Reaction $p+{}^{12}C \to {}^{13}C+\mu^+ + \nu_\mu$ and the weak form factors in the timelike region

S. L. Mintz
Physics Department, Florida International University, Tamiami Campus, Miami, Florida 33199
(Received 29 January 1986)

We present an approximate theoretical calculation of the cross section for the reaction $p+{}^{12}C \to {}^{13}C+\mu^+ + \nu_\mu$ for center of mass proton energies from $E_p = 1060$ MeV to $E_p = 1085$ MeV. We make use of data from $p+{}^{12}C \to {}^{13}C+\pi^+$ and $p+{}^{12}C \to \gamma + {}^{13}N$ reactions to obtain approximate values for the form factors involved via the partially conserved axial current hypothesis and conserved vector current hypothesis, respectively. We discuss the utility of this process in studying the role of anomalous thresholds in the nuclear partially conserved axial vector current.

I. INTRODUCTION

The process, $p + {}^{12}C \to {}^{13}C+\mu^+ + \nu_\mu$, is interesting for a number of reasons. Unlike most commonly studied weak nuclear processes, q^2 for this reaction is entirely timelike, thus enabling the weak nuclear form factors to be observed in this region. Additionally, low lying anomalous contributions to nuclear matrix elements involved in the partially conserved axial vector current could be more easily observed. Near threshold for this reaction the time component[1] of the axial current plays an important role and so may also be readily observed.

In this paper we present an approximate calculation for the cross section $\sigma(p+{}^{12}C \to {}^{13}C+\mu^+ + \nu_\mu)$ for proton center of mass energies from 1060 to 1085 MeV. The calculation makes use of an elementary particle model treatment whereby the axial current matrix elements are determined via the partially conserved axial vector current hypothesis (PCAC) from $p + {}^{12}C \to {}^{13}C+\pi^+$. The vector current matrix elements are determined from $p + {}^{12}C \to \gamma + {}^{13}N$ data via the conserved vector current hypothesis (CVC). This method complements a more standard treatment used to perform an earlier[2] approximate calculation.

In Sec. II of this paper we obtain the matrix elements of the axial vector and vector currents. In Sec. III we obtain the cross section $\sigma(p+{}^{12}C \to {}^{13}C+\mu^+ + \nu_\mu)$. In Sec. IV we discuss the cross section obtained here, and also the possibility of calculating an inclusive reaction cross section. We also consider the effect on the cross section of low-lying anomalous threshold states in the nuclear PCAC relation.

II. CURRENT MATRIX ELEMENTS

We examine the hadronic part of the axial current matrix element for the process

$$p+{}^{12}C \to {}^{13}C+\mu^+ + \nu_\mu\,, \quad \langle {}^{13}C \,|\, A^\mu \,|\, p, {}^{12}C \rangle\ .$$

We note that ${}^{13}C$, p, and ${}^{12}C$ are $\frac{1}{2}^-$, $\frac{1}{2}^+$, and 0^+ nuclei, respectively. Making use of Lehman-Symanzik-Zimmerman[3] (LSZ) notation, the structure of $\langle {}^{13}C \,|\, A^\mu \,|\, p, {}^{12}C \rangle$ may be given as

$$\langle {}^{13}C \,|\, A_\mu \,|\, p, {}^{12}C \rangle = \bar{u}_{f\alpha} C^{\mu}_{\alpha\beta}(p_f{}^{\nu}, p^{\nu'}, p_c^{\nu''}) u_{p\beta}\ , \tag{1}$$

where $p_{f\mu}$, p_μ, and $p_{i\mu}$ are the four-momenta of the ${}^{13}C$, p, and ${}^{12}C$ nuclei, respectively. The axial current must satisfy the usual relationships $P\mathbf{A}(0)P^{-1} = \mathbf{A}(0)$ and $PA_0(0)P^{-1} = -A_0(0)$ which then constrains Eq. (1). Because the initial state is a two-body state, the number of contributing form factors is eight, taking into account the possible helicity states[4] and parity. We may write the matrix element of Eq. (1) therefore in terms of eight independent form factors as

