R-matrix analysis of the $\beta^{\mp}$-delayed alpha spectra from the decay of ^{8}Li and ^{8}B

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The general question of the analysis of beta-delayed particle emission is discussed. It is emphasized that neither an integrated Fermi function nor a beta branching ratio is, in general, rigorously defined for beta decay to a specific level (resonance). As an example, spectra of breakup α particles observed following ^{8}Li$(\beta^-)^8$Be and ^{8}B$(\beta^+)^8$Be are analyzed by the many-level one-channel approximation of R-matrix theory. In addition, the $L=2$ α-α elastic scattering phase shifts are analyzed up to $E_\alpha=34$ MeV. It is found that satisfactory fits are obtained without introducing intruder states below 26-MeV excitation. Gamow-Teller matrix elements are extracted for decay to the 3.0-MeV first-excited state of ^{8}Be and for the doublet near 16 MeV. The width and location of the ^{8}Be 3.0-MeV state are also obtained. The results are compared to shell-model predictions.

I. INTRODUCTION

Gamow-Teller matrix elements are a valuable source of information on the strong interactions within nuclei. These interactions quench the axial vector coupling constant to $\sim 80\%$ of its free nucleon value. In order to investigate this phenomenon in detail it is desirable to have as full and accurate a set of measured GT (Gamow-Teller) matrix elements as is possible. For this reason a survey of those experimental GT matrix elements obtainable from beta decay in the light ($A \leq 21$) nuclei has been undertaken.[1] A similar survey was recently completed[2] for those states in $A=17$–39 nuclei which belong to the $(2s,1d)$ configurational space.

It become immediately apparent that for a special class of states GT matrix elements have not been extracted from the experimental data with as high a degree of accuracy as is possible; this class being that of beta-delayed particle emitters. That is, those decays which proceed to particle unbound states. In this paper we consider the archetypical examples of ^{8}Li$(\beta^-)^8$Be and ^{8}B$(\beta^+)^8$Be which are followed by the breakup of ^{8}Be into two α particles.

A great deal of very high quality data has been collected for the $\beta^{\pm}$ decays leading to ^{8}Be, all for the purpose of searching for second class currents. Wilkinson and Alburger[3] explored the dependence of $(ft)^+/(ft)^-$ on excitation energy in ^{8}Be, while Garvey and collaborators[4] investigated $\beta^{\pm}$-α angular correlations and also the related ^{4}He$(\alpha,\gamma)^8$Be reaction.[5,6] However, none of these data were analyzed to obtain GT matrix elements.

The data considered in this paper is that of Wilkinson and Alburger.[3] The aim is to extract GT matrix elements for all final states involved from the spectrum of breakup alphas. Important constraints on the parameters of ^{8}Be influencing the breakup alpha spectra are provided by α-α scattering data. Thus, we shall also consider the information provided by previous phase shift analysis of such data.

II. R-MATRIX ANALYSIS

A. General analysis of beta-decayed particle emission

In an important seminal paper, Barker[7] has presented an R-matrix analysis of the α-particle spectra following ^{8}Li and ^{8}B decay which is generally applicable to spectra of single-particle emission following β decay to particle unbound states. The following discussion is based on Appendix III of Barker's paper. We consider beta decay into an energy region of the continuum which, in general, consists of unresolved overlapping resonances which decay by single particle emission. The interference between these overlapping resonances takes us away from our usual approach to nuclear beta decay. Because of this interference, a partial half-life cannot rigorously be defined for a given level (resonance), nor can an integrated Fermi function f_β. Thus we cannot construct an ft value in the usual way, but most work directly in terms of more fundamental parameters; namely, the nuclear matrix elements.

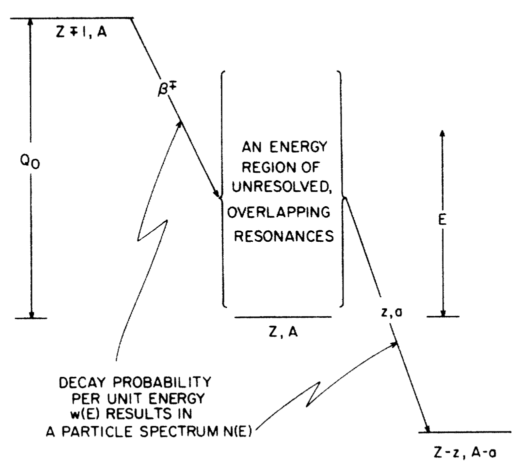

FIG. 1. Schematic of beta-delayed particle emission.

However, an energy region in the daughter nucleus can be chosen so that a branching ratio for this region is well defined. A partial mean life τ [partial half-life $t=(\ln 2)\tau$] can be assigned to the β decay into this region. Then, as indicated in Fig. 1, a beta decay probability per unit energy interval, $w(E)$, with E the excitation energy in the daughter nucleus, leads to a particle spectrum $N(E)$. The transition probability is

$$w=\int_0^\infty w(E)dE \equiv \tau^{-1} \quad (1)$$

and, defining

$$N=\int_0^\infty N(E)dE \ , \quad (2)$$

there is a one-to-one correspondence between $w(E)$ and $N(E)$:

$$N(E)=N\tau w(E) \ . \quad (3)$$

In Barker's treatment, the spectrum $N(E)$ is represented by a many-level one-channel approximation of R-matrix theory. The limitations on the applicability of this approximation are discussed by Barker[7] and by Barker, Hay, and Treacy.[8] It would appear that the limitation to one channel is quite well justified for most cases of beta-delayed particle emission in light nuclei and certainly for beta decay of mass 8 nuclei.

Barker writes:

$$N(E)=C^2N\tau f_\beta P_L \left\{ \frac{\left|\sum_\lambda [g_{\lambda F}\gamma_\lambda/(E_\lambda-E)]\right|^2+\left|\sum_\lambda [g_{\lambda G}\gamma_\lambda/(E_\lambda-E)]\right|^2}{\left|1-(S_L-B_L+iP_L)\sum [\gamma_\lambda^2/(E_\lambda-E)]\right|^2} \right\} \quad (4)$$

with the constant C^2 chosen to satisfy Eq. (1) and all particles assumed spinless. In Eq. (4), f_β is the integrated Fermi function for $E_{\beta^-}=Q_0-E$ or $E_{\beta^+}=Q_0-2mc^2-E$ with Q_0 being the mass difference between parent and daughter. It is assumed that the particle emission proceeds entirely via a single L wave (the generalization to several L waves and to nonzero spin is straightforward) with penetrability $P_L(E)$, shift factor $S_L(E)$, and boundary condition parameter B_L. The sums λ are over a specific set of resonances with resonance energy E_λ and reduced width γ_λ^2. If one chooses B_L appropriately, γ_λ^2 is simply related to the level width:

$$\Gamma_\lambda=2\gamma_\lambda^2 P_L(E_\lambda)/[1+\gamma_\lambda^2\dot{S}_L(E_\lambda)] \ , \quad (5)$$

where the dot indicates the energy derivative.[9] The $g_{\lambda F}$ and $g_{\lambda G}$, with F and G denoting Fermi and Gamow-Teller interactions, respectively, are the weak interaction equivalents of the strong interaction amplitude γ_λ. By analogy to the treatment of radiative capture, the $g_{\lambda x}$ ($x=F$ or G) must be proportional to $M_{\lambda x}$, i.e., the Fermi or GT matrix elements. (It is assumed that higher-order terms are negligible.) The constant of proportionality is derivable from first principles but is also quite simply obtained by taking $w(E)$ to the limit of a particle-bound isolated level (i.e., P_L, S_L, $S_L \to 0$). For such a level Eqs. (1)–(4) lead to

$$w_\lambda(P_L\to 0)=\tau_\lambda^{-1}=\pi C^2 f_\beta(g_{\lambda F}^2+g_{\lambda G}^2) \ , \quad (6)$$

