Low equation, pion-nucleon scattering, and Castillejo-Dalitz-Dyson pole

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We examine the p-wave πN scattering at medium energies by means of the Low equation with a view to determining the form factor of the πN interaction. Solutions of the equation with and without a Castillejo-Dalitz-Dyson (CDD) pole are used. The solution with no CDD pole corresponds to the old Chew-Low model, whereas the one with a CDD pole corresponds to the quark version of the Chew-Low model. The πN interaction form factor is determined so that the Δ resonance is well reproduced. We find that the solution with a CDD pole leads to a softer form factor but is not as soft as those expected from the nucleon size in the quark model. Using the solutions and form factors thus determined, we also examine the pionic contributions to the nucleon magnetic moment and the nucleon mass.

I. INTRODUCTION

In describing scattering processes the Low equation has some important advantages over the Schrödinger equation or the Lippmann-Schwinger equation. The Low equation involves only renormalized mass and coupling constant. It is essentially a dispersion relation for the scattering amplitude, and is independent of the details of the interaction. It is manifestly crossing symmetric. Unlike the Schrödinger equation, however, the Low equation is nonlinear and there are an infinite number of solutions which differ from each other with respect to the so-called Castillejo, Dalitz, and Dyson[1] (CDD) pole in the denominator of the scattering amplitude. It is believed that solutions with different numbers of CDD poles correspond to different Hamiltonians, which have different numbers of "elementary" particles.[2]

This multiplicity of solutions of the Low equation has an interesting implication regarding the mechanism of the Δ resonance in the p-wave πN scattering. In the old Chew-Low (CL) model[3] the Δ resonance is interpreted as due to the iteration of the process of $\pi N \to \pi\pi N \to \pi N$, and one chooses the solution of the Low equation with no CDD pole. From the quark model point of view, however, the Δ resonance is due to the presence of the bare Δ, which is as elementary as N, both consisting of three quarks. This leads to the quark version of the CL model.[4,5] Although the underlying Hamiltonians are different, the Low equation for πN scattering remains exactly the same between the old CL model and its quark version. In solving the Low equation for the quark CL model, however, one has to choose a solution with a CDD pole which corresponds to the bare Δ.

It is prohibitive to solve the Low equation as such for the CL model because it involves an infinite number of inelastic channels. As we will discuss in the following the equation can be reduced to a manageable form by specifying the inelasticity. The solution depends on the πNN vertex form factor $v(k)$ which one has to assume. This $v(k)$ can be determined by requiring that the solution reproduces the empirical phase shifts at low and medium energies, in particular the Δ resonance in the P_{33} channel. By assuming certain simple forms of $v(k)$ containing a cutoff momentum Λ, attempts have been made to determine the optimum value of Λ in the old CL model (i.e., by using solutions of the Low equation with no CDD pole). Bajaj and Nogami[6] assumed a straight cutoff, i.e., $v(k)=\theta(\Lambda-k)$, and found $\Lambda=6.8m_\pi$ where m_π is the pion mass. They emphasized the importance of taking account of crossing symmetry, inelasticity, and the nucleon recoil correction. Ernst and Johnson[7] did a more detailed analysis of the solution of the Low equation. They assumed the Gaussian form for $v(k)$, i.e.,

$$v(k)=\exp(-k^2/2\Lambda^2) \tag{1.1}$$

and obtained $2\Lambda^2=30m_\pi^2$, or $\Lambda=\sqrt{15}m_\pi$. It should be noted, however, that the calculation of Ref. 7 was done in the static limit. Bajaj and Nogami[6] showed that when the nucleon recoil effect is included one needs a larger value of Λ. They obtained $\Lambda=6.8m_\pi$ and $4.9m_\pi$ with and without the recoil correction, respectively. The latter value, $4.9m_\pi$, can be compared with $\sqrt{15}m_\pi$ of Ref. 7. If the recoil correction is incorporated into the calculation of Ernst and Johnson, Λ will probably be increased to $5m_\pi$–$6m_\pi$. If we interpret $v(k)$ of Eq. (1.1) as the Fourier transform of a density distribution $\rho(r)$, we obtain

$$\langle r^2\rangle^{1/2} \equiv \left[\int r^2\rho(r)d\mathbf{r}\right]^{1/2}=\sqrt{3}/\Lambda \,. \tag{1.2}$$

