$\pi^0\eta$ and $\pi^0\eta'$ mixing and the reaction $d+d \rightarrow \pi^0 + {}^4He$

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The isospin-forbidden reaction $d+d \rightarrow \pi^0 + {}^4He$ provides a very good test of the validity of charge symmetry. The forward differential cross section $d\sigma/d\Omega$ is estimated to be ≈ 0.12 pb/sr at a deuteron laboratory energy of 1.95 GeV. The assumed reaction mechanism is virtual η and η' production and $\pi^0\eta$ and $\pi^0\eta'$ mixing.

The reaction $d+d \rightarrow \pi^0 + {}^4He$ provides a very good test of the validity of charge symmetry. It is forbidden by charge symmetry since it requires $\Delta I = 1$ $[I(\pi)=1, I(d)=I({}^4He)=0]$. Therefore, if detected, this reaction provides a direct signal of charge symmetry breaking in nuclear interactions. Center of mass upper limits for this reaction have been set at the Joint Institute for Nuclear Research (JINR)[1] (<900 pb/sr at an incident deuteron laboratory energy of 404 MeV; 1 pb$=10^{-36}$ cm^2), Berkeley[2] (<97 pb/sr at 460 MeV), and Saclay[3] (<19 pb/sr at 787 MeV). Two proposed experiments[4,5] expect to achieve a detection sensitivity of at least 0.1 pb/sr, two orders of magnitude less than the present upper limit. Preliminary results from a recent Saclay experiment[6] indicate possible new upper limits for this reaction of 1 pb/sr at 800 MeV and 0.5 pb/sr at 1.35 GeV. It is of interest, then, to estimate the contributions of expected $\Delta I = 1$ sources in this reaction. In this paper, we demonstrate that $\pi\eta\eta'$ mixing predicts a cross section at the 0.1 pb/sr level, thus raising the possibility of detecting the charge asymmetric reaction $d + d \rightarrow \pi^0 + {}^4He$.

The reaction could proceed by an $I=1$ admixture in the ground state of ${}^4He$. That is, an $I=1$ excited state of ${}^4He$ could be connected to the ground state by a charge asymmetric operator such as the Coulomb interaction. Experimentally, no 0^+ $I=1$ excited state of ${}^4He$ has been found.[7] A hypothetical $I=1$ shell model excited state at ~ 35 MeV introduces an $I=1$ impurity probability of $<1.3\times 10^{-5}$ into the ${}^4He$ ground state.[8] Using isospin projection operators instead of perturbation theory yields an estimate of 5×10^{-5} for the probability of an $I=1$ impurity.[9] Because of the uncertainties in these calculations due to the unobserved excited 0^+ $I=1$ state, we neglect the $I=1$ impurity in the ${}^4He$ ground state in the following calculation.

We assume $\pi^0\eta$ and $\pi^0\eta'$ mixing as the driving force in this reaction. We envisage the process in Fig. 1, where the charge symmetric reaction $d + d \rightarrow X + {}^4He$ produces an η or η' meson which turns into a π^0 via the $\Delta I = 1$ $\langle \pi^0 | H_{em} | \eta \rangle$ and $\langle \pi^0 | H_{em} | \eta' \rangle$ transition matrix elements. The calculation of $d\sigma/d\Omega$ is simplified by the recent measurement of $d+d \rightarrow \eta + {}^4He$; the first successful detection of η production in this reaction.[10] This experimental number obviates the necessity of models for eta production and the concomitant uncertainties in eta-nucleon or eta-nucleus coupling constants. Instead, the reliability of our estimate lies entirely in the credibility of the magnitude of $\langle \pi^0 | H_{em} | \eta \rangle$, $\langle \pi^0 | H_{em} | \eta' \rangle$, and the semistrong mixing angle of η' and η. We regard the latter to be adequate.

