Separable representation of the Bonn nucleon-nucleon potential

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A separable representation of a boson-exchange nucleon-nucleon potential is constructed via the Ernst-Shakin-Thaler method. The resulting separable interactions provide for a satisfactory approximation of the on-shell as well as off-shell properties of the Bonn potential. Their form factors are composed of rational functions suitable for today's computer codes for few-body systems. First results obtained with these separable potentials for elastic nucleon-deuteron scattering are presented.

I. MOTIVATION

It has been learned over the last few years that investigations of nuclear few-body problems require a refined description of the underlying nucleon-nucleon (N-N) interaction. This is especially true for the three-nucleon (3-N) problem. Consequently we have been looking for means that would allow one to introduce all the characteristics of the N-N force (both the on-shell and off-shell behavior) into 3-N calculations. At nonrelativistic energies meson exchange today gives the best account of all known features of N-N dynamics. It is certainly reliable for long and intermediate internucleon distances, the domain which is most important for nuclear few-body processes.

In recent papers[1] we presented a separable approximation of the Paris N-N potential,[2] a model which is basically derived from $(\pi+2\pi+\omega)$ exchange.[3] It has already been very useful in studies of various observables of the 3-N system (see, e.g., Refs. 4–9). In fact, it allowed one for the first time to introduce features of modern meson-exchange theory in a reliable way into 3-N scattering calculations.

For future investigations on the 3-N system and also other few-body problems it is essential to compare predictions of different meson-exchange models (e.g., multipion exchange versus boson exchange) to each other in order to learn about details of the N-N interaction. For this purpose we construct in the present work a separable approximation to the Bonn potential in the version as it was published in Ref. 10. This is an energy-independent one-boson exchange (OBE) model derived in the framework of the Blankenbeckler-Sugar equation, which can also be used in the conventional nonrelativistic Lippmann-Schwinger (or Schrödinger) equation due to the minimal-relativity transformation. This version of the Bonn potential yields an overall satisfactory description of the up-to-date N-N phenomenology and it is therefore desirable to study its specific features in few-body problems.

II. BEST POTENTIALS

In this paper we present a separable EST (Ernst-Shakin-Thaler[11]) approximation of the Bonn potential[10] in the $^1S_0$ and $^3S_1$-$^3D_1$ partial-wave states, the ones that are most influential on 3-N observables. Furthermore, it is predominantly in these states that differences to other models occur in the Bonn potential. Discrepancies in higher partial waves are generally smaller and in addition their effect is considerably diminished in 3-N calculations, in particular for differential cross sections.

The EST method is presented in Ref. 11 and its application to the N-N interaction is discussed in Refs. 1 and 12. Here we follow exactly the same procedure as in the case of the Paris potential in Ref. 1. For example, our separable approximation to the Bonn potential has the form

TABLE I. Interpolation energies selected for the construction of the separable potentials BESTN.

State	Abbreviation and rank	Selected energies E_i (MeV) or ensembles $\alpha_i=\{E_i,l_i\}$
$^1S_0$	BEST1	$E_1=0$
	BEST3	$E_1=0,E_2=100,E_3=300$
$^3S_1$-$^3D_1$	BEST1	$\alpha_1=\{-2.225,-\}$
	BEST4	$\alpha_1=\{-2.225,-\}\alpha_3=\{125,0\}$
		$\alpha_2=\{200,2\}\alpha_4=\{450,2\}$

TABLE II. Parameters of the BESTN potentials in the $^1S_0$ partial wave. With respect to the p-p case, which is not contained in the original Bonn potential, see the discussion in the text.

