Charge-symmetry breaking in neutron-proton scattering: Isospin-mixing parameter

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An isospin-mixing parameter due to charge-symmetry breaking in the neutron-proton system is calculated in a new way, using the distorted-wave Born approximation. Three mechanisms of charge-symmetry breaking are investigated: one-photon exchange, rho-omega mixing, and the nucleon mass-difference effect in one-pion exchange. Numerical calculation of this parameter for a number of energies and angular momenta yields some interesting systematic behavior. In particular, for center-of-mass energies 100 MeV and greater, the largest values are obtained for total angular momentum $J=2$ (or 4). The mass-difference effect is the most dominant one for all energies greater than 100 MeV. The analyzing-power difference, which is measurable experimentally, is also calculated.

I. INTRODUCTION

Symmetry properties have always served an important role in the study of nuclear forces. In particular, the extent of the validity of isospin invariance (after the correction for the pure Coulomb force) is of interest. Charge independence (CI) has long been known[1] to be violated, primarily due to the mass difference of neutral and charged pions. With regard to the weaker symmetry, charge symmetry (CS), the situation is far less clear. One way of testing for charge-symmetry breaking (CSB) is to compare the low-energy scattering parameters, scattering length, and effective range, for neutron-neutron scattering with those for proton-proton scattering after correction for the Coulomb force. These results[2] are shown in Table I. Neutron-neutron effective-range parameters are not directly measurable but must be obtained from three-body processes, such as ^{2}H(n,2n)p or ^{2}H(π^-,γ)2n. The results from most recent analyses[3,4] of these two processes are given in the table. They do not agree within their error bars. It is argued in Ref. 4 that the discrepancy may be due to three-body interactions in the three-nucleon system in ^{2}H(n,2n)p. Leaving aside the question of three-body forces, nn and pp scattering lengths agree within the error bars. On the other hand, there is some question whether it is possible to extract the Coulomb contribution to these quantities in a model-independent way.[5] For this reason, it is of interest to investigate CSB in the neutron-proton system, using measurements of spin observables. Interest in theoretical expectations of the extent of CSB has been raised by experimental measurements under way at TRIUMF (Ref. 6) and Indiana University Cyclotron Facility (IUCF).[7] In addition, the discrepancy between the results from two recent theoretical calculations[8,9] suggests the need for further study of this problem.

By definition, charge symmetry requires invariance under charge reflection in the 1-2 plane of isospin space. A charge-symmetry operator $P_{\rm CS}$ can be defined as,

$$P_{\rm CS}=e^{i\pi T_2}=\prod_{i=1}^{A} e^{i\pi T_2(i)}\ , \tag{1}$$

where the charge corresponds to the three-axis, $T_2=\sum_{i=1}^{A} T_2(i)$. Charge symmetry requires that $P_{\rm CS}$ and the total Hamiltonian commute, i.e.,

$$[P_{\rm CS},H]=0\ . \tag{2}$$

This definition of CS implies invariance under change of all neutrons into protons and all protons into neutrons. This can be tested in finite nuclei (e.g., mirror nuclei) or in nucleon-nucleon scattering.

In this work,[10,11] the CSB phase parametrization is calculated in a new way, namely by deriving a formula for the isospin mixing parameter[9] $\bar{\gamma}_J$ expressed in terms of the CSB part of the nucleon-nucleon scattering T matrix. In the neutron-proton system, CSB is due to class-IV forces (as classified by Henley and Miller[12]), which break both CI and CS, mix total isospin, and act only in the n-p system. Using the distorted-wave Born approximation[8] (DWBA), the T-matrix elements due to certain class-IV

TABLE I. Experimental scattering length and effective range in nucleon-nucleon scattering.

