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Solar neutrinos from the decay of ^{8}B

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The spectrum of neutrinos produced by the beta decay of ^{8}B in the sun is computed including forbidden corrections to the weak interactions. The total cross section for absorption by ^{37}Cl of ^{8}B neutrinos is $(1.06\pm0.1)\times10^{-42}$ cm^2. The uncertainty is determined by calculating the cross section using different measured spectra for the alpha particles that result from the decay of ^{8}B to ^{8}Be and by varying the transition matrix elements consistent with the branching ratios measured using delayed protons emitted in the beta decay of ^{37}Ca.

I. INTRODUCTION

The "solar neutrino problem" consists of the well-known discrepancy between expectation and observation in the ^{37}Cl experiment.[1] Most of the predicted capture rate for the ^{37}Cl experiment is ascribed to neutrinos from the decay of ^{8}B in the sun. Other experiments are in progress, or are planned, which are based entirely on the predicted ^{8}B spectrum, including electron-neutrino scattering in water,[2] a heavy water detector,[3] a liquid argon experiment,[4] and a geophysical measurement in which ^{98}Mo (Ref. 5) is the neutrino absorber. Additional experiments are planned in which the ^{8}B flux is a significant contributor to the total predicted capture rate.[6–9] The event rate caused by ^{8}B neutrinos is thus an important factor in assessing the existing solar neutrino problem and in interpreting future experiments designed to resolve the problem. In order to interpret solar neutrino experiments, it is useful to have values accurate to a few percent for the absorption cross sections and for the shape of the neutrino spectrum.

The spectrum and absorption cross sections for ^{8}B neutrinos on various targets have been calculated previously[10,11] using the familiar allowed approximation of nuclear beta decay. Forbidden matrix elements, although small, are much more important for neutrinos from ^{8}B decay than for the other sources of solar neutrinos. The leading terms involving forbidden matrix elements contain one or two extra powers of energy with respect to the allowed transitions. Since the neutrinos from ^{8}B decay have a comparatively large maximum energy of about 14 MeV (much larger, e.g., than the 0.4 MeV maximum energy of the proton-proton neutrinos), the forbidden terms are relatively important for the ^{8}B case. Also, the logft value of the ^{8}B beta decay is rather large [log$ft\sim5.6$ (Ref. 12)], indicating that the allowed terms are smaller than usual for this decay.

In this paper, we reevaluate the spectrum of solar neutrinos from ^{8}B decay, taking into account forbidden corrections to the weak interactions[13] and making use of a more detailed measured alpha-particle spectrum[14,15] that results from the decay of ^{8}Be. For the convenience of other workers who are considering possible effects of neutrino oscillations on the measured capture rates, we present a numerical tabulation of the calculated spectrum of ^{8}B neutrinos that is produced in the central region of the sun.

The spectrum of neutrinos from ^{8}B decay is evaluated in Sec. II and the results are presented in Fig. 1 and Table I. The cross sections, including forbidden effects, for absorption by ^{37}Cl of neutrinos from ^{8}B decay are determined in Sec. III and are summarized in Table II and Eq. (5). The formulae for forbidden corrections to both the decay and the capture processes are given in the Appendix.

II. THE SPECTRUM

In previous calculations,[10,11] the spectrum of ^{8}B solar neutrinos was determined using the phase-space factor for allowed beta decay in conjunction with an empirical characterization of the broad excited state to which the decay predominantly occurs. In this approximation, the probability $\lambda(q)$ that a neutrino of energy q is produced is proportional to

$$\lambda_{\text{allowed}}(q)\propto pWF(-Z,A,W)(W_{\max}-W)^2\ , \tag{1a}$$

where p and W are the momentum and energy of the accompanying positron, and F is the well-known relativistic Fermi function for a final nucleus of charge Z equal to 4 and mass number A equal to 8. The neutrino energy is related to the maximum electron energy $W_{\max}$ by the equation

$$q\cong W_{\max}-W\ . \tag{1b}$$

The forbidden correction to beta decay can be expressed, for a particular transition, as a function of the energy of the positron (or electron) that is produced. We write symbolically:

$$\lambda_{\text{corrected}}(q)=\lambda_{\text{allowed}}(q)\times\text{forbid}(W)\ . \tag{2}$$

