Separable approximation to the NN Paris potential in the framework of the Bethe-Salpeter equation

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The Bethe-Salpeter equation is solved with a separable kernel for the most important nucleon-nucleon partial wave states. We employ the Ernst-Shakin-Thaler method in the framework of minimal relativity (Blankenbecler-Sugar equation) to generate a separable representation of the meson-theoretical Paris potential. These separable interactions, which closely approximate the on-shell and half-off-shell behavior of the Paris potential, are then cast into a covariant form for application in the Bethe-Salpeter equation. The role of relativistic effects is discussed with respect to on-shell and off-shell properties of the NN system.

I. INTRODUCTION

During the last decade interest has focused on covariant two-body equations with regard to relativistic three-body calculations. Because of the complexity of three-body calculations, up till now nobody has used the four-dimensional analog of the Lippmann-Schwinger (LS) equation, namely the Bethe-Salpeter (BS) equation, for the description of the two-particle subsystems. All "relativistic" calculations in three-body problems have been performed within the framework of three-dimensional reductions of the four-dimensional Bethe-Salpeter (BS) equation. Two-particle equations of that type, which are widely used, are the so-called Blankenbecler-Sugar (BBS) equation,[1] Gross equation, [2] Erkelenz-Holinde equation,[3] and Kadyshevsky equation.[4]

We have already shown in our previous works[5] that a reasonable description of NN and πN scattering can be performed in the framework of the BS equation. We have been able to reproduce NN and πN scattering data without introducing some reduction techniques to go from four to three dimensions.

In all of these calculations the free parameters of the four-dimensional Yamaguchi-type form factors were fitted in order to reproduce corresponding two-body phase shifts as well as possible. However, no attention at all was paid to the off-shell behavior of the two-body T matrices.

Since it is well known that important features of three-body systems (in the binding as well as in the scattering domain) are very sensitive to the underlying two-body off-shell behavior,[6] it is our goal in this paper to present a separable four-dimensional approach to the BS equation, which yields reasonable descriptions of the on-shell as well as off-shell properties of the two-nucleon system. Due to the fact that direct information on the off-shell behavior is scarce and insufficient for our purpose, we have chosen to regard the off-shell behavior of a meson theoretical potential as the basis for our calculations. Actually we took the Paris potential[7] as a reference model and followed a method which was given by Ernst, Shakin, and Thaler (EST).[8]

The EST method is established in three dimensions only; we have therefore divided our investigations into two steps. Firstly, we construct a three-dimensional separable potential (in the framework of the BBS equation), which reproduces the on- and half-off- shell properties of the Paris potential (see Sec. III) satisfactorily. Secondly, we construct a four-dimensional version of the obtained potential, by fixing the parameter β and varying the coupling strengths within the BS framework to refit the phase shift. We have seen that this second step has almost no influence on the off-shell behavior.

In Sec. II we briefly introduce the BS equation in its separable form. Section III shows the results obtained with the EST method in the NN system; in Sec. IV some concluding remarks will be given.

II. THE BS EQUATION WITH SEPARABLE INTERACTIONS

The BS equation,

$$T(q,q';s)=V(q,q')+\frac{i}{4\pi^3}\int d^4k\; V(q,k)G(k,s)T(k,q';s)\;, \qquad (2.1)$$

describes the relativistic two-particle scattering in terms of the T matrix, of an interaction V, the kernel of the integral equation, and the free two-particle Green's function G. Since the total angular momentum is conserved, we may decompose T and V into partial wave components obtaining the partial wave decomposed BS equation in momentum space

$$T_l(q_0,q,q_0',q';s)=V_l(q_0,q,q_0',q')+\frac{i}{4\pi^3}\int_{-\infty}^{+\infty}dk_0\int_0^{\infty}k^2dk\,V_l(q_0,q,k_0,k)G(k_0,k;s)T_l(k_0,k,q_0',q';s) \tag{2.2}$$

with

$$G(k_0,k;s)=[(k_0+a\sqrt{s})^2-E_1^2+i\varepsilon]^{-1}\times[(k_0-b\sqrt{s})^2-E_2^2+i\varepsilon]^{-1} . \tag{2.3}$$

Using the abbreviations $a=m_1/(m_1+m_2)$ and $b=m_2/(m_1+m_2)$, the relative momenta are given by

$$q=(q_0,q)=aq_2-bq_1 ,$$

m_1 and m_2 are the masses of the particles with momenta q_1 and q_2.

q, k, and q' are the initial, intermediate, and final relative momenta, s is the total energy squared in the center of mass system, and $E_i=(k^2+m_i^2)^{1/2}$; $i=1,2$.

The phase shift is connected to the fully on-shell T matrix by

$$T_l(p_0,p,p_0,p;s)=-\frac{8\pi\sqrt{s}}{p}e^{i\delta_l(p)}\sin\delta_l(p) .$$

For the interaction V_l we have chosen the following four-dimensional separable ansatz (to simplify the discussion we show the formulae just for a rank-1 separable potential)

$$V_l(q_0,q,k_0,k)=v_l(q_0,q)\lambda_l v_l(k_0,k) , \tag{2.4}$$

where λ_l is the coupling parameter and $v_l(q_0,q)$ is a relativistic generalization of the Yamaguchi form factor (vertex function)

$$v_l(q_0,q)=\frac{q^l}{(-q_0^2+q^2+\beta^2)^{l+1}} . \tag{2.5}$$

We have used "magic vectors" for a covariant treatment of the threshold behavior.[9] As a consequence the BS equation can then be solved in closed form to give

$$T_l(q_0,q,q_0',q')=v_l(q_0,q)v_l(q_0',q')/D_l(s) , \tag{2.6}$$

$$D_l(s)=\lambda_l^{-1}-\frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0\int_0^{\infty}k^2dk\,v_l^2(k_0,k)\times G(k_0,k;s) . \tag{2.7}$$

For the k_0 integration in the complex energy plane in Eq. (2.7), we investigate the four singularities of the Green's function at momenta k_0; in the lower k_0 plane,

$$k_{01}=-a\sqrt{s}+E_1-i\varepsilon ,$$

$$k_{03}=b\sqrt{s}+E_2-i\varepsilon ,$$

and in the upper k_0 plane,

$$k_{02}=-a\sqrt{s}-E_1+i\varepsilon ,$$

$$k_{04}=b\sqrt{s}-E_2+i\varepsilon ,$$

with $E_1=\sqrt{k^2+m_1^2}$ and $E_2=\sqrt{k^2+m_2^2}$, and in addition the singularities of the potential

$$k_{05}=\sqrt{k^2+\beta^2}-i\varepsilon \quad \text{lower } k_0 \text{ plane} ,$$

$$k_{06}=\sqrt{k^2+\beta^2}+i\varepsilon \quad \text{upper } k_0 \text{ plane} .$$

These potential poles are of second order for S waves and of fourth order for P and D waves.