To proceed, we note that η and η' mixing in the physical π^0 yields the amplitude

$$T_{\pi^0} = \frac{\langle \eta | H_{em} | \pi^0 \rangle}{m_\pi^2 - m_\eta^2} T_\eta + \frac{\langle \eta' | H_{em} | \pi^0 \rangle}{m_\pi^2 - m_{\eta'}^2} T_{\eta'} \tag{1a}$$

or

$$T_{\pi^0} = \lambda_\eta T_\eta + \lambda_{\eta'} T_{\eta'} , \tag{1b}$$

where T_η is the transition amplitude for $d+d \rightarrow \eta + {}^4He$. Alternatively, one arrives at the same equation by putting the η,η' propagator of Fig. 1 on the pion mass shell. T_η is related to the differential cross section in the center-of-mass (c.m.) system by[11]

$$\left. \frac{d\sigma}{d\Omega} \right|_\eta = (\text{phase space})_\eta \, | T_\eta |^2 . \tag{2}$$

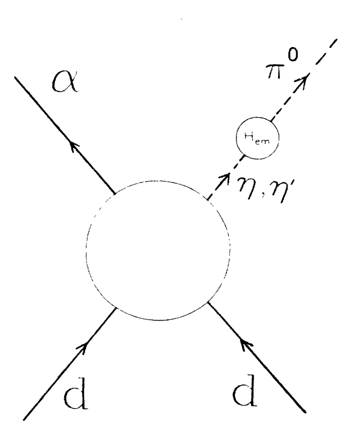

FIG. 1. Particle mixing diagram contributing to the reaction $d+d \rightarrow \pi^0 + {}^4He$.

Although only T_η is measured, we find $T_{\eta'}$ most easily by working in the strange-nonstrange quark basis where the physical η and η' states are described by

$$|\eta'\rangle = \sin\phi\,|\eta_{NS}\rangle + \cos\phi\,|\eta_S\rangle\,,$$
$$|\eta\rangle = \cos\phi\,|\eta_{NS}\rangle - \sin\phi\,|\eta_S\rangle\,. \tag{3}$$

Then $T_{\eta'}/T_\eta = \tan\phi$ because T_{η_S} is strongly suppressed, as neither the deuteron nor ^{4}He contain strange quarks. Then, at the same c.m. angle, the reaction $d+d\to\pi^0+{}^4$He is obtained from Eqs. (1) and (2) to be

$$\left.\frac{d\sigma}{d\Omega}\right|_{\pi^0} = \frac{(\text{phase space})_\pi}{(\text{phase space})_\eta}\lambda_\eta^2\left[1+\frac{\lambda_{\eta'}}{\lambda_\eta}\tan\phi\right]^2\left.\frac{d\sigma}{d\Omega}\right|_\eta\,. \tag{4}$$

The c.m. cross section $(d\sigma/d\Omega)_\eta$ was measured[10] to be 0.25 ± 0.10 nb/sr at $\theta_{\text{lab}}=6°$, with an incident deuteron kinetic energy of 1950 MeV. The phase space ratio is simply the ratio of the c.m. momenta of the π^0 to the η, which is 1.27. We will demonstrate below that values of

$$\langle\eta|H_{\text{em}}|\pi^0\rangle = -0.0042\ \text{GeV}^2\,,$$
$$\langle\eta'|H_{\text{em}}|\pi^0\rangle = -0.0047\ \text{GeV}^2 \tag{5}$$

are predicted theoretically[12] and are consistent with data. Then $\lambda_{\eta'}/\lambda_\eta \simeq +\frac{1}{3}$ and $[1+(\lambda_{\eta'}/\lambda_\eta)\tan\phi]^2$ in Eq. (4) is relatively insensitive to the semistrong mixing angle, which is well understood theoretically[13] to be near 45°. We use the phenomenological value $\phi\simeq 42°$ ($\tan\phi\simeq 0.9$), as in Refs. 12 and 14. This corresponds to a singlet-octet mixing angle of $-13°$. Then our prediction for the reaction $d+d\to\pi^0+{}^4$He at the same c.m. angle as the η-production reaction measurement becomes

$$\left.\frac{d\sigma}{d\Omega}\right|_{\pi^0} = 0.12\pm0.05\ \text{pb/sr}\,, \tag{6}$$

where we have used $m_\eta=0.548$ GeV, $m_{\eta'}=0.958$ GeV, and $m_\pi=0.135$ GeV.

