Valence proton orbit radii and total rms charge radii in the tin region

Rupayan Bhattacharya
City College, Calcutta-9, India and Saha Institute of Nuclear Physics, Calcutta-9, India
(Received 20 January 1986)

Orbit radii of $1g_{9/2}$, $2p_{1/2}$, and $2p_{3/2}$ proton orbitals of even isotopes of tin and their total charge radii have been investigated through an optimized average one-body potential. Fairly good agreement with the experimental results has been obtained. Constancy of orbit radii with separation energy, in contradiction to Moalem's observation, has been achieved.

In recent times the details of the nuclear interior have been revealed through several subtle experiments like sub-Coulomb nucleon transfer reactions and the magnetic electron scattering technique. In these types of experiments quite extensive results regarding rms orbit radii of shell model states in several nuclei have been obtained. There are mainly two types of theoretical approaches to explain the rich variety of data thus obtained—the mean field theory and the phenomenological theory. However, though mean field theories are rigorous in content, they fail to reproduce the valence orbit radii and total rms charge radii of the nuclei. It is therefore interesting to see how the picture emerges from the point of view of an average one body potential which brings out the location of the proton single-particle (-hole) states in the lead region with a fair amount of accuracy.

The motivation of the present paper lies in an extensive work done on the even isotopes of tin by Warwick *et al.*[1] One remarkable feature of that work is the near constancy of the orbit radii of $1g_{9/2}$, $2p_{1/2}$, and $2p_{3/2}$ orbitals of even tin isotopes, though the separation energy changes by ~ 4.5 MeV as we go from ^{112}Sn to ^{124}Sn. From our viewpoint, one possible explanation is that the increase in separation energy means an increase in the depth of the central potential, resulting in an inner shift of the wave function, thus diminishing the orbit radius. But the orbit radius is also a function of the mass number of the isotope. This mass dependence, having a positive gradient for increasing mass, will try to neutralize the squeezing effect.

The starting point of our calculation is the derivation of a set of average one body potential parameters in the lead region, because of the fact that the single particle shell model states have been extensively studied in that region. We have taken a potential of the form

$$V(r) = -V_0 f(r) + V_C(r) + \left[\frac{\hbar}{m_P}c\right]^2 V_s \frac{1}{r}\frac{df_{so}}{dr}(\mathbf{L}\cdot\mathbf{S}) , \quad (1)$$

where

TABLE I. Comparison of calculated rms point radii of proton orbitals with the experimental and HF values.

Nucleus	Orbit	Experimental radius (fm)	Calculated radius (fm) Present work	HF
^{112}Sn	$1g_{9/2}$	$4.95^{+0.17}_{-0.12}$	5.08	5.16±0.04
	$2p_{1/2}$	$4.51^{+0.17}_{-0.12}$	4.51	4.59±0.04
	$2p_{3/2}$	$4.54^{+0.15}_{-0.11}$	4.50	4.57±0.04
^{116}Sn	$1g_{9/2}$	$5.01^{+0.16}_{-0.11}$	5.11	5.18±0.04
	$2p_{1/2}$	$4.46^{+0.15}_{-0.11}$	4.51	4.60±0.04
	$2p_{3/2}$	$4.55^{+0.14}_{-0.10}$	4.50	4.57±0.04
^{118}Sn	$1g_{9/2}$	$5.00^{+0.15}_{-0.14}$	5.12	5.21±0.04
	$2p_{1/2}$	$4.52^{+0.14}_{-0.11}$	4.51	4.60±0.04
	$2p_{3/2}$	$4.46^{+0.14}_{-0.12}$	4.50	4.58±0.04
^{120}Sn	$1g_{9/2}$	$4.99^{+0.16}_{-0.13}$	5.14	5.22±0.04
	$2p_{1/2}$	$4.49^{+0.14}_{-0.11}$	4.51	4.63±0.04
	$2p_{3/2}$	$4.57^{+0.14}_{-0.11}$	4.51	4.59±0.04
^{124}Sn	$1g_{9/2}$	$5.02^{+0.14}_{-0.11}$	5.15	5.22±0.04
	$2p_{1/2}$	$4.58^{+0.14}_{-0.10}$	4.57	4.61±0.04
	$2p_{3/2}$	$4.50^{+0.13}_{-0.11}$	4.47	4.60±0.04

TABLE II. Comparison of separation energies and rms charge radii with the calculated values for protons.