If $\Lambda=5m_\pi$, for example, we obtain $\langle r^2\rangle^{1/2}=0.5$ fm.

The main purpose of this paper is to determine the form factor $v(k)$ for the quark CL model. We solve the Low equation by specifying the inelasticity, which we take from experiment, and determine $v(k)$ in the form of Eq. (1.1) by fitting the Δ resonance. The essential difference from the earlier attempts[6,7] is that the scattering amplitude for P_{33} contains a CDD pole. The position ω_c (energy) of the CDD pole is arbitrary, and we choose it in the range of 0.5–1.0 GeV, which leads to $\Lambda=3.3m_\pi$–$4.1m_\pi$ and $\langle r^2\rangle^{1/2}=0.6$–$0.8$ fm. This rms radius is somewhat smaller than the nucleon radius based on the quark model, the latter being typically 0.8–1.0 fm.[4,5] In other words the πN interaction form factor is not quite as soft as those expected from the quark model. Once the interaction form factor is determined one can calculate various pionic effects. We will examine the pionic contributions to the nucleon magnetic moment and nucleon mass.

Throughout this paper we confine ourselves to the p-wave πN interaction. We briefly summarize the Low equation and its solution with no CDD pole in Sec. II, and the solution with a CDD pole in Sec. III. Expressions for the pionic contributions to the nucleon magnetic moment and mass are given in Sec. IV. Results and discussion are given in Sec. V.

II. SOLUTION WITH NO CDD POLE

The Low equation for the CL model is written as[3]

$$h_\alpha(\omega)=\frac{\lambda_\alpha}{\omega}+\frac{1}{\pi}\int_{m_\pi}^{\infty}d\omega'\left[\frac{\mathrm{Im}h_\alpha(\omega')}{\omega'-\omega-i\epsilon}+\frac{\mathrm{Im}h_\alpha(-\omega')}{\omega'+\omega}\right], \tag{2.1}$$

where

$$\lambda_\alpha=\frac{2f^2}{3m_\pi^2}\begin{bmatrix}-4\\-1\\-1\\2\end{bmatrix}.$$

Here f^2 (≈ 0.08) is the renormalized πN coupling constant and m_π is the pion mass, and ω the pion energy. The suffix α ($=1,2,3,4$) refers to $(2I,2J)=(1,1)$, $(1,3)$, $(3,1)$, and $(3,3)$, respectively. The $h_\alpha(\omega)$ is related to the real and imaginary parts of the phase shifts, δ_α and η_α, by[8]

$$h_\alpha(\omega)=\frac{\eta_\alpha\sin\delta_\alpha}{k^3v^2}e^{i\delta_\alpha}, \tag{2.2}$$

where $v(k)$ is the πN interaction form factor, for which we assume Eq. (1.1). In order to include the nucleon recoil effect approximately we interpret ω to be

$$\omega=\omega_0+k^2/(2m_{\rm N}), \tag{2.3}$$

where k is the pion momentum, $m_{\rm N}$ the nucleon mass, and $\omega_0=(k^2+m_\pi^2)^{1/2}$. As we noted in Sec. I, Eq. (2.1) applies to the quark CL model as well as to the old CL model.

