Exit doorways and intermediate structure resonances

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When the energy resolution of a reaction is considerably large compared to the width and spacing of compound nuclear states, but small compared to optical model shape resonances, one observes the well-known intermediate structure resonances. In the case of compound nuclear processes, a dynamical account of intermediate structure resonances exists, where one assumes the existence of a doorway state through which the incident (or the final) state couples to the compound nuclear states. In the case of compound nuclear reactions the incident and the final states are, in general, different and so are the doorway states which lead the incident and the final states to compound nuclear states, the former doorway states being the usual entrance doorway states and the latter the exit doorway states. A new dynamical formalism is developed using a recently proposed nested doorway model, which explains how intermediate structure resonances arise because of these two types of doorway states. In this model the system develops through a series of doorway hallway states before entering the true compound nuclear state. The first stage of the doorway hallway states is the usual entrance doorway and the last stage is the exit doorway. Finally, we present experimental evidence of the occurrence of intermediate structure resonances due to isolated exit doorway states.

I. INTRODUCTION

Intermediate structure resonances in compound elastic and/or inelastic processes are often explained by the doorway state hypothesis which assumes that the system has simple modes of excitation called the doorway states which are the only states having strong coupling with the entrance (or the exit) channel.[1-3] The doorway state representing the "first collision"[4] of the projectile and the target has played a fundamental role in the analysis of compound elastic and inelastic processes.

In the case of nuclear reactions involving particle transfer one can generalize the above picture by introducing two types of doorway states—the entrance and the exit doorway states representing the "first" and the "last collisions," respectively. We shall see that the exit doorway states may play a very important role in the analysis of nuclear reactions. The importance of exit doorway states becomes more explicit if we consider the time reversed reaction. As the transition amplitude is time reversal invariant one can also study the time reversed process. In the time reversed picture the exit doorway state becomes the entrance doorway state and hence should be equally important as the entrance doorway state in the analysis of nuclear reactions.

In the doorway state hypothesis one assumes that the incident state passes through the doorway state to the complicated compound nuclear states. The compound nuclear resonances arising from compound nuclear states having a strong coupling with the doorway states will be more strongly excited than those having a weak coupling with the doorway states. If the compound nuclear states which couple strongly with a doorway state are located in a relatively narrow region one observes a smooth peak whose width Γ_d will be intermediate. Such a resonance will be called an intermediate structure resonance. If the energy resolution ΔE of the experiment is large compared to the spacing of doorway states one will have the poor resolution of the optical model.

Now let us consider the compound inelastic process where the incident and the final states are different. Then one can generalize the concept of usual doorway states and introduce two types of doorway states—the entrance and the exit doorway states. The entrance doorway states are the only states which are strongly coupled to the entrance channel. Analogously the exit doorway states are the only states which are strongly coupled to the exit channel. In the case of the compound elastic process these doorway states are the same. Both doorway states also couple to the complicated compound nuclear states. The entrance and the exit channel states, on the other hand, do not couple with the compound nuclear states. So in the time development of the nuclear process the system will pass through the entrance doorway states before forming the complicated compound nuclear states. Subsequently such compound nuclear states have to pass through the exit doorway state before decaying to the exit channel state.

The importance of exit doorway states has been conjuctured in recent theoretical analyses[5,6] which, however, lacked convincing examples. The purpose of the present paper is twofold. First, we present a dynamical formalism including the effect of entrance and exit doorway states. The present formalism is different from that of Ref. 5. In Ref. 5 we employed rearrangement scattering theory to show the plausibility of appearance of intermediate structure due to exit doorway states. In the present paper we use the nested doorway model of Ref. 7 in order to develop a dynamical formalism which clearly exhibits the time development of the system through various doorway hallway states and which explains the appearance of intermediate structure due to these states.

The formalism of the present paper has some similarity with the formalism of Ahmed and Beres,[6] though the details are different. The present formalism is more general than the formalism of Ref. 6 in that the present approach is easily generalized to the case of arbitrary number of nested doorways. The second purpose of the present paper is to provide some examples of exit doorways and stress their importance in the analyses of various types of nuclear reactions.

When there are many doorway states of both types one may observe intermediate structure resonances corresponding to both types of doorway states. Of course, both of these types of intermediate structure resonances may not be simultaneously observed in a reaction. Depending on the nature of couplings of these doorway states with the compound nuclear states on one hand and with the entrance and exit channel states on the other hand, one type of intermediate structure resonance may dominate over the other type. For the intermediate structure resonances to appear, the coupling of the entrance (exit) doorway states with the entrance (exit) channel should be strong and the coupling of the doorway states to the compound nuclear resonances should not be spread out.

Now we would like to see why and under what conditions the exit doorway states are expected to lead to intermediate structure. For our purpose we consider the shell-model approach[3] for simplicity, but our conclusions are independent of this model. In the case of nucleon-nucleus scattering the system evolves through two-particle—one-hole (2p-1h), 3p-2h, 4p-3h, ..., etc. states before forming the complicated compound nucleus. The 2p-1h state is the entrance doorway state d_1 and 3p-2h, 4p-3h, ... states are the hallway states[2] d_n, $n=2,3,\ldots$, respectively, where increasing n denotes increasing complexity in relation to the incident channel state i. So, schematically, the time evolution of the system is given by $i\rightarrow d_1 \rightarrow d_2 \rightarrow d_3 \cdots$, where the residual interaction is responsible for these transitions. In the case of compound elastic and inelastic processes the doorway state d_1 is connected by residual interaction to the exit channel and $i\rightarrow d_1\rightarrow f$ is one natural route for transition from the initial channel state i to the final channel state f. But in the case of nuclear reactions involving particle transfer, the residual interaction will, in general, fail to connect the doorway state d_1 to the exit channel f. But *some* of the hallway states in d_n may have a simple mode of excitation with respect to the exit channel f and be connected by residual interaction to f, whereas such states usually have a more complicated mode of excitation in relation to the initial channel state i. It is these small number of states in d_n, having strong coupling with the exit channel state f, which are called the exit doorway states. The exit (entrance) doorway state is expected to produce intermediate structure resonance when the coupling of d_n (d_1) to $f(i)$ is strong and the coupling of d_n (d_1) to the complicated compound nuclear states is not too spread out.

