Hybrid quark-hadron model of Λ nonmesonic decay: Finite nuclei

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Λ hypernuclear nonmesonic lifetimes are calculated in the framework of the hybrid quark-hadron model, for both infinite nuclear matter and for a finite nucleus ${}^{12}_{\Lambda}$C. For nuclear matter, an approximate solution of the Bethe-Goldstone equation is used for the initial state ΛN cluster wave function. For ${}^{12}_{\Lambda}$C a shell model wave function is used. An eikonal approximation is used to represent final state interactions. It is shown that with an effective weak quark Hamiltonian explicitly constructed to be consistent with the $\Delta I=\frac{1}{2}$ rule the ${}^{12}_{\Lambda}$C nonmesonic width is about 1.3 times the free Λ width, which is consistent with a recent experimental measurement.

I. INTRODUCTION

A free Λ decays via the weak process $\Lambda \rightarrow N\pi$ with a rate $\Gamma_{\text{free}} \simeq 2.50\times 10^{-12}$ MeV, with the outgoing nucleon having a momentum of about 101 MeV/c, which is below the nuclear matter Fermi momentum. Thus, in medium to heavy nuclei Pauli blocking results in the dominance of the nonmesonic mode $\Lambda N \rightarrow NN$, where each final state nucleon has a momentum of about 417 MeV/c in the center of mass frame. The rate of this nonmesonic hypernuclear decay, Γ_{nm}, is of fundamental interest. Due to the large momentum transfer involved, a satisfactory explanation of Γ_{nm} requires both an understanding of the underlying interactions and a good model for the description of the reaction mechanism for both long and short distances. As we demonstrate in this paper, this process provides a constraint on the nature of the effective $\Delta S=1$ Hamiltonian at short distance. Since we use weak quark interactions directly in our model, we are able to extract constraints on the effective $\Delta S=1$ quark Hamiltonian.

In previous work,[1] which we refer to as paper I, we presented preliminary results for the calculation of the ratio $\Gamma_{\text{nm}}/\Gamma_{\text{free}}$ in nuclear matter. The interested reader is referred to Ref. 1 for a more in depth discussion of the motivation of this work and its formulation within the hybrid quark-hadron (HQH) model, and to Refs. 2 and 3 for a description of the model itself. To summarize briefly, we will calculate the amplitude for the process $\Lambda N \rightarrow NN$ in two parts. One part is a description of the decay in the six quark or interior region, defined by the relative variable describing the separation of the centers of mass of the two baryons being less than a distance $r_0 \approx 1$ fm. Here we assume that quarks interacting via the exchange of weak vector bosons modified by gluons give the appropriate weak interaction. The other part is a calculation of the contribution from the hadronic or exterior region where the description is given in terms of a weak Hamiltonian for baryons and mesons. The HQH model ensures that there is no double counting.

The format of this paper is as follows. In Sec. II we discuss the interior (quark) calculation, with separate subsections set aside for the consideration of wave functions and Hamiltonians. In Sec. III we present the formulation for the exterior region, both for nuclear matter as well as ^{12}C. The results and conclusions are discussed in Sec. IV.

The work presented here represents a substantial improvement over that of paper I which, for one thing, was a nuclear matter calculation only. We will point out other differences as they occur in the following text. Finally, we remark that a more detailed description of the calculational techniques employed here can be found in Ref. 4.

II. QUARK CONTRIBUTION

In this section, we outline the calculation of the quark contribution to Γ_{nm}. We use a spectator model, where it is assumed that the process $\Lambda N \rightarrow NN$ can be described (in the interior region) as four spectator quarks and two interacting quarks undergoing the weak interaction depicted in Fig. 1. We begin by discussing the quark wave functions.