while the usual expression relating f_β, t, and the $M_{\lambda x}^2$ is

$$f_\beta t_\lambda=\frac{B}{M_{\lambda F}^2+M_{\lambda G}^2} \ , \quad (7)$$

where our definition of $M_{\lambda G}^2$ has R, the ratio of the GT and Fermi coupling constants, adsorbed into it.[10] A recent recommendation for the best values for B and R—due to Wilkinson[11]—is

$$B=6166(3)\ \text{sec}, \quad R^{1/2}=1.2635(57) \ . \quad (8)$$

Relating Eqs. (6) and (7) and incorporating the result into Eq. (4) we have

$$N(E)=\left[\frac{Nt}{6166\pi}\right] f_\beta P_L \left\{ \frac{\left|\sum_\lambda [M_{F\lambda}\gamma_\lambda/(E_\lambda-E)]\right|^2+\left|\sum_\lambda [M_{G\lambda}\gamma_\lambda/(E_\lambda-E)]\right|^2}{\left|1-(S_L-B_L+iP_L)\sum_\lambda [\gamma_\lambda^2/(E_\lambda-E)]\right|^2} \right\} . \quad (9)$$

$N(E)$ has the units of counts/(unit energy). In Eq. (9) t is in sec, and the energy unit in $N(E)$ must be the same as that for γ_λ^2 and $E_\lambda-E$.

Equation (9) is the basic formula for analyzing beta-delayed particle spectrum. The fact that it has an absolute normalization traceable to Eqs. (1) and (2) is of central importance and overcomes somewhat the ambiguities due to the interference effects between the overlapping resonances. A computer program was written to perform least-squares fits to Eq. (9) with any subset of the

λ-indexed parameters variable and the rest fixed. The integrated Fermi function $f_\beta(E)$ was evaluated using a subroutine based on the Wilkinson-Macefield series parametrization.[12] The Coulomb functions $P_L(E)$ and $S_L(E)$ were evaluated in a subroutine which called the Manchester Coulomb wave function routine[13] RCWFN. When needed $\dot{S}_L(E)$ was evaluated in this subroutine by numerical differentiation of $S_L(E)$. The best procedure for evaluating B_L and the effect of the uncertainty in it depends on the specific reaction under consideration. The procedure for ^{8}Li and ^{8}B decay will be discussed in Sec. II C.

B. A first orientation from α-α scattering

Before undertaking analysis of the beta-delayed α spectra we briefly consider the information available from α-α scattering. Barker *et al.*[7,8] performed least-squares fits to experimental $L=0$ and 2 phase shifts of

$$\delta_L(E) = -\phi_L + \arctan\left\{ \frac{P_L}{\sum_\lambda [\gamma_\lambda^2/(E_\lambda - E)]^{-1} - S_L + B_L} \right\}, \tag{10}$$

where ϕ_L is the hard-sphere phase shift. Data was included up to a c.m. energy of 17 MeV (above which energy the proton channel opens up and the phase shift becomes complex) and results were presented for values of the channel radius R_c of 5.5–9.0 fm. Three levels were included in both the $L=0$ and 2 fits and a good representation of the data was obtained for all R_c considered. The lowest-lying 0^+ and 2^+ levels are the 0^+ ground state and the 3-MeV state[14] as shown in Fig. 2. Barker found that hitherto unknown states with $E<14$ MeV and with widths comparable to the sum rule limit had to be invoked to obtain satisfactory fits. States of these characteristics would of necessity arise from $n\hbar\omega$ ($n=2,4,\ldots$) excitations of the s^4p^4 configuration.[15,16] If such intruder states are present below 17-MeV excitation it is indeed surprising that they have not manifested themselves in some other reaction.

One of our main interests in the present study is to reexamine the evidence from α-α scattering and beta decay which bears on the energy of this lowest-lying 2^+ intruder state. To this end we have repeated the $L=2$ α-α phase shift fits made by Barker[7] for $R_c=5.5$–9.0 fm and have extended them to $R_c=4.5$ fm. The data considered is essentially the same as detailed by Barker. We find that equally good fits are obtained for all R_c considered. The fit for $R_c=4.5$ fm is shown in Fig. 3. As R_c increases, the excitation energy of the intruder state decreases. This is shown in Fig. 4. Thus we see that it is not necessary to invoke a low-lying intruder state in order to explain the $L=2$ phase shifts. (The same conclusion can also be made for the $L=0$ phase shifts.) The value of R_c expected from the prescription usually used in R-matrix theory is

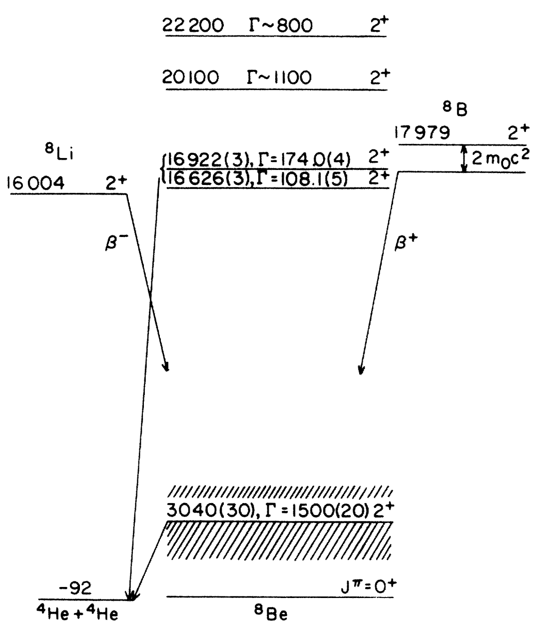

FIG. 2. Decay scheme for ^{8}Li and ^{8}Be beta emission to the known 2^+ levels of ^{8}Be. All data are from Ref. 14. Level energies and widths are in keV. The uncertainty in the last figure is given in parentheses.

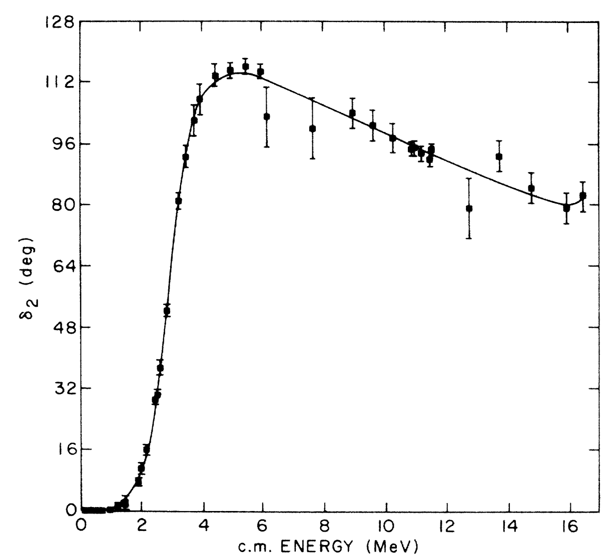

FIG. 3. Least squares fit of Eq. (10) to the experimental $L=2$ phase shifts for α-α scattering detailed in Ref. 7. [One further datum is included; namely, $\delta_2(16\ \text{MeV})=82°$ from A. D. Bacher *et al.*, Phys. Rev. Lett. **29**, 1331 (1972).] Six ^{8}Be 2^+ states were included in the fit; namely, the five shown in Fig. 2 and one further ($\lambda=2$) state. The positions and widths of the four higher-lying states of Fig. 2 were fixed and have very little effect on the phase shift for $E_{\text{c.m.}}<16$ MeV. (The effect of the 16-MeV doublet is just becoming noticeable at the very highest energy.) For the first-excited state ($\lambda=3$) and the intruder state ($\lambda=2$) the results of the fit are $E_3=3024(13)$ keV with $\gamma_3^2=1018(22)$ keV [corresponding to $\Gamma_3=1426(32)$ keV], and $E_2=37(5)$ MeV with $\gamma_2^2=3(1)$ MeV where E_λ is the excitation energy in ^{8}Be. The fit was performed with $B_2=S_2(E_3)$ and gave χ^2 (per degree of freedom)$=0.54$.