	β (fm^{-1})	$^1S_0$ (n-p) C (fm^0)	λ (MeV fm^{-1})	β (fm^{-1})	$^1S_0$ (p-p) C (fm^0)	λ (MeV fm^{-1})
BEST1	$\beta_{11}=1.1660253$	$C_{11}=-8.6258839$	$\lambda=-1.0$	$\beta_{11}=1.1660253$	$C_{11}=-8.5358323$	$\lambda=-1.0$
	$\beta_{12}=9.3107735$	$C_{12}=111.73977$		$\beta_{12}=9.3107735$	$C_{12}=110.57325$	
	$\beta_{13}=4.0014261$	$C_{13}=-299.47272$		$\beta_{13}=4.0014261$	$C_{13}=-296.34632$	
	$\beta_{14}=2.2374729$	$C_{14}=201.72876$		$\beta_{14}=2.2374729$	$C_{14}=199.62277$	
BEST3	$\beta_{11}=1.1660253$	$C_{11}=-852.43787$	$\lambda_{11}=-0.00044226264$	$\beta_{11}=1.1660253$	$C_{11}=-640.74125$	$\lambda_{11}=-0.00074843061$
	$\beta_{12}=9.3107735$	$C_{12}=11042.487$	$\lambda_{12}=-0.011774589$	$\beta_{12}=9.3107735$	$C_{12}=8300.1678$	$\lambda_{12}=-0.018327164$
	$\beta_{13}=4.0014261$	$C_{13}=-29594.867$	$\lambda_{13}=0.061410371$	$\beta_{13}=4.0014261$	$C_{13}=-22245.202$	$\lambda_{13}=0.088138854$
	$\beta_{14}=2.2374729$	$C_{14}=19935.491$	$\lambda_{22}=3.1909586$	$\beta_{14}=2.2374729$	$C_{14}=14984.660$	$\lambda_{22}=3.0556541$
	$\beta_{21}=2.1730577$	$C_{21}=-90.001143$	$\lambda_{23}=-8.3656108$	$\beta_{21}=2.1730577$	$C_{21}=-90.001143$	$\lambda_{23}=-7.8422754$
	$\beta_{22}=3.6724500$	$C_{22}=311.07512$	$\lambda_{33}=19.498347$	$\beta_{22}=3.6724500$	$C_{22}=311.07512$	$\lambda_{33}=17.645822$
	$\beta_{23}=2.8505222$	$C_{23}=886.10544$	$\lambda_{ij}=\lambda_{ji}$	$\beta_{23}=2.8505222$	$C_{23}=886.10544$	$\lambda_{ij}=\lambda_{ji}$
	$\beta_{24}=3.4952873$	$C_{24}=-1407.0247$		$\beta_{24}=3.4952873$	$C_{24}=-1407.0247$	
	$\beta_{31}=1.4186040$	$C_{31}=-9.8140402$		$\beta_{31}=1.4186040$	$C_{31}=-9.8140402$	
	$\beta_{32}=2.8440133$	$C_{32}=-23.813627$		$\beta_{32}=2.8440133$	$C_{32}=-23.813627$	
	$\beta_{33}=3.1753131$	$C_{33}=884.80934$		$\beta_{33}=3.1753131$	$C_{33}=884.80934$	
	$\beta_{34}=3.6674019$	$C_{34}=-1088.6312$		$\beta_{34}=3.6674019$	$C_{34}=-1088.6312$	

$$V_{LL'}(p',p)=\sum_{i,j=1}^{N} g_{Li}(p')\lambda_{ij}g_{L'j}(p) \quad (1)$$

for $L,L'=0,2$ in $^3S_1$-$^3D_1$ and $L=L'=0$ in $^1S_0$. The form factors are expressed as rational functions of the type

$$L=0:\; g_{0i}(p)=\sum_{n=1}^{4}\frac{C_{0in}p^{2(n-1)}}{(p^2+\beta_{0in}^2)^n}\,,$$
$$L=2:\; g_{2i}(p)=\sum_{n=1}^{4}\frac{C_{2in}p^{2n}}{(p^2+\beta_{2in}^2)^{n+1}}\,. \quad (2)$$

Again we provide approximations of rank-1 and rank-3 for the $^1S_0$ state and of rank-1 and rank-4 for the coupled $^3S_1$-$^3D_1$ state with interpolation energies chosen according to Table I. The parameters of the separable potentials obtained in this way (BEST) are listed in Tables II and III.

We remark that the Bonn potential is designed for the $T=0$ neutron-proton (n-p) $^1S_0$ state contrary to the Paris potential, which is for $T=1$, i.e., proton-proton (p-p) or neutron-neutron (n-n). In order to allow for a splitting of the $^1S_0$ state, often necessary in few-body applications, we provide in the same spirit as for the Paris potential in Ref. 1, also a separable parametrization of the Bonn potential for $^1S_0$ (p-p). This is achieved by shifting the on-shell