It is true that the density of states in d_n increases rapidly as increasing n. Hence it might be thought that isolated exit doorway states may not be observed when the exit doorway states belong to d_n where n is not too small. This reasoning is, however, not correct, because the exit doorway states are not all the states in d_n but are only a few states in d_n connected by residual interaction to the exit channel. Hence the density of the exit doorway states could be small even when n is not too small and one can observe the effect of isolated exit doorway states.

The plan of the paper is as follows. In Sec. II we present our formulation in the nested doorway model where only two types of doorway hallway states are present—the first one being the entrance doorway state and the second one being the exit doorway state. In Sec. III we present a generalization of the present formulation to the case of a hierarchy of doorway hallway states. In Sec. IV we present various examples where exit doorway states lead to intermediate structure resonances. Finally, Sec. V presents a brief discussion and concluding remarks.

II. FORMULATION IN A SIMPLE MODEL

The nested doorway model which we use for the study of entrance and exit doorway states assumes that the system passes through a chain of stages of increasing complexity denoted by d_n, $n=1,2,3,\ldots$, where increasing n denotes increasing complexity in relation to the incident state. d_1 is the usual doorway state having strong overlap with the incident channel state. Let us assume that d_N is the Nth doorway state which has strong overlap with the exit channel and is the exit doorway state. In this section we take $N=2$. This is the simplest model where the exit and the entrance doorway states are different. This model is physically as rich as the general model with $N>2$, though the mathematics is more complicated for a general N. So in this section we treat the $N=2$ case in order to explain our ideas and leave the general treatment for the next section.

We introduce the usual orthogonal projection operators P, d_1, d_2, and q such that

$$P+d_1+d_2+q=1\ , \tag{2.1}$$

with

$$d_1+d_2+q=Q \tag{2.2}$$

and the condition

$$Pd_1=d_1d_2=d_2q=d_1q=Pq=Pd_2=0\ .$$

The transition amplitude from the incident channel state ϕ_i to the final channel state ϕ_f is given by

$$T=\langle \phi_f\,|\,(V+VGV)\,|\,\phi_i\rangle\ , \tag{2.3}$$

where $G=(E-H+i0)^{-1}$ is the full resolvent operator and $H=(H_0+V)$ is the full Hamiltonian, V is the channel interaction, and H_0 is the kinetic energy operator of relative motion. In the following we shall not explicitly show the $i0$ part of the resolvent operators and we shall also suppress the channel levels i and f on the transition operator T.

Next let us introduce the operators V_n and G_n through the recursion relations[7]

$$V_n=V_{(n-1)}+V_{(n-1)}G_{(n-1)}V_{(n-1)} \tag{2.4}$$

and

$$G_n = d_n(E - d_n H_n d_n)^{-1} d_n \ , \tag{2.5}$$

with $n=0,1,2,3$. Here we take $d_0=P$ and $d_3=q$ in order to have a unified notation, also $V_0=V$ and $H_n=H_0+V_n$. Then T of Eq. (2.3) can be rewritten as[7]

$$T = \langle \phi_f | (V_3 + V_3 G_3 V_3) | \phi_i \rangle \ . \tag{2.6}$$

In explicit notation Eq. (2.6) reads

$$T = \langle \phi_f | V | \psi_i^{(+)} \rangle + \langle \psi_f^{(-)} | V G_1 V | \psi_i^{(+)} \rangle + \langle \psi_f^{(-)} | (V + V G_1 \mathscr{V}) G_2 (V + \mathscr{V} G_1 V) | \psi_i^{(+)} \rangle$$
$$+ \langle \psi_f^{(-)} | [V + V G_1 \mathscr{V} + (V + V G_1 \mathscr{V}) G_2 (\mathscr{V} + \mathscr{V} G_1 \mathscr{V})] G_3 [V + \mathscr{V} G_1 V + (\mathscr{V} + \mathscr{V} G_1 \mathscr{V}) G_2 (V + \mathscr{V} G_1 V)] | \psi_i^{(-)} \rangle \ , \tag{2.7}$$

where

$$\mathscr{V} = V + V G_0 V \ ,$$
$$| \psi_1^{(+)} \rangle = (1 + G_0 V) | \phi_i \rangle \ , \tag{2.8}$$

and

$$\langle \psi_f^{(-)} = \langle \phi_f | (1 + V G_0) \ .$$

Equation (2.7) contains four terms. The first term essentially represents the direct transition between the initial and final states, this term varies smoothly with energy and contributes to a nonresonant background. The second term corresponds to a transition from the initial to final state via the doorway space d_1 and contributes to resonant contributions corresponding to this space. The third term denotes a transition involving both the doorway spaces d_1 and d_2 and contributes to resonant contributions from these spaces. The last term represents the formation of compound nucleus and contributes to true compound nuclear fluctuations.

Now we introduce the doorway state hypothesis that the system passes through stages of increasing complexity which is mathematically expressed as

$$P_i H d_1 = d_1 H P_i \neq 0, \ \ d_1 H d_2 = d_2 H d_1 \neq 0,$$
$$d_2 H q = q H d_2 \neq 0, \ \text{and} \ d_2 H P_f = P_f H d_2 \neq 0$$

with all other similar matrix elements being zero;

$$d_1 H q = P_i H d_2 = P_i H q = d_1 H P_f = 0 \ .$$

Here P_i denotes the incident channel part of P space and P_f denotes the final channel part. Next we assume that $d_i \mathscr{V} d_j = d_i V d_j \equiv d_i H d_j$, $i,j,=1,2,3$, i.e., transition between two types of doorway states via the P space will lead to a process of higher order and is neglected in practice. This will lead to a mathematical simplification. Then Eq. (2.7) becomes

$$T = \langle \phi_f | V | \psi_i^{(+)} \rangle + \langle \psi_f^{(-)} | H_{Pd_2} G_2 H_{d_2 d_1} G_1 H_{d_1 P} | \psi_i^{(+)} \rangle + \langle \psi_f^{(-)} | H_{Pd_2} G_2 H_{d_2 d_3} G_3 H_{d_3 d_2} G_2 H_{d_2 d_1} G_1 H_{d_1 P} | \psi_i^{(+)} \rangle \ . \tag{2.9}$$