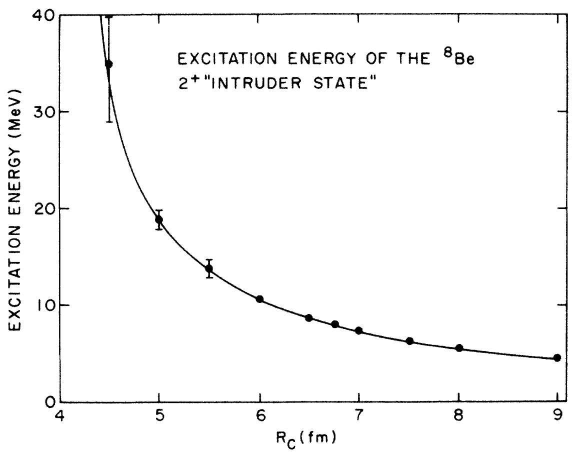

FIG. 4. Results of least squares fits to Eq. (10) similar to that of Fig. 3 as a function of the interaction radius R_c. The excitation energy of the intruder state as obtained from the fits is shown. Points without error bars ($R_c \geq 6$ fm) are from Ref. 7. The present results are in agreement with these and yield the values for $R_c < 6$ fm shown with error bars.

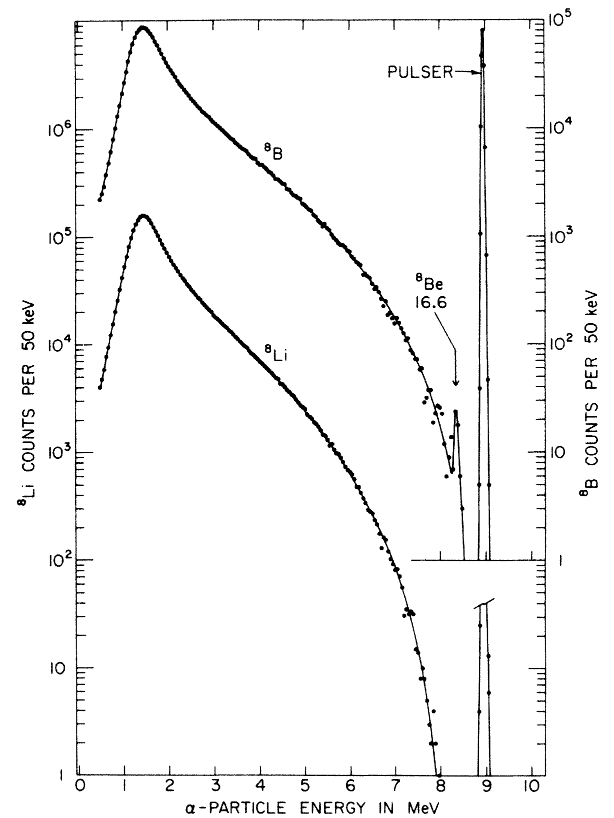

FIG. 5. The original α-breakup spectra of Wilkinson and Alburger (Ref. 3). The solid curves are drawn to guide the eye.

$$R_c \sim r_0(A_1^{1/3}+A_2^{1/3}) , \tag{11}$$

which for α-α scattering and $r_0=1.4$ fm (the value suggested by electron scattering) is $R_c \sim 4.5$ fm. In view of this, the very large R_c of $\sim$6.75–7.0 fm adopted by Barker appears quite surprising. We see that is the use of such a large value of R_c that leads to the postulation of low-lying intruder states.

We now turn to the beta decay data. Our aim is to obtain satisfactory fits to the α spectra for $R_c \sim 4.5$ fm. If this can be done there is no longer any need to invoke a low-lying 2^+ intruder state.

C. ^{8}Li(β^-)^{8}Be and ^{8}B(β^+)^{8}Be

1. The data

The decay schemes of ^{8}Li and ^{8}B are illustrated in Fig. 2. ^{8}Li and ^{8}B have $J^\pi=2^+$ ground states so that allowed decay can proceed to $J^\pi=1^+$, 2^+, or 3^+ states of ^{8}Be.[14] However 1^+ and 3^+ states of ^{8}Be cannot decay into the 2α channel, but since there are known 1^+ or 3^+ states below the β threshold in ^{8}Be, allowed decay is expected to 2^+ states only, with the consequence of a one-to-one correspondence between the beta spectrum $w(E)$ and the α spectrum $N(E)$.

The α-particle spectra of Wilkinson and Alburger[3] collected following ^{8}Li(β^-)^{8}Be and ^{8}B(β^+)^{8}Be are shown in Fig. 5. These are the original data obtained with the "thick" catcher described in Ref. 3. In principle, various experimental distortions should be considered in the comparison with theory. The four most important of these are outlined by Alburger, Donovan, and Wilkinson.[17] The only one of these with a significant influence on the extracted physical observables is that due to energy loss of the α particles in leaving the catcher foil. Correction for this energy loss was made by a perturbation of the energy calibration using the prescription applied by Alburger and Wilkinson[3] to the thin catcher data. The only other serious distortions are those due to the spreading due to the experimental energy resolution (FWHM $\sim$34 keV) and the electron-neutrino recoil (see the Appendix). These two distortions will worsen the agreement between experiment and theory (i.e., increase χ^2) but have an insignificant effect on the derived parameters. Corrections for these two effects were not made. For the ^{8}Li spectrum the total number of counts N is 3.60×10^6, while for ^{8}B $N=2.06\times10^6$. Because of potential use in other applications, the original data of Wilkinson and Alburger and the energy calibration (including energy loss effects) are given in the Appendix where we also illustrate the effect of the electron-neutrino recoil.

2. Treatment of the doublet at 16.6–16.9 MeV

It is well known that the ^{8}Be analog of the ^{8}Li and ^{8}B ground states with $J^\pi=2^+$, $T=1$ is mixed almost completely with a 2^+ $T=0$ state with the same LS (or SU$_3$) symmetry.[14] Thus, the two states at 16626(3) and 16922(3) keV with center-of-mass α widths of 108.1(5) and 74.0(4) keV, respectively, are almost equal mixtures of $T=0$ and 1.[14,18] Again we follow Barker's treatment[7] of these states. The physical states at 16626 and 16922 keV are designated by $\lambda=a$ and b, respectively; the shell-model (unmixed) $2^+,1$ and $2^+,0$ states are designated by $\lambda=1$ and 0, respectively. Two-state mixing is assumed so

$$\Psi_a=\alpha\Psi_0+\beta\Psi_1,\quad \Psi_b=\beta\Psi_0-\alpha\Psi_1 \tag{12}$$

with $\alpha^2+\beta^2=1$. Then, since α decay with $\Delta T=1$ is forbidden,

$$\begin{aligned}&\Gamma_a=\Gamma_b=\Gamma_0,\quad \Gamma_a/\Gamma_b=\alpha^2/\beta^2\,,\\&\Gamma_a/\Gamma_0=\alpha^2,\quad \Gamma_b/\Gamma_0=\beta^2\,;\end{aligned} \tag{13}$$

likewise we take

$$\gamma_a^2=2\alpha^2\bar{P}_2\Gamma_0,\quad \gamma_b^2=2\beta^2\bar{P}_2\Gamma_0,\quad \gamma_0^2=2\bar{P}_2\Gamma_0\,, \tag{14}$$

where $\bar{P}_2=[P_2(E_a)+P_2(E_b)]/2$. The beta-decay matrix elements M_{ax} and M_{bx} ($x=F$ or G) can be written in terms of M_{1F}, M_{0G}, M_{1G} (since $M_{0F}=0$). With these assumptions and definitions Eq. (9) becomes for ^{8}Li and ^{8}B decay