In order to make use of the CDD technique[1] we introduce the denominator function $g_\alpha(\omega)$ by

$$g_\alpha(\omega)\equiv\frac{\lambda_\alpha}{\omega h_\alpha(\omega)}=\frac{\lambda_\alpha k^3v^2}{\omega\eta_\alpha\sin\delta_\alpha}e^{-i\delta_\alpha}. \tag{2.4}$$

Equation (2.1) is satisfied if $g_\alpha(\omega)$ satisfies[7,9]

$$g_\alpha(\omega)=1+\frac{\omega}{\pi}\int_{m_\pi}^{\infty}\frac{d\omega'}{\omega'}\left[\frac{\mathrm{Im}g_\alpha(\omega')}{\omega'-\omega-i\epsilon}-\frac{\mathrm{Im}g_\alpha(-\omega')}{\omega'+\omega}\right]. \tag{2.5}$$

The $\mathrm{Im}g_\alpha(\omega)$ in Eq. (2.5) is determined from Eq. (2.4) as

$$\mathrm{Im}g_\alpha(\omega)=-\lambda_\alpha k^3v^2/(\omega\eta_\alpha). \tag{2.6}$$

The $\mathrm{Im}g_\alpha(-\omega)$ can be evaluated by means of the crossing relation

$$\frac{1}{g_\alpha(-\omega)}=-\frac{1}{\lambda_\alpha}\sum_\beta A_{\alpha\beta}\frac{\lambda_\beta}{g_\beta(\omega)}, \tag{2.7}$$

where

$$A=\frac{1}{9}\begin{bmatrix}1&-4&-4&16\\-2&-1&8&4\\-2&8&-1&4\\4&2&2&1\end{bmatrix}. \tag{2.8}$$

More explicitly we use

$$g_\alpha(-\omega)=-\frac{\lambda_\alpha f_\alpha(\omega)}{|f_\alpha(\omega)|^2} \tag{2.9}$$

with

$$f_\alpha(\omega)=\sum_\beta A_{\alpha\beta}\lambda_\beta\frac{g_\beta(\omega)}{|g_\beta(\omega)|^2}. \tag{2.10}$$

Let us now describe how we solve the Low equation. There are two inputs. One is η_α which is related to the inelasticity. For this we substitute the imaginary part of the experimental phase shift.[10] The other is the form factor $v(k)$ of Eq. (1.1). In this way the only free parameter that we have is the Λ in $v(k)$, which we determine by requiring that the Δ-resonance energy $\omega_\Delta=296$ MeV $=2.12m_\pi$ be fitted.

If we ignore the term with $g_\alpha(-\omega)$, i.e., in the no-crossing approximation, Eq. (2.5) can be solved immediately.[1,2] Starting with this no-crossing solution, we solve Eq. (2.5) by iteration.[7] In each step of the iteration we incorporate the crossing contribution by means of Eqs. (2.9) and (2.10). Once $g_\alpha(\omega)$ is determined, the Δ-resonance energy is determined from $\mathrm{Re}g_4(\omega)=0$, and the width Γ by[6]

$$\Gamma=\frac{2\,\mathrm{Im}g_4(\omega_\Delta)}{\mathrm{Re}[g_4'(\omega_\Delta)]}, \tag{2.11}$$

where $g_4'(\omega)=dg_4(\omega)/d\omega$.

III. SOLUTION WITH A CDD POLE

For $\alpha=4$, i.e., for P_{33}, it can easily be shown from Eqs. (2.4) and (2.5) that $\mathrm{Im}g_4(z)<0$ for $\mathrm{Im}(z)>0$; hence $g_4(z)$ is a generalized R function.[11] Then, according to CDD, the same Low equation (2.1) can be satisfied even if we replace Eq. (2.5) for $\alpha=4$ with

$$g_4(\omega)=\text{rhs of Eq. (2.5)}-\sum_i\frac{c_i\omega}{\omega_i(\omega_i-\omega)}-\sum_j\frac{c_j\omega}{\omega_j(\omega_j+\omega)}, \tag{3.1}$$

where c_i and ω_i ($>m_\pi$) are all arbitrarily positive constants. The poles in the added terms are referred to as CDD poles. At these poles, $h_4(\omega)$ and hence the elastic cross section vanishes. The same prescription does not apply to the channels other than $\alpha=4$, because $g_\alpha(\omega)$ with $\alpha\neq 4$ is not a generalized R function.