		Scattering length a	Effective range r
nn	^{2}H(π^-,γ)2n	$-16.9\ \ \pm0.6$	2.65 ± 0.18 (Ref. 3)
nn	^{2}H(n,2n)p	$-18.6\ \ \pm0.5$	2.83 ± 0.16 (Ref. 4)
pp		-7.828 ± 0.008	2.80 ± 0.02
pp, corrected for Coulomb		$-17.1\ \ \pm0.2$	2.84 ± 0.03

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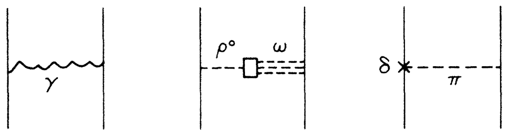

FIG. 1. Charge-symmetry breaking mixing mechanisms considered in this work.

forces, one-photon exchange, ρ^0-ω mixing, and nucleon mass difference (see Fig. 1), are computed numerically, and the mixing coefficients extracted. Because DWBA takes more completely into account the isospin-invariant part on the nucleon-nucleon interaction, the $\bar{\gamma}_J$ have more complicated J dependence than do those obtained by the plane-wave Born approximation.[9]

II. CALCULATION

Experimentally, CSB can be measured[6,7] in the neutron-proton system by comparing the analyzing powers from scattering of polarized neutrons from unpolarized protons (A_n) and unpolarized neutrons from polarized protons (A_p). Any nonzero difference, ΔA, between these analyzing powers is due to CSB in the nucleon-nucleon interaction. This analyzing-power difference can be expressed[8] as

$$\Delta A = A_n(\theta_n) - A_p(\theta_p) = \tfrac{1}{2}\,\mathrm{Re}\{\mathrm{Tr}[T_{\mathrm{CS}}^{\dagger}(\boldsymbol{\sigma}_n - \boldsymbol{\sigma}_p)\cdot\hat{\mathbf{n}}\,\delta T]\}/\sigma_0 \,, \tag{3}$$

where δT is the CSB part of the nucleon-nucleon T matrix, written as

$$T = T_{\mathrm{CS}} + \delta T \,. \tag{4}$$

Here T_{CS} is the T matrix obtained under the assumption that charge symmetry is valid, σ_0 is the differential cross section summed over spins, and $\hat{\mathbf{n}}$ is a unit vector perpendicular to the scattering plane.

From scattering theory the relation between S- and T-matrix elements is

$$\langle \hat{\mathbf{k}}'S'\nu' \,|\, S(k) \,|\, \hat{\mathbf{k}}S\nu\rangle = \delta_{SS'}\delta_{\nu\nu'}\delta_{\hat{\mathbf{k}}'\hat{\mathbf{k}}} - 2\pi i \rho_\epsilon \langle \hat{\mathbf{k}}'S'\nu' \,|\, T(k) \,|\, \hat{\mathbf{k}}S\nu\rangle \,. \tag{5}$$

If isospin is mixed, then by parity conservation and the generalized Pauli principle, $(-1)^{l+I+S} = -1$, spin is also mixed, $S' \neq S$. The S-matrix elements can be written as

$$\langle \hat{\mathbf{k}}'S'\nu' \,|\, S(k) \,|\, \hat{\mathbf{k}}S\nu\rangle = \sum_{l'lJ} \langle \hat{\mathbf{k}}'l'S'\nu' \,|\, Y^J \,|\, \hat{\mathbf{k}}lS\nu\rangle S^J_{l'S',lS}(k) \,, \tag{6}$$

where

$$\langle \hat{\mathbf{k}}'l'S'\nu' \,|\, Y^J \,|\, \hat{\mathbf{k}}lS\nu\rangle = \sum_{m'mM} Y_{l'}^{m'}(\hat{\mathbf{k}}')Y_l^{m\,*}(\hat{\mathbf{k}})\langle l'S'm'\nu' \,|\, JM\rangle\langle lmS\nu \,|\, JM\rangle \,. \tag{7}$$

Using the Stapp or "bar" phase shift parametrization,[13] the symmetric unitary matrix $S^J_{l'S',lS}$ has the form, for $l' = l = J$,

$$S^J_{JS',JS} = \begin{bmatrix} \cos(2\bar{\gamma}_J)e^{2i\bar{\delta}_{JJ0}} & i\sin(2\bar{\gamma}_J)e^{i(\bar{\delta}_{JJ0}+\bar{\delta}_{JJ1})} \\ i\sin(2\bar{\gamma}_J)e^{i(\bar{\delta}_{JJ0}+\bar{\delta}_{JJ1})} & \cos(2\bar{\gamma}_J)e^{2i\bar{\delta}_{JJ1}} \end{bmatrix} , \tag{8}$$

where use has been made of the phase-shift identity

$$\delta_{JJ0} + \delta_{JJ1} = \bar{\delta}_{JJ0} + \bar{\delta}_{JJ1} \,. \tag{9}$$

