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Theoretical estimates of photoproduction cross sections for neutral subthreshold pions in carbon-carbon collisions

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Using the Weizsacher-Williams method of virtual quanta, total cross section estimates for the photoproduction of neutral subthreshold pions in carbon-carbon collisions at incident energies below 300 MeV/nucleon are made. Comparisons with recent experimental data indicate that the photoproduction mechanism makes an insignificant contribution to these measured cross sections.

Recently, there has been considerable experimental and theoretical activity in the area of subthreshold pion production in nucleus-nucleus collisions. (By subthreshold, we refer to collisions at energies below the free nucleon-nucleon pion production threshold of 290 MeV/nucleon.) Cross section measurements[1,2] for pion production have been made at energies down to 35 MeV/nucleon. Even before these measurements, however, speculations concerning possible subthreshold pion production were made.[3] Within the past decade numerous theories have arisen to describe this phenomenon. Proposals include nucleon-nucleon collision models,[4,5] statistical and thermal models,[6–10] collective models,[11–14] and a time-dependent Hartree-Fock model.[15] Some of the models have successfully described portions of the data—but none have yet been able to describe all of the data. This is not surprising, however, since different pion production mechanisms may be operative at any particular energy. In addition to the aforementioned mechanisms, an additional source of pions might be photoproduction by the interacting Coulomb fields of the colliding nuclei. It is known, for example, that tagged photon beams can be used to produce pions from nuclear targets.[16] In addition, it has been recently noted that dissociation of projectile nuclei by the virtual photon field of target nuclei have cross sections which are a significant fraction of their inclusive fragmentation cross sections.[17–20] In this paper, total photoproduction cross sections for neutral subthreshold pions in carbon-carbon collisions will be estimated using methods analogous to those employed to describe Coulomb dissociation processes in nucleus-nucleus collisions.[19]

Proceeding, we assume that the total photoproduction cross section for pion species π is given by

$$\sigma_{\mathrm{em}}(\pi) = \int_{E_0(\pi)} \sigma_\gamma(E,\pi) N(E)\, dE, \tag{1}$$

where $E_0(\pi)$ is the photonuclear threshold, $\sigma_\gamma(E,\pi)$ is the photonuclear reaction cross section for pion production in one of the nuclei, and $N(E)$ is the virtual photon number spectrum produced by the other nucleus in the collision pair. For photoproduction of neutral pions on carbon, experimental values for the total reaction cross sections, $\sigma_\gamma(E,\pi)$, from Ref. 16 were used. These are displayed in Fig. 1. The virtual photon number spectrum calculated using the Weizsacher-Williams method of virtual quanta,[21,22] is given by

$$N(E) = \frac{2Z^2\alpha}{\pi E \beta^2}\left\{ x K_0(x) K_1(x) - \frac{\beta^2}{2}[K_1^2(x) - K_0^2(x)] \right\}, \tag{2}$$

where $N(E)$ is the number of virtual photons per unit photon energy, E, Z is the nuclear proton number, β is the velocity of the nucleus in units of c, and α is the electromagnetic fine structure constant. The parameter x in Eq. (2) is defined as

$$x = \frac{E b_{\min}}{\gamma\beta(\hbar c)}, \tag{3}$$

where γ is the usual relativistic factor and the minimum impact parameter is

$$b_{\min} = R_1 + R_2, \tag{4}$$

with R_1 and R_2 being the 10% charge density radii of the colliding nuclei (3.33 fm for carbon[23]). The modified Bessel functions of the second kind, $K_0(x)$ and $K_1(x)$, are calculated using polynomial approximations.[24] Typical photon number spectra are displayed in Fig. 2. Two trends are apparent: (1) as E increases, $N(E)$ decreases, and (2) as the

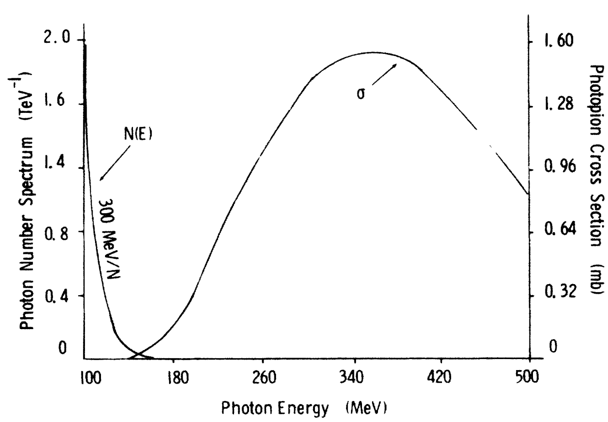

FIG. 1. Virtual photon number spectrum, $N(E)$ (300 MeV/nucleon shown), and photopion cross section as a function of photon energy. Note that the photon number spectrum is very small in the region of maximum cross section strength.

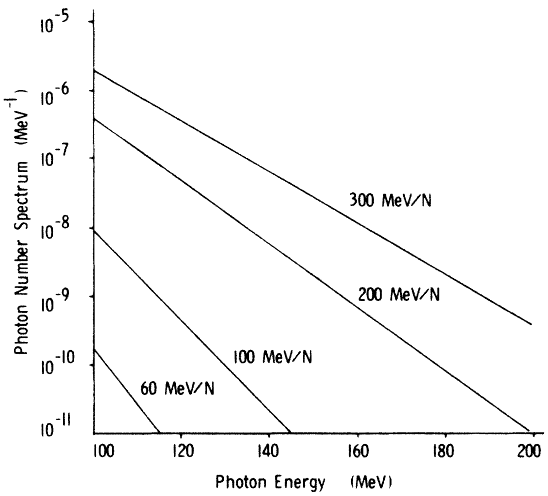

FIG. 2. Virtual photon number spectrum vs photon energy for a variety of incident carbon laboratory kinetic energies. The rapid decrease in photon number with decreasing carbon kinetic energy is striking.

incident carbon kinetic energy increases (i.e., β increases), $N(E)$ increases rapidly. Both trends occur because of the behavior of the Bessel functions. From page 374 of Ref. 24, both $K_0(x)$ and $K_1(x)$ are observed to decrease rapidly as their arguments are increased (due to E increasing or β decreasing). Pilkuhn and collaborators[17] have developed an alternative methodology for calculating the virtual photon spectrum which has been shown to yield comparable results.[20]

Table I lists theoretical estimates of neutral pion photoproduction cross sections for carbon-carbon collisions, obtained from (1) by summing contributions from both nuclei.

TABLE I. Total cross sections for neutral pion production in subthreshold carbon-carbon collisions.

Incident kinetic energy (MeV/nucleon)	Total cross sections (μb) Theoretical	Experimental
34	2.6×10^{-15}	0.022
60	1.5×10^{-11}	1.7
74	2.1×10^{-10}	8.5
84	9.3×10^{-10}	18.9
100	6.2×10^{-9}	...
150	3.1×10^{-7}	...
200	3.3×10^{-6}	...
250	1.8×10^{-5}	...
300	6.3×10^{-5}	...

For comparison experimental data from Refs. 1 and 2 are also listed. As is readily apparent, the theoretical estimates are orders of magnitude smaller than the experimental measurements and clearly indicate that pion photoproduction is of *negligible importance* in subthreshold nucleus-nucleus collisions. The reason for this is clearly shown in Fig. 1, where it is noted that the virtual photon number density (300 MeV/nucleon shown) is very small in the region where the photonuclear reaction cross section has its main strength. In summary, a simple model for estimating pion photoproduction cross sections in carbon-carbon collisions has demonstrated that Coulomb interactions are an insignificant source of subthreshold pions.

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