The meaning of these solutions with CDD poles has been clarified for the Lee and Lee-type models in which the crossing term is absent. Dyson[2] suggested for those models that for each CDD pole there corresponds a bare (elementary) particle in the system. The same Low equation applies to many different physical systems, but there is a one-to-one correspondence between the solutions and the physical systems. Kumar and Nogami[2] explicitly demonstrated this correspondence for the Lee-type models. No such analysis seems to have been done for more realistic models with crossing terms, but we assume that Dyson's interpretation of CDD poles holds in general.

Since we are interested only in Δ for which only one underlying bare baryon is expected, we take only one CDD pole. That is, we take

$$g_4(\omega)=1+\frac{\omega}{\pi}\int_{m_\pi}^{\infty}\frac{d\omega'}{\omega'}\left[\frac{\mathrm{Im}g_\alpha(\omega')}{\omega'-\omega-i\epsilon}-\frac{\mathrm{Im}g_\alpha(-\omega')}{\omega'+\omega}\right]-\frac{c\omega}{\omega_c(\omega_c-\omega)} \tag{3.2}$$

together with Eq. (2.5) for $\alpha=1$, 2, and 3. Equations (2.6)–(2.11) remain valid as such. In addition to Λ in the interaction form factor we now have two new parameters c and ω_c. We solve Eqs. (3.2) and (2.5) in the same way as in Sec. II, i.e., by iteration. For the parameters we arbitrarily assume ω_c, and determine c and Λ by fitting ω_Δ and Γ.

IV. OTHER PIONIC EFFECTS

Once $g_\alpha(\omega)$ and $v(k)$ are determined, the CL model enables us to calculate practically any other pionic effects.[3,12] The pionic contribution to the nucleon magnetic moment (in nuclear magnetons) is given by $\mu_\pi\tau_3$ where

$$\mu_\pi=\frac{8f^2}{3\pi}\frac{m_{\rm N}}{m_\pi^2}\int_0^\infty dk\frac{v^2k^4}{\omega_0^2\omega^2}-\frac{m_{\rm N}}{27\pi^3}\int_0^\infty dk\frac{v^2k^4}{\omega_0^2\omega}\int_0^\infty dk'\frac{k'(2\omega+\omega')}{\omega_0'\omega'(\omega+\omega')^2}\mathrm{Im}\left[\frac{\lambda_1g_1}{|g_1|^2}-\frac{\lambda_2g_2}{|g_2|^2}-\frac{\lambda_3g_3}{|g_3|^2}+\frac{\lambda_4g_4}{|g_4|^2}\right]. \tag{4.1}$$

Here $\omega_0=(k^2+m_\pi^2)^{1/2}$ and ω is defined by Eq. (2.3). The first term in the right-hand side of Eq. (4.1) is the Born term.

The nucleon self-energy due to the πN interaction is given by

$$\Delta E_\pi=-\frac{3f^2}{\pi m_\pi^2}\int_0^\infty dk\frac{v^2k^4}{\omega_0\omega}+\frac{1}{12\pi^3}\int_0^\infty dk\frac{v^2k^4}{\omega_0}\int_0^\infty dk'\frac{\omega+2\omega'}{\omega_0'(\omega+\omega')^2}\mathrm{Im}\left[\frac{\lambda_1g_1}{|g_1|^2}+\frac{2\lambda_2g_2}{|g_2|^2}+\frac{2\lambda_3g_3}{|g_3|^2}+\frac{4\lambda_4g_4}{|g_4|^2}\right]. \tag{4.2}$$

In the static limit there is no distinction between ω and ω_0, and the above two formulas are reduced to the corresponding formulas of Ref. 12. The factor ω_0 stems from $(2\omega_0)^{-1/2}$ which is associated with the creation or annihilation operator of the pion, whereas ω is from the energy denominator.

