Determination of an effective radius from the gamma-ray multiplicities in fusion reactions

A. B. Balantekin and P. E. Reimer
Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831
(Received 30 September 1985)

The gamma-ray multiplicity and the fusion cross-section data, together, are used to determine the effective radius employed in a previously developed inversion procedure. Implications on the relationship between the gamma-ray multiplicity and the average angular momentum are pointed out.

Recently, considerable attention has been directed to the experimental study of the heavy-ion reactions near and below the Coulomb barrier. In particular, fusion cross sections[1-3] and the spin distributions for the compound nucleus[4,5] have been studied. The enhancement and strong isotopic dependences observed in the subbarrier fusion of heavy nuclei[1] require a full coupled-channels treatment of the fusion problem.[6-8] Numerical implementation of such a program is very difficult since many channels need to be included. On the other hand, the fusion of light nuclei can be studied to a large extent[9,10] within the one-dimensional barrier penetration picture. In this simple description, one assumes that the system first penetrates the real, one-dimensional, local energy-independent barrier formed by nuclear and Coulomb interactions and then is completely absorbed into the fusion channel.

Since the heavy-ion physics is expected to be amenable to a semiclassical description, the one-dimensional barrier penetration is usually analyzed in terms of the Wentzel-Kramers-Brillouin (WKB) approximation. In this approximation, the cross section at the center-of-mass energy E is given by

$$\sigma=\sum_{L=0}^{\infty}\sigma_L \ , \tag{1a}$$

with

$$\sigma_L=\frac{\pi\hbar^2}{2mE}(2L+1)T_L(E) \ , \tag{1b}$$

where

$$T_L(E)=\{1+\exp[2S_L(E)]\}^{-1} \tag{2a}$$

and

$$S_L(E)=\int_{r_1}^{r_2}\left[\frac{2m}{\hbar^2}[V(r)-E]+\frac{L(L+1)}{r^2}\right]^{1/2}dr \ . \tag{2b}$$

In Eq. (2b) the turning points r_1 and r_2 are complex above the barrier and real below the barrier. Also, $V(r)$ is the nuclear plus Coulomb potential.

As a further approximation, the angular momentum quantum number dependence in Eq. (2) can be reproduced by a shift in the energy:[9]

$$T_L(E)=T_0\left[E-\frac{L(L+1)\hbar^2}{2mR^2(E)}\right] \ , \tag{3}$$

where the effective moment of inertia $mR^2(E)$ is a function of energy. Using Eq. (3), one can directly obtain the s-wave penetrability from the experimentally measured fusion cross section, using[9]

$$S_0(E)=\frac{1}{2}\ln\left\{\left[\frac{d}{dE}\left[\frac{E\sigma}{\pi R^2}\right]\right]^{-1}-1\right\} \ . \tag{4}$$

In writing the above equations, we have not specified the energy dependence of R. It is usually taken to be a constant[11] equal to the value of the position r_B of the potential maximum $B=V(r_b)$. In a more elaborate treatment,[9] R has been taken to be a linear combination of r_B and the Coulomb turning point:

$$R(E)=\eta r_B+(1-\eta)\frac{Z_1Z_2e^2}{E} \ , \tag{5}$$

where η was empirically adjusted to be 0.5. The purpose of this Brief Report is to study the energy dependence of $R(E)$ in general, and furthermore, to point out that the spin distribution data of the compound nucleus formed at near-barrier energies can be used to determine unambiguously this effective moment of inertia.

We assume that the average angular momentum $\langle L\rangle$,

$$\langle L\rangle=\left[\sum_{L=0}^{\infty}L\sigma_L\right]\Big/\sigma \ , \tag{6}$$

where σ and σ_L are given by Eq. (1), can be determined reasonably accurately from the gamma-ray multiplicities data as was done in Refs. 4 and 5. Using Eqs. (1), (2), and (3), Eq. (6) can be rewritten as

$$\langle L\rangle=\frac{m}{\hbar^2}\frac{R^4(E)}{E\sigma(E)}\int_{-\infty}^{E}dE'\frac{E'\sigma(E')}{R^2(E')}\times\left[\frac{mR^2(E)}{\hbar^2}(E-E')+\frac{1}{4}\right]^{-1/2} \ . \tag{7}$$

Equation (7) holds at energies both below and above the barrier. Consequently, if the average angular momentum and the cross section are known over a certain energy range, then $R(E)$ can also be determined solving Eq. (7) iteratively. Furthermore, if this range includes the barrier energy B, since $S_0(B)=0$, one can find B unambiguously by solving[9]

$$\frac{d}{dE}\left[\frac{E\sigma(E)}{\pi R^2(E)}\right]\Bigg|_{E=B}=\frac{1}{2} \ . \tag{8}$$

In order to study the behavior of $R(E)$, we have generated the average angular momenta and cross sections from a universal Woods-Saxon potential for ^{12}C-^{12}C (Ref. 12) and the phenomenological potential of Ref. 13 for ^{64}Ni-^{64}Ni (the same potentials used to test the inversion procedure of Ref. 9). The effective radii $R(E)$ obtained by solving Eq. (7)

for these systems are shown in Fig. 1. As we expect for ^{64}Ni-^{64}Ni, where many partial waves are involved, the effective radius $R(E)$ is consistent with Eq. (5) or with the actual position of the maximum of the barrier ($r_B = 10.64$ fm, the dashed line), whereas for ^{12}C-^{12}C, where there are fewer partial waves, $R(E)$ is smaller than that given by either Eq. (5) or the actual barrier position ($r_B = 7.81$ fm, the dashed line).