$$N(E)=\left[\frac{Nt}{6166\pi}\right]f_\beta P_2 \times\left\{\frac{\Sigma_F^2+\Sigma_G^2}{\left|1-(S_2-B_2+iP_2)\sum_\lambda[\gamma_\lambda^2/(E_\lambda-E)]\right|^2}\right\}; \tag{15a}$$

the Fermi term is

$$\Sigma_F^2=2\alpha^2\beta^2\gamma_0^2\left[\frac{0.296}{(16.626-E)(16.992-E)}\right]^2, \tag{15b}$$

where $0.296=16.992-16.626$ MeV and we have taken

$$M_{1F}^2=T(T+1)-T_3(T_3+1)=2\,.$$

The GT term is

$$\Sigma_G^2=\left\{M_{0G}\gamma_0\left[\frac{\alpha^2}{(16.626-E)}+\frac{\beta^2}{(16.922-E)}\right]+\left[\frac{0.296\alpha\beta M_{1G}\gamma_0}{(16.626-E)(16.922-E)}\right]+\sum_{\lambda=\text{rest}}\frac{M_{\lambda G}\gamma_\lambda}{E_\lambda-E}\right\}^2. \tag{15c}$$

The Fermi term and the term in M_{1G} are both very small for $E<14$ MeV because they are proportional to the energy separation (296 keV) between the doublet. The matrix element M_{1G} is predicted by shell-model calculations[15] to be very small compared to M_{0G}. Thus, the term in M_{1G} was kept fixed at zero but the uncertainties in other resonance parameters due to the uncertainty in this term were evaluated by taking the Cohen-Kurath 6—16(2BME) value for M_{1G} together with $\alpha\beta=\pm\sqrt{\alpha^2\beta^2}$; i.e., the main uncertainty in this term is assumed to be the ambiguity in the relative sign of α and β.

3. Fitting the α spectra

From the known level scheme of ^{8}Be (Fig. 2) and a cursory inspection of the α spectra of Fig. 5 it is clear that the minimal set of resonances needed to explain the α spectra is the 2^+ level at ~ 3 MeV ($\lambda=3$) and the doublet at 16.6—16.9 MeV. As shown in Fig. 2, the next higher-lying known 2^+ levels in ^{8}Be above the 16-MeV doublet lie at 20.1 and 22.2 MeV. It was found that these states have only a small effect on the α spectra. Nevertheless they were included in the fit with fixed excitation energies and reduced widths (extracted from the level widths assuming $\Gamma_\alpha=\Gamma$). The GT matrix elements were assumed to be the values predicted by the 6—16(2BME) s^4p^4 calculation of Cohen and Kurath[15] but these values were varied when assessing the uncertainties in parameters of other levels.

We have seen that at least one additional level is needed to explain the $L=2$ α-α phase shifts in the beta-delayed α spectra. The influence of higher-lying 2^+ resonances is largely determined by the product of the Fermi function f_β and the α penetrability P_L and for a given resonance is quite insensitive to its position and width. This insensitivity is illustrated in Fig. 6 which shows, for ^{8}B decay, the energy dependence of a *single* isolated resonance of the indicated position and width. Because of this insensitivity higher-lying 2^+ levels can be adequately parametrized by a single resonance—with reduced width to be determined. The same situation applies to ^{8}Li decay. We refer to this resonance as the "intruder state."

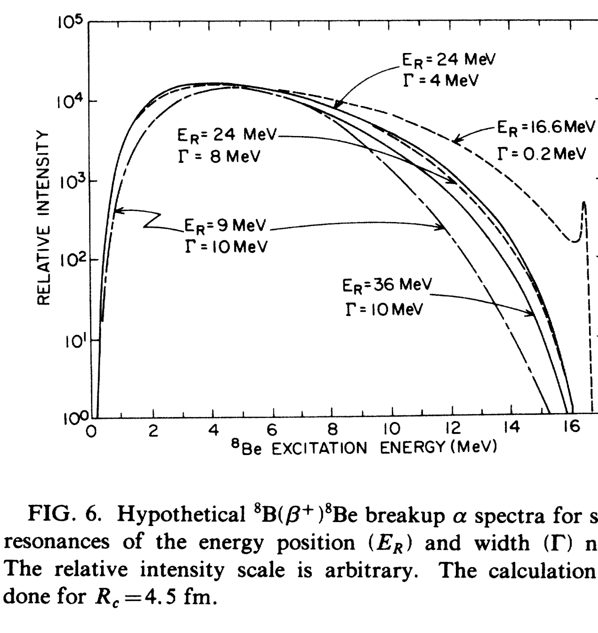

FIG. 6. Hypothetical ^{8}B(β^+)^{8}Be breakup α spectra for single resonances of the energy position (E_R) and width (Γ) noted. The relative intensity scale is arbitrary. The calculation was done for $R_c=4.5$ fm.

4. *Results of the fits*

The least-square fits to the ^{8}Li and ^{8}Be spectra for $R_c = 4.50$ fm are shown in Figs. 7 and 8. The parameters of the fits are summarized in Table I. Uncertainties are not given because the fitting errors are small compared to those resulting from uncertainties in the R-matrix theory itself and in its parameters. The χ_D^2 in Table I is χ^2 normalized to the degrees of freedom. The large values (compared to unity) for these χ_D^2 values is assumed to result from the aforementioned experimental distortions of the α spectra.

Also shown in Figs. 7 and 8 is an attempt to isolate the effects of the three contributions to the Gamow-Teller strength. The three curves, labeled by the resonance(s) they represent, were generated by evaluating $N(E)$ from Eq. (15a) with only one $M_{\lambda G}$ different from zero. Note that this isolation is only approximate because the effect of the other resonances is still felt through the denominator of Eq. (15a). It is apparent from Figs. 7 and 8 and Table I that the fits for ^{8}Li and ^{8}B are very similar. It is also apparent that the Gamow-Teller matrix element for the 16-MeV doublet, M_{0G}, is determined from a considerably larger range of excitation energy than the naive expectation of just the sharp resonance effect in ^{8}B(β^+). In fact it is determined just about as well from ^{8}Li(β^-) as from ^{8}B(β^+).

As already discussed, the results are insensitive to the excitation energy (E_2) of the intruder state. The results of Table I are for E_2 fixed at 37 MeV—the value obtained from the fit to the α-α phase shifts (Fig. 3). The parameters derived from the fits to the phase shifts are also included in Table I.

Similar fits were obtained every 0.5 fm from $R_c = 4.0$ to 8.5 fm with the excitation energy of the intruder state constrained to lie above 26 MeV; the quality of the fits was essentially independent of R_c. For each value of R_c the ^{8}Li and ^{8}B results agreed with each other just about as well as for $R_c = 4.50$ fm (Table I). The extracted GT matrix element for the 3.0-MeV state decreased slowly with increasing R_c and is well parametrized (for ^{8}B decay) by $-M_{3G} = 0.0614 + 0.412/R_c$.