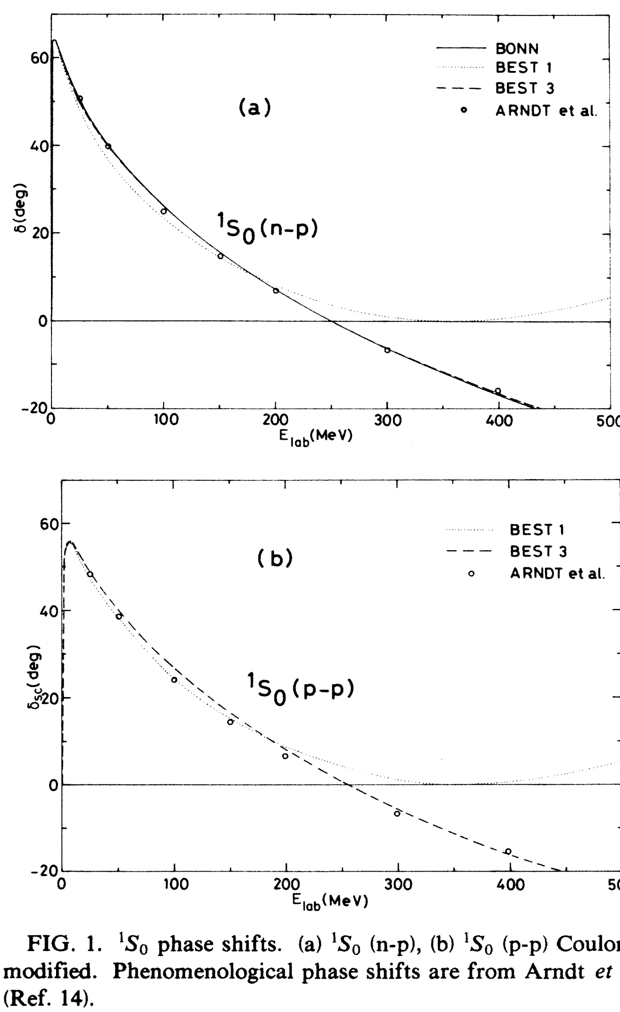

FIG. 1. $^1S_0$ phase shifts. (a) $^1S_0$ (n-p), (b) $^1S_0$ (p-p) Coulomb modified. Phenomenological phase shifts are from Arndt *et al.* (Ref. 14).

TABLE III. Parameters of the BESTN potentials in the coupled $^3S_1$-$^3D_1$ partial-wave state.

	β (fm^{-1}) $L=0$	C (fm^0) $L=0$	λ (MeV fm^{-1})	β (fm^{-1}) $L=2$	C (fm^0) $L=2$
BEST1	$\beta_{11}=1.565\,038\,6$	$C_{11}=-17.116\,545$	$\lambda=-1.0$	$\beta_{11}=1.559\,310\,8$	$C_{11}=-20.353\,445$
	$\beta_{12}=3.639\,772\,4$	$C_{12}=212.833\,87$		$\beta_{12}=2.529\,059\,3$	$C_{12}=-59.978\,368$
	$\beta_{13}=5.539\,557\,5$	$C_{13}=-1122.3589$		$\beta_{13}=3.438\,590\,9$	$C_{13}=440.154\,48$
	$\beta_{14}=5.324\,741\,5$	$C_{14}=936.063\,29$		$\beta_{14}=3.787\,058\,9$	$C_{14}=-397.3917$
BEST4	$\beta_{11}=1.565\,038\,6$	$C_{11}=-44.865\,871$	$\lambda_{11}=-0.213\,057\,04$	$\beta_{11}=1.559\,310\,8$	$C_{11}=-53.350\,43$
	$\beta_{12}=3.639\,772\,4$	$C_{12}=557.879\,94$	$\lambda_{12}=-0.205\,217\,9$	$\beta_{12}=2.529\,059\,3$	$C_{12}=-157.215\,24$
	$\beta_{13}=5.539\,557\,5$	$C_{13}=-2941.926$	$\lambda_{13}=0.066\,257\,482$	$\beta_{13}=3.438\,590\,9$	$C_{13}=1153.7325$
	$\beta_{14}=5.324\,741\,5$	$C_{14}=2453.6082$	$\lambda_{14}=0.093\,122\,02$	$\beta_{14}=3.787\,058\,9$	$C_{14}=-1041.6428$
	$\beta_{21}=2.268\,383\,0$	$C_{21}=-119.470\,57$	$\lambda_{22}=0.078\,209\,203$	$\beta_{21}=1.153\,513\,6$	$C_{21}=4.074\,203\,3$
	$\beta_{22}=2.669\,700\,4$	$C_{22}=744.486\,01$	$\lambda_{23}=0.153\,093\,76$	$\beta_{22}=1.380\,357\,9$	$C_{22}=96.922\,681$
	$\beta_{23}=3.355\,896\,1$	$C_{23}=-2428.1168$	$\lambda_{24}=-0.079\,833\,542$	$\beta_{23}=1.840\,864\,7$	$C_{23}=-163.958\,72$
	$\beta_{24}=3.556\,299\,4$	$C_{24}=2069.4241$	$\lambda_{33}=-0.027\,954\,372$	$\beta_{24}=4.836\,640\,9$	$C_{24}=107.430\,36$
	$\beta_{31}=2.163\,110\,1$	$C_{31}=-115.546\,26$	$\lambda_{34}=-0.138\,391\,57$	$\beta_{31}=2.243\,635\,6$	$C_{31}=96.019\,654$
	$\beta_{32}=2.896\,594\,2$	$C_{32}=234.373\,03$	$\lambda_{44}=0.212\,647\,64$	$\beta_{32}=2.811\,369\,1$	$C_{32}=-1965.208$
	$\beta_{33}=2.642\,911\,8$	$C_{33}=699.624\,64$	$\lambda_{ij}=\lambda_{ji}$	$\beta_{33}=3.247\,804\,5$	$C_{33}=4681.8829$
	$\beta_{34}=3.427\,306\,6$	$C_{34}=-1122.048$		$\beta_{34}=3.856\,345\,8$	$C_{34}=-3199.2136$
	$\beta_{41}=4.233\,279\,0$	$C_{41}=-369.793\,87$		$\beta_{41}=1.098\,083\,3$	$C_{41}=0.078\,517\,541$
	$\beta_{42}=4.469\,867\,6$	$C_{42}=1640.1431$		$\beta_{42}=1.875\,527\,7$	$C_{42}=134.035\,37$
	$\beta_{43}=3.825\,420\,6$	$C_{43}=5639.2656$		$\beta_{43}=2.733\,232\,2$	$C_{43}=-494.860\,65$
	$\beta_{44}=3.142\,184\,6$	$C_{44}=-6590.7957$		$\beta_{44}=3.813\,053\,3$	$C_{44}=502.535\,21$