Closing the contour in the lower plane, we are able to perform the k_0 integration as the sum of four residues; the second integral in Eq. (2.7) is evaluated numerically. By fitting the free parameters λ and β to the phase shifts, one has to be careful, since the values of the variable β are constrained to avoid integration singularities. On the one hand β has to be less than two times the nucleon mass. In addition there exists a lower limit for β which depends on the scattering energy. For example, for an energy range up to 350 MeV the lower bound $\beta\approx\frac{1}{10}$ m^2 for NN scattering.

III. NN SCATTERING PHASE SHIFTS AND HALF-OFF-SHELL FUNCTIONS

The EST mechanism leads to the construction of a separable potential that reproduces the on- as well as off-shell behavior of some arbitrary potential—in our case the Paris potential.[7] One selects some energy points (interpolation points) in a particular channel (the number of energy points determines the rank of the constructed separable potentials) and the EST method guarantees that the T matrix of the Paris potential and the T matrix of the separable potential approximation are identical at the chosen energy point.[10]

It is well known that within a covariant three-dimensional equation the parameters of the potential are close to the ones obtained from a fully relativistic four-dimensional calculation. Therefore it is obvious to use such a semirelativistic equation to handle the EST method, suggesting that the resulting parameters of the separable potential lead to smaller corrections in the Bethe-Salpeter equation as would happen in the framework of the LS equation.

As mentioned in Sec. I, we use the BBS equation as a starting point of our investigations, since in comparison to other three-dimensional relativistic equations, the BBS equation is a symmetric reduction of the BS equation.

As a consequence, the creation of a four-dimensional interaction is much easier to perform by using an unsymmetric reduction of the BS equation. To show the accuracy of this method we have calculated the NN $^1S_0$, $^3S_1$-$^3D_1$, $^1P_1$, $^3P_0$, and $^3P_1$ phase shifts and the corresponding half-off-shell functions.

For the $^1S_0$ p-p wave we have used a rank-3 potential; in terms of the EST method this corresponds to three interpolation energies $E_1=0$, $E_2=100$, and $E_3=500$ MeV.

The appropriate separable interaction, where each vertex function consists of the sum of five terms, is given by

TABLE I. Parameters of the separable interaction in the p-p $^1S_0$ partial wave. The coupling strengths λ_i^{BBS} are results of an application of the EST method by using the BBS equation; λ_i^{BS} are the refitted values within the BS framework.

β (fm^{-1})	C_{1i} (fm^0)	C_{2i} (fm^0)	C_{3i} (fm^0)	λ (fm^{-4})
$\beta_1=1.1000$	$C_{11}=-35.741609$	$C_{21}=48.037979$	$C_{31}=-81.570514$	$\lambda_1^{\mathrm{BBS}}=-1.$ $\lambda_1^{\mathrm{BS}}=-1.083$
$\beta_2=1.9965421$	$C_{12}=1032.6580$	$C_{22}=116.71817$	$C_{32}=2886.0197$	$\lambda_2^{\mathrm{BBS}}=-1.$ $\lambda_2^{\mathrm{BS}}=-1.52659$
$\beta_3=2.8295473$	$C_{13}=-8225.6013$	$C_{23}=-3390.4081$	$C_{33}=-18329.260$	$\lambda_3^{\mathrm{BBS}}=1.$ $\lambda_3^{\mathrm{BS}}=0.538$
$\beta_4=3.6238004$	$C_{14}=17430.832$	$C_{24}=8110.8219$	$C_{34}=35577.737$	
$\beta_5=4.3904282$	$C_{15}=-10568.521$	$C_{25}=-4923.0097$	$C_{35}=-20594.912$	

$$V_0(q_0,q,k_0,k)=\sum_{i=1}^{3} v_i(q_0,q)\lambda_i v_i(k_0,k) \ , \tag{3.1}$$

$$v_i(q_0,q)=\sum_{j=1}^{5}\frac{C_{ij}}{(-q_0^2+q^2+\beta_j^2)} \ . \tag{3.2}$$

The parameters of the potential are given in Table I; in Fig. 1 we show the p-p $^1S_0$ scattering phase shift obtained within the BS equation, whereby we show the phase shifts, obtained without and with a change of the coupling strengths. In addition, the $^1S_0$ phase within the BBS equation is presented. To show the quality of our calculation we compare in Fig. 1 our results also with the $^1S_0$ phase shift of the Paris potential and with the phenomenological data of Arndt *et al.*[11]

In order to use the BS equation instead of the BBS equation, we had to refit the coupling strengths, λ_1, λ_2, and λ_3. All other parameters we could keep fixed since the phase shifts are not as sensitive on them as on the coupling strengths. We had to raise λ_1 and λ_2 to get more attraction and to lower λ_3, which is responsible for the repulsion, to get less repulsion (see Table I); this is also evident if one looks at the dashed-point line in Fig. 1.

Figure 2 shows the half-off-shell function

$$f(q_0,q,s)=\frac{T(q_0,q,p_0,p;s)}{T(p_0,p,p_0,p,s)} \tag{3.3}$$

at the energy $E_{\mathrm{lab}}=100$ MeV. As shown in Fig. 2, good agreement of the BS description of the $^1S_0$ partial wave with the corresponding property of the Paris potential is obtained. The results of half-off-shell functions at the two additional interpolation energies ($E=0$ MeV, $E=500$ MeV) are of the same accuracy as compared in Fig. 2; they are still reasonable at the whole energy domain.

For the $^1P_1$, $^3P_0$, $^3P_1$ channel we have constructed a rank-2 separable potential (each vertex function consists again of five terms)

$$V_1(q_0,q,k_0,k)=\sum_{i=1}^{2} v_i(q_0,q)\lambda_i v_i(k_0,k) \ , \tag{3.4}$$

$$v_i(q_0,q)=\sum_{j=1}^{5}\frac{C_{ij}q}{(-q_0^2+q^2+\beta_j^2)^2} \ . \tag{3.5}$$

Table II gives the potential parameters, the corresponding phase shifts are shown in Figs. 3–5. For the $^1P_1$ and $^3P_1$ state the interpolation energies were chosen at $E_1=50$ and $E_2=150$ MeV.

To obtain a good fit to the phase shifts in the framework of the BS equation, both coupling strengths λ_1 and λ_2 had to be lowered by a small amount (for both cases, see Table II). The half-off-shell behavior, however, is in good approximation to the corresponding Paris potential properties (Figs. 6 and 8); the reproduction is especially accurate at the interpolation energies.