Here $H_{Pd_2}=PHd_2$, $H_{d_2d_3}=d_2Hd_3$, etc. With the introduction of the doorway state hypothesis the resolvent operators G_1, G_2, and G_3 of Eq. (2.9) are essentially given by

$$G_0 = P(E - H_{PP})^{-1} P \ ,$$
$$G_1 = d_1 (E - H_{d_1 d_1} - W^{(0)}_{d_1 d_1})^{-1} d_1 \ ,$$
$$G_2 = d_2 (E - H_{d_2 d_2} - W^{(0)}_{d_2 d_2} - W^{(1)}_{d_2 d_2})^{-1} d_2 \ , \tag{2.10}$$

and

$$G_3 = q (E - H_{qq} - W^{(2)}_{qq})^{-1} q \ ,$$

where

$$W^{(i)} = H G_i H \ .$$

Here $W^{(0)}_{d_1 d_1} = d_1 W^{(0)} d_1$, $W^{(1)}_{d_2 d_2} = d_2 W^{(1)} d_2$, etc. Now using the following easily verified identity

$$\mathscr{G}_2 = G_2 + G_2 H_{d_2 d_3} G_3 H_{d_3 d_2} G_2 \ , \tag{2.11}$$

with

$$\mathscr{G}_2 = d_2 (E - H_{d_2 d_2} - W^{(0)}_{d_2 d_2} - W^{(1)}_{d_2 d_2} - \widetilde{W}_{d_2 d_2})^{-1} d_2 \tag{2.12}$$

and

$$\widetilde{W}_{d_2 d_2} = H_{d_2 q} (E - H_{qq})^{-1} H_{q d_2} \ ,$$

Eq. (2.9) can be rewritten as

$$T = \langle \phi_f | V | \psi_i^{(+)} \rangle + \langle \psi_f^{(-)} | H_{Pd_2} \mathscr{G}_2 H_{d_2 d_1} G_1 H_{d_1 P} | \psi_i^{(+)} \rangle . \tag{2.13}$$

It should be noted that in the present model Eq. (2.9) is an identity, where the first term is the direct transition amplitude, the second term gives the resonant contributions from the doorway states, and the last term is the resonant contribution from the compound nuclear processes. The second term of Eq. (2.9) has two resolvent operators G_1 and G_2 defined by Eq. (2.10). The operator G_1 has rapid variations in energy associated with the doorway space d_1 or the entrance doorway state, whereas the operator G_2

has the rapid variations of the exit doorway space d_2. In Eq. (2.10) the operator $W^{(i)}$ is a complex interaction for the doorway states. It is obvious that $W^{(i)}$ is nonlocal, complex, and energy dependent. The imaginary parts of W's will provide the doorway states with various types of widths.

The intermediate structure will correspond to an energy averaged transition amplitude. The q space fluctuations contained in the resolvent operator G_3 of Eqs. (2.9) and (2.10) are assumed to be very rapid. In order to observe the intermediate structure corresponding to doorway states d_1 and d_2 one has to perform an energy average of (2.9) with respect to an interval I which is very large compared to the width of q space fluctuations and is at the same time very small compared to the widths of fluctuations of doorway spaces d_1 and d_2. We also assume that the resonances in the d_1 and d_2 spaces are isolated such that, for example, a particular resonance in d_1 space has no overlap with a resonance in d_1 space or d_2 space.

We shall perform the q space averaging of the transition amplitude (2.9) with respect to a Lorentz weight factor of width I. The result is to introduce[1] an imaginary term $iI/2$ in the energy denominator of G_3. Consequently the energy averaged transition operator T is given by

$$\langle T\rangle_I=\langle\phi_f|V|\psi_i^{(+)}\rangle+\langle\psi_f^{(-)}|H_{Pd_2}G_2H_{d_2d_1}G_1H_{d_1P}|\psi_i^{(+)}\rangle$$
$$+\langle\psi_f^{(-)}|H_{Pd_2}G_2H_{d_2d_3}(E-H_{qq}-W_{qq}^{(2)}+iI/2)^{-1}H_{d_3d_2}G_2H_{d_2d_1}G_1H_{d_1P}|\psi_i^{(+)}\rangle\ . \quad (2.14)$$

Using the identity (2.11), Eq. (2.14) can be rewritten as

$$\langle T\rangle_I=\langle\phi_f|V|\psi_i^{(+)}\rangle+\langle\psi_f^{(-)}|H_{Pd_2}(E-H_{d_2d_2}-W_{d_2d_2}^{(0)}-W_{d_2d_2}^{(1)}-\overline{W}_{d_2d_2})^{-1}H_{d_2d_1}(E-H_{d_1d_1}-W_{d_1d_1}^{(0)})^{-1}H_{d_1P}|\psi_i^{(+)}\rangle\ , \quad (2.15)$$

where

$$\overline{W}_{d_2d_2}=H_{d_2q}(E-H_{qq}+iI/2)^{-1}H_{qd_2}\ .$$

The last term on the right-hand side of Eq. (2.15) contains two energy denominators, corresponding to propagation in the doorway spaces d_1 and d_2. This expression is very useful in studying intermediate resonances when the exit doorway state dominates a reaction and the entrance doorway space resolvent operator can be treated as a background. When the entrance doorway space resolvent operator dominates a reaction the following form is very useful for the study of intermediate structures

$$\langle T\rangle_I=\langle\phi_f|V|\psi_i^{(+)}\rangle+\langle\psi_f^{(-)}|H_{Pd_2}(E-H_{d_2d_2}-W_{d_2d_2}^{(0)}-\overline{W}_{d_2d_2})^{-1}H_{d_2d_1}(E-H_{d_1d_1}-W_{d_1d_1}^{(0)}-\hat{W}_{d_1d_1})^{-1}H_{d_1P}|\psi_i^{(+)}\rangle\ , \quad (2.16)$$

where

$$\hat{W}_{d_1d_1}=H_{d_1d_2}(E-H_{d_2d_2}-W_{d_2d_2}^{(0)}-\overline{W}_{d_2d_2})^{-1}H_{d_2d_1}\ .$$

Equation (2.16) is an identical rewriting of Eq. (2.15) and can be easily verified by considering the Neumann series expansions of the resolvent operators of these two equations. In these two extreme cases when one type of doorway state dominates a reaction Eqs. (2.15) and (2.16) can be used to define the width and position of isolated resonances. But if in a nuclear reaction both types of doorway states are important and have similar widths because of interference effects a unique definition of resonance parameters becomes difficult.

