the procedure outlined here for (e,e′) can readily be applied to the analysis of the inelastic scattering of other probes such as neutrinos, kaons, etc.

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