The adopted resonance parameters from ^{8}Li and ^{8}B decay are listed in Table II. The uncertainties assigned in this table contain contributions derived from uncertainties assigned to the following quantities: R_c, E_2, γ_2, B_2, $\alpha\beta M_{1G}$, the α-spectra energy calibration, and the differences between the ^{8}Li and ^{8}B results. It is in the nature of the R-matrix approach that some of these uncertainties are very roughly and subjectively assigned. A ± 0.5 fm uncertainty in R_c was assumed. The uncertainty due to the unknown sign of the contribution from the Gamow-

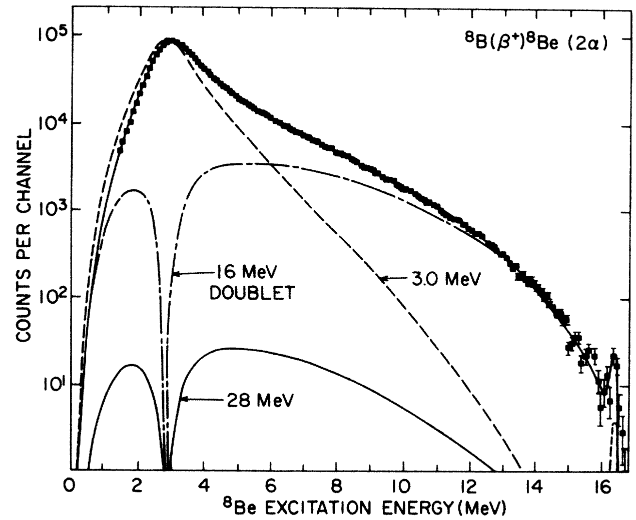

FIG. 8. The least-squares fit to the ^{8}B(β^+)^{8}Be breakup α spectrum for $R_c = 4.5$ fm is illustrated by the solid curve through the experimental points. The three contributors to the total spectra are approximately represented by the appropriately labeled curves (E_λ) as explained in the text.

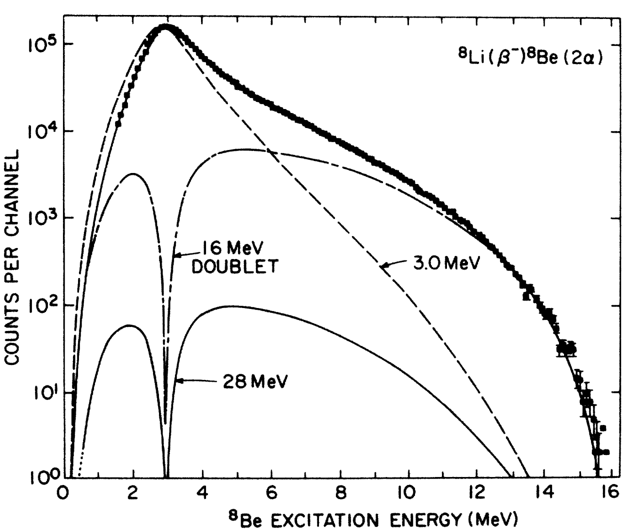

FIG. 7. The least-squares fit to the ^{8}Li(β^-)^{8}Be breakup α spectrum for $R_c = 4.5$ fm is illustrated by the solid curve through the experimental points. The three contributors to the total spectra are approximately represented by the appropriately labeled curves (E_λ) as explained in the text.

TABLE I. Results of least-squares R-matrix analysis at $R_c = 4.50$ fm. All quantities are defined in the text. Parameters in parentheses were held fixed.

Quantity	Units	^{8}Li(β^-)	^{8}B(β^+)	α-α
χ_D^2		1.75	1.68	0.54
E_3	keV	3103	3134	3024
γ_3	keV$^{1/2}$	+35.0	+34.2	31.9
Γ_3	keV	1738	1670	1426
M_{3G}	μ_N	−0.163	−0.152	
M_{0G}	μ_N	+2.69	+2.64	
M_{2G}	μ_N	−0.226	−0.211	
E_2	MeV	(37.0)	(37.0)	37.0
γ_2	keV$^{1/2}$	+80.2	+75.6	56.8

TABLE II. Resume of final parameters determined for ^{8}Li-^{8}B decay. All quantities are discussed in the text.

Quantity	Units	Adopted value
E_3	keV	3120 ± 130
Γ_3	keV	1700 ± 130
γ_3	keV$^{1/2}$	34.6 ± 1.3
M_{0G}	μ_N	2.7 ± 1.1[a]
M_{3G}	μ_N	-0.163 ± 0.011[a]
$M_{3G}\gamma_3/M_{0G}\gamma_0$		-0.46
S_3		0.77
S_0/S_3		0.015
BR_3	%	$\sim80(^8\text{B});91(^8\text{Li})$
BR_0	%	$\sim9(^8\text{B});7(^8\text{Li})$
BR_2	%	$\sim3(^8\text{B});2(^8\text{Li})$

[a]For ^{8}Li(β^-), see the text.

TABLE III. Results of least-squares R-matrix analysis at $R_c=6.75$ fm following Barkers's (Ref. 7) treatment.

Quantity	Units	^{8}Li(β^-)	^{8}B(β^+)
χ_D^2		1.87	1.85
E_3	keV	2709	2722
Γ_3	keV	1480	1460
γ_3	keV$^{1/2}$	18.4	18.2
M_{3G}	μ_N	$+0.116$	$+0.104$
M_{0G}	μ_N	$+1.84$	$+1.68$
M_{8G}	μ_N	-0.195	-0.179
γ_8	keV$^{1/2}$	25.3	25.7
Γ_8	keV	6500	6670
γ_{33}	keV$^{1/2}$	48.0	48.2

Teller strength of the lowest 2^+, $T=1$ state was found to be negligible. A ±50 keV uncertainty in excitation energy seems about reasonable. It was found that M_{0G} is very sensitive to the contributions from higher-lying resonances. Thus, it cannot be obtained very accurately.

In Table II, S_λ refers to the α-spectroscopic factor defined as the ratio of γ_λ^2 to the sum-rule limit $\theta^2=1.5\hbar^2/MR_c$ which, in the present case, is $31.35/R_c^2$ MeV. We list S_0/S_3 since it can be predicted theoretically. The sign of $M_{3G}\gamma_3/M_{0G}\gamma_0$ is also of theoretical interest. As already discussed, branching ratios to individual resonances are not rigorously defined in the present case. Those listed in Table II correspond to the relative areas under the curves in Figs. 7 and 8. Gamow-Teller matrix elements are given for ^{8}Li(β^-)^{8}Be decay. The relationship of these to analogous ^{8}B(β^+)^{8}Be values are discussed in Sec. II C 6 below.

It is clear from the results presented that higher-lying resonances contribute very little to the $\beta^\pm$ decay in a direct way. Thus, the intruder state resonance has only $\sim0.07\%$ of the Gamow-Teller strength of the 16-MeV doublet. On the other hand, γ_2 is quite large. It corresponds to ~3.7 times the sum rule limit. Both these parameters seem reasonable. The Gamow-Teller matrix element represents a sum, $\Sigma M_{\lambda G}\gamma_\lambda/(E_\lambda-E)$, over all higher-lying (i.e., $E_\lambda>20$ MeV) resonances and the signs of the $M_{\lambda G}\gamma_\lambda$ would be expected to be essentially random. By contrast, γ_2 must represent the sum $\Sigma\gamma_\lambda^2/(E_\lambda-E)$ in the denominator of Eq. (15a) and this sum is coherent. Thus, we expect a large effective γ_2 and are not surprised at a small effective M_{2G}.[19]

5. *Comparison to Barker's treatment*

The significant difference between the treatment of Barker and his colleagues[7,8,20] and the present one is that they assumed intruder states (i.e., from $n\hbar\omega$ excitations with $n=2,4,\ldots$) of 0^+ and 2^+ at ~7 and 9 MeV, respectively. Since the data available to Barker was inferior to that used here, we have performed a fit with $R_c=6.75$ fm to the ^{8}Li and ^{8}B data assuming a 2^+ level at 8.1 MeV ($\lambda=8$) with Gamow-Teller strength to be determined and a further level at 33.6 MeV ($\lambda=33$) with no GT strength but with γ_{33} variable. These are the resonance energies found by Barker for $R_c=6.75$ fm in his original fit. The results of these fits are summarized in Table III. It is seen that the fits are just about as good as those of Table I and probably could be improved by minor adjustments. As far as the 3.0-MeV 2^+ state is concerned, the major differences between the two treatments are the resonance energy (a rather strong function of R_c) and the relative sign of $M_{3G}\gamma_3$. We return to this latter point in Sec. II C 7 below.