First, let us consider the case where the exit doorway states lead to isolated resonances and the entrance doorway states lead to a nonresonant background. Then Eq. (2.15) is appropriate for our purpose. The exit doorway state resonances are given by the poles of the complex propagator

$$(E-H_{d_2d_2}-W_{d_2d_2}^{(0)}-W_{d_2d_2}^{(1)}-\overline{W}_{d_2d_2})^{-1}\ .$$

For the sake of simplicity we consider as in Ref. 1 the case of an isolated exit doorway state ψ_{d_2} satisfying

$$(E_{d_2}-H_{d_2d_2})\psi_{d_2}=0\ , \quad (2.17)$$

where E_{d_2} is a real energy eigenvalue. Then, as in Ref. 1, Eq. (2.15) can be written as

$$\langle T\rangle_I=\langle\phi_f|V|\psi_i^{(+)}\rangle+\langle\psi_f^{(-)}|H_{Pd_2}|\psi_{d_2}\rangle(E-E_{d_2}-\langle\psi_{d_2}|W_{d_2d_2}^{(0)}|\psi_{d_2}\rangle-\langle\psi_{d_2}|W_{d_2d_2}^{(1)}|\psi_{d_2}\rangle-\langle\psi_{d_2}|\overline{W}_{d_2d_2}|\psi_{d_2}\rangle)^{-1}$$
$$\times\langle\psi_{d_2}|H_{d_2d_1}(E-H_{d_1d_1}-W_{d_1d_1}^{(0)})^{-1}H_{d_1P}|\psi_i^{(+)}\rangle\ , \quad (2.18)$$

for $E \approx E_{d_2}$. Let

$$\langle \psi_{d_2} | W^{(0)}_{d_2 d_2} | \psi_{d_2} \rangle = \Delta^{\uparrow}_{d_2} - \tfrac{1}{2} i \Gamma^{\uparrow}_{d_2} ,$$
$$\langle \psi_{d_2} | W^{(1)}_{d_2 d_2} | \psi_{d_2} \rangle = \Delta^{\uparrow}_{d_2 \to d_1} - \tfrac{1}{2} i \Gamma^{\uparrow}_{d_2 \to d_1} , \quad (2.19)$$

and

$$\langle \psi_{d_2} | \overline{W}_{d_2 d_2} | \psi_{d_2} \rangle = \Delta^{\downarrow}_{d_2} - \tfrac{1}{2} i \Gamma^{\downarrow}_{d_2} .$$

From Eqs. (2.18) and (2.19) it is clear that this particular exit doorway state is located at the energy

$$E = E_{d_2} + \Delta^{\uparrow}_{d_2} + \Delta^{\uparrow}_{d_2 \to d_1} + \Delta^{\downarrow}_{d_2} , \quad (2.20)$$

and has the width

$$\Gamma = \Gamma^{\uparrow}_{d_2} + \Gamma^{\uparrow}_{d_2 \to d_1} + \Gamma^{\downarrow}_{d_2} . \quad (2.21)$$

$\Gamma^{\downarrow}_{d_2}$ is the spreading width acquired by the doorway state because of its coupling to the compound nuclear states, $\Gamma^{\uparrow}_{d_2}$ is the escape width because of its coupling to the exit channel, and $\Gamma^{\uparrow}_{d_2 \to d_1}$ is the contribution to the width because of the coupling between d_1 and d_2 spaces. Of course, it is assumed that the last matrix element in Eq. (2.18) involving the propagator in the d_1 space is a smooth function of energy.

When we use Eq. (2.15) to describe the entrance doorway state effect we face a problem. The entrance doorway space propagator of Eq. (2.15) has only one W and hence only one contribution to width, e.g., the entrance doorway state has only escape width to the entrance channel and no spreading width due to its coupling to d_2 space. The effect of coupling between d_1 and d_2 spaces is contained in the exit doorway state propagator of Eq. (2.15). But in Eq. (2.16) the coupling between d_1 and d_2 spaces contributes to the width of the entrance doorway state. This is why for isolated resonances in the entrance doorway space d_1 Eq. (2.16) is appropriate for the study of intermediate structure resonances. Now we assume that the exit doorway states lead to a nonresonant background. The entrance doorway state resonances are given by the poles of the complex propagator

$$(E - H_{d_1 d_1} - W^{(0)}_{d_1 d_1} - \widehat{W}^{(0)}_{d_1 d_1})^{-1} .$$

For the sake of simplicity we consider an isolated entrance doorway state ψ_{d_1} satisfying

$$(E_{d_1} - H_{d_1 d_1}) \psi_{d_1} = 0 , \quad (2.22)$$

where E_{d_1} is a real energy eigenvalue. Then for $E \simeq E_{d_1}$ Eq. (2.16) can be written as

$$\langle T \rangle_I = \langle \phi_f | V | \psi_i^{(+)} \rangle + \langle \psi_f^{(-)} | H_{P d_2} (E - H_{d_2 d_2} - W^{(0)}_{d_2 d_2} - \overline{W}_{d_2 d_2})^{-1} H_{d_2 d_1} | \psi_{d_1} \rangle$$
$$\times (E - E_{d_1} - \langle \psi_{d_1} | W^{(0)}_{d_1 d_1} | \psi_{d_1} \rangle - \langle \psi_{d_1} | \widehat{W}^{(0)}_{d_1 d_1} | \psi_{d_1} \rangle)^{-1} \langle \psi_{d_1} | H_{d_1 P} | \psi_i^{(+)} \rangle . \quad (2.23)$$

Let

$$\langle \psi_{d_1} | W^{(0)}_{d_1 d_1} | \psi_{d_1} \rangle = \Delta^{\uparrow}_{d_1} - \tfrac{1}{2} i \Gamma^{\uparrow}_{d_1} \quad (2.24)$$

and

$$\langle \psi_{d_1} | \widehat{W}^{(0)}_{d_1 d_1} | \psi_{d_1} \rangle = \Delta^{\downarrow}_{d_1} - \tfrac{1}{2} i \Gamma^{\downarrow}_{d_1} .$$

From Eqs. (2.23) and (2.24) it is clear that this particular entrance doorway state is located at an energy

$$E = E_{d_1} + \Delta^{\uparrow}_{d_1} + \Delta^{\downarrow}_{d_1} \quad (2.25)$$

and has the width

$$\Gamma = \Gamma^{\uparrow}_{d_1} + \Gamma^{\downarrow}_{d_1} . \quad (2.26)$$

Here $\Gamma^{\downarrow}_{d_1}$ is the spreading width acquired by the doorway state because of its coupling to the d_2 space, and $\Gamma^{\uparrow}_{d_1}$ is the escape width because of its coupling to the entrance channel. When there is an isolated doorway state Eqs. (2.20) and (2.21) [Eqs. (2.25) and (2.26)] give its energy and width if it is an exit (entrance) doorway state. Such a unique definition of width becomes difficult if we have an entrance and exit doorway state close to a particular energy.

