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Thermalization in ultrarelativistic heavy ion collisions: Parton cascade approach

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A parton cascade model is formulated for ultrarelativistic heavy ion collisions. The model is used to determine the degree of thermalization, as well as energy and baryon number densities in central collisions of heavy ions at ultrarelativistic energies. It is shown that the energy densities required for quark-gluon plasma formation can easily be reached at $\sqrt{s_{\rm NN}}$ less than 100 GeV.

One of the important questions regarding an accelerator based search for the quark-gluon plasma has been whether the energy densities achievable in an ultrarelativistic heavy ion collision will be sufficiently large to overcome the latent heat associated with the transition. Cross sections both for hadronic processes such as NN scattering, and for the Coulomb-like scattering among their parton constituents argue that thermalization of the total center of mass energy available in a high energy collision is not possible. Estimates of the degree of thermalization based on analysis of cosmic ray data,[1] inside-outside cascade approach,[2] and other models[3] confirm this conclusion.

In order to provide a numerical simulation of the time evolution of ultrarelativistic collisions, we wish to formulate a parton cascade model (classical, independent parton-parton collisions) based on perturbative QCD. Such a formulation will have obvious limitations in its applicability to such nonperturbative problems as pion multiplicities and entropy production in the hadronization region. Nevertheless, questions such as the degradation of the hard momentum components of the hadronic wave functions, large $p_\perp$ phenomena, etc., are within its scope. The application with which we wish to concern ourselves here (others will be presented elsewhere) is the mass and bombarding energy dependence of energy and baryon number densities in the central region.[4]

Although details of the code will be presented elsewhere, at least an outline of the ingredients is required here. The calculations presented are for equal mass target and projectile and are viewed in the center of mass frame. All results presented are for zero impact parameter. For the initialization, the quarks and gluons are randomly assigned a fraction x of the center of mass momentum per nucleon according to the distributions[5]

$$xq(x)=1.10(1+3.7x)(1-x)^{3.19} \ ,$$
$$xg(x)=2.62(1+3.5x)(1-x)^{5.9} \ ,$$

where $q(x)dx$ and $g(x)dx$ are the number of quarks or gluons in an x interval dx. Three quarks, five gluons, and no antiquarks were assigned to each nucleon, so that the x region $0.02 \lesssim x \leq 1$ was covered for the partons. For this x range, the integrated antiquark distribution was sufficiently small that their number was set equal to zero in the initialization. The perpendicular momenta $p_\perp$ were assigned according to a normal distribution[6] with variance of 180 MeV/c. Nucleons are assigned initial positions randomly throughout the volume of the target and projectile. Partons are randomly assigned positions within a sphere of radius 1 fm around the nucleon positions. These positions are then Lorentz contracted with respect to the center of mass frame chosen for this study. Next, the parton positions are smeared out[7] with a normal distribution as well, with variance equal to the reduced de Broglie wavelength squared. However, for the x region considered here, this is not a large effect.

The partons follow classical trajectories until they collide. What might be called "internal" parton collisions (defined here as $\sqrt{s} < 2$ GeV) in the thermalization phase of the reaction are ignored. The distance of closest approach of two trajectories is defined as the point where the product of the relative velocity and separation of a given parton pair is zero. As is usual in the cascade approach, if this separation distance is within an area specified by the cross section chosen for the pair, a collision occurs. The cross sections chosen are the lowest order $2\rightarrow 2$ processes from QCD summarized in Ref. 8. Only the dominant term at small four-momentum transfer t was kept; to avoid the Coulomb-like singularity, the cross section was put equal to a constant below a cutoff t determined by a cutoff momentum p_0 (set equal to $\frac{1}{2}$ GeV/c).