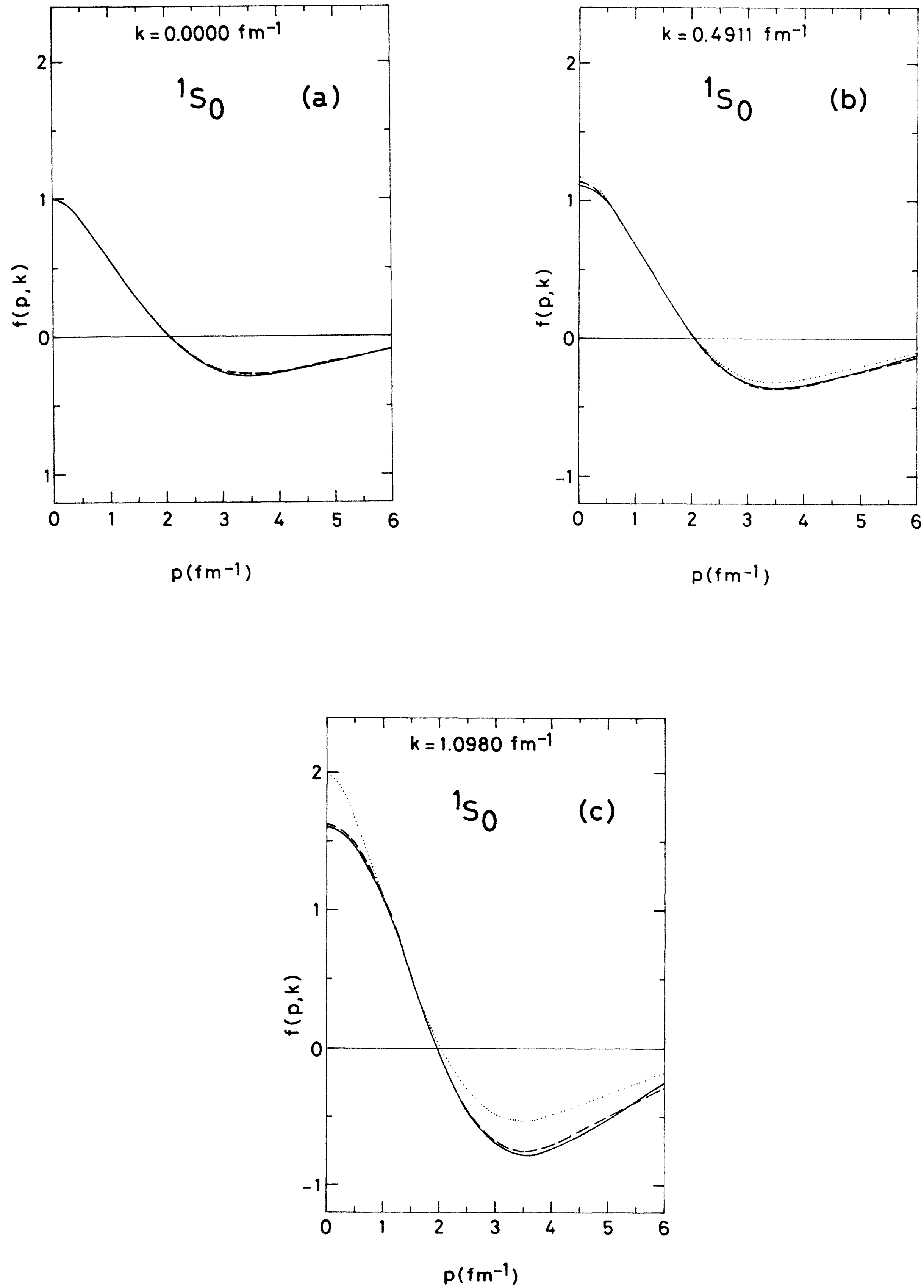

FIG. 2. Noyes-Kowalski half-off-shell functions for $^1S_0$ (n-p). (a) $E_{\rm lab}=0$ MeV, (b) $E_{\rm lab}=20$ MeV, (c) $E_{\rm lab}=100$ MeV. In (a) BEST1 and BEST3 are identical. Same description as in Fig. 1.