The key elements in our estimate in Eq. (6) are the transition matrix elements $\langle\eta|H_{\text{em}}|\pi^0\rangle$ and $\langle\eta'|H_{\text{em}}|\pi^0\rangle$, which are difficult to extract cleanly from experimental measurements. Nevertheless, the latter numbers can be found in a relatively model-independent way by relating the strength of quark annihilation diagrams to meson masses. This calculation was carried out in Ref. 12 and yielded Eq. (5). The theoretical value $\langle\eta|H_{\text{em}}|\pi^0\rangle \simeq -0.0042$ GeV^2 compares well with the "experimental value" obtained in Ref. 14 from the decay $\eta\to3\pi^0$,

$$|\langle\eta|H_{\text{em}}|\pi^0\rangle|_{\text{exp}}\simeq 0.0045\pm0.0003\ \text{GeV}^2\,. \tag{7}$$

A recent independent[15] estimate from all $\eta\to3\pi$ decays, which incorporates new data on the $\eta\to\gamma\gamma$ decay width,[16] suggests that

$$|\langle\eta^0|H_{\text{em}}|\pi^0\rangle|_{\text{exp}}\simeq 0.0042\pm0.0010\ \text{GeV}^2\,, \tag{8}$$

in even better agreement with Eq. (5). Finally, the value for $\langle\eta|H_{\text{em}}|\pi^0\rangle$ of Eq. (5) provides a consistent explanation[14] for the decay ratios

$$R=\Gamma(\psi'\to\psi\pi^0)/\Gamma(\psi'\to\psi\eta)$$

and

$$r=\Gamma(\psi\to\eta'\gamma)/\Gamma(\psi\to\eta\gamma)\,.$$

Another estimate[17] of these decay ratios yields a slightly better fit using a value of ϕ ($=35.3°$) which appears too low. In any event, substitution of the values of Ref. 17 into Eq. (4) yields the same prediction for $d+d\to\pi^0+{}^4$He as that of Eq. (6). We regard the consistent description of four decays as strong evidence for the reliability of $\langle\eta|H_{\text{em}}|\pi^0\rangle$ used in Eqs. (4) and (6).

The predicted cross section, however, also depends on the value of $\langle\eta'|H_{\text{em}}|\pi^0\rangle$. Both transition matrix elements were estimated in Ref. 12 from the quark Hamiltonian density $H_{\text{em}}=H_{jj}+H_{tad}(\lambda_3)$. The photon exchange and quark annihilation contributions to the purely electromagnetic H_{jj} were determined by relating them to meson masses. The calculation was sharpened in Ref. 14 by the observation that the π^+-π^0 mass shift sets a model independent scale for the annihilation contribution to H_{jj} (it was only estimated in Ref. 12). These calculations yield a value of $\langle\eta'|H_{\text{em}}|\pi^0\rangle$ nearly equal to that of $\langle\eta|H_{\text{em}}|\pi^0\rangle$ [Eq. (5)]. An alternative realization[18] of the same scheme incorporates the phenomenological parameters of quantum-chromodynamics-based, but nonrelativistic, constituent quark models to estimate the same contributions to H_{jj}. A recent variant[19] of this calculation finds $\langle\eta|H_{\text{em}}|\pi^0\rangle$ similar to that of Eq. (5), but predicts $\langle\eta'|H_{\text{em}}|\pi^0\rangle$ to be one half the magnitude (and same sign) as $\langle\eta|H_{\text{em}}|\pi^0\rangle$, consistent with the earlier estimate.[18] The latter prediction appears ruled out by the recently measured[20] decay of $\eta'\to3\pi^0$. As this decay includes both $\eta\pi^0$ and $\eta'\pi^0$ mixing contributions, this experiment provides the first empirical constraint on $\langle\eta'|H_{\text{em}}|\pi^0\rangle$. An analysis of this decay[21] found that the approximately equal mixing parameters of Eq. (5) yielded exactly the experimental decay rate. On the other hand, a value of $\langle\eta'|H_{\text{em}}|\pi^0\rangle$ one half that of $\langle\eta|H_{\text{em}}|\pi^0\rangle$ implies[21] a decay width a factor of 2.5 below the experimental value.

We close by analyzing an earlier prediction of the reaction $d+d\to\pi^0+{}^4$He at somewhat lower energies.[22] This prediction assumed the π^0 to be produced from a charge asymmetric process $NN\to NN\pi^0$ involving $\Delta(3,3)$ excitation. Then the charge asymmetry comes from a two-nucleon mechanism involving, for example, photon exchange, $\eta\eta'\pi^0$ mixing, and $\rho^0\omega$ mixing deep inside the analogous four-point function similar to that of Fig. 1. In the calculation of Ref. 22, the dominant term due to $\pi^0\eta$ mixing depends strongly on the coupling $g^2_{\eta NN}$, which is uncertain even to order of magnitude.[12] For a large, but not unreasonable, value of $g^2_{\eta NN}$, the prediction of these mechanisms lies between 0.1 and 0.01 pb/sr for the deuteron laboratory energy in the range 500–700 MeV. The estimate presented in the present paper is, however, at the higher energy 1950 MeV, far from the $\Delta(3,3)$ resonance, and where the cross section predictions of Ref. 22 appear to be negligible.