A similar behavior is found for the $^3P_0$ wave (interpolation energies: $E_1=$ 50 MeV, $E_2=$ 350 MeV); agreement with the NN-Paris potential is satisfactory. The absolute

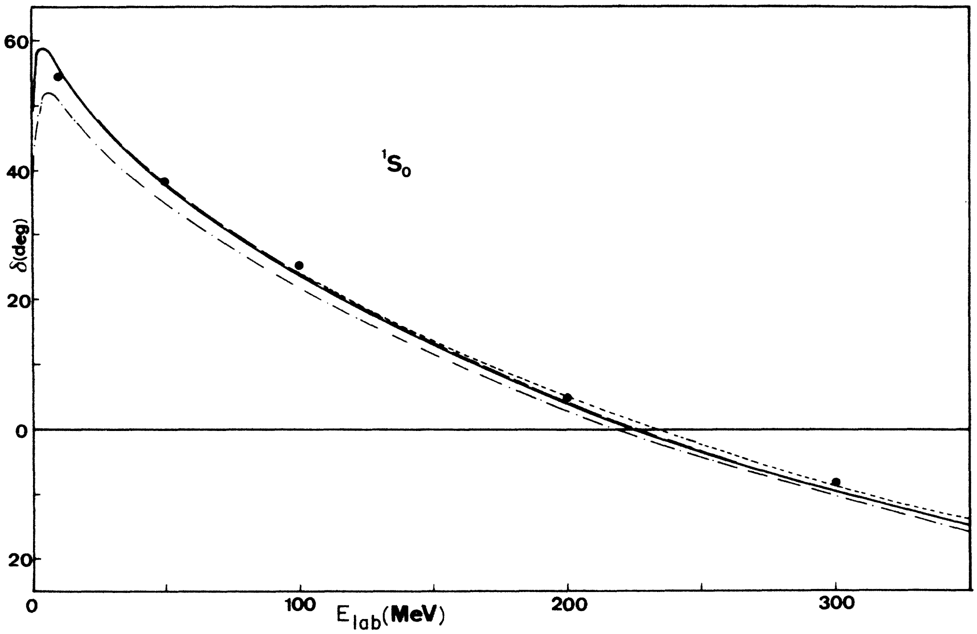

FIG. 1. Scattering phase shifts of the NN system. The solid line represents the calculation within the BS equation with refitted coupling strengths λ_i^{BS}, the dashed dotted line shows the result of the BS investigation without refitting λ_i^{BBS}. The dashed line describes the NN-Paris potential calculation and the dotted line shows the results of the EST method applied in the BBS framework. The circles indicate the experimental values of Ref. 11.

value of the coupling strength λ_1 (λ_1 is responsible for the attraction) had to be raised by a very small amount. In contradiction to λ_1, the coupling parameter λ_2, which is responsible for the repulsion in this partial wave, had to be lowered to obtain satisfactory results in comparison to the experimental data (see Table II). The changes of the coupling parameters are obvious by looking at Fig. 4, where the dashed-dotted line shows less attraction and too

TABLE II. Parameters of the separable interactions in the $^1P_1$, $^3P_0$, and $^3P_1$ partial waves. The parameters λ_1^{BS} and λ_i^{BBS} are defined in the caption of Table I.

β (fm^{-1})	C (fm^0)		λ (fm^{-4})
	$^1P_1$		
$\beta_1=0.7$	$C_{11}=-0.696\,129\,28$	$C_{21}=3.875\,983\,8$	$\lambda_1^{BBS}=1.$
$\beta_2=1.4$	$C_{12}=27.246\,204$	$C_{22}=-387.301\,46$	$\lambda_2^{BBS}=1.$
$\beta_3=2.1$	$C_{13}=538.285\,92$	$C_{23}=2945.2732$	$\lambda_1^{BS}=0.950\,000$
$\beta_4=2.8$	$C_{14}=-1848.8997$	$C_{24}=-6658.5553$	$\lambda_2^{BS}-=0.906\,813\,1$
$\beta_5=3.5$	$C_{15}=1980.6184$	$C_{25}=-4992.4279$	
	$^3P_0$		
$\beta_1=0.8$	$C_{11}=2.471\,662\,7$	$C_{21}=-8.912\,079\,7$	$\lambda_1^{BBS}=-1.$
$\beta_2=1.6$	$C_{12}=-211.318\,36$	$C_{22}=666.771\,57$	$\lambda_2^{BBS}=1.$
$\beta_3=2.4$	$C_{13}=-313.923\,85$	$C_{23}=-7266.8332$	$\lambda_1^{BS}=-1.052\,00$
$\beta_4=3.2$	$C_{14}=3228.2158$	$C_{24}=20\,055.583$	$\lambda_2^{BS}=0.850\,415$
$\beta_5=4.0$	$C_{15}=-2695.7289$	$C_{25}=-13\,461.244$	
	$^3P_1$		
$\beta_1=1.0$	$C_{11}=-0.361\,859\,04$	$C_{21}=-5.646\,626\,3$	$\lambda_1^{BBS}=1.$
$\beta_2=1.866\,066$	$C_{12}=1088.8383$	$C_{22}=511.773\,53$	$\lambda_2^{BBS}=1.$
$\beta_3=2.687\,875\,4$	$C_{13}=-9490.2618$	$C_{23}=-3661.759$	$\lambda_1^{BS}=0.900\,625$
$\beta_4=3.482\,202\,2$	$C_{14}=21\,167.417$	$C_{24}=10\,752.023$	$\lambda_2^{BS}=0.960\,000$
$\beta_5=4.256\,699\,6$	$C_{15}=-13\,723.185$	$C_{25}=-8810.46$	

much repulsion. The relatively small changes of the coupling strengths in comparison to the S waves are caused by the fact that in the S-channel short-range effects are more effective.

Figure 7 shows the half-off-shell function of the $^3P_0$ n-p partial wave where discrepancies occur for off-shell momenta $q>2$ fm^{-1} within the BBS equation. This is mainly due to the fact that we have chosen the second interpolation energy at $E_2=350$ MeV; as a consequence, agreement of the half-off-shell function with the NN-Paris potential property improves at higher energies. The corresponding half-off-shell function, obtained within the BS equation is, in the whole off-shell-momenta domain, in good agreement with the results of the NN-Paris potential.