After some calculation,[11] one can get for $S_f = \nu' = 0$, $S_i = \nu = 1$, the $\bar{\gamma}_J$ in terms of the CSB part of the T-matrix element as

$$\delta T = -\frac{e^{i\phi}}{2\sqrt{2\pi^2}kM}\sum_J \frac{(2J+1)}{\sqrt{J(J+1)}} e^{i(\delta_{JJ0}+\delta_{JJ1})}\bar{\gamma}_J P_J^1(\cos\theta) . \tag{10}$$

On the other hand, in DWBA the CSB T-matrix element can be written as[14]

$$\delta T = \sum_J \frac{8}{\sqrt{2\pi}}(2J+1)G_J e^{i(\delta_{JJ0}+\delta_{JJ1})} e^{i\phi} P_J^1(\cos\theta) \,, \tag{11}$$

where

$$G_J = \int_0^\infty R_{JJ0}(r)R_{JJ1}(r)f(r)r^2 dr \,. \tag{12}$$

Here $f(r)$ is the radial behavior of the class-IV potential for the particular contribution to the CSB and the R are radial wave functions for nucleon-nucleon scattering by the charge-symmetric interaction. Hence, by combining Eqs. (10) and (11), one obtains the CSB mixing parameter in DWBA as

$$\bar{\gamma}_J = -16\pi Mk\sqrt{J(J+1)}G_J \,, \tag{13}$$

where k and M are momentum and (average) nucleon mass, respectively. To compute G_J, the Paris potential[15] and Arndt *et al.* phase-shift analysis[16] are used in the calculation of the distorted wave functions. After direct application of Feynman rules to Fig. 1, the various class-IV forces are calculated as[14]

$$f_\gamma(r) = -\frac{e^2}{4\pi}\frac{K_n}{4M^2}\left[\frac{1}{r}\frac{d}{dr}\left[\frac{1}{r}\right] + \text{form-factor contribution}\right] \tag{14}$$

for one-photon exchange (direct electromagnetic effect), where $e^2/(4\pi) = \frac{1}{137}$ and $K_n = -1.91$ is the neutron anomalous magnetic moment;

$$f_{\rho\omega}(r) = -\frac{g_\rho g_\omega}{4\pi}\frac{K_n}{4M^2}\frac{\langle\omega \,|\, H_{\mathrm{em}} \,|\, \rho^0\rangle}{m_\omega^2 - m_\rho^2}\frac{1}{r}\frac{d}{dr}\frac{1}{r} \times (e^{-m_\rho r} - e^{-m_\omega r}) \tag{15}$$

TABLE II. Isospin-mixing parameter $\bar{\gamma}_J$ at center-of-mass energies 100, 200, 300, and 400 MeV and six angular momenta.

Orbital angular momentum $L=J$	Mixing parameter $\bar{\gamma}_J$ em effect	ρ^0-ω mixing	Mass difference
c.m. energy=100 MeV			
1	0.8354×10^{-2}	0.3299×10^{-2}	-0.9798×10^{-2}
2	0.1230×10^{-1}	0.3018×10^{-2}	$+0.1233\times10^{-1}$
3	0.7640×10^{-2}	0.2392×10^{-3}	-0.4479×10^{-2}
4	0.6565×10^{-2}	0.5340×10^{-4}	$+0.2836\times10^{-2}$
5	0.5104×10^{-2}	0.5407×10^{-5}	-0.1460×10^{-2}
6	0.4396×10^{-2}	0.8311×10^{-6}	$+0.8787\times10^{-3}$
c.m. energy=200 MeV			
1	0.7975×10^{-2}	0.5816×10^{-2}	-0.1203×10^{-1}
2	0.1234×10^{-1}	0.6280×10^{-2}	$+0.1612\times10^{-1}$
3	0.9721×10^{-2}	0.1183×10^{-2}	-0.8231×10^{-2}
4	0.9223×10^{-2}	0.4670×10^{-3}	$+0.6361\times10^{-2}$
5	0.7153×10^{-2}	0.7894×10^{-4}	-0.3627×10^{-2}
6	0.6201×10^{-2}	0.2043×10^{-4}	$+0.2487\times10^{-2}$
c.m. energy=300 MeV			
1	0.7248×10^{-2}	0.7250×10^{-2}	-0.1268×10^{-1}
2	0.1112×10^{-1}	0.7894×10^{-2}	$+0.1657\times10^{-1}$
3	0.1030×10^{-1}	0.2410×10^{-2}	-0.1049×10^{-1}
4	0.1078×10^{-1}	0.1294×10^{-2}	$+0.9235\times10^{-2}$
5	0.8593×10^{-2}	0.2976×10^{-3}	-0.5617×10^{-2}
6	0.7705×10^{-2}	0.1024×10^{-3}	$+0.4177\times10^{-2}$
c.m. energy=400 MeV			
1	0.6454×10^{-2}	0.7899×10^{-2}	-0.1252×10^{-1}
2	0.9080×10^{-2}	0.7866×10^{-2}	$+0.1473\times10^{-1}$
3	0.1055×10^{-1}	0.3682×10^{-2}	-0.1216×10^{-1}
4	0.1168×10^{-1}	0.2343×10^{-2}	$+0.1148\times10^{-1}$
5	0.9588×10^{-2}	0.6717×10^{-3}	-0.7337×10^{-2}
6	0.8612×10^{-2}	0.2710×10^{-3}	$+0.5605\times10^{-2}$