A. Six-quark wave functions

It is an ansatz of the hybrid quark model[2,3] that we can write the wave function for the six quarks of the ΛN system as

$$\phi^{6q}_{(E)}=a(E)\sum_m C_m \phi^{6q}_m(\mathbf{r}_1,\ldots,\mathbf{r}_6)\,. \tag{1}$$

Here E is the total energy, the ϕ^{6q}_m constitute some complete set of orthonormal six-quark states, and $a(E)$ is the amplitude, with its square related to the six-quark probability—that is, the fraction of time that a two-baryon system spends in the interior region. Drawing an analogy to the ordinary nuclear shell model we use products of single particle (quark) wave functions:

$$\phi^{6q}_{(E)}=a(E)\sum_m C_m \prod_{i=1}^{6} \phi^{i}_m(\mathbf{r}_i)\,. \tag{2}$$

For our purposes, we have chosen to use massless quark

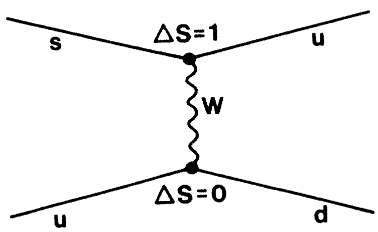

FIG. 1. Lowest-order short-range $\Delta S=1$ interaction.

MIT bag wave functions,[5] truncating our model space after the $p_{1/2}$ state, with the additional proviso that the spectator quarks are all in the $S_{1/2}$ state. Thus, our most general six-quark wave function is

$$\phi^{6q}(E)=a(E)(S_{1/2})^4\left[\alpha(S_{1/2})^2+\beta(p_{3/2})^2+\gamma(p_{1/2})^2+\frac{\delta}{\sqrt{2}}(p_{3/2}p_{1/2}+p_{1/2}p_{3/2})\right] . \quad (3)$$

It is hoped that with proper choices of the expansion coefficients α, β, γ, and δ we can adequately represent the strong interaction dynamics (QCD) we ignored in arriving at Eq. (3). Finally, we point out that this wave function is much improved over that of paper I, where our model space consisted only of the $S_{1/2}$ state.

B. Effective quark weak interaction Hamiltonian

In the absence of strong interactions, the interior Hamiltonian is taken as a standard $\Delta S=1$ Cabibbo form, corresponding to Fig. 1 for low energy W exchanges,

$$H_{\rm int}^{w}=\frac{G_w}{\sqrt{2}}\sin\theta_c\cos\theta_c\bar{u}\gamma_\mu(1-\gamma_5)s\bar{d}\gamma^{N}(1-\gamma_5)u+cc . \quad (4)$$

This is just the result of the combination of $V-A$ theory with the Cabibbo hypothesis concerning the relative strengths of $\Delta Q=1$ weak currents.[6]

It is possible to derive effective quark Hamiltonians which include strong interaction corrections through the use of renormalization group techniques. It was the large discrepancy between the Cabibbo model [Eq. (4)] and the experimental $\Delta I=\frac{1}{2}$ rule which motivated this theory for the $\Delta S=1$ effective Hamiltonian.[7] Using the values of Gilman and Wise,[8] based on the one loop gluonic corrections and renormalization group evolution, one obtains

$$H_{\rm int}^{w}=\frac{G_w}{\sqrt{2}}\sin\theta_c\cos\theta_c[c_1\bar{u}\gamma_\mu(1-\gamma_5)s\bar{d}\gamma^\mu(1-\gamma_5)u+c_2\bar{d}\gamma_\mu(1-\gamma_5)s\bar{u}\gamma^\mu(1-\gamma_5)u] , \quad (5)$$

where the constants c_1 and c_2 have the values

$$c_1=1.51 , \quad c_2=-0.856 . \quad (6)$$

In arriving at Eq. (5), we have used only leading operators with the same $(V-A)(V-A)$ chiral structure. Note the first term in Eq. (5) has the same form as the uncorrected interaction of Eq. (4), while the second term has the appearance of a strangeness changing neutral current. This effective Hamiltonian has a greatly improved $\Delta I=\frac{3}{2}$ to $\Delta I=\frac{1}{2}$ ratio, but still does not give the $\Delta I=\frac{1}{2}$ rule.