To investigate the coupled $^3S_1$-$^3D_1$ channel we have constructed a rank-4 approximation to the Paris potential. For the interpolation we have chosen the following energies:

$$S \text{ wave: } E_1=E_d,\quad E_2=100 \text{ MeV},$$

$$D \text{ wave: } E_3=125 \text{ MeV},\quad E_4=425 \text{ MeV},$$

where E_d is the binding energy of the deuteron. The separable interaction is given by

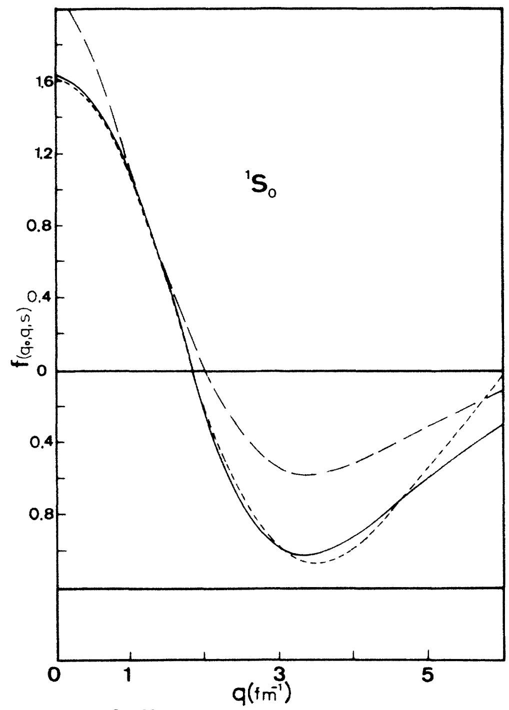

FIG. 2. Half-off-shell functions of the NN system at $E_{\text{lab}}=100$ MeV. The solid line shows the result of the BS investigation with refitted coupling strengths λ_i^{BS}, the dotted line represents the results of the EST method applied in the BBS framework, and the dashed line corresponds to the calculation of the NN-Paris potential.

$$V_{02}(q_0,q,k_0,k)=(v_{01}(q_0,q)\ v_{02}(q_0,q)\ v_{03}(q_0,q)\ v_{04}(q_0,q))\Delta_0\Lambda\Delta_2\begin{bmatrix} v_{21}(k_0,k)\\ v_{22}(k_0,k)\\ v_{23}(k_0,k)\\ v_{24}(k_0,k)\end{bmatrix} \tag{3.6}$$

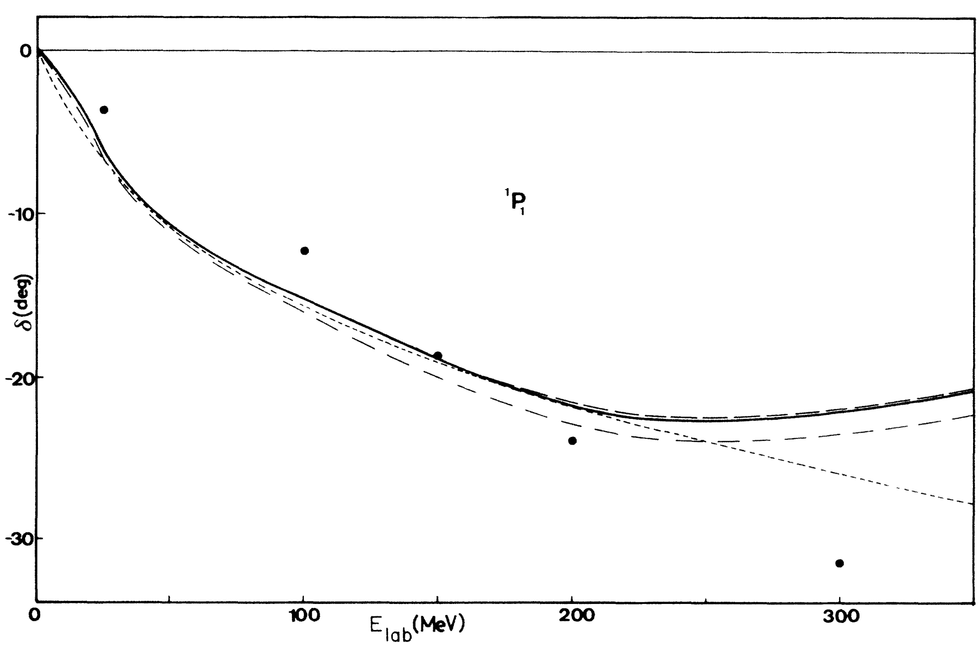

FIG. 3. Same caption as in Fig. 1.

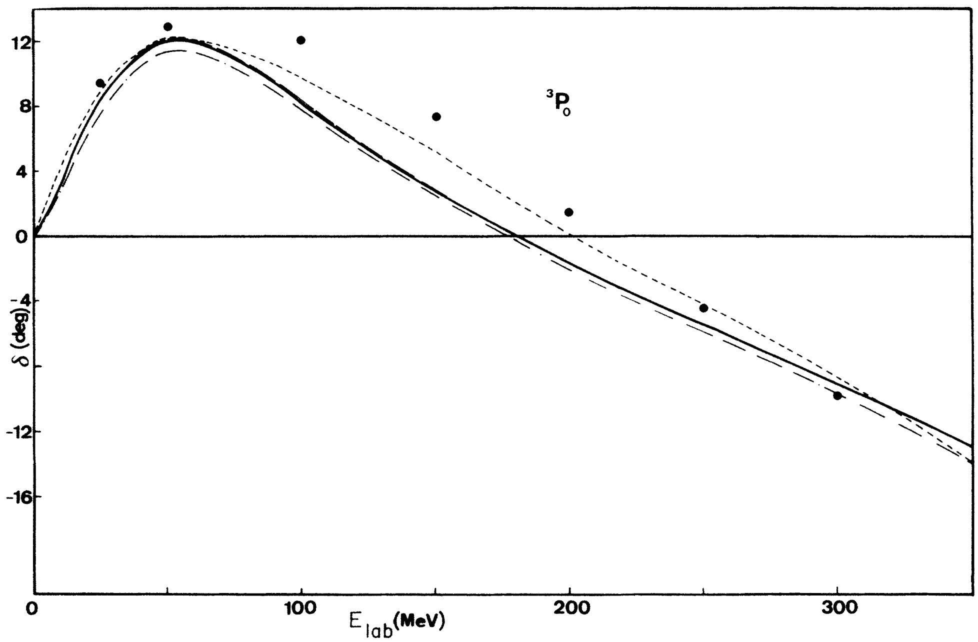

FIG. 4. Same caption as in Fig. 1.

with

$$\Delta_0=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}, \tag{3.7}$$

$$\Delta_2=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}, \tag{3.8}$$

$$\Lambda=\begin{bmatrix}\lambda_{11}&\lambda_{12}&\lambda_{13}&\lambda_{14}\\\lambda_{21}&\lambda_{22}&\lambda_{23}&\lambda_{24}\\\lambda_{31}&\lambda_{32}&\lambda_{33}&\lambda_{34}\\\lambda_{41}&\lambda_{42}&\lambda_{43}&\lambda_{44}\end{bmatrix}. \tag{3.9}$$

Each vertex function consists in this two-channel problem of six terms

$$v_{0i}(q_0,q)=\sum_{j=1}^{6}\frac{C_{ij}}{(-q_0^2+q^2+\beta_{ij}^2)}, \tag{3.10}$$