$$\begin{aligned}\langle {}^{13}C \,|\, A_\mu \,|\, {}^{12}C, p \rangle = \bar{u}_f(&\gamma_\mu F_1 + Q_\mu F_2 + d_\mu F_3 + q_\mu F_4 \\ &+ \sigma_{\mu\nu} Q^\nu F_5 + \sigma_{\mu\nu} d^\nu F_6 + \sigma_{\mu\nu} q^\nu F_7 \\ &+ \epsilon_{\mu\nu\rho\sigma} q^\nu Q^\rho d^\sigma \gamma_5 F_8) u_p\ ,\end{aligned} \tag{2}$$

where $Q_\mu = p_{f\mu} + p_\mu$, $d_\mu = p_{f\mu} - p_\mu$, and $q_\mu = d_\mu - p_{i\mu}$.

However, we will be looking at processes for which $q = (q_0, \mathbf{q} \sim 0)$, so that we are mostly concerned with representing $\langle {}^{13}C \,|\, A_0(0) \,|\, {}^{12}C, p \rangle$. For this only two form factors are necessary.[5] From Eq. (2) we find that $F_1\gamma_\mu$ and $q_\mu F_4$ are a sufficient choice where we have neglected terms of the order $\mathbf{p}_\alpha^2/M_\alpha^2$, where $\mathbf{p}_\alpha$ is a nuclear momentum and M_α is the nuclear mass. We thus write, relabeling F_4 as F_2, $\langle {}^{13}C \,|\, A^\mu \,|\, {}^{12}C, p \rangle$ as

$$\langle {}^{13}C \,|\, A_\mu(0) \,|\, {}^{12}C, p \rangle \cong \bar{u}_f(\gamma_\mu F_1 + q_\mu F_2) u_p\ , \tag{3}$$

although we know that only the timelike part will be important for our calculation here.

It is perhaps worthwhile to comment on Eq. (3). It looks very much like an axial vector current for the nucleon case except that it is missing a γ_5. This is because p and ${}^{13}C$ have opposite parity. As a result the large terms in both Eqs. (3) and (2) are the time components instead of the usual case of large space components for the axial current matrix element. The authors of Ref. 2 noted this effect but here it is immediately apparent and due to the spin parity assignments of the participating nuclei.

The simple structure of Eq. (3) is, however, somewhat misleading because the form factors F_1 and F_2 are functions of four independent scalars q^2, d^2, $Q \cdot q$, and η^2, where $\eta^\mu = \epsilon^{\mu\nu\rho\sigma} p_\nu p_{i\rho} p_{f\sigma}$ and

$$\eta^2=p^2p_f^2[(m^2+m_i^2+2EE_i)\sin^2\theta+2p^2] \tag{4}$$

in the center of mass frame. However both d^2 and $Q\cdot q$ are slowly varying in our range of interest and the form factors here are effectively functions of q^2 and η^2. This dependence on two scalars was also noted by the authors of Ref. 2 but here it is forced from this type of treatment.

It is thus only necessary to obtain F_1 and F_2. We do this by making use of data available for the related process $p+{}^{12}C\rightarrow{}^{13}C+\pi^+$. We note that

$$\partial_\mu A^\mu=f_\pi m_\pi^2\phi_\pi , \tag{5}$$

which leads to the relation

$$\langle{}^{13}C\,|\,\partial_\mu A^\mu\,|\,{}^{12}C,p\rangle=\frac{f_\pi m_\pi^2}{m_\pi^2-q^2}\langle{}^{13}C\,|\,J_\pi\,|\,{}^{12}C,p\rangle , \tag{6}$$

where we have made use of $(\Box^2+m_\pi^2)\phi_\pi=J_\pi$. The q^2 at which the pion production takes place is necessarily $q^2=m_\pi^2$. In Eq. (3) the pion contributes[6] only to F_2 so that if we take the limit indicated