For the sake of comparison we can take the form of the Hamiltonian given by Eq. (5) as an ansatz and determine the values for c_1 and c_2 which preserve the $\Delta I=\frac{1}{2}$ rule. It is easy to see that the Hamiltonian can be written as

$$H_{\rm int}^{w}=\frac{G_w}{\sqrt{2}}\sin\theta_c\cos\theta_c[\tfrac{1}{2}(c_1+c_2)Q_{3/2}+\tfrac{1}{2}(c_1-c_2)Q_{1/2}] , \quad (7)$$

where Q_J is a pure $I=J$ operator. The ratio of the amplitudes is given by

$$R=\frac{A_{1/2}}{A_{3/2}}=\frac{c_1-c_2}{c_1+c_2} . \quad (8)$$

The Gilman-Wise results of Eq. (6) give $R=3.6$, whereas experiment gives a value of $R^{\rm exp}\approx 20$.

In the original paper of Galliard and Lee[7] the $\Delta I=\frac{1}{2}$ rule is obtained by calculating the ratio of the θ_8 operator ($I=\frac{1}{2}$, octet) to the $I=\frac{3}{2}$ operator θ_{27}. They find

$$\frac{c_1-c_2}{c_1+c_2}=\frac{\sqrt{5}}{2}\frac{K^{0.48}}{K^{-0.24}} , \quad (9)$$

where

$$K=1+\frac{g^2}{4\pi}\frac{b}{2\pi}\ln\frac{M_w}{\mu}$$

(see also Desplanques *et al.*[9]). The Gilman-Wise values give $K_0=5.17$. If one arbitrarily uses the functional form of Eq. (9) to fit the experimental value of $R=20$, then $K=54.7$. As a result we have

$$c_1+c_2=(1.51-0.859)\left[\frac{K_0}{K}\right]^{0.24}=0.37 . \quad (10)$$

This should not be considered to be a renormalization group calculation since the one loop basis is not justified, but simply a procedure for picking the coefficients c_1 and c_2 for the most important terms in the effective Hamiltonian. Calculation of the quark contribution to the nonmesonic rate gives[4]

$$\frac{\Gamma_Q^{\rm nm}}{\Gamma_{\rm free}}=2.73(c_1+c_2)^2P_i^{6q}F(\alpha,\beta,\gamma,\delta) , \quad (11)$$

where P_i^{6q} is the initial six-quark probability (see Sec. III) and $F(\alpha,\beta,\gamma,\delta)$ is a complicated function of the expansion coefficients of the wave function of Eq. (3). This leads to the result

$$\Gamma_Q^{\rm nm}/\Gamma_{\rm free}=2.73P_i^{6q}F(\alpha,\beta,\gamma,\delta)\times\begin{Bmatrix}1.0 \text{ uncorrected [Cabibbo, } H_{\rm int}^{w}(1)]\\ 0.42 \text{ strong interaction corrected, } H_{\rm int}^{w}(2)\\ 0.14\ \Delta I=\frac{1}{2} \text{ rule, } H_{\rm int}^{w}(3)\end{Bmatrix}. \quad (12)$$

As a final point we should emphasize that Eq. (12) is a phenomenological result incorporating the $\Delta I=\frac{1}{2}$ rule, guided by renormalization group results including "penguins." For this low energy region the entire procedure of obtaining effective Hamiltonians and the $\Delta I=\frac{1}{2}$ rule is an area of active research.

III. PIONIC CONTRIBUTION

In this section we calculate the contribution to the Λ nonmesonic decay rate from the exterior—one-pion exchange (OPE) region. Additionally, it is from the baryonic wave functions for this region that we calculate the quantity P_i^{6q} present in Eq. (12). Given a two-baryon bound wave function for the initial Λ-N cluster $\Psi_{\Lambda N}(r)$, where r is the relative variable, then conservation of probability requires

$$P_i^{6q}=1-\int_{r_0}^{\infty}|\Psi_{\Lambda N}(r)|^2r^2dr\ , \quad (13)$$

where we have neglected isospin and angular momentum labels for clarity. Clearly Eq. (13) can be used only for normalizable $\Psi_{\Lambda N}$. In this paper, we present results from a nuclear matter calculation, where P_i^{6q} cannot be calculated using Eq. (13) and must be obtained from other means, and also from a hypernuclear ^{12}C shell model calculation, where P_i^{6q} is directly calculable. For the final scattering states the six-quark amplitudes are also determined by probability conservation, using the methods of Ref. 3.