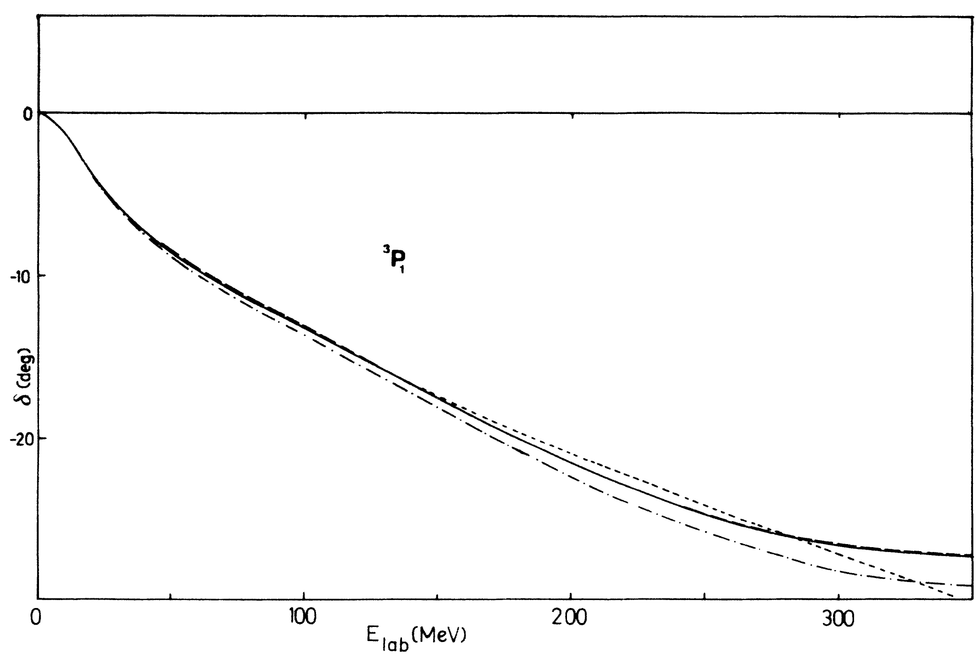

FIG. 5. Same caption as in Fig. 1.

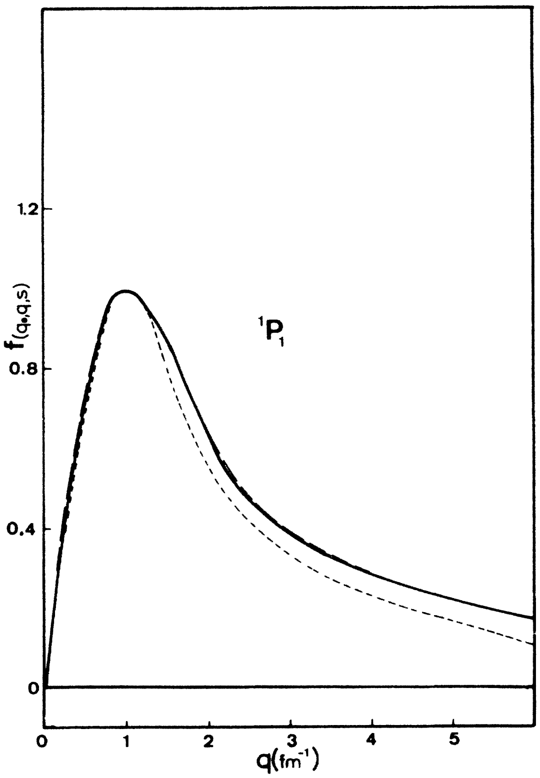

FIG. 6. Same caption as in Fig. 2.

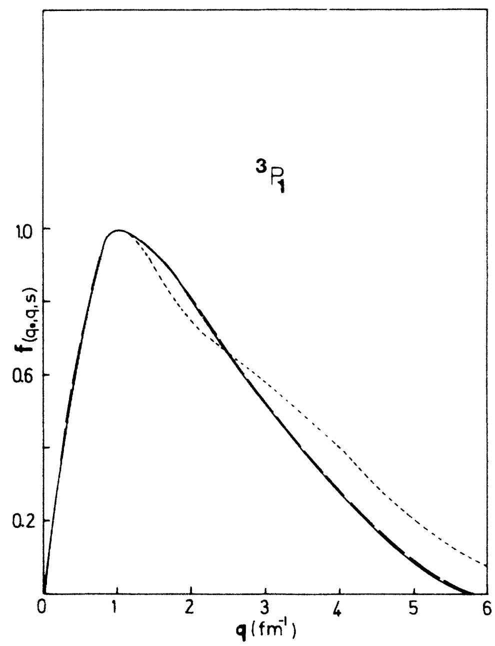

FIG. 8. Same caption as in Fig. 2.

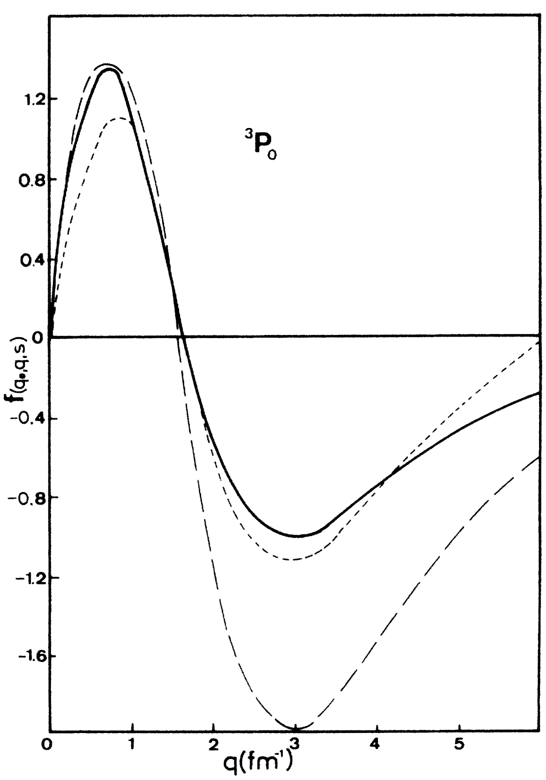

FIG. 7. Same caption as in Fig. 2.

$$v_{2i}(q_0,q)=\sum_{j=1}^{6}\frac{C_{ij}q^2}{(-q_0^2+q^2+\beta_{ij}^2)^2}\ . \tag{3.11}$$

The additional term with respect to the other channels avoids higher orders than two in the denominator of the vertex functions responsible for the D state. As a consequence the integration about the fourth component is much simpler as it would be if one could have orders for one degree higher. The separable interaction [(3.6)—(3.11)] of the $^3S_1$-$^3D_1$ NN channel yields to a fairly good description of the phase shifts in the scattering domain up to 350 MeV. While the fits to the phases and the mixing parameters are shown in Figs. 9—11, the parameters of the potential are listed in Table III.