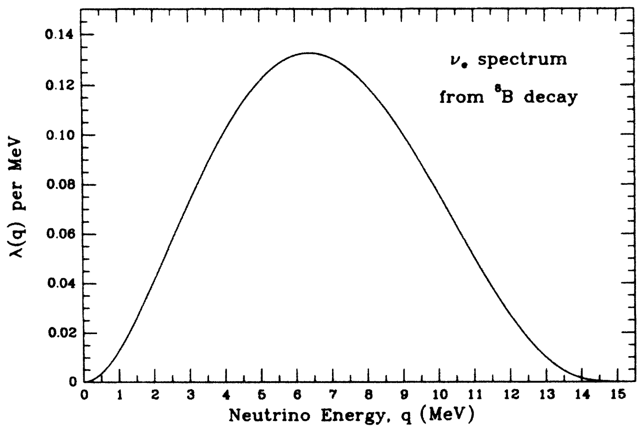

FIG. 1. The spectrum of neutrinos from the beta decay of ^{8}B. The function $\lambda(q)$ is the probability per MeV that a neutrino is emitted with an energy q.

The function forbid(W) is given explicitly in the Appendix. The formalism[13] that was used in obtaining the forbidden corrections is based upon symmetry arguments that are in general use in particle physics.

The ground state of ^{8}B decays primarily to a broad 2^+ excited state in ^{8}Be at about 3 MeV excitation energy. The spectrum of neutrinos can be calculated[10,11] with the aid of the observed spectrum of alpha particles, which allows us to average over the shape of the broad final state. The resulting spectrum, $\langle \lambda(q) \rangle$, can be written

$$\langle \lambda(q) \rangle = \int dq_{\max} P(q_{\max}) [\lambda(q)]_{q_{\max}} \, . \tag{3}$$

In a well-known paper, Wilkinson and Alburger[14] measured the spectrum of alpha particles from the decay of ^{8}B. Details of the alpha-particle spectrum have recently been made available by Warburton.[15] We have also used, for comparison purposes, the alpha-particle spectra measured earlier by Clark *et al.*[16] and by Farmer and Class.[17] As we shall see in the following section, the ^{8}B neutrino spectrum and absorption cross sections, calculated with all three alpha-particle spectra, are in good agreement with each other.

We note in passing that the superallowed decay of ^{8}B to the excited state of ^{8}Be near 16.6 MeV excitation energy has persistently been overestimated in the literature.[12] The ratio of the phase space factors $f(Z, W_{\max})$ to the excited analog state at 16.6 MeV and to the broad 2^+ state near 2.9 MeV is only 10^{-7}. Thus the branching ratio to the analog state must be less than 0.01%. This result is consistent with the measured alpha-particle spectrum of Wilkinson and Alburger.[14,15]

Figure 1 shows the normalized spectrum of neutrinos

TABLE I. The spectrum of solar neutrinos from the decay of ^{8}B. The neutrino energy q is in MeV and $P(q)$ is the probability that a neutrino with energy q is emitted between $q \pm 0.05$ MeV.