A. Nuclear matter

1. Potential

The lowest order $\Delta S=1$ process in the external region is shown in Fig. 2. For the weak vertex, the phenomenological form

$$H_w=iG_w\overline{\Psi}_N(1+\lambda\gamma_5)\tau\cdot\Psi_\Lambda\phi_\pi \quad (14)$$

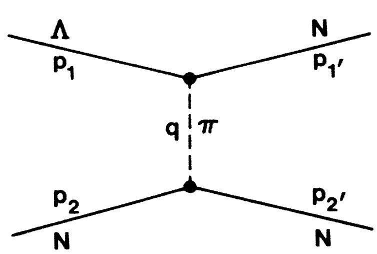

FIG. 2. Lowest-order long-range $\Delta S=1$ interaction.

was used, where λ is an empirical constant[10] taken to be $\lambda=-6.87$. Ψ_Λ represents the lambda spurion field, which is a direct product of the usual isosinglet Λ field with the $|I,I_3\rangle=|-\frac{1}{2},-\frac{1}{2}\rangle$ isospinor.

Using the usual pseudoscalar coupling for the strong vertex, and making the standard nonrelativistic reduction gives[1,4]

$$V(r)=[V_0(r)\sigma_1\cdot\sigma_2+V_1(r)\sigma_2\cdot\hat{r}+V_2(r)S_{12}]\tau_1\cdot\tau_2\ , \quad (15)$$

where

$$V_0(r)=\frac{G_sG_w}{4\pi}\frac{\lambda\tilde{m}^3}{12\overline{M}^2}\frac{e^{-\tilde{m}r}}{\tilde{m}r}\ ,$$
$$V_1(r)=\frac{G_sG_w}{4\pi}\frac{i\tilde{m}^2}{2M}\frac{e^{-\tilde{m}r}}{\tilde{m}r}\ , \quad (15a)$$
$$V_2(r)=\frac{G_sG_w}{4\pi}\frac{\lambda\tilde{m}^3}{4\overline{M}^2}\frac{e^{-\tilde{m}r}}{\tilde{m}r}\ .$$

In Eq. (15a), $\overline{M}$ is the average baryon mass for the incoming legs, $\overline{M}=\frac{1}{2}(M_\Lambda+M_N)$, and $\tilde{m}=107$ MeV is an effective pion mass, accounting for the fact that energy as well as momentum is transferred in the process of Fig. 2.

2. Correlations

For the initial state wave function, the ΛN cluster is taken to be in a relative S state. Denoting this wave function as $R_i(qr)$ we have, in the absence of initial state interactions,

$$R_i(qr)=j_0(qr)\ ,$$

where q is the relative ΛN momentum. To examine the effects of correlations, the wave function from Adams's paper[11] is used

$$R_i(qr)=\left[j_0(qr)+(1.072)\frac{\sin(qr_c)S_i(1.633r)}{qr}\right]\theta(r-r_c)\ , \quad (16)$$

where $S_i(x)$ is the sine-integral function. This represents an approximate solution of the Bethe-Goldstone equation for a potential consisting solely of a hard core of radius $r_c=0.4$ fm.

The form given in Eq. (16) causes S-S, S-P, and S-D transitions. The effect of final state interactions is estimated by using an eikonal approximation along with the central part of a ^{12}C-P optical potential.[12] Assuming back to back emergence of the final state nucleons, one obtains the relative NN wave function

$$R_{f_L}(Qr)=\frac{i^{-L}}{2}\int_{-1}^{1}\exp\left\{i\left[\mathbf{Q}\cdot\mathbf{r}-\frac{m}{Q}\int_{-\infty}^{z/2}U_{\rm opt}(x/2,y/2,z'/2)dz'\right]\right\}P_L(\cos\theta)d(\cos\theta) \tag{17}$$

for the Lth partial wave, where Q is the final relative NN momentum. The effects of the distorting potential can be examined by setting $U_{\rm opt}$ to zero, in which case $R_{f_L}(Qr)\to j_L(Qr)$.