for ρ^0-ω mixing, where we take $\langle\omega|H_{\rm em}|\rho^0\rangle=-3.4\times10^3$ MeV^2; $g_\rho^2/(4\pi)=2.4$, $g_\omega^2/(4\pi)=18.4$;

$$f_\delta(r)=-(-1)^{J+1}\frac{g_\pi^2}{4\pi}\frac{\delta}{2M^2}\frac{1}{r}\frac{d}{dr}\left[\frac{1}{r}e^{-m_\pi r}\right], \quad (16)$$

for nucleon mass difference (MD) contribution to one-pion exchange, where

$$\delta=(m_{\rm n}-m_{\rm p})/(m_{\rm n}+m_{\rm p}),\ \ g_\pi^2/(4\pi)=14.5 .$$

The phase factor $(-1)^{J+1}$ in Eq. (16) results from the spin-isospin dependence of the MD term, which is different from the other two effects.

III. RESULTS AND DISCUSSION

The mixing parameter $\bar{\gamma}_J$ has been computed for angular momenta J up to 6 and for a number of center-of-mass energies. Results for center-of-mass energies 100, 200, 300, and 400 MeV are shown in Table II; for more complete results, see Ref. 11. From these results, one sees that the isospin mixing is a small quantity, which reflects the fact that CSB is a weak violation. Mass difference and direct electromagnetic effects give rise to larger viola-

TABLE III. Comparison of isospin-mixing parameter $\bar{\gamma}_J$ obtained from the direct electromagnetic effect with plane-wave and distorted-wave Born approximation, at 100 MeV (c.m.).

$J=L$	$\bar{\gamma}_J$ (em with PWBA)	$\bar{\gamma}_J$ (em with DWBA)	$\Delta\bar{\gamma}_J/\bar{\gamma}_J$
1	0.2024×10^{-1}	0.8354×10^{-2}	58.7%
2	0.1169×10^{-1}	0.1230×10^{-1}	5.2%
3	0.8265×10^{-2}	0.7640×10^{-2}	7.6%
4	0.6402×10^{-2}	0.6565×10^{-2}	2.5%
5	0.5227×10^{-2}	0.5104×10^{-2}	2.4%
6	0.4417×10^{-2}	0.4396×10^{-2}	0.5%