q	$P(q)$	q	$P(q)$	q	$P(q)$	q	$P(q)$	q	$P(q)$
0.1	0.000 15	3.1	0.075 96	6.1	0.132 01	9.1	0.098 57	12.1	0.025 41
0.2	0.000 59	3.2	0.079 00	6.2	0.132 31	9.2	0.096 28	12.2	0.023 42
0.3	0.001 31	3.3	0.081 98	6.3	0.132 49	9.3	0.093 94	12.3	0.021 50
0.4	0.002 28	3.4	0.084 91	6.4	0.132 57	9.4	0.091 56	12.4	0.019 65
0.5	0.003 49	3.5	0.087 78	6.5	0.132 54	9.5	0.089 15	12.5	0.017 87
0.6	0.004 92	3.6	0.090 60	6.6	0.132 40	9.6	0.086 69	12.6	0.016 17
0.7	0.006 57	3.7	0.093 34	6.7	0.132 15	9.7	0.084 21	12.7	0.014 56
0.8	0.008 41	3.8	0.096 02	6.8	0.131 80	9.8	0.081 70	12.8	0.013 03
0.9	0.010 44	3.9	0.098 62	6.9	0.131 34	9.9	0.079 17	12.9	0.011 58
1.0	0.012 63	4.0	0.101 14	7.0	0.130 78	10.0	0.076 61	13.0	0.010 22
1.1	0.014 97	4.1	0.103 59	7.1	0.130 12	10.1	0.074 04	13.1	0.008 95
1.2	0.017 45	4.2	0.105 95	7.2	0.129 36	10.2	0.071 46	13.2	0.007 77
1.3	0.020 06	4.3	0.108 23	7.3	0.128 49	10.3	0.068 87	13.3	0.006 68
1.4	0.022 78	4.4	0.110 41	7.4	0.127 54	10.4	0.066 27	13.4	0.005 69
1.5	0.025 60	4.5	0.112 50	7.5	0.126 48	10.5	0.063 68	13.5	0.004 79
1.6	0.028 51	4.6	0.114 50	7.6	0.125 33	10.6	0.061 09	13.6	0.003 99
1.7	0.031 50	4.7	0.116 40	7.7	0.124 09	10.7	0.058 51	13.7	0.003 27
1.8	0.034 55	4.8	0.118 20	7.8	0.122 77	10.8	0.055 94	13.8	0.002 65
1.9	0.037 67	4.9	0.119 90	7.9	0.121 35	10.9	0.053 38	13.9	0.002 12
2.0	0.040 82	5.0	0.121 50	8.0	0.119 85	11.0	0.050 85	14.0	0.001 67
2.1	0.044 01	5.1	0.123 00	8.1	0.118 27	11.1	0.048 34	14.1	0.001 30
2.2	0.047 23	5.2	0.124 39	8.2	0.116 61	11.2	0.045 85	14.2	0.001 00
2.3	0.050 46	5.3	0.125 67	8.3	0.114 87	11.3	0.043 40	14.3	0.000 75
2.4	0.053 70	5.4	0.126 85	8.4	0.113 06	11.4	0.040 99	14.4	0.000 57
2.5	0.056 95	5.5	0.127 92	8.5	0.111 18	11.5	0.038 61	14.5	0.000 42
2.6	0.060 18	5.6	0.128 88	8.6	0.109 23	11.6	0.036 28	14.6	0.000 31
2.7	0.063 39	5.7	0.129 72	8.7	0.107 22	11.7	0.034 00	14.7	0.000 22
2.8	0.066 59	5.8	0.130 46	8.8	0.105 14	11.8	0.031 76	14.8	0.000 16
2.9	0.069 75	5.9	0.131 09	8.9	0.103 01	11.9	0.029 59	14.9	0.000 12
3.0	0.072 88	6.0	0.131 60	9.0	0.100 81	12.0	0.027 47	15.0	0.000 08

from ^{8}B beta decay that was calculated using Eqs. (1)–(3) and the Wilkinson-Alburger spectrum of alpha particles. Table I gives the numerical form of the spectrum in sufficient detail that it can be used in calculating the effects of neutrino oscillations on solar neutrino experiments.

III. THE CROSS SECTIONS

In the allowed approximation, the total cross section for absorption of ^{8}B neutrinos by ^{37}Cl leading to ^{37}Ar can be calculated using the formulae given in Ref. 11. The corrections due to forbidden contributions are presented in the Appendix. We adopt for the moment the standard values for the ratios of the matrix elements that are given in the Appendix [see Eqs. (A12) and (A13)].