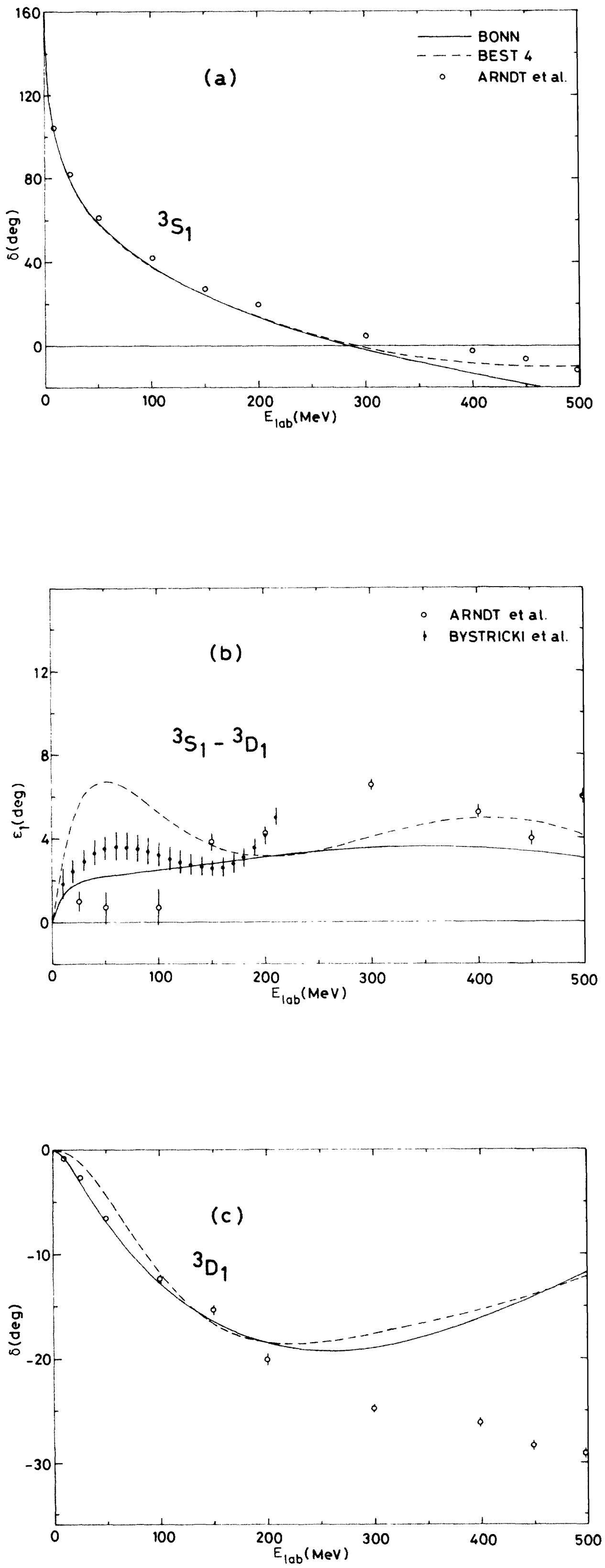

FIG. 3. (a) $^3S_1$ phase shift, (b) mixing parameter ϵ_1, (c) $^3D_1$ phase shift. Due to the uncertain phenomenological evidence on ϵ_1 the results of Bystricki *et al.* (Ref. 18) are added in (b).