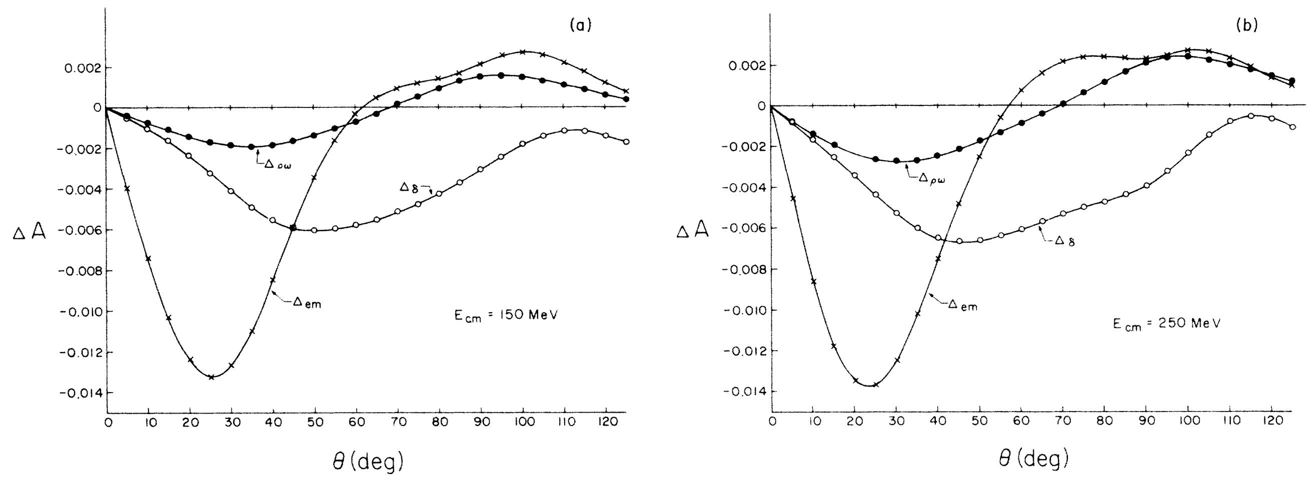

FIG. 2. Analyzing-power difference $\Delta A(\theta)$ calculated with the three mechanisms shown in Fig. 1. (a) $E_{\text{c.m.}}=150$ MeV; (b) $E_{\text{c.m.}}=250$ MeV.

tions than does ρ^0-ω mixing. As the energy increases, beyond about 100 MeV, the mass-difference effect gives the dominant contribution. The results indicate that the strongest CS violation occurs at c.m. energy 300 MeV, dominated by the MD effect. In contrast to the usual behavior of scattering quantities decreasing with total angular momentum, the mixing parameter does not monotonically decrease, but has a maximum at $J=2$, which beyond c.m. energy 300 MeV shifts to $J=4$ for the em effect. This behavior may be due to the fact that, in the distorting potential, the Paris potential, the tensor force has particularly strong coupling among the even states, particularly $L=0$ with $L=2$. The isospin mixing cannot occur in $L=0$ states, but the strong $L=0$ interaction can still have an indirect effect through the tensor force.

On the other hand, in the plane-wave Born approximation,[9] there can be no such effect of the distorting potential, and the $\bar{\gamma}_J$ do, in fact, decrease monotonically with increasing J. This is seen in Table III where, for the direct electromagnetic effect, the DWBA and PWBA values of $\bar{\gamma}_J$ are compared at c.m. energy 100 MeV. The largest difference occurs for $J=1$, as expected since the effect of the distorted waves should be stronger at lower angular momenta. This large discrepancy in the $J=1$ contribution could be the major reason for the discrepancy in the PWBA results of Ref. 9 with the DWBA ones of Ref. 8.

Figure 2 shows the calculated values of the contributions to $\Delta A(\theta)$ from each of the three CSB effects studied, as obtained using Eq. (1) and the formulas given by Hoshizaki.[17] Again, the dominance of the MD effect at all but the smallest angles is evident. The experiment[6] is done at c.m. energy 240 MeV and angle around 70°–75° where the analyzing power passes through zero. Under those conditions, almost all the CSB contribution comes from the nucleon mass-difference effect in one-pion exchange.

IV. CONCLUSION

Charge-symmetry breaking due to certain class-IV forces is examined. A formula for the isospin mixing parameter $\bar{\gamma}_J$ is derived in a new way using the DWBA. The results show that $\bar{\gamma}_J$ is a small quantity of the order of 1.6×10^{-2} for the largest MD effect. At c.m. energies higher than 50 MeV, $\bar{\gamma}_J$ has a somewhat unexpected behavior, in that it peaks at $J=2$ ($J=4$ for energy higher than 350 MeV). The effect of distortion in the DWBA approach used here is very important, as can be seen by comparison with the PWBA results of Gersten.[9] The optimum energy at which to measure CSB should be high enough that the effect is large, but not so high that inelastic effects, which have been neglected in this work, become important. The calculated values of $\bar{\gamma}_J$ are a maximum at c.m. energy 300 MeV, but a somewhat lower energy may be more suitable, to avoid inelastic effects.

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