Finally, the effect of smearing of the pion vertex is calculated in the standard way. Modification of the propagator

$$\frac{1}{q^2+\tilde{m}^2}\to\frac{1}{q^2+\tilde{m}^2}\frac{\Lambda_\pi^2}{q^2+\Lambda_\pi^2} \tag{18}$$

results in a corresponding change in the potential

$$V(r)\to\frac{\Lambda_\pi^2}{\tilde{m}^2-\Lambda_\pi^2}[V(r,\Lambda_\pi)-V(r,\tilde{m})]\ . \tag{19}$$

We expect that the correlations discussed here, which represent short distance corrections, will have a relatively small effect on our results. This is because the short distance part of the problem has already been projected out of the baryon space and into the quark regime.

The final ingredient of the nuclear matter calculation is the initial state six-quark probability

$$P_i^{6q}=0.15\ , \tag{20}$$

which is compatible with previous calculations using this model[13] and with a study of inclusive electron scattering from ^{3}H$_e$.[14] In the final nuclear case, we calculate P_i^{6q}.

B. Finite nucleus, $^{12}_{\Lambda}$C

In this section, we sketch the methodology involved in a finite nucleus calculation for $^{12}_{\Lambda}$C. We use a weak coupling model with the Λ taken in the lowest oscillator state

$$\Psi(^{12}_{\Lambda}{\rm C})=\Psi_{\rm core}(^{11}{\rm C})\otimes\Phi_{S_{1/2}}(\Lambda)\ . \tag{21}$$

Recent hypernuclear shell model calculations[15] show that this is a satisfactory model. A pure configuration wave function is used for the shell model core

$$\Psi_{\rm core}(^{11}{\rm C})=[(S_{1/2})^4(P_{3/2})^7]^{3/2^-,1/2}\ , \tag{22}$$

where the superscripts refer to the quantum numbers J^π and T. Our model space consists of only the $S_{1/2}$ and $P_{3/2}$ orbits.

In the process ΛN→NN the active nucleon can be from the p shell or the s shell. In the former, the resulting nucleus is in a $0\hbar\omega$ state. In the latter, the nucleus is in a $1\hbar\omega$ state and care must be taken to subtract out contributions from spurious (unphysical) components of the wave function. The amplitude for the pionic contribution now takes the form

$$T_{i_f}^{\rm N}=\sqrt{S}\,t_{i_f}\ , \tag{23}$$

where the spectroscopic factor S depends only on nuclear structure, and the two-body amplitude t_{i_f} depends on the reaction mechanism.

1. Structure calculation

The problem encountered here is identical to that of single nucleon pickup. We calculate the C^2S factor for p shell pickup using the techniques of Brussaard and Glaudemans.[16] The results, for all possible (J_fT_f) in our model space, are shown in Table I.

Experiments using the (p,2p) reaction[17] on p-shell nuclei demonstrate that states resulting from s-shell pickup appear as a broad bump at high excitation energy. Thus it is reasonable to concentrate all remaining strength, after subtraction of spurious components, into a single state for each possible (J_fT_f). Using results of Millener,[18] one obtains the spectroscopic factors of Table II.