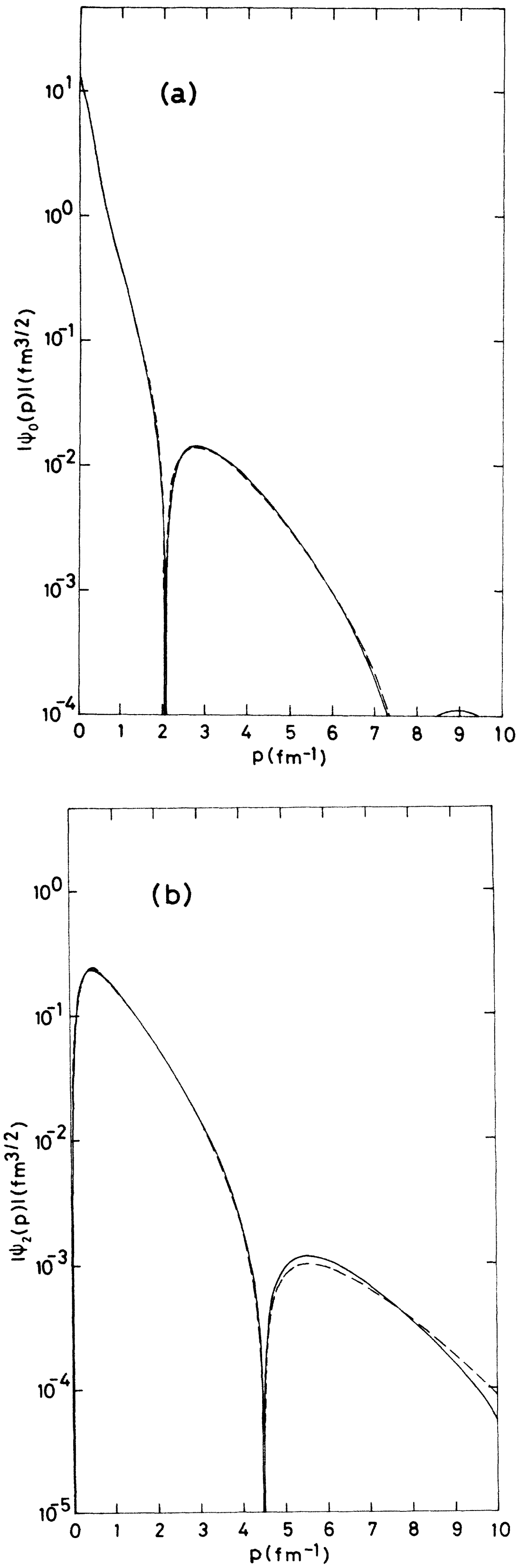

FIG. 4. Momentum-space deuteron wave functions (a) $\psi_0(p)$ and (b) $\psi_2(p)$ for the S and D states, respectively. The results for BEST1 and BEST4 are identical. Same description as in Fig. 3.