Suppose now that we have two doorway states ψ_{d_1} and ψ_{d_2} given by Eqs. (2.17) and (2.22) such that $E \simeq E_{d_1} \simeq E_{d_2}$. In this case one can either work with Eq. (2.15) or (2.16). Suppose we choose Eq. (2.15) which can now be written as

$$\langle T \rangle_I = \langle \phi_f | V | \psi_i^{(+)} \rangle + \langle \psi_f^{(-)} | H_{P d_2} | \psi_{d_2} \rangle (E - E_{d_2} - \langle \psi_{d_2} | W^{(0)}_{d_2 d_2} | \psi_{d_2} \rangle - \langle \psi_{d_2} | W^{(1)}_{d_2 d_2} | \psi_{d_2} \rangle - \langle \psi_{d_2} | \overline{W}_{d_2 d_2} | \psi_{d_2} \rangle)^{-1}$$
$$\times \langle \psi_{d_2} | H_{d_2 d_1} | \psi_{d_1} \rangle (E - E_{d_1} - \langle \psi_{d_1} | W^{(0)}_{d_1 d_1} | \psi_{d_1} \rangle)^{-1} \langle \psi_{d_1} | H_{d_1 P} | \psi_i^{(+)} \rangle . \quad (2.27)$$

Equation (2.27) has two energy denominators corresponding to two types of doorway states but the physical interpretation of widths and energies becomes difficult. As in Eq. (2.18) the exit doorway state energy and width also gets modified in Eq. (2.27) because of coupling to d_1, P, and q spaces. But the entrance doorway state of Eq. (2.27) gets modified only because of its coupling to the P space; it does not feel the effect of coupling to the d_2 space. The coupling between d_1 and d_2 spaces only appears in one of the denominators and not both. The same problem will remain if we choose Eq. (2.16) to define the doorway state widths and spacings. This problem is related to the difficulty one faces in uniquely defining widths and spacings of overlapping resonances. A similar problem also appears in the formulation of Ahmed and Beres [see, Eq. (32f) of Ref. 6]. Apart from this difficulty of interpretation of resonance parameters Eq. (2.27) gives a useful parametrization of the average transition amplitude in terms of entrance and exit doorways. Writing

$$\langle \psi_{d_2} | (W^{(0)}_{d_2d_2} + W^{(1)}_{d_2d_2} + \overline{W}_{d_2d_2}) | \psi_{d_2} \rangle = \Delta_{d_2} - i\Gamma_{d_2}/2$$

and

$$\langle \psi_{d_1} | W^{(0)}_{d_1d_1} | \psi_{d_1} \rangle = \Delta_{d_1} - i\Gamma_{d_1}/2 ,$$

it is easy to realize that Eq. (2.27) takes the following form

$$\begin{aligned}\langle T \rangle_I = &\langle \phi_f | V | \psi_i^{(+)} \rangle + \langle \psi_f^{(-)} | H_{Pd_2} | \psi_{d_2} \rangle \\ &\times (E - \mathscr{E}_{d_2} + i\Gamma_{d_2}/2)^{-1} \langle \psi_{d_2} | H_{d_2d_1} | \psi_{d_1} \rangle \\ &\times (E - \mathscr{E}_{d_1} + i\Gamma_{d_1}/2)^{-1} \langle \psi_{d_1} | H_{d_1P} | \psi_i^{(+)} \rangle ,\end{aligned} \tag{2.28}$$

where $\mathscr{E}_{d_2} = E_{d_2} + \Delta_{d_2}$ and $\mathscr{E}_{d_1} = E_{d_1} + \Delta_{d_1}$. Equation (2.28) presents a useful parametrization when there is one isolated entrance and an exit doorway state near the energy E. The first term on the right-hand side of Eq. (2.28) is the direct amplitude which is supposed to be approximated usually by a distorted wave amplitude in a calculation. The last term of Eq. (2.28) exhibits the intermediate resonances arising out of entrance and exit doorway states. This term is supposed to be parametrized in a calculation. It is not difficult to generalize the above expression to the case where many doorway states of each type are present near the energy E. Introducing the doorway states by $\psi^{(j)}_{d_1}$ and $\psi^{(k)}_{d_2}$ satisfying

$$(E - E^{(j)}_{d_1})\psi^{(j)}_{d_1} = 0$$

and

$$(E - E^{(k)}_{d_2})\psi^{(k)}_{d_2} = 0 ,$$

where j and k level the doorway states, the eigenfunction expansion of the propagators can be performed again and one can write the final result for the average transition amplitude. Modifications necessary in this case are easily achieved by putting the levels j and k over the entrance and exit doorway states and finally summing over these levels. For example, Eq. (2.28) becomes

$$\begin{aligned}\langle T \rangle_I = \langle \phi_f | V | \psi_i^{(+)} \rangle + \sum_k \sum_j &\langle \psi_f^{(-)} | H_{Pd_2} | \psi^{(k)}_{d_2} \rangle (E - \mathscr{E}^{(k)}_{d_2} + i\Gamma^{(k)}_{d_2}/2)^{-1} \\ &\times \langle \psi^{(k)}_{d_2} | H_{d_2d_1} | \psi^{(j)}_{d_1} \rangle (E - \mathscr{E}^{(j)}_{d_1} + i\Gamma^{(j)}_{d_1}/2)^{-1} \langle \psi^{(j)}_{d_1} | H_{d_1P} | \psi_i^{(+)} \rangle .\end{aligned} \tag{2.29}$$

The doorway state term of Eq. (2.29) can still be parametrized when the summation does not involve too many terms near energy E. But the calculation of average cross section becomes very complicated when there are many overlapping resonances which may lead to statistical fluctuations in the cross section.[7] Overlapping or not, Eq. (2.29) gives the parametrization of the average transition amplitude in terms of doorway state resonance parameters.