The scattered partons were distributed in t and mass (m), the latter according to m^{-2}. Once a parton has been scattered off shell, it decays with a lifetime given by $\hbar/\alpha m$. At the time step when this parton decays, it produces two collinear partons which carry off fractions z and $(1-z)$ of its summed energy and momentum $E+|p|$. The z distributions are taken from Altarelli and Parisi,[9] with the singularity at $z=1$ avoided by only allowing decays between z_c and $1-z_c$, where $z_c=p_0/(E+|p|)$. Lastly, the QCD constant α_s was set equal to $\frac{1}{2}$ throughout. Run on a mainframe at 5×10^6 instructions/sec, one event of a $(A=50)+(A=50)$ central collision typically required 0.5 central-processor-unit hours. The code is now being vectorized.

A sample of the results obtainable from the model is shown in the figures. The first question which we wish to address is the thermalization time scale. We will concentrate on what happens in the central region in coordinate space (defined here as a cylinder 1 fm in length, 3 fm in radius centered at the center of mass). The antiquark energy density is a good measure of the degree of thermalization since it is initially zero. Figure 1 shows that the energy density of antiquarks rises rapidly as the nuclei begin to overlap and reaches a maximum at 4×10^{-24} sec for a central $(A=50)+(A=50)$ collision at $\sqrt{s_{\rm NN}}=40$ GeV. Although the number of both quarks and antiquarks is increasing, their difference is decreasing, also as shown in Fig. 1, lead-

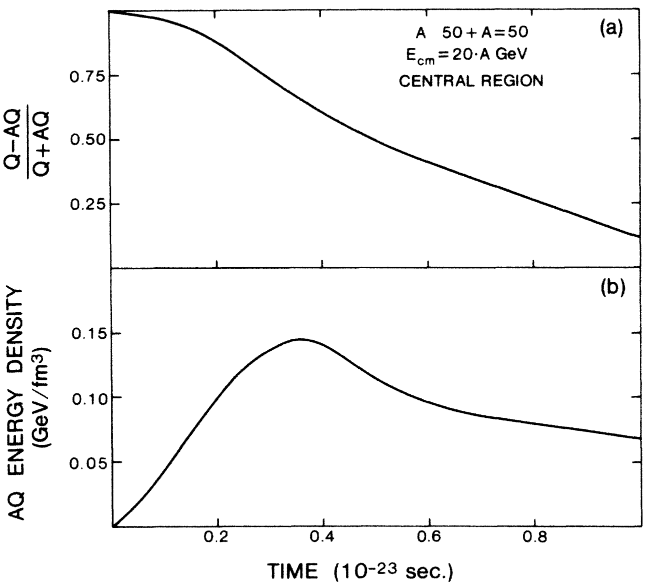

FIG. 1. Time evolution of a central $(A=50)+(A=50)$ collision at $\sqrt{s_{NN}}=40$ GeV. See text for the definition of the central region. (a) Comparison of quark (Q) and antiquark (AQ) number densities. (b) Antiquark energy density.

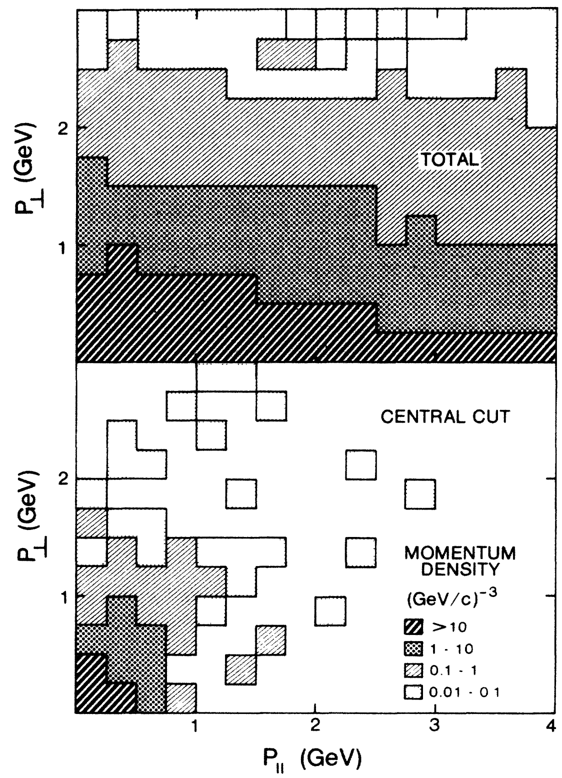

FIG. 2. Momentum distribution of gluons in the same reaction as Fig. 1 after 6×10^{-24} sec. Distributions for all of the gluons, and for only those subject to a central cut in coordinate space, are shown.

