Brief Reports

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Determination of the neutron and proton effective charges in the quadrupole operator of nuclear collective models

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We discuss the reliability of the determination of the neutron and proton effective charges in nuclear collective models from experimental effective neutron and proton matrix elements for the 0_1^+ to 2_1^+ transition.

With the discovery of a 1^+ state in ^{156}Gd, which is strongly excited in inelastic electron scattering experiments,[1] interest in collective F-spin vector excitations in nuclei has grown. In a number of papers the properties of magnetic dipole excitations were studied.[1-6] Subsequently, attention was also paid to electric quadrupole and magnetic octupole isovector excitations.[7-13]

For the study of electric quadrupole isovector excitations, the neutron and proton effective charges e_ν and e_π in the $E2$ transition operator are an essential ingredient. These effective charges can be determined from the study of the $E2$ transitions from the 0_1^+ ground state to the first excited 2_1^+ state as a function of neutron and proton number.[3,7,8,12] The 0_1^+ to 2_1^+ states are predominantly symmetric in the neutron and proton degrees of freedom or, in the language of the interacting boson model (IBM),[14] they have maximal F spin.[15] As a consequence, the effective proton matrix element[16] for the 0_1^+ to 2_1^+ transition [i.e., the square root of $B(E2;0_1^+\rightarrow 2_1^+)$] has the form[7,8,10]

$$M_\pi = f(N)(e_\pi N_\pi + e_\nu N_\nu) \ , \tag{1}$$

where N_π and N_ν are the number of valence pairs (bosons) of protons and neutrons, and N is the total, $N=N_\pi+N_\nu$. The function $f(N)$ can be easily evaluated for the three dynamical symmetries of the IBM:[7,8,10]

$$f(N)=\left(\frac{5}{N}\right)^{1/2}, \quad U_5 \text{ or vibrational limit} \ , \tag{2a}$$

$$f(N)=\left(\frac{N+4}{N}\right)^{1/2}, \quad SO_6 \text{ or } \gamma\text{-unstable rotor limit} \ , \tag{2b}$$

$$f(N)=\left(\frac{2N+3}{N}\right)^{1/2}, \quad SU_3 \text{ or deformed rotor limit} \ . \tag{2c}$$

This function is plotted for the three limits in Fig. 1. We

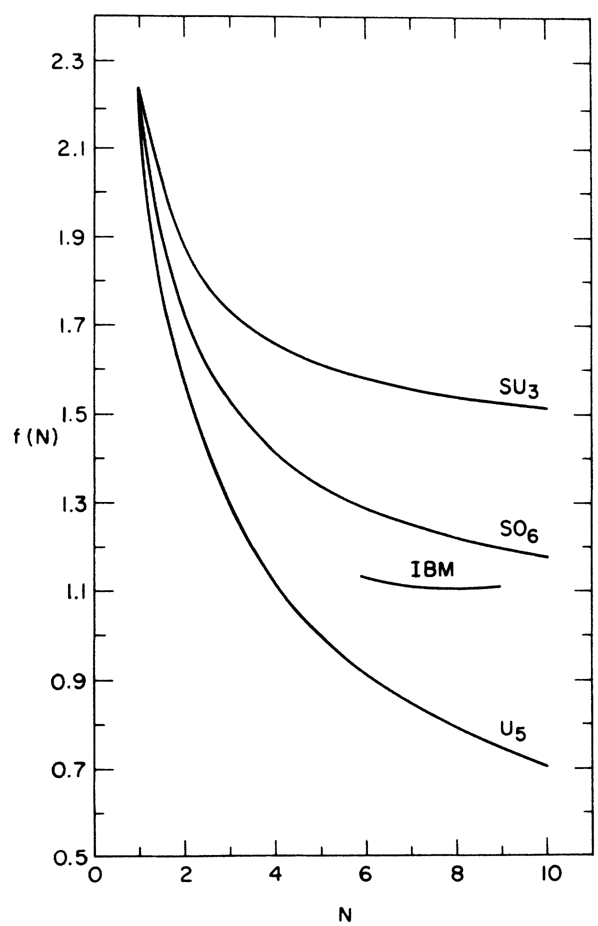

FIG. 1. The function $f(N)$ vs boson number N for the three symmetry limits given in Eq. (2) and for the IBM Hamiltonian given in Eq. (3).

note that the dependence on N varies greatly in the three limits. This means that this function depends very much on the structure of the nucleus being considered. Consequently, the boson effective charges extracted will be sensitive to the nuclear structure. In this Brief Report we study the dependence of e_ν and e_π on the function $f(N)$. Also, we show that the ratio e_ν/e_π can be determined independent of the specific form assumed for $f(N)$.