III. GENERALIZATION IN A MULTISTEP MODEL

In this section we generalize the formulation of the last section where the exit doorway state has a complex mode of excitation in relation to the entrance doorway state. That means it is not the first hallway state in the time evolution of the system. In general it is the Nth hallway state where N is not too large. The q space or the space of compound nuclear states appears after the exit doorway state.

We introduce the usual orthogonal projection operators $P, d_1, d_2, d_3, \ldots, d_N, q$ such that[7]

$$P + \sum_{n=1}^{N} d_n + q = 1 \tag{3.1}$$

and the usual orthogonality relations between different projection operators exist. Equations (2.3)–(2.5) are now valid with $n = 0,1,2,\ldots,(N+1)$. In terms of these operators T of Eq. (2.3) can be written as

$$T = \langle \phi_f | (V_{(N+1)} + V_{(N+1)} G_{(N+1)} V_{(N+1)}) | \phi_i \rangle . \tag{3.2}$$

Now we introduce the doorway state hypothesis of Sec. II that $d_i V d_j = d_i H d_j \neq 0$ only when $|(i-j)| = 1$. Also we introduce the condition that $d_i \mathscr{V} d_j = d_i V d_j$. Using Eqs. (2.4) and (2.5) recursively in Eq. (3.2) and using the doorway state hypothesis it is easy to realize that T of Eq. (3.2) takes the following form

$$\begin{aligned}T = &\langle \phi_f | V | \psi_i^{(+)} \rangle + \langle \psi_f^{(-)} | H_{Pd_N} G_N H_{d_N d_{(N-1)}} G_{(N-1)} \cdots G_2 H_{d_2d_1} G_1 H_{d_1P} | \psi_i^{(+)} \rangle \\ &+ \langle \psi_f^{(-)} | H_{Pd_N} G_N H_{d_N d_{(N+1)}} G_{(N+1)} H_{d_{(N+1)} d_N} G_N H_{d_N d_{(N-1)}} \cdots G_2 H_{d_2d_1} G_1 H_{d_1P} | \psi_i^{(+)} \rangle ,\end{aligned} \tag{3.3}$$

where the notation is the same as Eq. (2.9). With the doorway state hypothesis the resolvent operators G_1, G_2, and G_3 of Eq. (3.3) are essentially given by

$$G_0=P(E-H_{PP})^{-1}P\ ,$$

$$G_i=d_i(E-H_{d_id_i}-W^{(i-1)}_{d_id_i})^{-1}d_i\ ,\quad i=1,2,\ldots,(N-1)\ , \tag{3.4}$$

$$G_N=d_N(E-H_{d_Nd_N}-W^{(N-1)}_{d_Nd_N}-W^{(0)}_{d_Nd_N})^{-1}d_N\ ,$$

and

$$G_{(N+1)}=q(E-H_{qq}-W^{(N)}_{qq})^{-1}q\ ,$$

where

$$W^{(i)}=HG_iH\ .$$

Here $d_{(N+1)}=q$ and the notation is the same as in Eq. (2.10). Now we have the identity analogous to (2.11), e.g.,

$$\mathcal{G}_N=G_N+G_NH_{d_Nd_{(N+1)}}G_{(N+1)}H_{d_{(N+1)}d_N}G_N\ , \tag{3.5}$$

with

$$\mathcal{G}_N=d_N(E-H_{d_Nd_N}-W^{(0)}_{d_Nd_N}-W^{(N-1)}_{d_Nd_N}-\widetilde{W}_{d_Nd_N})^{-1}d_N$$

and

$$\widetilde{W}_{d_Nd_N}=H_{d_Nq}(E-H_{qq})^{-1}H_qd_N\ .$$

Using the identity (3.5), Eq. (3.3) can be written as

$$T=\langle\phi_f\,|\,V\,|\,\psi_i^{(+)}\rangle+\langle\psi_f^{(-)}\,|\,H_{Pd_N}\mathcal{G}_NH_{d_Nd_{(N-1)}}G_{(N-1)}\cdots G_2H_{d_2d_1}G_1H_{d_1P}\,|\,\psi_i^{(+)}\rangle\ . \tag{3.6}$$

The system now develops in the following sequence starting from the entrance channel state i: $i\rightarrow d_1\rightarrow d_2\rightarrow\cdots\rightarrow d_N\rightarrow f$. The d_N state is the only state which couples to the compound nuclear states $q=d_{(N+1)}$ in the following way: $d_N\rightarrow q\rightarrow d_N$. In the usual shell model terminology d_n corresponds to $(n+1)$ particle n hole states. Entrance (exit) doorway states are those states in d_1 (d_N) space which are strongly coupled to the initial (final) channel state. Density of such states is supposed to be low. Starting from the d_1 space the successive doorway states will first have an increasing density of states, and then this density reduces until we reach the space d_N which is expected to have a low density. Equation (3.6) has propagators for different spaces. The concept of isolated doorway state is useful for $n=1,2$ or for $n=(N-1)$, N. For all other n the density of states is expected to be high and isolated resonances are unlikely to appear. The q or the $d_{(N+1)}$ space propagator $G_{(N+1)}$ which explicitly appears in Eq. (3.3) and which is hidden in $\mathcal{G}_N$ of Eq. (3.6) contains among others the compound nuclear resonances. If we would like to study intermediate structure due to entrance and exit doorway states we would have to perform the usual Lorentz average over the q space rapid fluctuations with respect to a function of width I. The result can be easily derived from Eqs. (2.15) and (3.6), and one has for the energy averaged transition amplitude

$$\langle T\rangle_I=\langle\phi_f\,|\,V\,|\,\psi_i^{(+)}\rangle+\langle\psi_f^{(-)}\,|\,H_{Pd_N}\overline{\mathcal{G}}_NH_{d_Nd_{(N-1)}}G_{(N-1)}\cdots G_2H_{d_2d_1}G_1H_{d_1P}\,|\,\psi_i^{(+)}\rangle\ , \tag{3.7}$$

with

$$\overline{\mathcal{G}}_N=d_N(E-H_{d_Nd_N}-W^{(0)}_{d_Nd_N}-W^{(N-1)}_{d_Nd_N}-\overline{W}_{d_Nd_N})^{-1}d_N\ ,$$