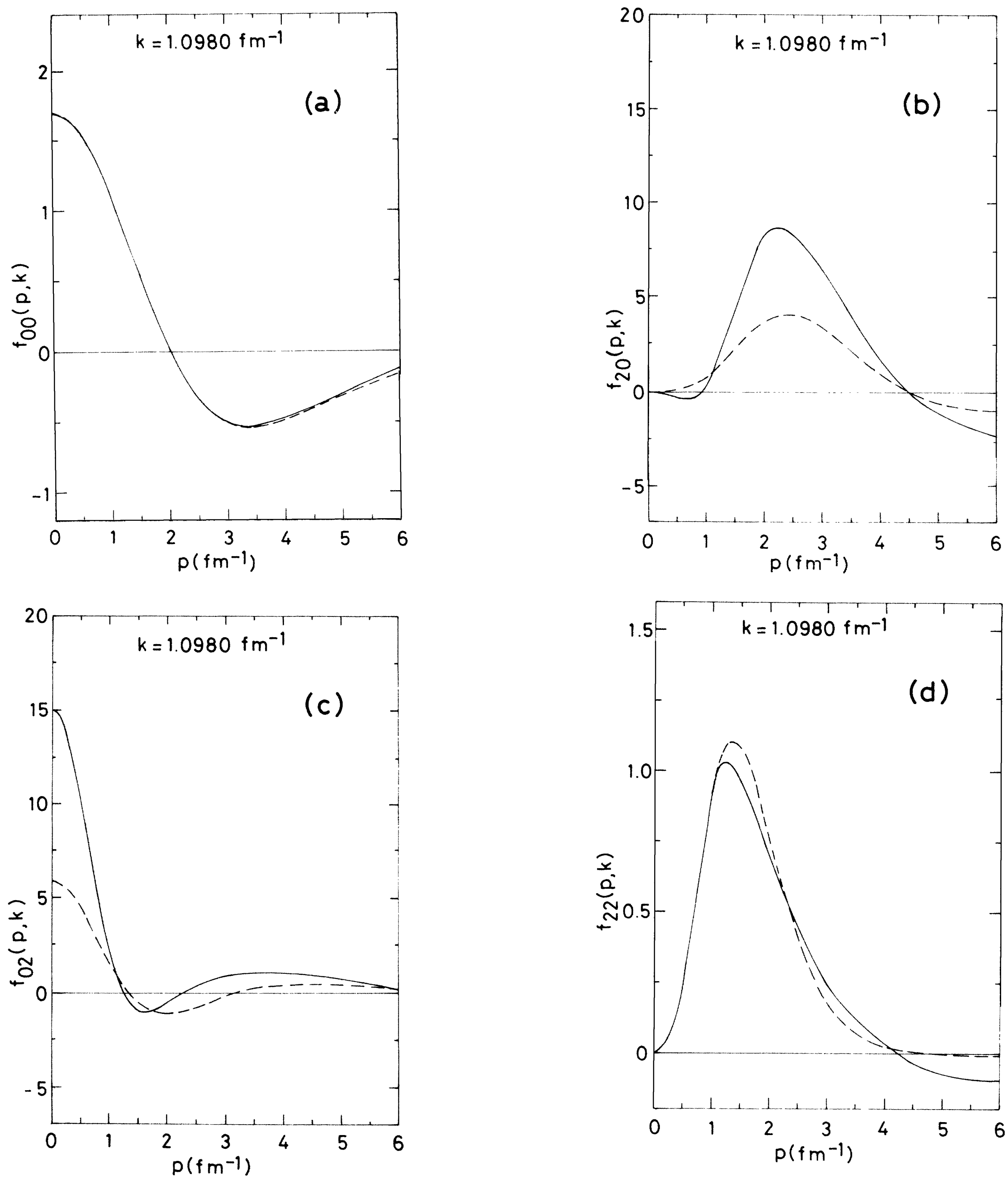

FIG. 5. Half-off-shell functions in $^3S_1$-$^3D_1$ for (a) $^3S_1 \rightarrow {}^3S_1$, (b) $^3S_1 \rightarrow {}^3D_1$, (c) $^3D_1 \rightarrow {}^3S_1$, and (d) $^3D_1 \rightarrow {}^3D_1$ transitions at $E_{lab} = 100$ MeV. Same description as in Fig. 3.

TABLE IV. Effective-range parameters in the $^1S_0$ state.

	$^1S_0$ (n-p)		$^1S_0$ (p-p) Coulomb modified		$^1S_0$ (p-p) Purely nuclear	
	a (fm)	r (rm)	a_{sc} (fm)	r_{sc} (fm)	a_s (fm)	r_s (fm)
Bonn	−23.75[a]	2.69[a]				
BEST1	−23.75	2.73	−7.82	2.89	−17.85	2.76
BEST3	−23.75	2.71	−7.82	2.89	−17.85	2.77
Expt. (Ref. 13)	−23.748±0.010	2.75±0.05	−7.8098±0.0023	2.767±0.010		

[a]Given by the authors (Ref. 10).

TABLE V. Triplet effective-range parameters and deuteron properties.

	a_t (fm)	r_t (fm)	E_D (MeV)	Q_D (fm^2)	p_d (%)	η	A_s
Bonn	5.49[a]	1.86[a]	2.225[a]	0.2856[a]	4.58[a]	0.0267[a]	0.9008[a]
BEST1	5.49	1.86	2.225	0.2855	4.58	0.0267	0.8950
BEST4	5.48	1.85	2.225	0.2855	4.58	0.0267	0.8950
Expt.	5.424±0.004	1.759±0.005	2.2246±0.00005	0.286±0.0015		0.0271±0.0004	
(Refs. 13 and 15—17)						0.02628±0.00047	

[a]Given by the authors (Ref. 10).

behavior from n-p to p-p, while preserving the typical off-shell behavior. In the Paris case we had to do it in the other way around (see the corresponding discussion in Ref. 1).