In view of existing experimental data, to be discussed below, we consider the Pd isotopes ($Z=46$; $104 \leq A \leq 110$). We use an IBM Hamiltonian of the form

$$H = \epsilon N_d + \kappa Q \cdot Q \ , \tag{3a}$$

where N_d is the number operator for the quadrupole bosons

$$N_d = d_\pi^\dagger \cdot \tilde{d}_\pi + d_\nu^\dagger \cdot \tilde{d}_\nu \ , \tag{3b}$$

Q is the quadrupole operator

$$Q = (s_\pi^\dagger \tilde{d}_\pi + d_\pi^\dagger s_\pi) + (s_\nu^\dagger \tilde{d}_\nu + d_\nu^\dagger s_\nu) \ , \tag{3c}$$

$s_\tau^\dagger$ is the monopole boson for neutrons ($\tau=\nu$) or protons ($\tau=\pi$), and $d_{\mu\tau}^\dagger$ is the quadrupole boson. The Hamiltonian (3) is intermediate between U_5 and SO_6, depending on the ratio ϵ/κ, and has been shown to be appropriate for the Pd isotopes.[17] In the present calculation we take the single-boson energy ϵ and the interaction strength κ from Ref. 18. In Fig. 1 we show the resulting function $f(N)$. We see that this more realistic $f(N)$ lies between the U_5 and SO_6 limits and that its N dependence resembles the SO_6 limit more than the U_5 limit. In fact, for this range of N, $f(N)$ is roughly constant, which is consistent with Ref. 12.

This result can be understood by deriving, in leading order perturbation theory, the expressions for $f(N)$ close to the U_5 and SO_6 limits. We find

$$f(N) = \left[\frac{5}{N}\right]^{1/2}\left[1 - \frac{\kappa(N-1)}{\epsilon}\right] \quad \text{near the } U_5 \text{ limit} \ , \tag{4a}$$

$$f(N) = \left[\frac{N+4}{N}\right]^{1/2}\left[1 - \frac{1}{2}\left[\frac{\epsilon}{4\kappa}\right]^2 \frac{(N-1)(N+3)}{(N+1)^4}\right], \tag{4b}$$

near the SO_6 limit. Near the U_5 limit the correction is of *first* order in κ/ϵ and introduces an N dependence very different from the zeroth-order expression (2a). In Fig. 1 this $f(N)$ is plotted for the parameters given in Ref. 18 and the results are indistinguishable from the exact numerical solution of (3). This means that, even though a nucleus is near the vibrational limit, the function $f(N)$ can be very different than that given by the exact U_5 limit. On the other hand, near the SO_6 limit the correction to $f(N)$ is only of *second* order in ϵ/κ, indicating that the zeroth-order expression (2b) is rather stable against deviations from the exact SO_6 Hamiltonian.

We shall now extract the boson effective charges for the Pd isotopes using the data of recent $\pi^\pm$ scattering experiments, which determine both M_π and the effective neutron matrix element,[19]

$$M_\nu = f(N)(e_\nu N_\pi + e_\pi N_\nu) \ . \tag{5}$$

In this extraction we assume that e_ν and e_π are mass independent, which is reasonable for nuclei like the Pd isotopes.[9,20] In Fig. 2 the results of this calculation are shown. Good agreement with the experimental matrix elements M_ν and M_π is obtained if the function $f(N)$ is taken from the SO_6 limit [Eq. (2b)] or if $f(N)$ is determined from the numerical calculation [which is almost identical to the perturbed-U_5 expression (4a)]. However, comparing the vibrational-(U_5) limit with the experiment in Fig. 2, we find that the trend is wrong.