Nucleus	Orbit	Separation energy (MeV) Expt.[a]	Calc.	HF	rms radius (fm) Expt.[b]	Calc.
^{112}Sn	$1g_{9/2}$	7.51	7.083	8.87		
	$2p_{1/2}$	8.05	9.067	12.29	4.586±0.005	4.600
	$2p_{3/2}$	8.32	10.543	14.11		
^{116}Sn	$1g_{9/2}$	9.27	8.771	10.51		
	$2p_{1/2}$	9.61	10.651	13.86	4.619±0.005	4.625
	$2p_{3/2}$	9.87	12.119	15.74		
^{118}Sn	$1g_{9/2}$	10.01	9.585	11.13		
	$2p_{1/2}$	10.33	11.418	14.58	4.634±0.005	4.638
	$2p_{3/2}$	10.60	12.880	16.46		
^{120}Sn	$1g_{9/2}$	10.67	10.381	11.84		
	$2p_{1/2}$	10.98	12.169	15.39	4.646±0.005	4.652
	$2p_{3/2}$	11.27	13.624	17.17		
^{124}Sn	$1g_{9/2}$	12.10	11.919	13.27		
	$2p_{1/2}$	12.42	13.623	16.63	4.670±0.005	4.692
	$2p_{3/2}$	12.76	15.064	18.53		

[a]The values are the separation energies of Gove and Wapstra (Ref. 7) plus the excitation energy.
[b]Experimental values are taken from Ficenec *et al.* (Ref. 8).

$$f(r)=\left\{1+\left[\exp\left[\frac{r-r_0(A-1)^{1/3}}{a}\right]\right]\right\}^{-1} \quad (2)$$

and $V_C(r)$ is the Coulomb potential. First, we have optimized the six parameters of this potential by the χ^2-minimization technique. For nine proton states near the Fermi surface of ^{208}Pb we got $\chi^2=0.22$ [$\chi^2/N=0.024$]. The parameters were reported earlier[2] in another context. Keeping the geometrical shape invariant, we found the central depth for tin isotopes following the prescription of Bohr and Mottelson,[3]

$$V_p=V_0+\left[\frac{N-Z}{A}\right]K\,, \quad (3)$$

where $K=33$ was found, after a thorough search, to be optimum.

The strength of the spin orbit potential is assumed to have proportional dependence on V_p. This method has been found to be suitable for the evaluation of the shell gap at the newly found doubly closed shell nucleus ^{146}Gd.[4] The potential parameters thus obtained have been used to calculate the rms charge radii and orbit radii of even isotopes of tin. The Perey prescription of nonlocality was used for all the cases discussed.

It is apparent from Tables I and II that an average one body potential suitably optimized in the Pb region when interpolated to the Sn region with a suitable interpolation formula reproduces the orbit radii of $2p_{1/2}$, $2p_{3/2}$, and $1g_{9/2}$ proton orbitals and their separation energies with a fair amount of accuracy. In this respect we should comment that though a Hartree-Fock (HF) calculation[5] shows the trend of orbit radii in the right direction, in the case of separation energies the performance is poor. Again, we must point out that neither in the case of separation energies nor in the case of orbit radii did we make any adjustment anywhere. Gaining confidence in the case of orbit radii we then proceeded further to calculate the total rms charge radii of the even isotopes of tin. Necessary precautions for the finite proton form factor, the c.m. correction, etc., were taken into account for all cases. From Table II we find excellent agreement between the calculated values and the experimental values. About the systematic variation of the rms radius with separation energy, closer ex-

TABLE III. Comparative studies of variation of separation energies with rms orbit radii.

Orbit	Nucleus	Separation energy (MeV)	rms orbit radius (fm)
$1g_{9/2}$	^{112}Sn	7.51	4.95
	^{116}Sn	9.27	5.01
	^{118}Sn	10.01	5.00
	^{120}Sn	10.67	4.99
	^{124}Sn	12.10	5.02
$2p_{1/2}$	^{112}Sn	8.05	4.51
	^{116}Sn	9.61	4.46
	^{118}Sn	10.33	4.52
	^{120}Sn	10.98	4.49
	^{124}Sn	12.42	4.58
$2p_{3/2}$	^{112}Sn	8.32	4.54
	^{116}Sn	9.87	4.55
	^{118}Sn	10.60	4.46
	^{120}Sn	11.27	4.57
	^{124}Sn	12.76	4.50

amination of the experimental data reveals that an appreciable change is observed whenever the separation energy decreases with the increasing mass number. However, as the separation energy increases with increasing mass number, the change in rms orbit radius is not so dramatic (Table III). We conclude by saying that in the tin region the general systematic feature of $\partial r/\partial \epsilon = -0.08$ fm MeV^{-1} observed by Moalem[6] is absent and the problem of the dependence of orbit radius on mass and separation energy needs careful investigation.

The author thanks Dr. S. Sen for his interest in this problem.

[1]A. Warwick, R. Chapman, J. L. Durell, and J. N. Mo, Nucl. Phys. **A391**, 9 (1982).
[2]P. Mukherjee, R. Bhattacharya, and C. Samanta, Phys. Rev. C **28**, 1396 (1983).
[3]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. I.
[4]P. Mukherjee, R. Bhattacharya, and I. Mukherjee, Phys. Rev. C **24**, 1810 (1981).
[5]X. Campi, D. W. L. Sprung, and J. Martorell, Nucl. Phys. **A223**, 541 (1974).
[6]A. Moalem, Phys. Rev. C **26**, 1757 (1982).
[7]N. B. Gove and A. H. Wapstra, Nucl. Data Tables **A11**, 127 (1972).
[8]J. R. Ficenec, L. A. Fajardo, W. P. Trower, and I. Sick, Phys. Lett. **42B**, 213 (1972).