In this paper we have combined a new measurement of the charge symmetric reaction $d+d\to\eta+{}^4$He with theoretical estimates of the $\Delta I=1$ transitions $\langle\eta\,|\,H_{em}\,|\,\pi^0\rangle$ and $\langle\eta'\,|\,H_{em}\,|\,\pi^0\rangle$ to predict a c.m. cross section of about 0.12 pb/sr at 2 GeV for the charge asymmetric reaction $d+d\rightarrow\pi^0+{}^4He$. If this reaction is detected, if $I=1$ impurities in the ${}^4He$ ground state are truly negligible, and if the production of η' mesons is observed in charge symmetric reactions, then one can turn the problem around and suggest that these reactions may provide the cleanest method yet of measuring the transition matrix elements $\langle\eta\,|\,H_{em}\,|\,\pi^0\rangle$, $\langle\eta'\,|\,H_{em}\,|\,\pi^0\rangle$, and the semistrong mixing angle ϕ. The structure of Eq. (4) shows clearly that just the measurement of the reaction $d+d\rightarrow\pi^0+{}^4He$ alone would yield a good estimate of the transition matrix element $\langle\eta\,|\,H_{em}\,|\,\pi^0\rangle$.

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[1]Yu. K. Akimov *et al.*, Zh. Eksp. Teor. Fiz. 41, 708 (1961) [Sov. Phys. JETP **14**, 512 (1962)].

[2]J. A. Poirier and M. Pripstein, Phys. Rev. **122**, 1917 (1961); **130**, 1171 (1963).

[3]J. Banaigs *et al.*, Phys. Lett. **53B**, 390 (1974).

[4]B. M. Preedom, in Proceedings of the Charge Symmetry Breaking Workshop, edited by N. E. Davison, J. P. Svenne, and W. T. H. van Oers, TRIUMF Report Tri-81-3, 1981.

[5]Proposal No. 105, Laboratoire National Saturne, F. Plouin, Spokesman.

[6]F. Plouin and L. Goldzahl, private communication; L. Satta, Contribution to Workshop on Nuclear and Particle Physics at Intermediate Energies with Hadrons, Trieste, 1985.

[7]S. Fiarman and W. E. Meyerhof, Nucl. Phys. **A206**, 1 (1973).

[8]W. M. MacDonald, Phys. Rev. **101**, 271 (1956); in *Nuclear Spectroscopy,* edited by F. Ajzenberg-Selove (Academic, New York, 1960), Part B, p. 932.

[9]S. B. Khadkikar and C. S. Warke, Nucl. Phys. **A130**, 577 (1969).

[10]J. Banaigs *et al.*, Phys. Rev. C **32**, 1448 (1985).

[11]See, for example, M. D. Scadron, *Advanced Quantum Theory* (Springer, Heidelberg, 1979), p. 183; or H. Pilkuhn, *Relativistic Particle Physics* (Springer, Heidelberg, 1979), p. 38.

[12]S. A. Coon and M. D. Scadron, Phys. Rev. C **26**, 562 (1982); see also P. C. McNamee, M. D. Scadron, and S. A. Coon, Nucl. Phys. **A249**, 483 (1975).

[13]N. Isgur, Phys. Rev. D **12**, 3770 (1975).

[14]M. D. Scadron, Phys. Rev. D **29**, 2076 (1984); Rep. Prog. Phys. **44**, 213 (1981).

[15]Ll. Ametller, C. Ayala, and A. Bramon, Phys. Rev. D **30**, 674 (1984).

[16]A. Weinstein *et al.*, Phys. Rev. D **28**, 2896 (1983).

[17]Ll. Amettler, C. Ayala, and A. Bramon, Phys. Lett. **141B**, 414 (1984).

[18]N. Isgur, Phys. Rev. D **21**, 779 (1980).

[19]C. Y. Cheung, Phys. Lett. **138B**, 245 (1984).

[20]F. Binon *et al.*, Yad. Fiz. 39, 1429 (1984) [Sov. J. Nucl. Phys. **39**, 903 (1984)].

[21]S. A. Coon, B. H. J. McKellar, and M. D. Scadron (unpublished).

[22]C. Y. Cheung, Phys. Lett. **119B**, 47 (1982).