$$\overline{W}_{d_Nd_N}=H_{d_Nq}(E-H_{qq}+iI/2)^{-1}H_{qd_N}\ .$$

Making an eigenfunction expansion of the propagators in Eq. (3.7) as in Sec. II it is easy to realize that one propagator will correspond to resonance for a particular space. Isolated resonances are most likely to appear in space d_1 or d_N. For $N>n>1$ the density of resonances is expected to be large. Intermediate structure corresponding to isolated exit doorways is most likely to be visible when the width of such resonances are small and the strength is not weak. Another obvious condition for easy identification of entrance and exit doorway states happens when the intermediate stages $(N>n>1)$ lead to resonances of large width and low strength. In such cases the remaining propagators of Eq. (3.7) can be grouped into a matrix element which can be treated as a smooth function and one has from Eq. (3.7),

$$\langle T\rangle_I=\langle\phi_f\,|\,V\,|\,\psi_i^{(+)}\rangle+\langle\psi_f^{(-)}\,|\,H_{Pd_N}\overline{\mathcal{G}}_N\overline{H}_{d_Nd_1}G_1H_{d_1P}\,|\,\psi_i^{(+)}\rangle\ , \tag{3.8}$$

where

$$\overline{H}_{d_Nd_1}\equiv H_{d_Nd_{(N-1)}}G_{(N-1)}\cdots G_3H_{d_3d_2}G_2H_{d_2d_1} \tag{3.9}$$

essentially acts as a nonresonant smooth object. Equation (3.8) now has two propagators—G_1 corresponding to the entrance doorway state and G_N corresponding to the exit doorway state. Equation (3.8) is similar to Eq. (2.15) and the treatment following Eq. (2.15) of Sec. II also applies to Eq. (3.8).

In a certain reaction apart from the entrance and exit doorway states some of the other propagators of Eq. (3.9) may lead to a resonant contribution. Then the last term of Eq. (3.8) can conveniently be written as a function containing several resonant energy denominators. It is worthwhile to remember that for $N=2$ Eq. (3.8) reduces to Eq. (2.15) where one has only two energy denominators.

We conclude the present section with a brief comparison of our formulation with those of Refs. 5 and 6. In Ref. 5 using rearrangement scattering theory we showed the pausibility of the appearance of exit doorway states within a dynamical formulation. There no attempt was made to relate the exit and entrance doorway states in the time evolution of the system. The work of Ahmed and Beres[6] is the first one which shows a relation between exit and entrance doorway states. The present work exploits some of the ideas of Ref. 6 but the details are different. Ahmed and Beres[6] use the formulation of Beres and collaborators in their study of doorway states. We use the nested doorway model of Ref. 7 in the present study, which can be considered as a generalization of the ideas of Beres and collaborators to the case of a hierarchy of doorway hallway states. The first doorway in this space of doorway hallway states is called the entrance doorway and the last one is called the exit doorway.

IV. EXAMPLES OF EXIT DOORWAY STATES

In this section we would like to provide evidence for intermediate structure resonances arising from exit doorway states. There are only a few examples of intermediate structure resonances which can convincingly be attributed to the usual (entrance) doorway states. They are (a) the delta resonances bound in a nuclei as encountered in the intermediate energy pion-nucleus systems, (b) the giant resonances, (c) the isobaric analog resonances, (d) the isolated resonances as encountered in low-energy neutron-nucleus scattering, and (e) the resonances of a subsystem as encountered in heavy ion reactions. Now we shall present evidence where the above resonances appear as exit doorway states and lead to identifiable intermediate structure resonances.

For our purpose we chose elastic and inelastic scatterings of two nuclear fragments, which exhibit pronounced intermediate structure resonance due to the entrance doorway state. Then we studied nuclear reactions leading to the above two nuclear fragments in the final state, where the above entrance doorway state may appear as the exit doorway state whose effect is established by identifying the same intermediate structure resonance as above.

A. Delta resonances

A striking example of the exit doorway state is to be found in the pion production process described by $AB \to \pi C$, where each of A or B is a nucleon or a nucleus and C is a nucleus; specific examples are $NN \to \pi {}^2H$, $N {}^2H \to \pi {}^3H$, etc., where N stands for a nucleon. There is a pronounced peak[8–10] in such pion production cross sections and simple kinematics relate this peak to the formation of a Δ resonance bound in the nucleus immediately before the pion production. This Δ, which subsequently decays as $\Delta \to \pi N$, is the exit doorway state for pion production. Notice that this doorway state is not present in the elastic channel. For the sake of completeness we perform the simple kinematic calculation for the resonance energy.

Let us consider the process $AB \to \pi C$ with B at rest initially and let m_1, m_2, m_3, and m_π be the rest masses of A, B, C, and π, respectively. Let us relate the center of mass energy of this process to that of $\pi C \to X$ where C is at rest initially. Then we have

$$m_1^2 + m_2^2 + 2E_1 m_2 = m_3^2 + m_\pi^2 + 2E_\pi m_3 ,$$

where E_1 and E_π are full energies of A and π in these two processes. Then the kinetic energy T_1 of A is given by

$$T_1 = (m_3^2 + m_\pi^2 + 2E_\pi m_3 - m_1^2 - m_2^2)/(2m_2) - m_1 . \quad (4.1)$$

We know that the process $\pi C \to X$ resonates[11] at $E_\pi \cong 290$ MeV because of the formation of Δ as an entrance doorway state. This energy is fairly independent of the mass of C so long as C is a light nucleus.[11] Equation (4.1) gives the incident energy of A in the process $AB \to \pi C$ necessary for the formation of the exit doorway state Δ. For $NN \to \pi {}^2H$ and $N {}^2H \to \pi {}^3H$ the incident proton energy necessary for the formation of the exit doorway state can be easily calculated from Eq. (4.1) to be $T_1 \cong 590$ and 440 MeV, respectively, for[11] $E_\pi \cong 290$ MeV which are to be compared with the experimental results:[8,10] $T_1 \cong 600$ MeV and $T_1 \cong 450$ MeV, respectively. This confirms the production of exit doorway states in the pion production process. Using this idea of exit doorway states one can formulate an isobar exit doorway model for pion production in the same way as Kisslinger and Wang[11] formulated the isobar doorway model for the pion-nucleus system.