Table II gives the calculated capture cross sections to different excited states of ^{37}Ar using a particular set of phenomenologically determined nuclear matrix elements that are called model A in Ref. 11. We first evaluate the effect of including forbidden corrections only for the ^{8}B neutrino spectrum. The third and the fourth columns of Table II compare cross sections that are calculated in the *allowed* approximation, with and without the forbidden corrections to the ^{8}B neutrino spectrum. The forbidden contributions to the neutrino *spectrum* increase the total capture rate by about 3%. The forbidden corrections to both the capture cross sections and the neutrino spectrum are included in the values listed in the second column of Table II, labeled "All." The contributions of forbidden terms to the capture cross section increase the total rate by about 1% with respect to the values listed in the third column.

The results of the calculation of forbidden corrections for the absorption cross sections are used here only to estimate the uncertainty due to forbidden effects, not the best-estimate value. Forbidden effects are included (of necessity) in the experimental matrix elements determined from positron decay that are the basis of model A, but with (in many cases) different signs and coefficients for the constituent terms (see the Appendix). The last column of Table II contains the cross sections calculated previously by Bahcall,[11] using the alpha-particle decay spectrum reported by Clark *et al.*[16] without forbidden corrections. The difference between the present result and the previously calculated value is about 1%.

In the earlier calculation, the alpha-particle spectrum was read from a graph in the paper of Clark *et al.*, and it was assumed, because of the lack of resolution in the graph, that the peak in the alpha-particle spectrum corresponded to an excitation energy of 2.9 MeV in ^{8}Be. (The peak actually occurs at an excitation energy of about 3.08 MeV in the Wilkinson-Alburger data.) We have repeated the calculations of the absorption cross sections using digitized forms of the data of Clark *et al.*[16] and Farmer and Class.[17] The results obtained for the total cross sections by using these alpha-particle spectra, phenomenological values for the transition probabilities, and forbidden corrections to both the neutrino spectrum and the absorption cross section are

$$\sigma_{\text{Clark}} = 1.07 \times 10^{-42}\ \text{cm}^2\ ,\quad \text{model A} \tag{4a}$$

and

$$\sigma_{\text{FC}} = 1.00 \times 10^{-42}\ \text{cm}^2\ ,\quad \text{model A}\ , \tag{4b}$$

where FC denotes Farmer and Class. These results agree with the more accurate value given in Table II:

TABLE II. Absorption cross sections for ^{8}B neutrinos incident on ^{37}Cl. All cross sections are given in units of 10^{-46} cm^2. The labels on the columns of cross sections correspond to different ways of calculating the results: All [includes forbidden corrections to *both* absorption cross sections and the neutrino spectrum with the Wilkinson-Alburger (Refs. 14 and 15) α-particle spectrum]; WA + spec [Wilkinson-Alburger (Ref. 14) α-particle spectrum + forbidden correction to spectrum]; WA (Wilkinson-Alburger data; *no* forbidden correction); and RMP 78 [original calculation by Bahcall (Ref. 11)].

Excitation energy (MeV)	σ All	σ WA + spec	σ WA	σ RMP 78
0.0	620	595	584	596
1.41	433	414	405	418
2.80	445	424	413	429
3.53	367	349	340	354
3.84	423	402	390	405
4.40	130	124	120	126
4.50	240	228	221	231
4.66	140	133	129	135
4.95	259	245	237	249
4.98	6173	6284	6085	6375
5.12	832	788	762	799
5.32	158	150	145	153
5.45	81	76	74	78
6.02	129	122	118	126
Total	1.04	1.03×10^4	1.00×10^4	1.05×10^4

$$\sigma_{\mathrm{WA}}=1.04\times 10^{-42}\ \mathrm{cm}^2\ ,\quad \text{model A}\ , \tag{4c}$$

where WA denotes Wilkinson and Alburger.

We have made a number of calculations in which we have varied individually the dimensionless ratios of matrix elements that contribute to the forbidden correction. We have varied independently the experimentally determined ratios for the ^{8}B decay within the quoted errors [see Eqs. (A8) and (A9) of the Appendix] and have also changed the dimensional estimates of Eq. (A5) by a factor of 2 (in both directions). The total cross section is relatively insensitive to these variations; the range of values obtained differed by only about 1%. The reason is that the forbidden correction to the decay spectrum involves six terms and the correction for the capture cross section involves eight other terms, with different phases between the various elements. The total correction is of order of a few percent, distributed over many (often canceling) terms. Random changes in individual ratios of forbidden matrix elements are unlikely to produce large variations in the total cross section.