ing to a baryon number number depleted region. Hence, a time scale of $\frac{1}{2}\times10^{-23}$ sec is indicated for the peaks in the energy density. It should be remarked that the antiquark energy density shown in Fig. 1 is somewhat underpredicted, in that several reactions which should contribute to it have been omitted since they involve lower powers of t than those retained here.

To see whether the gluons, which carry much of the momentum, are reasonably thermalized, we show their distribution in momentum space in Fig. 2. The reaction is the same as in Fig. 1 with the reaction stopped at 6×10^{-24} sec. The top part of Fig. 2 shows the distribution of all of the gluons (an average of 1500 per event; 40 events were summed over) in the directions perpendicular and parallel to the collision axis. One can see that there is a strong longitudinal component to the distribution. However, if one makes a central cut in coordinate space, the distribution is much more isotropic. The cut used in the lower part of Fig. 2 is a cylinder 2 fm in length and 1 fm in radius centered on the center of mass. This region should be at rest with respect to the center of mass frame chosen for the calculations. Indeed, it can be checked numerically that the net velocity of the partons in this cell is very small. Enlarging the coordinate space cut in the perpendicular direction introduces more of a perpendicular component to the distribution, corresponding to radial flow.

Having established that the central region is fairly thermalized, the major question is what is the energy density. For the reaction shown in Figs. 1 and 2, the energy density associated with the gluons in the central region is 5 GeV/fm^3. In this code, chemical equilibrium has not been established among the species by 5×10^{-24} sec, and most of the energy is carried by the gluons. However, this value is still more than the few GeV/fm^3 which it is argued is required for plasma formation.[3] Larger energy densities can be achieved by either increasing the energy or mass. Exam-

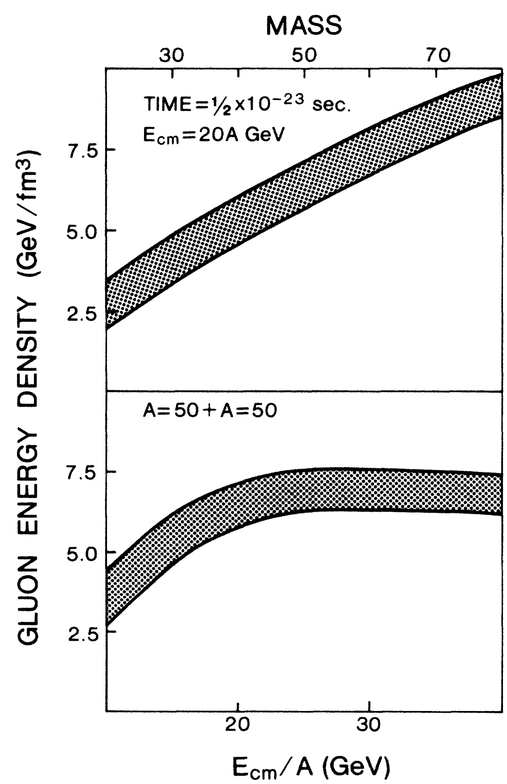

FIG. 3. Mass and bombarding energy dependence of the gluon energy density in the central region.

ples are shown in Fig. 3 for the gluon energy density in the same central region as defined in Fig. 1, with the reaction stopped at 5×10^{-24} sec. The top part of the figure illustrates the mass dependence (equal projectile and target masses) for a $20A$ GeV+$20A$ GeV central collision. Calculations were carried out to $A=100+A=100$, beyond which execution time became prohibitively long (3 central-processor-unit hours/event at 5×10^{6} instructions/sec). The cross hatched region represents the uncertainty caused by using only a finite number of events. By using large nuclei, energy densities in excess of 10 GeV/fm^3 can be achieved. In the calculations performed so far, the dependence of the energy density on bombarding energy is not so pronounced. For an $(A=50)+(A=50)$ collision, the energy density appears to flatten out above $20A$ GeV in the center of mass, as shown in the lower portion of Fig. 3.