6. *Mirror asymmetry in ^{8}Li-^{8}B decay*

In mirror Gamow-Teller decay the positron emitter often has an ft value larger than that of the negatron emitter, i.e., $\delta=(ft)^+/(ft)^--1$ is often significantly greater than zero.[21,22] The data of Fig. 5 were originally collected[3] as part of a study of this effect in ^{8}Li-^{8}B decay and its relevance to the possible existence of second class currents (SCC). In brief, second-class currents are expected to cause a linear dependence of δ on $W_0^+ + W_0^-$ where $W_0^\pm$ is the end point energy for the $\beta^\pm$ mirror decay. In ^{8}Li-^{8}B decay with the broad final states forming a continuum of excitation energies, the almost unique opportunity exists to search for such a dependence in a single mirror system. Analysis of the data, of which Fig. 5 represents a part, yielded a constant value for $(ft)^+/(ft)^-$ over the ^{8}Be excitation range ~1.4–8.8 MeV.[3] This result was then interpreted to yield a limit on the second-class current coupling constant.[3] Deviation of δ from unity can also be caused by nuclear effects, so that $\delta=\delta^{\text{SCC}}+\delta^{\text{nucl}}$. The most important nuclear effect is expected to be due to the binding energy difference between the proton in β^+ decay and the neutron in β^- decay.[21,22] This difference will lead to different spatial overlap between the initial and final state for $\beta^\pm$ decay. In the analysis of Wilkinson and Alburger[3] it was implicitly assumed that ^{8}Li-^{8}B decay proceeds 100% to the first 2^+ state. If Barker's analysis or the present one is correct this is not the case. In the present analysis, for instance, as E varies from 1.4 to 8.8 MeV the yield changes from predominantly to the first 2^+ state to predominantly to the 16.6-MeV doublet. At the least, this complicates the interpretation of the δ vs

$W_0^+ + W_0^-$ dependence. In the general case, one would expect a dependence of $\delta^{\rm nucl}$ on the final state (as well as on E for unbound final states). Thus, the existence of significant branching to states other than the first 2^+ state weakens the definiteness with which the Wilkinson-Alburger experiment[3] can be said to delimit the strength of second-class currents.

Subsequent to the results of Ref. 3, studies of $\beta^{\pm}$-α angular correlations have been performed which sharply limit the strength of second-class currents. These experiments are discussed in Ref. 4. If one assumes negligible effects from second-class currents then the Wilkinson-Alburger analysis[3] limits the effect of the final state on $\delta^{\rm nucl}$ and is consistent with no dependence on the final state. This is a possible constraint to be considered in any future calculations of $\delta^{\rm nucl}$ which, incidentally, has not yet been calculated in a totally satisfactory way. With δ assumed to be independent of E the question remains as how to extract a value of $\delta^{\rm expt}$ from experiment. The conventional approach of calculating f^+ and f^- and combining these with half-lives and branching ratios is not adequate because of interference effects as already discussed. Rather $\delta_\lambda^{\rm expt}$ should be obtained as $(M_{\lambda G}^-/M_{\lambda G}^+)^2$. In our analysis $\delta_3^{\rm expt}$ so obtained is quite insensitive to R_c and other effects and we can quote $\delta_3^{\rm expt}=+0.13(1)$. A useful value of the asymmetry $\delta_0^{\rm expt}$ for the 16-MeV doublet cannot be evaluated in this manner since the M_{0G} are not determined with sufficient accuracy. However, the Wilkinson-Alburger analysis[3] implies $\delta_0^{\rm expt}=\delta_3^{\rm expt}$. Assuming this equality one can fit the ^{8}Li and ^{8}B spectra with exactly the same λ-indexed parameters in Eq. (15a) but with Σ_G^2 for the ^{8}Li case multiplied by a variable $1+\delta^{\rm nucl}$ to be determined. This procedure yields an overall $\delta^{\rm expt}=+0.16(3)$, the larger uncertainty being due to the increased sensitivity to resonances at higher excitation energies. The GT matrix elements listed in Table II pertain to ^{8}Li$(\beta^-)^8$Be for which we expect small or negligible nuclear overlap effects. To obtain the best ^{8}B$(\beta^+)^8$Be values, we recommend multiplication by $1+\delta^{\rm expt}=1.14$.

7. *Comparison to Cohen-Kurath predictions*

In this subsection we address the question as to how well the results of our least-squares fit agree with expectations of the shell model. Comparison of our results to the (6—16)2BME predictions of Cohen and Kurath is made in Table IV. Excitation energies are rather poorly predicted. Nevertheless the identification of the experimental and theoretical levels with each other is straightforward and unambiguous. This is so because their symmetries are so different. The three lowest-lying J^π, $T=2^+$, 0 states are predicted to have SU$_3$ symmetries which are predominantly $(\lambda\mu)$LS$=(40)20$, (21)12, and (21)22, respectively, while the lowest 2^+, 1 state is predicted to be mainly (21)12. The nearly degenerate $T=0$ and 1 2^+ states can mix strongly because of their similar symmetries. Beta decay to the lowest 2^+ state proceeds mainly via a small (21)12 admixture in the latter. Assuming pure s^4p^4 configurations, the α-spectroscopic factor is directly proportional to the intensity of $(\lambda\mu)=(40)$ symmetry in the wave function. This leads to the theoretical predictions for $S(2_1^+)$ and $S(2_2^+)/S(2_1^+)$—which is labeled S_0/S_3 in Table III—and R_G—which is equal to $M_{3G}\gamma_3/M_{0G}\gamma_0$ (Table II).

TABLE IV. Comparison of the ^{8}Li-^{8}B decay results to the shell-model predictions of Cohen and Kurath (Ref. 15). All quantities are explained in the text.

Quantity	Units	Present analysis	Cohen-Kurath (6—16)2BME
$E(2_1^+,0)$	MeV	3.12(13)	3.41
$E(2_2^+,0)$	MeV	$\sim$16.8	14.43
$E(2_1^+,1)$	MeV	$\sim$16.8	15.80
$E(2_3^+,0)$	MeV	20.1[a]	17.66
$S(2_1^+)$		0.77	0.98
$S(2_2^+)/S(2_1^+)$		0.015(5)[b]	0.01
$\lvert M_G(2_1^+,0)\rvert$	μ_N	0.163(11)[c]	0.188
$\lvert M_G(2_2^+,0)\rvert$	μ_N	2.7(11)[c]	1.94
R_G		$-0.46(8)$[b]	-0.95

[a]From Ref. 14.
[b]The uncertainty is that in R_c only, namely ± 0.5 fm.
[c]^{8}Li(β^-), see the text.

We have defined the spectroscopic factor S as the ratio of γ^2 to the sum rule limit. The latter is only roughly defined as the single-particle value. In this region of A, it is empirically found[23] that a more realistic single-particle reduced width is $\sim$0.7 times the sum rule limit. Thus, the observed value of $S(2_1^+)$ is in satisfactory agreement with expectations. The agreement in magnitude for $S(2_2^+)/S(2_1^+)$ and R_G can be considered as satisfactory. We also remark that all three quantities are rather strong functions of R_c and the agreement worsens with increasing R_c. The sign of R_G is of vital importance.[24] This sign is a strong prediction of the theory, resulting from quite fundamental properties of the effective interaction. Experimentally, it is also well determined. It is reassuring that this sign is correctly predicted.