One must emphasize that solving Eqs. (7) and (8) together yields a unique value for the barrier maximum B, rather than a fit; especially since the cross section decreases very rapidly below the barrier, one does not need to know $\sigma(E)$ more than a couple of MeV below the barrier to get a reasonably accurate solution to Eq. (8). Also, as can be seen from this equation, a knowledge of $\langle L \rangle$ well below the barrier is not necessary to determine B. We have found B to be 6.30 and 97.83 MeV for ^{12}C-^{12}C and ^{64}Ni-^{64}Ni, respectively, compared to the actual values of 6.30 and 97.74 MeV (cf. Fig. 1).

When one or both of the heavy ions are strongly deformed, the fusion cross sections have to averaged[14] over all orientation angles and the position and height of the barrier change considerably as the orientation angle changes. Consequently, in such cases it is more important to consider the energy dependence of $R(E)$. The effects of the deformation on fusion cross sections have been studied in Ref. 2 for ^{16}O + ^{148,150,152,154}Sm. Recently, fusion cross sections and the spin distributions of the compound nuclei have been studied, and the average angular momenta have been deduced in Refs. 4 and 5 for three different projectiles (α, ^{12}C, ^{16}O) on ^{154}Sm. Using the data from Refs. 2, 4, and 5, we have solved Eq. (7) for ^{12}C-^{154}Sm and ^{6}O-^{154}Sm systems. The resulting effective radii are shown in Figs. 2(a) and 2(b). Equation (8) gives the barrier height for these systems to be 44.3 MeV for ^{12}C-^{154}Sm and 59.2 MeV for ^{16}O-^{154}Sm, respectively, to be compared with 44.6 and 59.0 MeV obtained in Ref. 5 by *fitting* the data. It is also interesting to note that when we repeated the fits in Ref. 5 using the formalism of Ref. 11 [except for changing r_B to $R(E)$], we obtained a slightly larger deformation parameter ($\beta = 0.28$) for ^{154}Sm than was deduced in Ref. 5.

We have shown that, given the fusion cross section and the average angular momentum, one can calculate the effective radius in Eq. (3). Once $R(E)$ is known, it can be used to determine the value of the maximum, and also if the data at subbarrier energies are available, the thickness[9] of the potential. The average angular momentum is usually obtained from[4,5] the gamma-ray multiplicity M_γ assuming a linear relationship

$$\langle L \rangle = \alpha M_\gamma - \delta \ . \tag{9}$$

Instead of using Eqs. (7) and (8) to determine B, one can argue that the value of B is accurately determined from other experiments and the consistency between two values of B is indirect evidence that the form of Eq. (9) and the values of the parameters α and δ used to determine $\langle L \rangle$ are correct. In light of the above results, we conclude that the parametrization of $\langle L \rangle$ given in Refs. 4 and 5 is quite accurate.

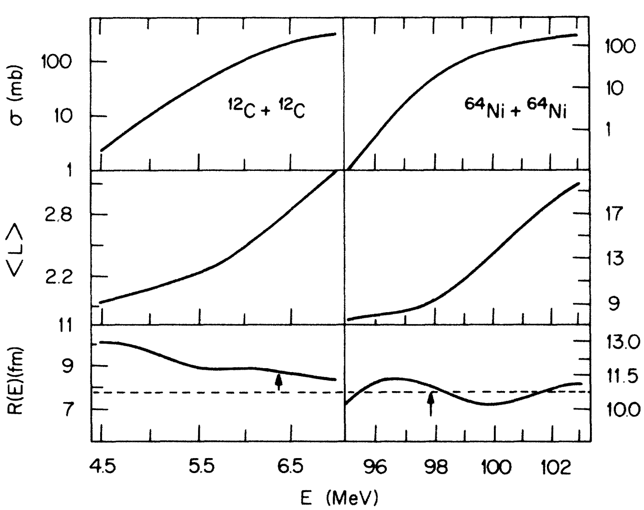

FIG. 1. Results obtained using model potentials. The nuclear interaction for the ^{12}C-^{12}C system is the Woods-Saxon potential of Ref. 11, $V = -V_0\{1+\exp[(r-r_0)/a]\}^{-1}$, with $V_0 = 50$ MeV, $a = 0.4$ fm, and $r_0 = 5.82$ fm. The nuclear interaction for the ^{64}Ni-^{64}Ni system is the potential given in Ref. 12, Eqs. (17)–(20). Upper and middle portions are the cross section and average angular momenta determined using the WKB formalism. The lower portion shows the effective radius $R(E)$ (solid line) obtained by solving Eq. (7) using $\sigma(E)$ and $\langle L \rangle(E)$ given above. The dashed line is the actual position of the barrier maximum, and the arrow identifies B found by solving Eq. (8).