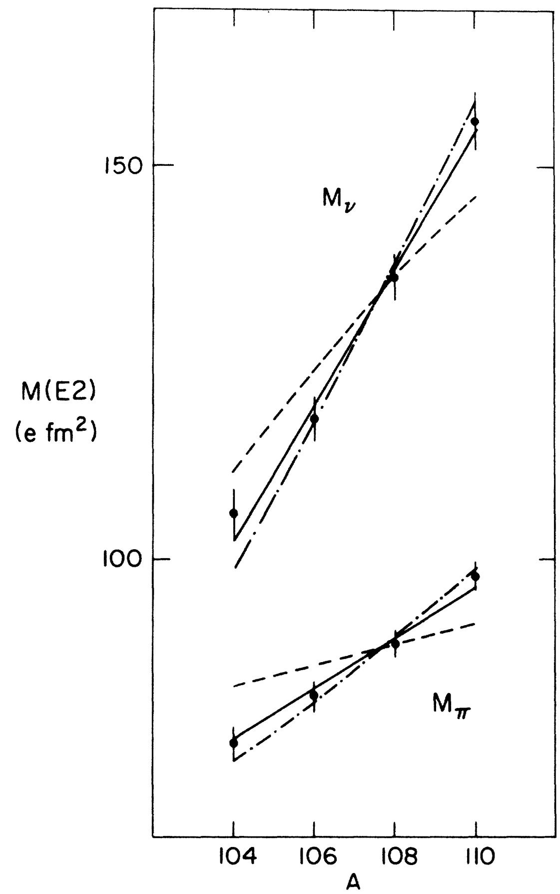

FIG. 2. Effective proton (M_π) and neutron (M_ν) matrix elements calculated in the U_5 limit (dashed line), in the SO_6 limit (solid line), and with the IBM Hamiltonian given in Eq. (3) (dashed-dot line). The experimental data are taken from Ref. 19.

The boson effective charges e_ν and e_π, resulting from the different fits, are given in column (a) of Table I. Again, the U_5 limit differs greatly from the other calculations, but the ratio e_ν/e_π is the same for all. The latter can be understood by noting that, by virtue of Eqs. (1) and (5), we have

$$\frac{e_\nu}{e_\pi} = \frac{M_\pi N_\nu - M_\nu M_\pi}{M_\nu N_\nu - M_\pi N_\pi} \ . \tag{6}$$

Hence, from the experimental values of M_ν and M_π, the ratio e_ν/e_π can be extracted *independent* of the form assumed for $f(N)$.

It is also instructive to fit the effective proton matrix element M_π *alone*, as is done when only $B(E2)$ data are available. In that case, it is possible to find agreement with the data (within the experimental errors) for all functions $f(N)$ considered here (i.e., U_5, SO_6, and intermediate). The extracted boson effective charges differ significantly from the previous fit to both M_ν and M_π only in the U_5 limit (see Table I). In the latter case the boson effective charges are very different and even have $e_\nu > e_\pi$.

TABLE I. Extracted boson effective charges for the U_5 and SO_6 limits and for the intermediate IBM Hamiltonian in Eq. (3).

	e_ν (a)	e_ν (b)	e_π (a)	e_π (b)	e_ν/e_π (a)	e_ν/e_π (b)
U_5	10.4	15.4	25.2	10.9	0.41	1.42
SO_6	6.8	7.2	16.4	15.2	0.41	0.47
IBM	7.5	6.4	18.1	21.4	0.41	0.30

[a]Fit to M_ν and M_π. [b]Fit to M_π alone.

In conclusion, we find that the interacting boson model of nuclei can reproduce the effective proton and neutron matrix elements in the Pd isotopes. However, the results are very sensitive to the nuclear structure. In particular, the exact vibrational limit will not give a good fit, but one need only look at the first-order correction to this limit to reproduce the matrix elements. Furthermore, we find that using the simple vibrational formula to extract the boson effective charges may give misleading results if only the effective proton matrix element for the 0_1^+ to 2_1^+ transition is fitted. From the effective proton and neutron matrix elements, an accurate estimate of the ratio e_ν/e_π can be made which does not depend on the detailed nuclear structure but only on the assumption of maximal F spin for the 0_1^+ and 2_1^+ states.

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