The quality of the separable BESTN approximations can be estimated from the comparison of their on-shell and off-shell properties with the ones of the original Bonn potential. Table IV gives the low-energy parameters in the $^1S_0$ state. It is clear that a satisfactory reproduction of the Bonn results is guaranteed in $T=0$, while the experimental data are well fitted for the $T=1$ proton-proton. The same is true for the phase shifts in Fig. 1 at least for BEST3. The rank-1 approximation, of course, cannot but deviate with increasing energy. The half-off-shell properties are shown in Fig. 2 at three different energies. From that it is evident that also the off-shell behavior of the Bonn potential is closely approximated; at low energies by both BEST1 and BEST3 and over a wider energy range only by BEST3.

For the $^3S_1$-$^3D_1$ state the triplet effective-range parameters and deuteron properties are summarized in Table V. Since the bound state was taken as interpolation energy in both cases BEST1 and BEST4, they yield a correct reproduction of the deuteron as described by the Bonn potential.

The on-shell behavior in $^3S_1$-$^3D_1$ can be seen from Fig. 3. While the rank-1 approximation is only good for $^3S_1$ at rather low energies (not shown in Fig. 3), the BEST4 approximation is satisfactory in the whole energy range. The off-shell behavior can be estimated from the deuteron wave function, whose shape in momentum space is well reproduced up to $p \approx 10$ fm^{-1} (Fig. 4), or from the half-off-shell functions in Fig. 5. Notice that the energy $E_{lab}=100$ MeV lies apart from the interpolation energies chosen for BEST4 (cf. Table I).

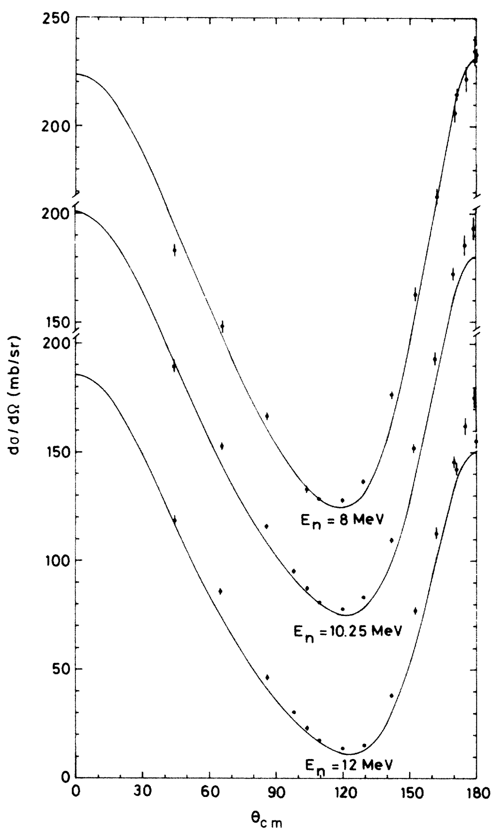

FIG. 6. Neutron-deuteron differential cross section at $E_n=8$, 10.25, and 12 MeV. The solid line is the result with BEST3 in $^1S_0$ (n-p charge independent), BEST4 in $^3S_1$-$^3D_1$, and P as well as D waves substituted from the separable model of Ref. 19. Open circles are experimental data from the Karlsruhe group (Ref. 20), while full circles are data from Uppsala (Ref. 21). For the latter the points at $\theta=180°$ are extrapolated values.

III. n-d ELASTIC SCATTERING

We give an example for the application of the BEST potentials in N-d scattering by the elastic n-d differential cross section at $E_n=8$, 10.25, and 12 MeV. In order to make the results comparable to our former study[6] employing the PEST approximations of the Paris potential we use BEST3 in $^1S_0$, BEST4 in $^3S_1$-$^3D_1$, and supplement the higher N-N partial waves again from the phenomenological separable potentials developed by Doleschall.[19] Figure 6 shows the angular dependence of the differential cross section in comparison to experimental data. Quite a satisfactory agreement is achieved at each energy considered. As compared to the corresponding calculation with the PEST interactions (Ref. 6) the present result lies somewhat higher at very forward and very backward angles. This, however, does not yet allow one to draw a definite conclusion on qualities of either the Paris or Bonn potentials. Further extensive investigations of various N-d observables are required to disentangle the effects from different properties of these interactions, like, e.g., low-energy parameters, D-state probability, off-shell behavior. Finally such studies can be expected to yield quantitative evidence to which extent meson-exchange theory as a dynamical concept for the N-N interaction can be employed. The necessary calculations are made feasible by the now existing separable approximations to the most advanced meson theoretical models.

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