V. RESULTS AND DISCUSSION

For the πN coupling constant we use $f^2=0.086$.[13] For η_α we substitute the empirical data.[10] In the solution with no CDD pole there is only one parameter Λ, which we determine such that $\mathrm{Re}g_4(\omega_\Delta)=0$. The resonance width Γ may not be produced because there is no more free parameter.

When a CDD pole is included there are three parameters in the solution, Λ, c, and ω_c. Mathematically c and ω_c can be any arbitrary positive numbers, but we choose ω_c in the range of 0.5–1.0 GeV. Since $h_4(\omega)$ and hence the elastic cross section vanishes at $\omega=\omega_c$, we cannot choose ω_c too low. On the other hand the model that we are dealing with is meant for low and medium energies, and hence it would be sensible to limit the range of the parameters to $\lesssim 1$ GeV; hence $\omega_c=0.5$–1.0 GeV. In our calculation, we assume a certain value of ω_c and then determine Λ and c by requiring that $\mathrm{Re}g_4(\omega_\Delta)=0$ with $\omega_\Delta=2.12m_\pi$ and $\Gamma=0.84m_\pi$. In solving the equation we iterated about eight times; $\mathrm{Re}g_\alpha(\omega)$ converged within the first four digits.

We present the results in two tables. The crossing contribution, inelasticity, and nucleon recoil effect are ignored in Table I, whereas they are all included in Table II. In both tables, the row with $c=0$ is for the case with no CDD pole. In this case, as we noted above, the empirical width $\Gamma=0.84m_\pi$ is not reproduced.[14] When a CDD pole is included, Γ can always be fitted.

In both tables one can see that Λ is reduced when a CDD pole is included, and this effect is more pronounced for larger ω_c. In Table II, Λ can be as small as $\sim 3.3m_\pi$; the rms radius defined by Eq. (1.2) can be as large as 0.75 fm. This is still somewhat smaller than the nucleon size usually expected from the quark model (i.e., 0.8–1.0 fm).[4,5] The form factor $v(k)$ that we obtained is therefore not as soft as those used in Refs. 4 and 5. A consequence of this not very soft form factor is that the pionic contributions to the magnetic moment and nucleon self-energy

TABLE I. Results without the crossing contribution, inelasticity, and nucleon recoil effect. The first row with $c=0$ is for the solution with no CDD pole. The empirical value for the width of the Δ resonance is $\Gamma=0.84m_\pi$, which cannot be fitted when $c=0$. μ_π is in units of nuclear magnetons.

ω_c (GeV)	c (MeV)	Λ/m_π	Γ/m_π	μ_π Born	μ_π Total	$-\Delta E_\pi/m_N$ Born	$-\Delta E_\pi/m_N$ Total
	0	11.46	1.44	3.98	3.99	8.07	8.37
0.5	194	5.53	0.84	1.51	1.52	0.87	0.90
0.6	472	3.80	0.84	0.85	0.85	0.27	0.28
0.7	817	3.12	0.84	0.61	0.61	0.144	0.148
0.8	1223	2.79	0.84	0.49	0.50	0.100	0.103
0.9	1693	2.60	0.84	0.44	0.44	0.080	0.083
1.0	2230	2.49	0.84	0.40	0.40	0.069	0.071

are not very small. This is rather disturbing. In the tables the Born term and the entire contribution are separately entered. Note that for the magnetic moment the Born term constitutes almost the entire effect. This is because of the cancellation in the last term of Eq. (4.1).