B. Giant dipole resonances

Giant dipole resonances[12] are usually observed in gamma-nucleus systems as a 1p-1h doorway state when the incident photon excites one nucleon to a different shell. Such resonances are observed, for example, in γ-^{16}O, γ-^{13}C, and γ-^{208}Pb systems among many others. To identify such giant resonances as an exit doorway state we should consider reactions leading to final states γ-^{16}O, γ-^{13}C, and γ-^{208}Pb, etc.—states which have strong coupling with the giant resonances which will act as exit doorway states.

It is known that the giant dipole resonance of ^{16}O is observed in many photoproduction reactions involving ^{16}O, such as,[13] ^{12}C(^{4}He,γ)^{16}O, ^{13}C(^{3}He,γ)^{16}O, ^{14}N(^{2}H,γ)^{16}O, ^{15}N(^{1}H,γ)^{16}O among others. The giant dipole resonance has a simple mode (1p-1h) of excitation with respect to ^{16}O and acts as the exit doorway state responsible for the giant peak in the cross section. If all the doorway-hallway spaces other than the giant resonance exit doorway space were nonresonant then the intermediate structure resonance in all four above reactions would have appeared at the same excitation energy. A small variation in the observed position of the resonances in the above reactions suggests the phenomenologically verified fact that the contributions of the above doorway-hallway spaces are not completely nonresonant.[13] Some of these spaces could give rise to a resonance which may interfere with the giant resonance giving it a small displacement. This then explains the position of the main giant resonance in these four reactions.

The parametrization of the transition matrix in terms of two resonances as done, for example, in the discussion of secondary doorway state[14] could have anticipated the existence of the effect of exit doorway states But to the best of our knowledge such an anticipation was never made, though the two-resonance parametrization has sometimes been used.

The existence of the giant dipole resonance in ^{16}O as an exit doorway state is not an exception. In fact all giant resonances can be observed as exit doorway states; examples being the giant dipole resonance of ^{13}C observed[15] as exit doorway in $^{12}C(n,\gamma)^{13}C$ and $^{11}B(^{2}H,\gamma)^{13}C$ and that of ^{209}Pb observed[15] as exit doorway in $^{208}Pb(n,\gamma)^{209}Pb$.

C. Isobaric analog resonances

Isobaric analog resonances are usually observed in proton-nucleus systems as 2p-1h doorway states (in the continuum), which are isobaric analog of isolated low lying states of a neighboring (parent) nucleus. Isospin selection rule is responsible for the observed intermediate structure resonance corresponding to the analog states.[3] In order to identify such an analogy resonance as an exit doorway state we should consider reactions leading to proton-nucleus final states, which have strong coupling with analog states which will act as exit doorway states.

In fact isobaric analog resonances have been observed as exit doorway states in $(^{2}H,^{1}H)$ and $(^{3}H,^{1}H)$ reactions on lead isotopes. For example, Hamburger[16] studied the reaction $^{207}Pb(^{2}H,^{1}H)^{208}Pb$ and identified the analog states of the parent nucleus ^{209}Pb in the yield curve of this reaction. In this case the isobaric analog states act as exit doorway states. More interesting is the study of Armstrong and Bernstein[17] who analyzed the reaction $^{208}Pb(^{3}H,^{1}H)^{210}Pb$ and identified the analog states of the parent nucleus ^{211}Pb which is far from the stability line. This reaction involves two-nucleon transfer and because of interference effect with other doorway-hallway resonances is less likely to show the resonances corresponding to the isobaric analog exit doorway states than the one-nucleon transfer reaction. The elastic $^{208}Pb(^{3}H,^{3}H)^{208}Pb$ process did not, as expected, show[17] the analog resonances and this confirms our conclusion of isobaric analog exit doorway states in $^{208}Pb(^{3}H,^{1}H)^{210}Pb$.

D. Isolated low energy resonances

Isolated doorway states have been observed in low energy neutron-nucleus scattering, for example, in n-^{56}Fe scattering[3,18] where 2p-1h excitations serve as isolated entrance doorways responsible for intermediate structure. These same low lying resonances have been identified[19] in $^{57}Fe(\gamma,n)^{57}Fe$ by Jackson and Strait, where the low lying 2p-1h excitations serve as exit doorways. Similarly, the well-known doorway state discovered by Farrel *et al.* in studies of the reaction $^{206}Pb(n,n')$ has been observed as an exit doorway state by Baglan *et al.* in studies of the reaction $^{207}Pb(\gamma,n)$ near threshold.[19]

E. Subsystem resonances in heavy-ion reactions

It has been argued that many of the resonances observed in heavy ion systems are due to the existence of a resonating subsystem.[5,6,20] Such subsystem resonances may act as exit doorway states. For example, in $^{12}C(^{16}O,^{8}Be)^{20}Ne$ it has been suggested[5] that just before forming the $(^{8}Be\text{-}^{20}Ne)$ final state the system passes through the exit doorway state consisting of ^{8}Be and ^{16}O in resonance while ^{16}O is bound to ^{4}He. (In the pion production process this phenomenon is similar to the pion forming a Δ resonance with a nucleon bound in the nucleus.) In this way it was possible to identify the $(^{8}Be\text{-}^{16}O)$ resonances as exit doorway states in the $^{12}C(^{16}O,^{8}Be)^{20}Ne$ reaction.[5] Another conjecture of such an exit doorway state is provided by Ahmed and Beres[6] for the reaction $^{12}C(^{12}C,^{4}He)^{20}Ne^{*}$. Further experiments are needed in order to confirm the presence of exit doorway states in heavy ion reactions. Sugimitsu *et al.*[21] have promised to do such experiments recently.

V. DISCUSSION

It is obvious that though the exit doorway states always existed their importance was never realized in the analysis of nuclear reactions. This is because the usual doorway state formulation[1] was really intended for the elastic and inelastic processes, though attempt[1] has been made to use it for reactions. In such a formulation the exit doorway states belong to a hallway space containing many overlapping resonances. It was never realized that the exit channel could select some isolated states of these many overlapping hallway states as isolated exit doorway states which could lead to intermediate structure resonance.

In this paper we developed a formalism for the compound inelastic process including the effect of entrance and exit doorway states. The present approach does not depend on the use of a special model for a nuclear reaction, for example, a shell model or an alpha particle model, etc. The doorway states are states of definite angular momentum, parity, etc. Hence the observed intermediate structure corresponding to an exit or entrance doorway state will have well-defined quantum numbers. In conclusion we would like to stress that exit doorway states deserve more attention in the analysis of nuclear reactions.

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