2. Reaction calculation

Obtaining useful relative wave functions for use in a reaction calculation is a two-step process. First, a transformation from the JJ basis implied by Eq. (21) to the LS basis is performed. Then the Moshinky techniques are used to separate the relative and center of mass motions. To avoid complications we use only those components with center of mass angular momentum $L=0$, since only these have appreciable overlap with the final state center of mass wave function. The results are

$$\begin{aligned}(S_{1/2}P_{3/2})^2&=\sqrt{1/2}P_{3/2}\ ,\\(S_{1/2}P_{3/2})^1&=\sqrt{1/3}\,{}^1P_1+\sqrt{1/6}\,{}^3P_1\ ,\\(S_{1/2}S_{1/2})^1&={}^3S_1\ ,\\(S_{1/2}S_{1/2})^0&={}^1S_0\ ,\end{aligned} \tag{24}$$

where the quantities on the left-hand side are the ΛN states expressed in shell model quantum numbers, and those on the right-hand side are the same states expressed in relative variables and quantum numbers, using the standard spectroscopic notation. The lack of normalization regarding the two $l=1$ states reflects the inclusion of only $L=0$ components, as discussed above.

TABLE I. Spectroscopic factors for p-shell pickup from ^{11}C.

(J_fT_f)	S	$(C^2S)_{\rm p}$	$(C^2S)_{\rm n}$
(0,1)	0.75	0.25	0.50
(2,1)	3.75	1.25	2.50
(1,0)	0.75	0.75	
(3,0)	1.75	1.75	

TABLE II. Spectroscopic factors for s-shell pickup from ^{11}C.

$(J_f T_f)$	S_{tot}	S_{sp}	$(C^2 S_{ph})_p$	$(C^2 S_{ph})_n$
(1,0)	0.375	0.075	0.30	
(1,1)	1.125	0.227	0.30	0.60
(2,0)	0.625	0.175	0.45	
(2,1)	1.875	0.223	0.55	1.10

TABLE III. Initial state six-quark probabilities for ΛN cluster in $^{12}_{\Lambda}$C.

	Uncorrelated	Correlated
P_i^{6q} (s waves)	0.078	0.051
P_i^{6q} (p waves)	0.007	0.005

3. *Correlation function and the six-quark probability*

Using harmonic oscillator wave functions in Eq. (24), we are now in a position to calculate the initial state six-quark probability from Eq. (13). To address the problem of a lack of two-body correlations in the independent particle model we multiply our two-body densities by a phenomenological correlation function due to Miller and Spencer[19]

$$f(r)=1-e^{-\alpha r^2}(1-\beta r^2) . \tag{25}$$

We use the values $\alpha=1.1$ fm^{-2} and $\beta=0.68$ fm^{-2}, known to give good results in calculations of parity violating nuclear matrix elements.[20] The six-quark probabilities calculated for both the correlated and uncorrelated cases are presented in Table III.

The result for uncorrelated S waves $P_i^{6q}=0.078$ is to be compared to Eq. (20), the nuclear matter value. Presently it is not known how to reconcile this difference.

IV. RESULTS AND CONCLUSIONS

A. Nuclear matter

1. *Pionic contribution*

The results for the pionic contribution to the nonmesonic Λ decay rate in nuclear matter are given in Table IV. In succeeding rows, the additional effects indicated are turned on. The last row represents our "best" value, $\Gamma_\pi^{nm}=0.77\Gamma^{free}$.

These pionic results can be checked with the nuclear matter calculation of McKellar and Gibson,[21] who use a conventional hadronic model, by taking $r_0=0$. There is good agreement with the pionic contribution. They also show that tensor correlation effects are small, and thus our forms should be satisfactory.

2. *Quark contribution*

One needs a reliable effective quark Hamiltonian in order to determine the values of the expansion coefficients in Eq. (3); unfortunately, no such Hamiltonian is available to give a reliable approximation for the quark wave function. We have averaged over different reasonable sets of these coefficients,[4] based on our knowledge of the S-wave content in various models of NN wave functions.[22] Using the value of P_i^{6q} from Eq. (20) we perform the rather tedious calculation and get

$$\Gamma_Q^{nm}=\begin{cases}5.19\Gamma^{free}\ [H^w(1)]\\ 2.18\Gamma^{free}\ [H^w(2)]\ \text{(nuclear matter)},\\ 0.73\Gamma^{free}\ [H^w(3)]\end{cases} \tag{26}$$

for the three candidate weak interactions described in Sec. II. Combining with the pion contribution gives

$$\Gamma^{nm}=\begin{cases}9.96\Gamma^{free}\ [H^w(1)]\\ 5.54\Gamma^{free}\ [H^w(2)]\ \text{(nuclear matter)}.\\ 3.0\Gamma^{free}\ [H^w(3)]\end{cases} \tag{27}$$

For a discussion of the relative phase between the pionic and quark contributions, see Ref. 4.