To obtain a reasonable description of the $^3S_1$-$^3D_1$ channel within the Bethe-Salpeter equation also, we have varied the coupling strengths λ_{ii} ($i=1$—4). The bound state energy at -2.225 MeV was obtained by changing λ_{11} while the other three parameters were fitted to reproduce the scattering data (see Table III). The coupled channel is at least reasonably well described, although the D-wave phase shift shows some discrepancy in the low energy domain. Since this feature can also be found for the $^3D_1$ phase shift of the NN-Paris potential, one can conclude that this is no particular problem of our method.

Concerning the mixing parameter ε_1 (Fig. 11), it is demonstrated that a proper description as compared to the experimental data and the results of Ref. 13, where the exchange of mesons for the NN interaction was chosen, has been obtained.

The half-off-shell behaviors of the $^3S_1$-$^3D_1$ are shown in Figs. 12—15; as in all other cases, again a good approxi-

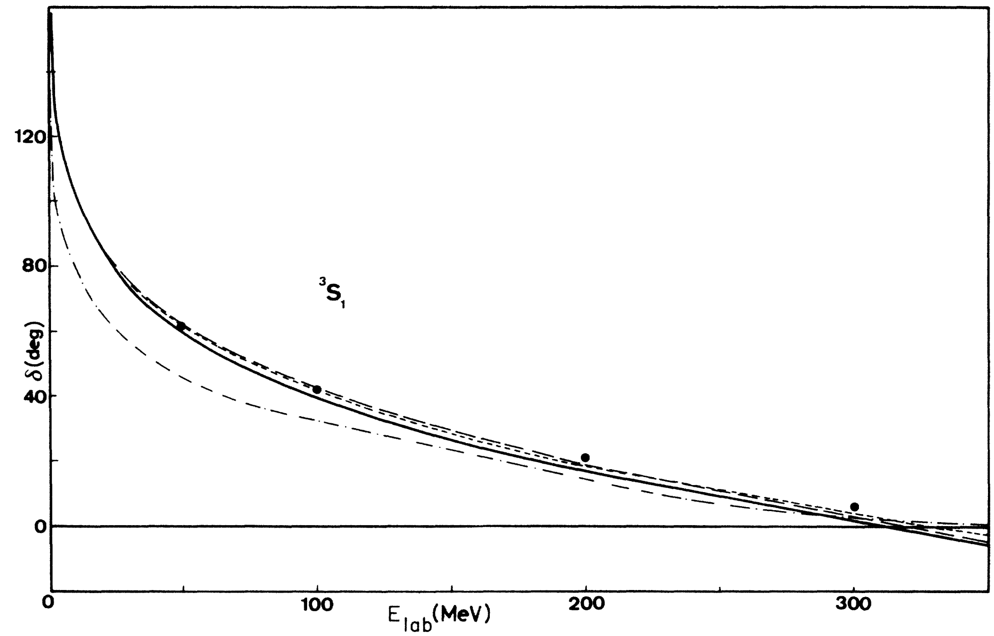

FIG. 9. Same caption as in Fig. 1.

mation to the properties of the NN-Paris potential has been achieved.

IV. CONCLUSION

We have presented a separable representation of the NN-Paris potential within the fully relativistic BS equation, which incorporates the most important features of the scattering domain in the N-N case.

Concerning the scattering phase shifts we have obtained satisfactory agreement with the experimental data of the phase shift analysis of Ref. 11. While these on-shell data are similar to results obtained in an earlier investigation (Ref. 5) of a separable approximation of the Bethe-Salpeter equation, we have presented in this paper a separable interaction which also reproduces the half-off-shell characteristics of one of the most popular meson-theoretical NN potentials, the NN-Paris potential. This

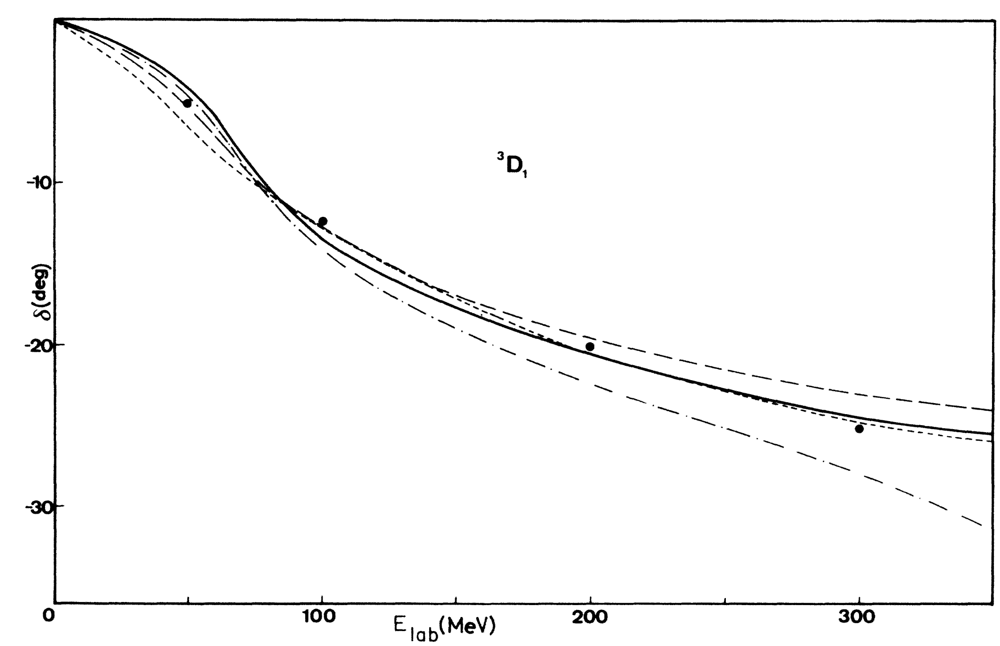

FIG. 10. Same caption as in Fig. 1.