In Ref. 11, cross section calculations were carried out using seven different sets of slightly different nuclear matrix elements, all of which were consistent with the measurements of Sextro *et al.*[18] or Poskanzer *et al.*[19] on delayed protons following the beta decay of ^{37}Ca. We have made similar calculations with different sets of matrix elements and have verified that the relative effect of varying the transition strengths is the same as found in Ref. 11. The average value of the total cross section that was obtained by Bahcall[11] using the different sets of phenomenological matrix elements was a factor of (1.08/1.05) larger than was calculated assuming the matrix elements of model A. We therefore adopt as our *best estimate* for the absorption cross section a value that is 3% larger than is given by the results shown in Table II (computed with nuclear model A of Ref. 11). We find

$$\sigma_{\mathrm{BE}}=(1.06\pm 0.1)\times 10^{-42}\ \mathrm{cm}^2\ , \tag{5}$$

where BE denotes best estimate.

The uncertainty shown in Eq. (5) is a best guess for the magnitude of the uncertainty due to forbidden effects ($\sim 1\%$) and to the unknown systematic effects in two sets of experiments: (1) the measurement of the alpha-particle spectrum resulting from the decay of ^{8}B; and (2) the relative strengths of some matrix elements in the beta decay of ^{37}Ca. We adopt an estimate of ±35% for the uncertainty resulting from the ^{8}B spectrum, based upon the spread of values given in Eq. (4). We assume ±3% for the uncertainty resulting from the imprecisely known relative strengths of matrix elements in the decay of ^{37}Ca, based upon the range of values for cross sections computed with the seven different models of the decay strengths described in Ref. 11. The combined root-mean-squared uncertainty from these two causes is 5%. In the spirit of conservatism described in Ref. 11, we double this estimated error to obtain the final uncertainty quoted in Eq. (5).

IV. SUMMARY

We have recalculated the neutrino spectrum from ^{8}B decay in the sun and have used this spectrum to determine the total absorption cross section, Eq. (5), for ^{8}B neutrinos incident on ^{37}Cl. Forbidden corrections to the decay spectrum and to the absorption cross section, both discussed in the Appendix, are of order of a few percent (see Table II). The spectrum and absorption cross sections have been calculated in a variety of ways making use of the existing data. All of the determinations of the neutrino spectrum and the total absorption cross section are in good agreement with each other. The best estimate obtained here agrees to better than one percent with the value derived previously by Bahcall,[11] who used less accurate alpha-particle data and did not include forbidden corrections.

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APPENDIX

The form of forbidden corrections to allowed beta decay is given in several sources. We shall quote here the result using the notation of Holstein and Treiman,[20] which is equivalent to that of Behrens and Jänecke.[21] We find for electron/positron (upper sign/lower sign) transitions[22]

$$\begin{aligned}\mathrm{forbid}(W)=\frac{1}{a_1^2+c_1^2}\Bigg\{ & a_1^2+c_1^2-\frac{2}{3}\frac{W_{\max}}{M}(c_1^2+c_1d\pm c_1b)+\frac{2}{3}\frac{W}{M}(5c_1^2\pm 2c_1b)-\frac{m_e^2}{3MW}(2c_1^2+c_1d\pm 2c_1b)\\ & +\tfrac{2}{9}c_1c_2(11m_e^2+10WW_{\max}-20W^2-2m_e^2W_{\max}/W)\\ & +\tfrac{2}{3}a_1a_2(m_e^2+4WW_{\max}-4W^2+2m_e^2W_{\max}/W)-c_1c_2\left[\frac{9}{2}\left(\frac{\alpha Z}{R}\right)^2\mp\frac{2}{3}\frac{\alpha ZW_{\max}}{R}\pm\frac{20}{3}\frac{\alpha ZW}{R}\right]\\ & -a_1a_2\left[\frac{9}{2}\left(\frac{\alpha Z}{R}\right)^2\pm 2\frac{\alpha ZW_{\max}}{R}\pm 4\frac{\alpha ZW}{R}\right]+\frac{\sqrt{10}\alpha Z}{6MR}(2c_1b\pm c_1d\pm c_1^2)\Bigg\}\ ,\end{aligned} \tag{A1}$$