B. Finite nucleus $^{12}_{\Lambda}$C

1. *Pionic contribution*

Using the shell model calculation as discussed in Sec. III, we obtain the results for the pionic contribution to Γ^{nm} in $^{12}_{\Lambda}$C presented in Table V. As before, the last row is our best value, $\Gamma_\pi^{nm}=0.41\Gamma^{free}$.

2. *Quark contribution*

Basically, the only aspect for the quark contribution which is different for the finite nucleus calculation is the reduction of P_i^{6q} given in Table III. The results are

$$\Gamma_Q^{nm}=\begin{Bmatrix}1.76\Gamma^{free}\ [H^w_{int}(1)]\\ 0.74\Gamma^{free}\ [H^w_{int}(2)]\\ 0.24\Gamma^{free}\ [H^w_{int}(3)]\end{Bmatrix}(^{12}_{\Lambda}\text{C}) . \tag{28}$$

TABLE IV. Pionic contribution to Γ^{nm} for nuclear matter.

	S-S	S-P	S-D	$\Gamma_\pi^{nm}/\Gamma^{free}$
No short range corrections	6.1×10^{-5}	0.18	0.81	0.99
Fermi averaging	7.7×10^{-5}	0.19	0.85	1.04
Initial state correlations	1.3×10^{-4}	0.19	0.92	1.12
Pion form factor $\Lambda_\pi^2=20m_\pi^2$	1.2×10^{-3}	0.17	0.76	0.93
Final state correlations	9.8×10^{-4}	0.11	0.66	0.77

TABLE V. Pion contributions to Γ^{nm} in ${}^{12}_{\Lambda}$C.

	$\Gamma_{\pi}^{nm}/\Gamma^{free}$
No correlations	0.48
Miller-Spencer correlations	0.49
Pion form factor $\Lambda_{\pi}^2=20$	0.45
Eikonal distortions	0.41

Combining the quark and pion contributions, we obtain for our final results

$$\Gamma^{nm}=\begin{Bmatrix}3.87\Gamma^{free}\ [H_{int}^{w}(1)]\\ 2.25\Gamma^{free}\ [H_{int}^{w}(2)]\\ 1.28\Gamma^{free}\ [H_{int}^{w}(3)]\end{Bmatrix}({}^{12}_{\Lambda}\text{C})\ . \qquad (29)$$

Our final result for ${}^{12}_{\Lambda}$C of $\Gamma^{nm}\approx 1.3\Gamma_{\Lambda}^{free}$ can be compared to a recent experimental measurement at BNL (Ref. 23) which gives

$$\Gamma_{expt}^{total}({}^{12}_{\Lambda}\text{C})=(1.25\pm0.18)+\Gamma_{\Lambda}^{free}\ .$$

To the extent that the total rate is dominated by the nonmesonic rate, the agreement is excellent.

In conclusion, the HQH model is seen to give a quite satisfactory description of a Λ lifetime in a finite nucleus. Since the crucial parts of the quark wave functions are determined by parameters which have been fixed in previous work, the main uncertainty within the model is the parametrization of the weak effective $\Delta S=1$ quark Hamiltonian. Our principal result is that if we choose the coefficients of this Hamiltonian by following the general prescription of the renormalization group calculations, but pick the parameters to fit the $\Delta S=\frac{1}{2}$ rule, we greatly improve the theoretical result, bringing it into satisfactory agreement with experiment. We conclude that a six-quark cluster model not only provides a convenient description for the short-range part of the nuclear weak interaction, but that the parameters of the HQH model have been sufficiently determined so that one can gain new information about the weak quark Hamiltonian from nonmesonic Λ hypernuclear decay.

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