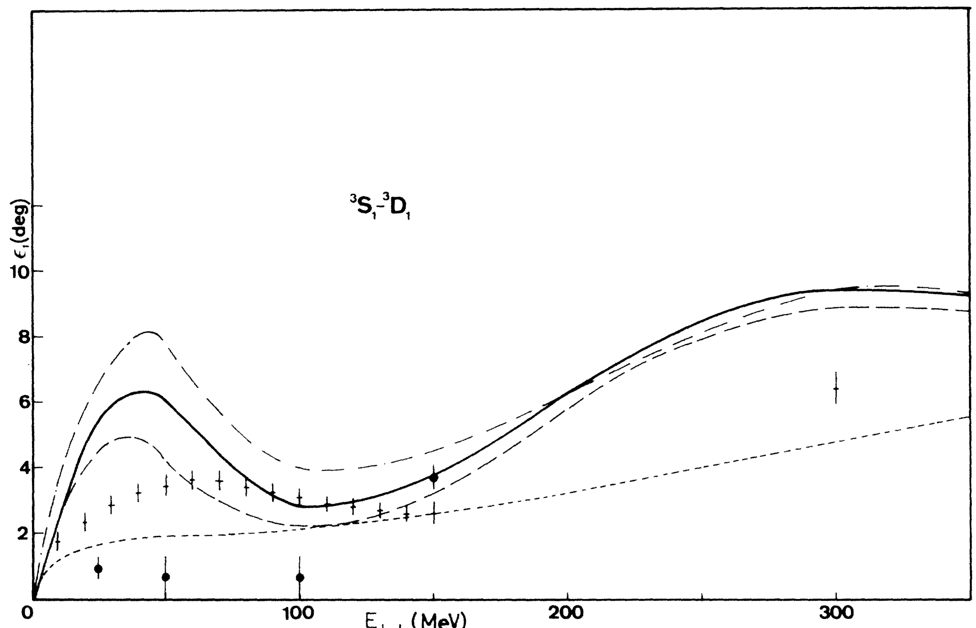

FIG. 11. Mixing parameter ε_1 of the $^3S_1$-$^3D_1$ coupled state. The solid line represents the calculation within the BS equation with refitted coupling strengths λ_i^{BS}, the dashed dotted line shows the result of the BS investigation without refitting λ_i^{BBS}. The dashed line describes the NN-Paris potential calculation and the dotted line shows the results of the EST method applied in the BBS framework. The experimental values (ɸ) are taken from Ref. 11 and (+) from Ref. 13.

TABLE III. Parameters of the separable interaction in the coupled $^3S_1$-$^3D_1$ partial wave state. The parameters λ_i^{BS} and λ_i^{BBS} are defined in the caption of Table I.

β (fm^{-1})	C (fm^0)	λ (MeV fm^{-1})	β (fm^{-1})	C (fm^0)
$l=0$			$l=2$	
$\beta_{11}=1.0$	$C_{11}=15.659\,827$	$\lambda_{11}^{BBS}=-0.227\,567\,47$	$\beta_{11}=1.0$	$C_{11}=-3.025\,362$
$\beta_{12}=1.866\,066$	$C_{12}=-1673.2143$	$\lambda_{12}=-0.144\,155\,10$	$\beta_{12}=1.866\,066$	$C_{12}=-390.330\,49$
$\beta_{13}=2.687\,875\,4$	$C_{13}=19\,597.873$	$\lambda_{13}=0.062\,892\,03$	$\beta_{13}=2.687\,875\,4$	$C_{13}=2615.2074$
$\beta_{14}=3.482\,202\,2$	$C_{14}=-98\,548.033$	$\lambda_{14}=0.003\,740\,14$	$\beta_{14}=3.482\,202\,2$	$C_{14}=-11\,652.333$
$\beta_{15}=4.256\,699\,6$	$C_{15}=229\,712.92$	$\lambda_{21}=-0.144\,155\,1$	$\beta_{15}=4.256\,699\,6$	$C_{15}=17\,488.783$
$\beta_{16}=4.7$	$C_{16}=-151\,345.61$	$\lambda_{22}^{BBS}=0.113\,981\,21$	$\beta_{16}=4.7$	$C_{16}=-7066.9205$
		$\lambda_{23}=0.085\,399\,964$		
$\beta_{21}=0.8$	$C_{21}=49.801\,112$	$\lambda_{24}=-0.078\,046\,366$	$\beta_{21}=0.7$	$C_{21}=2.813\,554\,3$
$\beta_{22}=1.492\,852\,8$	$C_{22}=-2072.196$	$\lambda_{31}=0.062\,892\,029$	$\beta_{22}=1.306\,246\,2$	$C_{22}=-311.483\,49$
$\beta_{23}=2.150\,300\,3$	$C_{23}=6637.7315$	$\lambda_{32}=0.085\,399\,963$	$\beta_{23}=1.881\,512\,7$	$C_{23}=4450.8549$
$\beta_{24}=2.785\,761\,8$	$C_{24}=1773.5598$	$\lambda_{33}^{BBS}=-0.028\,782\,675$	$\beta_{24}=2.437\,541\,5$	$C_{24}=-16\,680.498$
$\beta_{25}=3.405\,359\,7$	$C_{25}=-20\,763.725$	$\lambda_{34}=-0.033\,827\,124$	$\beta_{25}=2.979\,689\,7$	$C_{25}=25\,938.388$
$\beta_{26}=4.012\,602\,2$	$C_{26}=14\,768.319$	$\lambda_{41}=0.003\,740\,141\,4$	$\beta_{26}=3.511\,026\,9$	$C_{26}=-14\,081.186$
		$\lambda_{42}=-0.078\,046\,364$		
$\beta_{31}=1.0$	$C_{31}=-25.801\,849$	$\lambda_{43}=-0.033\,827\,124$	$\beta_{31}=0.6$	$C_{31}=2.349\,516\,7$
$\beta_{32}=1.866\,066$	$C_{32}=2461.253$	$\lambda_{44}^{BBS}=0.178\,714\,37$	$\beta_{32}=1.2$	$C_{32}=-304.5283$
$\beta_{33}=2.687\,875\,4$	$C_{33}=-24\,133.622$	$\lambda_{11}^{BS}=-0.244\,153\,6$	$\beta_{33}=1.8$	$C_{33}=4578.6541$
$\beta_{34}=3.482\,202\,2$	$C_{34}=48\,591.603$	$\lambda_{22}^{BS}=0.133\,642\,3$	$\beta_{34}=2.4$	$C_{34}=-12\,876.067$
$\beta_{35}=4.256\,699\,6$	$C_{35}=-25\,019.227$	$\lambda_{33}^{BS}=-0.0268$	$\beta_{35}=3.0$	$C_{35}=5247.3456$
$\beta_{36}=4.7$	$C_{36}=-1532.4468$	$\lambda_{44}^{BS}=0.175$	$\beta_{36}=3.6$	$C_{36}=4576.4304$
$\beta_{41}=0.8$	$C_{41}=-26.818\,673$		$\beta_{41}=0.6$	$C_{41}=-1.593\,546\,2$
$\beta_{42}=1.6$	$C_{42}=2433.8912$		$\beta_{42}=1.119\,639\,6$	$C_{42}=120.3399$
$\beta_{43}=2.4$	$C_{43}=-31\,119.533$		$\beta_{43}=1.612\,725\,2$	$C_{43}=-1295.0393$
$\beta_{44}=3.2$	$C_{44}=114\,977.15$		$\beta_{44}=2.089\,321\,3$	$C_{44}=3342.7168$
$\beta_{45}=4.0$	$C_{45}=-171\,593.24$		$\beta_{45}=2.554\,019\,8$	$C_{45}=-1195.6997$
$\beta_{46}=4.7$	$C_{46}=86\,586.506$		$\beta_{46}=2.009\,451\,7$	$C_{46}=-1184.5975$