where here M is the *nuclear* mass,

$$a_1 = g_V \left\langle \beta \left\| \sum_i \tau_i^\pm \right\| \alpha \right\rangle \tag{A2}$$

is the usual Fermi matrix element, and

$$c_1 = g_A \left\langle \beta \left\| \sum_i \tau_i^\pm \sigma_i \right\| \alpha \right\rangle \tag{A3}$$

is the Gamow-Teller matrix element. The corresponding structure functions c_2, a_2 are related to the q^2 dependence of these matrix elements,

$$\begin{aligned} a_2 &= \frac{g_V}{6} \left\langle \beta \left\| \sum_i \tau_i^\pm r_i^2 \right\| \alpha \right\rangle , \\ c_2 &= \frac{g_A}{6} \left\langle \beta \left\| \sum_i \tau_i^\pm \sigma_i r_i^2 \right\| \alpha \right\rangle + \frac{g_A}{6\sqrt{10}} \sqrt{16\pi/5} \left\langle \beta \left\| \sum_i \tau_i^\pm r_i^2 C_{12;1}^{nn';k} \sigma_{in} Y_2^{n'}(r_i) \right\| \alpha \right\rangle . \end{aligned} \tag{A4}$$

For our approximate estimations here, we neglect the $[\sigma \times Y_2]_1$ matrix element contributing to c_2 and assume that the weak charge is uniformly distributed over the nuclear volume, yielding

$$\frac{a_2}{a_1} \approx \frac{c_2}{c_1} \approx \tfrac{1}{10} R^2 , \tag{A5}$$

where R is the nuclear radius. [Of course, for the dominant transitions in the ^{8}B decay, $a_1 = 0$, which causes two of the terms in Eq. (A1) to vanish.] Finally,

$$\frac{1}{A} b = g_M \left\langle \beta \left\| \sum_i \tau_i^\pm \sigma_i \right\| \right\rangle + g_V \left\langle \beta \left\| \sum_i \tau_i^\pm L_i \right\| \alpha \right\rangle \tag{A6}$$

is the weak magnetism term and

$$\frac{1}{A} d = g_A \langle \beta \| i \tau_i^\pm \sigma_i \times L_i \| \alpha \rangle + g_A \left\langle \beta \left\| \frac{i}{2} \sum_i \tau_i^\pm (\{\sigma_i \cdot r_i, p_i\} + \{\sigma_i \cdot p_i, r_i\}) \right\| \alpha \right\rangle \tag{A7}$$

is the axial tensor, with A being the mass number. We use the value for weak magnetism determined from the conserved vector current (CVC) hypothesis and the measurement of the $M1$ width of the 2^+ analog state in ^{8}Be (Ref. 23),

$$\frac{b}{Ac_1} = 7.7 \pm 1.0 , \tag{A8}$$

while for the axial tensor we employ the value which is measured in β-α correlation experiments on ^{8}B and ^{8}Li (Ref. 24),

$$\frac{d}{Ac_1} = 1.9 \pm 1.3 . \tag{A9}$$

In the case of the neutrino capture reaction, the same forms can be employed, but now with the substitution

$$W_{\max} \to W_e - W_\nu , \quad W \to W_e , \tag{A10}$$

where W_ν and W_e are the neutrino and electron energies relevant to the capture process and the upper (lower) sign is employed for neutrino (antineutrino) capture. Thus, for example, for the weak magnetism contribution to the ^{37}Cl capture, we find

$$F_{\text{forbid}}(W_e, W_\nu) = \frac{1}{a_1^2 + c_1^2} \left[a_1^2 + c_1^2 + \frac{2}{3M} c_1 b (W_e + W_\nu) - 2 \frac{m_e^2}{2MW_e} c_1 b + \cdots \right] , \tag{A11}$$