To summarize, using the parton cascade approach, we have demonstrated that the central region in coordinate space achieves approximate thermalization after $\frac{1}{2}\times10^{-23}$ sec in an ultrarelativistic collision. The region is baryon number depleted, particularly when comparing the net baryon number density with the gluon number density. The energy densities achievable appear to be quite adequate for the formation of the quark-gluon plasma state, particularly for projectile/target masses in excess of 100 and $\sqrt{s_{NN}}$ above 40 GeV.

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[2]L. McLerran and K. Kajantie, Nucl. Phys. **B240**, 261 (1983).

[3]For reviews, see *Quark Matter '84,* edited by K. Kajantie (Springer, Berlin, 1985).

[4]A preliminary version of this work was presented in the Proceedings of the Workshop on Experiments for a Relativistic Heavy Ion Collider, Upton, New York, edited by P. Haustein and G. Woody (unpublished).

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[7]Suggested by R. Hwa and L. McLerran (private communication).

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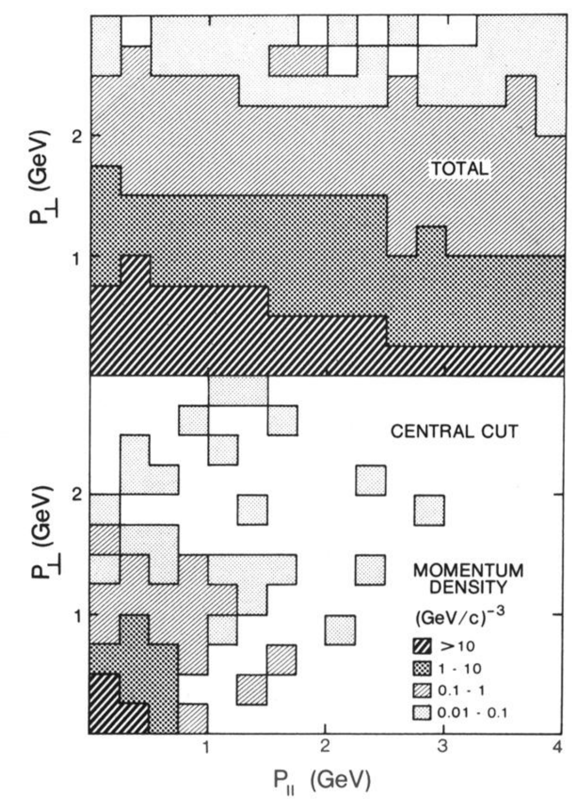
[9]G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).



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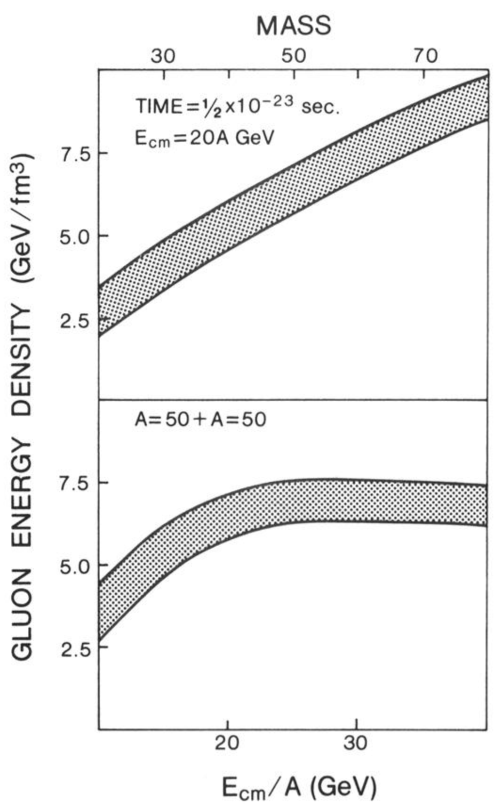

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