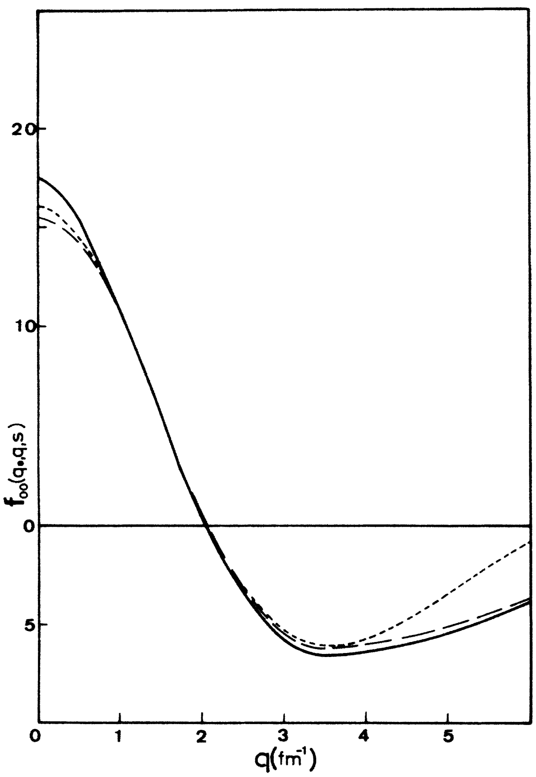

FIG. 12. Same caption as in Fig. 2.

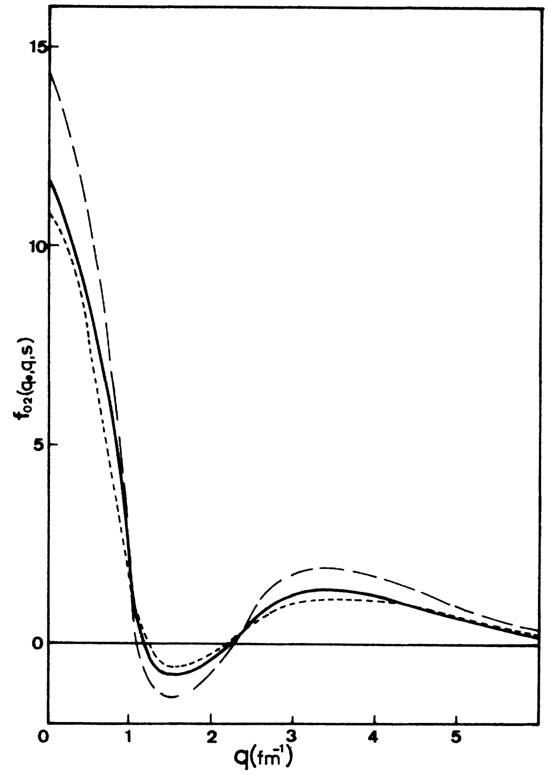

FIG. 14. Same caption as in Fig. 2.

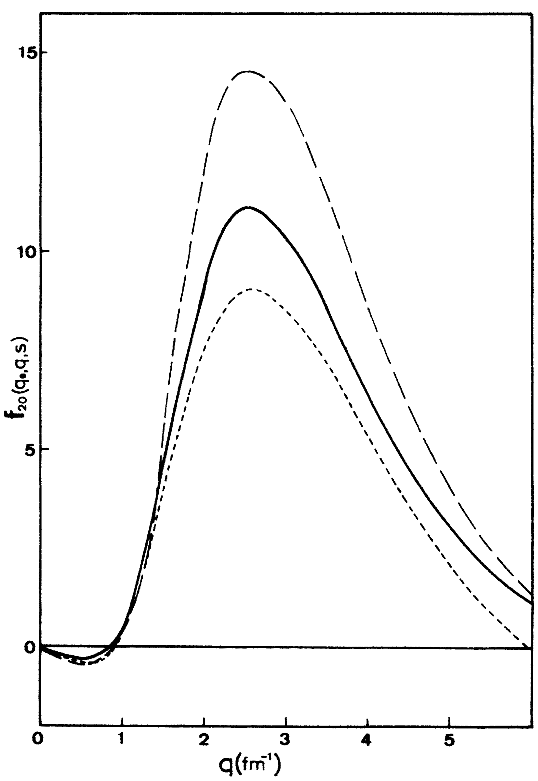

FIG. 13. Same caption as in Fig. 2.

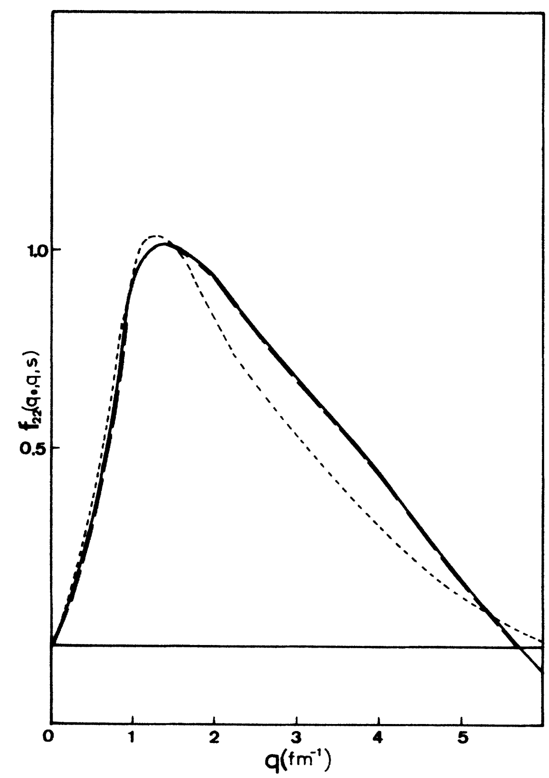

FIG. 15. Same caption as in Fig. 2.

separable potential is in favor over most of the other widely used separable potentials, which show a rather unreasonable half-off-shell behavior.

The difference of phase shifts, obtained in the framework of the BS equation with and without refitted coupling strengths (solid lines and dashed-dotted lines in our figures), is particularly large. It shows that the effects from the coupling between the positive and negative energy states cannot be neglected in describing the NN interaction as is done in the BBS equation. By changing the coupling strengths of the separable interaction, obtained from the use of the EST method with the BBS equation, a reasonable description of both, on-shell as well as half-off-shell properties of the NN-Paris potential had been obtained.

Because of this paper, fully relativistic three-body calculations should be possible.

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