and the remaining contributions can be evaluated similarly. For the estimates used in calculating the forbidden corrections for the ^{37}Cl absorption cross sections, we neglect the orbital contribution to weak magnetism, yielding

$$\frac{b}{Ac_1} = \frac{g_M}{g_A} \approx 4 , \tag{A12}$$

and choose a plausible value for the axial tensor

$$\frac{d}{Ac_1} \approx 2 . \tag{A13}$$

In addition, for the case of the neutrino capture reaction, one should also include the possibility of capture to forbidden levels. For first forbidden transitions, for example, the capture cross section has the same form as that for an allowed spectrum but with

$$a_1^2+c_1^2+\frac{2}{3M}c_1b(W_e+W_\nu)+\cdots \tag{A14}$$

replaced by

$$\left[s_1+y_1\left[\frac{\alpha Z}{2MR}+\frac{W_e-W_\nu}{3M}\right]\right]^2+\left[t_1+u_1\left[\frac{\alpha Z}{2MR}+\frac{W_e-W_\nu}{3M}\right]+z_1\left[\frac{\alpha Z}{2MR}-\frac{W_e+W_\nu}{3M}\right]\right]^2 \\ +\left[\frac{1}{18}z_1^2+\frac{2}{q}u_1^2\right]\frac{W_\nu^2+W_e^2}{M^2}+\frac{2}{q}u_1z_1\frac{W_\nu^2-W_e^2}{M^2}+v_1^2\frac{W_\nu^2+W_e^2}{6M^2}+\cdots\ , \tag{A15}$$

where

$$\begin{aligned} s_1&=-ig_A\left\langle\beta\left|\left|\sum_i\tau_i^\pm\gamma_5\right|\right|\alpha\right\rangle\ ,\\ t_1&=-ig_v\left\langle\beta\left|\left|\sum_i\tau_i^\pm\boldsymbol{\alpha}\right|\right|\alpha\right\rangle\ ,\\ y_1&=g_A\left\langle\beta\left|\left|\sum_i\tau_i^\pm\boldsymbol{\sigma}_i\cdot\mathbf{r}_i\right|\right|\alpha\right\rangle Am_N\ ,\\ u_1&=g_v\left\langle\beta\left|\left|\sum_i\tau_i^\pm\mathbf{r}_i\right|\right|\alpha\right\rangle Am_N\ ,\\ z_1&=g_A\left\langle\beta\left|\left|i\sum_i\tau_i^\pm\boldsymbol{\sigma}_i\times\mathbf{r}_i\right|\right|\alpha\right\rangle Am_N\ ,\\ v_1&=g_A\left\langle\beta\left|\left|i\sum_i\tau_i^\pm(\sigma_ir_j+\sigma_jr_i-\tfrac{2}{3}\delta_{ij}\boldsymbol{\sigma}\cdot\mathbf{r})\right|\right|\alpha\right\rangle Am_N\ . \end{aligned} \tag{A16}$$

Thus there are two distinct scales for matrix elements

$$s_1,t_1\sim\frac{P_F}{m_N} \tag{A17}$$

where $P_F\sim200$ MeV/c is the Fermi momentum of a typical nucleus, and

$$y_1,u_1,z_1,v_1\sim Am_NR\ , \tag{A18}$$

where R is the nuclear radius. Since

$$\frac{P_F}{m_N}\sim0.2 \tag{A19}$$

is the same order of magnitude (for $W_\nu\sim10$ MeV, $R\sim4$ fm) as

$$W_\nu R\sim0.2\ , \tag{A20}$$

we estimate the capture rate of a typical forbidden transition compared to that for the superallowed case as

$$\frac{\sigma(\text{forbidden})}{\sigma(\text{allowed})}\sim(0.2)^2\ . \tag{A21}$$

Thus an allowed neutrino capture cross section can be increased by several percent because of forbidden transitions. Of course, a detailed wave function calculation can give somewhat different values depending upon the particular levels involved. In the absence of a reliable detailed calculation, Eq. (A21) should provide a reasonable although crude estimate.

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