By contrast, the sign of R_G is positive if, as in Barker's treatment, an intruder state is assumed at $\sim$8-MeV excitation. We take this as an argument against this assumption. Briefly, since the lowest 2^+, 0 shell-model state has an intrinsically large α width, an intruder state carrying no intrinsic GT strength of its own would not be likely to affect the sign of $\gamma_3 M_{3G}$. A weaker statement can be made for the susceptibility of the sign of $\gamma_0 M_{0G}$ to change via mixing of the s^4p^4 wave functions with that of an intruder state. We note, however, that if such a mixing took place then the two-state mixing assumed for the 16-MeV doublet by Barker (and adopted here) is invalid. Finally, the predicted and observed GT matrix elements are in fair agreement, especially if the shell-model predictions are quenched by the $\sim$0.8 factor found to pertain[2] in the (s,d) shell.

III. SUMMARY

The many-level, one-channel R-matrix approximation developed by Barker[7,8] is shown to provide an excellent representation of the high-quality α-spectra collected by Wilkinson and Alburger[3] in 1971. Our treatment assumes only those 2^+ states below the $\beta^{\pm}$ thresholds which are identifiable with states of s^4p^4 predicted, for example, by Cohen and Kurath.[15] It is shown that a good approximation for the effects of higher-lying 2^+ states is to invoke a single level somewhere above 26 MeV, the exact resonance energy of which is not very important. The accuracy of this approach is scrutinized with some care and uncertainties are assigned to cover ambiguities in the theory and our ignorance of the structure of higher-lying levels. Good fits were obtained for $R_c = 4.0-8.5$ fm.

Fits were also made with Barker's assumption of an intruder 2^+ state between 3 and 16 MeV. The fits for $R_c = 6.75$ fm made assuming such an intruder state are just as good as those for our assumptions. *The question as to the existence or nonexistence of a 2^+ intruder state below 17-MeV excitation cannot be decided on the basis of the goodness-of-fit to the ^{8}Li and ^{8}B α spectra alone.*

The interaction radius R_c. The classic approach[25] to choosing the interaction radius in R-matrix theory is to fix it just large enough so that no nuclear interactions occur for $r > R_c$. As R_c is increased from this value, usually taken as that of Eq. (11), some strength will be lost to the interior. Thus, the γ_λ and $M_{\lambda X}$ would be expected to decrease with increasing R_c as is observed. We are thus prejudiced toward an R_c near 4.5 fm.

We have seen that an R-matrix fit to the α-α shifts reveals a one-to-one relationship (Fig. 4) between the excitation energy of the lowest-lying intruder state and R_c. Since there does not seem to be any other evidence for a 2^+ intruder state below 17 MeV, we favor $E > 17$ MeV for it and thus, from Fig. 4, $R_c \lesssim 5.0$ fm.

How do the results of our fits bear on the choice of R_c? With our assumptions, the relative sign of R_G—the ratio of the product of the reduced width amplitude γ and the GT matrix element for the 3.0-MeV state and for the $T=0$ part of the 16-MeV doublet—is found experimentally to be negative in agreement with expectations of the s^4p^4 calculations (see Table IV). If a 2^+ intruder state is assumed between 3.0 and 16 MeV then this sign is positive (see Table III). We take this as a fairly strong argument against the presence of such an intruder state, and thus for $R_c \sim 4.5$ fm. As R_c increases, R_G, $S(2_1^+)$, and $S(2_1^+)/S(2_2^+)$ all decrease, thus widening the disagreement with the predictions (Table IV). Thus, this comparison also favors a low value of R_c.

The 2^+ level parameters. We have not as yet discussed the comparison between the results of the $\beta^{\pm}$ α spectra and α-α phase shift fits which is summarized in Table I. We first comment on the reduced width of the intruder state. We have emphasized that this parameter is an effective one since the effects of all higher-lying states is represented by this one state. Then, since the effect of higher-lying states is differently weighed in the $\beta^{\pm}$ and α-α cases, there is no *a priori* reason for γ_2 to be the same in the two cases. That it is rather similar would suggest that indeed the bulk of the effect is actually due to one state somewhere near 25–45 MeV in excitation.

The one unexplained problem in these fits is the discrepancy between the $\beta^{\pm}$ and α-α fits for the resonance energy and width of the 3.0-MeV resonance. This discrepancy persisted despite attempts to resolve it by (a) varying the energy calibration of the $\beta^{\pm}$ α spectra and (b) fixing E_3 and γ_3 in the $\beta^{\pm}$ fits. (It should be noted that the spreads in the $M_{\lambda G}$ produced by these exercises were incorporated into the uncertainties quoted for these matrix elements.) One suspects that the difficulty lies in the model dependence with which E_3 and γ_3 are extracted from the $\beta^{\pm}$ α spectra. This model dependence is evident in Figs. 7 and 8 where it can be seen that interference effects have a strong influence on the position and width of the 3.0-MeV resonance. In any case it is expected that the position and width of such a broad resonance extracted by R-matrix theory will depend somewhat on how it is observed.

The $\beta^{\pm}$ Gamow-Teller matrix elements. The GT matrix elements for ^{8}Li and ^{8}B decay to the 3.0-MeV resonance were extracted with an accuracy of $\sim 8\%$. The uncertainty is due mainly to ignorance of the proper interaction radius and to unknown effects of higher-lying resonances. The value of 0.163(11) obtained for ^{8}Li decay corresponds to $\log ft = 5.37$ [Eq. (7)]. We emphasize that log ft cannot be calculated in the conventional way by combining an f value with a partial half-life. This is so because neither is rigorously defined. However, an approximate ft value can be formed from an approximate branching ratio and an approximate $\bar{f}$ value (obtained by averaging over the resonance, e.g., the 3.0-MeV curve of Fig. 7). For ^{8}Li decay to the ^{8}Be 3.0-MeV resonance we find in this way ($R_c = 4.5$ fm)

$$\log \bar{f}t = \log[(2.809\times 10^5)(0.838)/(0.90)] = 5.42$$

in fair agreement with the rigorous value of 5.37. Note that $\bar{f}$ differs considerably from the value of 4.5×10^5 one finds for $E = 3.10$ MeV.

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APPENDIX A

The energy calibration used for the Wilkinson-Alburger data is

$$E_\alpha(\text{keV}) = +66.0 + 48.204I + 0.0025I^2 , \quad \text{(A1)}$$

$$E(\text{keV}) = +40.0 + 96.408I + 0.0050I^2 , \quad \text{(A2)}$$

for ^{8}B decay and

TABLE V. The α-spectra of Wilkinson and Alburger collected following $^8Li(\beta^-)$ and $^8B(\beta^+)$ decay.