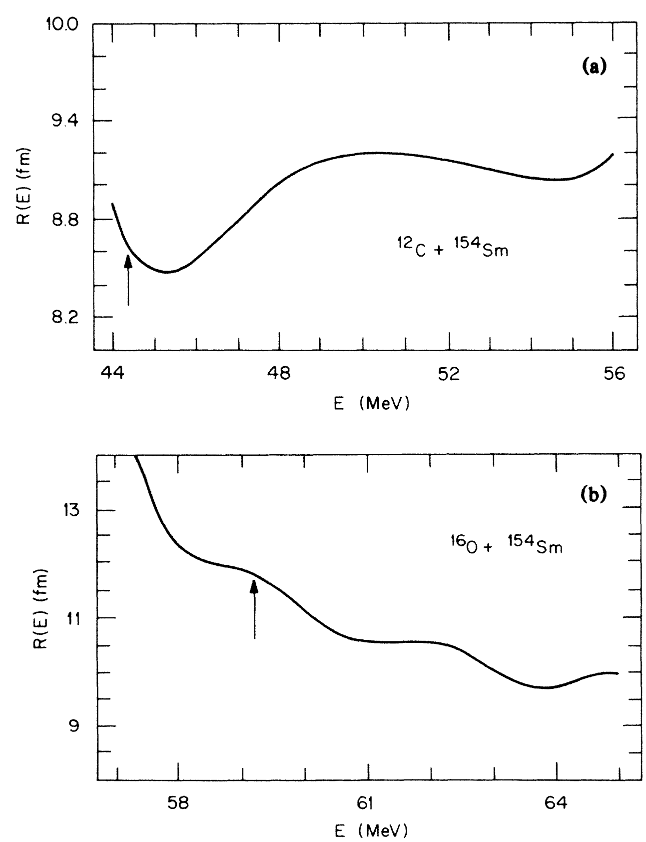

FIG. 2. (a) The effective radius for the ^{12}C-^{154}Sm system obtained by solving Eq. (7). The data are taken from Ref. 5. The arrow identifies the maximum value of the potential found by solving Eq. (8). (b) Same as (a) except for the ^{16}O-^{154}Sm system.

We would like to thank M. Beckerman, G. Bertsch, and R. Satchler for useful discussions. One of us (P.E.R.) wishes to thank the members of the Theory Group of Oak Ridge National Laboratory for their hospitality during his stay as a student research participant. The research was sponsored by the Division of Nuclear Physics, U.S. Department of Energy, under Contract No. DE-AC05-840R21400, with Martin Marietta Energy Systems, Inc.

---

[1]M. Beckerman, Phys. Rep. (to be published).

[2]R. G. Stokstad, Y. Eisen, S. Kaplanis, D. Pelte, U. Smilansky, and I. Tserruya, Phys. Rev. C **21**, 2427 (1980).

[3]J. Thomas, Y. T. Chen, S. Hinds, K. Langanke, D. Meredith, M. Olson, and C. A. Barnes, Phys. Rev. C **31**, 1980 (1985).

[4]R. Vandenbosch, B. B. Back, S. Gil, A. Lazzarini, and A. Ray, Phys. Rev. C **28**, 1161 (1983).

[5]S. Gil, R. Vandenbosch, A. J. Lazzarini, D. K. Lock, and A. Ray, Phys. Rev. C **31**, 1752 (1985).

[6]M. J. Rhoades-Brown and P. Braun-Munzinger, Phys. Lett. **136B**, 19 (1984); M. J. Rhoades-Brown, P. Braun-Munzinger, M. Prakash, and S. Sen, in *Fusion Reactions Below the Coulomb Barrier,* edited by S. G. Steadman (Springer-Verlag, Berlin, 1985), p. 162.

[7]S. Landowne, in *Fusion Reactions Below the Coulomb Barrier,* edited by S. G. Steadman (Springer-Verlag, Berlin, 1985), p. 182.

[8]A. B. Balantekin and N. Takigawa, Ann. Phys. (N.Y.) **160**, 441 (1985); N. Takigawa and K. Ikeda (unpublished).

[9]A. B. Balantekin, S. E. Koonin, and J. W. Negele, Phys. Rev. C **28**, 1565 (1983).

[10]M. Inui and S. E. Koonin, Phys. Rev. C **30**, 175 (1984); J. Thomas and K. Langanke (unpublished).

[11]C. Y. Wong, Phys. Rev. Lett. **31**, 766 (1973).

[12]S. Trentalonge, Ph.D. dissertation, California Institute of Technology, 1982 (unpublished).

[13]H. J. Krappe, J. R. Nix, and A. J. Sierk, Phys. Rev. C **20**, 992 (1979).

[14]R. G. Stokstad and E. E. Gross, Phys. Rev. C **23**, 281 (1981).