In their cloudy bag model, Théberge *et al.*[4] examined the P_{33} scattering by solving the Lippmann-Schwinger equation. They included the bare Δ contribution via $\pi N\rightarrow\Delta\rightarrow\pi N$, but not its crossing counterpart $\pi N\rightarrow\pi\pi\Delta\rightarrow\pi N$. Also they did not take account of the inelasticity nor the nucleon recoil correction. Hence their results would correspond to those of our Table I with a CDD pole. In Table I the cutoff Λ can be smaller than in Table II. For example, Λ can be $\sim 2.5m_\pi$ which corresponds to $\langle r^2\rangle^{1/2}\approx 1.0$ fm. It is interesting that this value of Λ is close to the value determined in the cloudy bag model. However, because of the lack of the crossing contribution, etc., in Table I, we think that the results of Table II should be preferred.

There are a few points in our calculation with which we are rather uncomfortable. We do not know how to remedy them but it would be useful to note them. In fact these are the problems inherent in all calculations of this type including the very early attempt of solving the Low equation by Salzman and Salzman.[15] The solutions that we obtained describe the P_{33} channel very well, the P_{13} and P_{31} channels fairly well, but the P_{11} amplitude is a failure. The empirical phase δ_1 for P_{11} changes its sign from negative to positive and grows towards the Roper resonance (1440), but the calculated δ_1 remains negative throughout. This is simply because of the repulsive driving term in the P_{11} channel. Presumably the Roper resonance could be associated with something like a CDD pole. However $g_1(\omega)$ within our framework of Eqs. (2.4) and (2.5) is not a generalized R function and hence we cannot use the CDD technique as such. One can force δ_1 to change its sign at a certain energy by using a subtracted dispersion relation.[9] We tried that but found that the convergence of the iteration was very slow. We are not sure that that is the right way of treating the P_{11} channel, and hence, we do not present those results in this paper.

Another problem is concerned with crossing symmetry. Although crossing symmetry is well satisfied for g_4, it is only approximately satisfied for other g's. This may sound strange because all g's are coupled, but this problem is inherent since Ref. 15. It is possible to improve on this point by introducing channel dependent form factors[9] but we feel that we need more physical insight into this problem. We took η_α from the empirical data, which means to assume that the empirical η_α in all channels are reproduced by the solution of the Low equation. This may be too much to demand of the model.

We included only one CDD pole which we assume to correspond to a bare Δ. The quark model, however, predicts many more baryon excited states. This would mean that we should choose a solution with correspondingly many CDD poles. If we do so it may turn out that the form factor becomes even softer. However, this does not necessarily mean that the pionic contributions to the magnetic moment, etc., are reduced. To include many bare baryon excited states means to take account of various virtual quark excitations. Such effects tend to enhance the pionic contributions as one can illustrate for the nucleon self-energy.[16]

TABLE II. The same as for Table I except that the crossing contribution, inelasticity, and nucleon recoil effect are all included.

ω_c (GeV)	c (MeV)	Λ/m_π	Γ/m_π	μ_π Born	μ_π Total	$-\Delta E_\pi/m_N$ Born	$-\Delta E_\pi/m_N$ Total
	0	4.45	1.05	1.35	1.36	0.80	0.83
0.5	48	4.13	0.84	1.19	1.20	0.61	0.64
0.6	130	3.93	0.84	1.09	1.11	0.51	0.54
0.7	266	3.78	0.84	1.02	1.03	0.44	0.46
0.8	461	3.61	0.84	0.94	0.95	0.38	0.40
0.9	740	3.45	0.84	0.87	0.88	0.33	0.34
1.0	1075	3.32	0.84	0.82	0.83	0.28	0.30

Finally, although the Low equation combined with the CDD technique enables us to obtain various exact, model insensitive results, it is not very useful in investigating the details of the interaction. For example, the relation between the CDD pole parameters and the underlying interaction is not clear. For Lee-type models the relation has been explicitly worked out by Kumar and Nogami.[2] It would be interesting if a similar analysis can be done for more realistic models.

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