		$^8Li(\beta^-)^8Be$						$^8B(\beta^+)^8Be$			
I	$N(I)$	I	$N(I)$	I	$N(I)$	I	$N(I)$	I	$N(I)$	I	$N(I)$
1	(0)	59	22 220	117	981	1	(0)	59	13 675	117	931
2	(3)	60	21 045	118	968	2	(2)	60	13 107	118	879
3	(14)	61	20 177	119	897	3	(9)	61	12 310	119	848
4	(44)	62	18 867	120	814	4	(27)	62	11 780	120	843
5	(108)	63	17 826	121	737	5	(63)	63	11 201	121	813
6	(224)	64	17 316	122	684	6	(127)	64	10 883	122	751
7	(412)	65	16 622	123	659	7	(227)	65	10 243	123	740
8	(699)	66	15 783	124	623	8	(380)	66	9 760	124	661
9	(1 115)	67	15 016	125	562	9	(597)	67	9 357	125	628
10	(1 694)	68	14 431	126	479	10	(899)	68	8 880	126	587
11	(2 480)	69	13 687	127	478	11	(1 307)	69	8 538	127	572
12	(3 525)	70	13 119	128	421	12	(1 847)	70	8 112	128	552
13	(4 893)	71	12 443	129	374	13	(2 553)	71	7 936	129	442
14	(6 665)	72	11 865	130	339	14	(3 466)	72	7 468	130	453
15	(8 938)	73	11 232	131	293	15	(4 637)	73	7 182	131	433
16	(11 839)	74	10 759	132	282	16	(6 129)	74	6 836	132	419
17	15 548	75	10 198	133	274	17	8 078	75	6 633	133	364
18	20 127	76	9 731	134	239	18	10 160	76	6 466	134	327
19	25 830	77	9 425	135	215	19	13 332	77	6 223	135	339
20	33 103	78	8 796	136	177	20	16 850	78	5 841	136	307
21	42 201	79	8 339	137	130	21	21 889	79	5 535	137	263
22	53 540	80	8 169	138	162	22	27 380	80	5 376	138	227
23	66 693	81	7 663	139	157	23	34 580	81	5 348	139	252
24	82 426	82	7 333	140	120	24	43 097	82	4 863	140	188
25	99 424	83	6 913	141	102	25	52 082	83	4 663	141	197
26	117 067	84	6 712	142	92	26	61 866	84	4 664	142	174
27	132 633	85	6 363	143	81	27	70 531	85	4 414	143	157
28	145 536	86	6 060	144	83	28	78 155	86	4 250	144	178
29	155 627	87	5 740	145	71	29	83 929	87	4 033	145	159
30	159 809	88	5 553	146	56	30	87 165	88	3 907	146	140
31	158 810	89	5 221	147	31	31	87 574	89	3 715	147	128
32	154 474	90	4 969	148	35	32	85 836	90	3 424	148	109
33	145 942	91	4 807	149	32	33	81 929	91	3 370	149	113
34	135 820	92	4 374	150	34	34	77 178	92	3 273	150	89
35	125 911	93	4 246	151	32	35	71 379	93	3 105	151	82
36	114 012	94	4 095	152	15	36	65 140	94	3 060	152	73
37	104 749	95	3 811	153	14	37	59 950	95	2 788	153	73
38	95 225	96	3 662	154	8	38	54 825	96	2 715	154	59
39	86 815	97	3 459	155	10	39	50 497	97	2 540	155	60
40	79 542	98	3 307	156	8	40	45 591	98	2 415	156	29
41	72 387	99	3 137	157	5	41	42 134	99	2 396	157	32
42	66 753	100	2 930	158	3	42	38 740	100	2 297	158	38
43	61 222	101	2 738	159	2	43	35 765	101	2 279	159	38
44	56 719	102	2 626	160	4	44	33 129	102	2 034	160	19
45	52 731	103	2 545	161	2	45	31 067	103	1 972	161	23
46	48 964	104	2 279	162	0	46	28 612	104	1 887	162	27
47	45 122	105	2 131	163	0	47	26 660	105	1 798	163	26
48	42 482	106	2 081	164	0	48	25 135	106	1 757	164	23
49	40 169	107	1 928	165	0	49	23 481	107	1 589	165	12
50	37 144	108	1 853	166	0	50	22 090	108	1 532	166	6
51	35 484	109	1 745	167	0	51	20 821	109	1 421	167	9
52	32 823	110	1 633	168	0	52	19 408	110	1 386	168	14
53	31 170	111	1 474	169	0	53	18 498	111	1 255	169	7
54	29 401	112	1 468	170	0	54	17 546	112	1 321	170	24
55	27 648	113	1 337	171	0	55	16 671	113	1 210	171	18
56	26 714	114	1 178	172	0	56	15 836	114	1 168	172	6
57	24 872	115	1 215	173	0	57	14 850	115	1 054	173	3
58	23 489	116	1 066	174	0	58	14 228	116	987	174	1

$$E_\alpha(\text{keV}) = +52.0 + 48.452I + 0.0017I^2 \,, \quad \text{(A3)}$$

$$E(\text{keV}) = +12.0 + 96.904I + 0.0034I^2 \,, \quad \text{(A4)}$$

for ^{8}Li decay. In Eqs. (A1)–(A4), I is the channel number, E_α the α energy, and E the ^{8}Be excitation energy. The uncertainty in E is estimated as $\sim$30 keV, roughly independent of E.

The experimental data are listed in Table V. For both spectra the original data extended from channels 11-174 but for channels 11-16 the distortion due to energy losses was too severe to be corrected adequately. Thus fits were performed to channels 17-174. The numbers in channels 1-16 (in parenthesis) of Table V are the calculated results of Figs. 7 and 8. The total summed counts over channels 1-174 is 3597 314 for ^{8}Li(β^-) and 2060 131 for ^{8}B(β^+).

The experimental data are subjected to a smearing due to both the experimental energy resolution—which had a full-width-at-half-maximum (FWHM) of $\sim$34 keV—and to the spread in α energies due to the electron neutrino recoil. This recoil converts the delta function in α-particle energy $E_{\alpha 0}$ for given ^{8}Be excitation energy into a spectrum $P(E_\alpha)$. Wilkinson[2,17] has derived an expression for $P(E_\alpha)$ appropriate for Gamow-Teller decay and valid when $E = pc$ is an adequate approximation for the betas. This expression is

$$P(E_\alpha) = \tfrac{5}{6}(1 - 3\phi + 2\phi^2)(2E_{\alpha 0}E_m)^{-1/2} \,, \quad \text{(A5)}$$

where E_m is the maximum recoil energy imparted to the recoil ^{8}Be for $E = 2E_{\alpha 0} - 91.78$ keV and $\phi = E_m^{-1}(E_\alpha^{1/2} - E_{\alpha 0}^{1/2})^2$. The $P(E_\alpha)$ distribution extends from $\phi = -\frac{1}{2}$ to $+\frac{1}{2}$, peaks at $\phi = 0$, i.e., $E_\alpha = E_{\alpha 0}$, and is symmetric with half-maximum values at $\phi_{1/2} = \pm(3 - 5^{1/2})/4$. If $E_{\alpha 02}$ is defined as the α-particle energy for a given $E_{\alpha 0}$ at $\phi_{1/2}$, the full-width-at-half-maximum (FWHM) is $\Delta_{1/2} = 2(E_{\alpha 02} - E_{\alpha 0})$. This parameter is plotted versus E in Fig. 9 for ^{8}B β^+ decay. Because the observed α spectra are slowly varying with energy compared to this distribution it has small effect on the present application. For instance, the FWHM of this distribution at $E = 3$ MeV is $\sim$170 keV which is about $\frac{1}{10}$ of the width of the 3-MeV level. Thus its effect on the extracted level width is $\sim$0.5%. The experimental resolution also has negligible effect on the spectrum except at highest values of E. Here the rapidly decreasing integrated Fermi function narrows the anomaly due to the 16-MeV doublet in ^{8}B decay so that there the experimental resolution has a noticeable effect (but not on the number of counts in that energy region and thus not on the derived GT matrix elements).

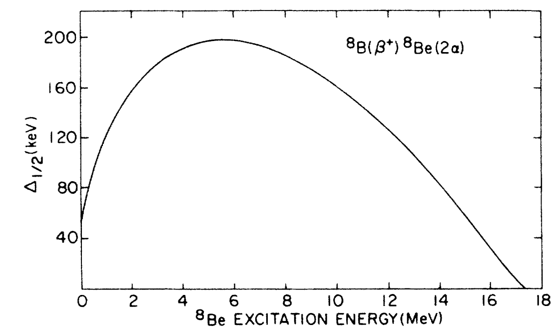

FIG. 9. The full-width-at-half-maximum, $\Delta_{1/2}$, of the distribution in α energy due to the spread in recoil energies associated with the $\beta^\pm$-γ recoil and the 2α breakup.

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[9]Actually, it is not possible to choose B_L so as to render Eq. (5) applicable to more than one level. In practice, therefore, there is an uncertainty in the extracted physical observables due to ambiguities in the choice of B_L. This uncertainty can be evaluated by varying B_L over a reasonable range of values. The reader is referred to Ref. 8 for a detailed discussion.

[10]Our M_F^2 and M_G^2 are then the same as the $B(F)$ and B(GT) of Ref. 2.

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