$$\lim(m_\pi^2-q^2)\langle{}^{13}C\,|\,\partial A_\mu\,|\,{}^{12}C,p\rangle =f_\pi m_\pi^2\langle{}^{13}C\,|\,J_\pi\,|\,{}^{12}C,p\rangle,\quad q^2\rightarrow m_\pi^2 \tag{7}$$

we obtain a relation involving only F_2,

$$i\bar{u}_f m_\pi^2F_2'u=f_\pi m_\pi^2\langle{}^{13}C\,|\,J_\pi\,|\,p,\,{}^{12}C\rangle , \tag{8}$$

where F_2' denotes the coefficient of the pion pole. However, we wish to determine F_1 because its contribution to $\sigma(p+{}^{12}C\rightarrow{}^{13}C+\mu^++\nu_\mu)$ is a large one. We do this by using a form of PCAC due originally to Nambu, namely that $\langle f\,|\,\partial_\mu A^\mu\,|\,i\rangle=0$ in the limit in which m_π^2 goes to zero. This leads to the relation

$$F_2=-\frac{F_1(mp_f\cdot q+m_fp\cdot q)}{(p\cdot p_f+mm_f)q^2} \tag{9}$$

or that

$$F_2=-\frac{F_1(mp_f\cdot q+m_fp\cdot q)}{(p\cdot p_f+mm_f)(q^2-m_\pi^2)} \tag{10}$$

if the pion is allowed to develop a mass. This relationship is a fairly good one for the nucleon but is not as good for nuclei because of anomalous thresholds. The question of the accuracy of Eq. (10) has been examined by Kim and Primakoff who find[7] that

$$F_P(q^2,i\leftrightarrow f)=-\frac{F_A(q^2,i\leftrightarrow f)}{q^2-m_\pi^2}[1+\epsilon(q^2;i\leftrightarrow f)] , \tag{11}$$

where F_P, the pseudoscalar form factor, corresponds to F_2, and F_A, the axial current form factor, corresponds to F_1 in our notation. The quantity $\epsilon(q^2;i\leftrightarrow f)$ represents the correction to the form of PCAC given by Eq. (10). These authors[7] estimate

$$\epsilon(-0.74m_\mu^2,{}^{12}B\leftrightarrow{}^{12}C)\cong-0.15$$

and find that

$$\lim_{A\rightarrow\infty}\epsilon(-m_\mu^2,i(A)\leftrightarrow f(A))\cong-0.29 . \tag{12}$$

Because of the order of approximation in the calculation presented here, an error even substantially larger than that indicated in Eq. (12) is acceptable and we therefore shall assume Eq. (10). We are then able to calculate the differential cross section in terms of F_1 and obtain

$$\frac{d\sigma}{d\Omega}=\frac{p_\pi(mE_iE_\pi+mEE_\pi-mm_\pi^2+m_fEE_\pi-m_fp\cdot p_\pi)^2F_1^2}{16\pi^2f_\pi^2p(E+E_i)^2(E_iE_\pi+EE_\pi-m_\pi^2+mm_f)} . \tag{13}$$

Data[8] are available for the reaction ${}^{12}C(p,\pi^+){}^{13}C$ in the range of $E_p=1094.7$ to 1138.3 MeV where E_p is the total relativistic energy of the proton. We are interested in the lower part of this range. From the data we find that F_1^2 may be parametrized as follows:

$$F_1^2=1.5\times10^{-5}(1-0.101\theta^2)/(1+6.5\theta^2) , \tag{14}$$

where θ is the angle of the outgoing pion. We can also parametrize F_1^2 in terms of our scalars, but Eq. (14) is more convenient. Thus Eqs. (10) and (14) completely determine Eq. (3) and it is only necessary to determine the vector current.

The matrix element $\langle{}^{13}C\,|\,V_\mu\,|\,p,{}^{12}C\rangle$ may be written as in the axial case in terms of eight independent form factors.

$$\langle{}^{13}C\,|\,V_\mu\,|\,p,{}^{12}C\rangle=\bar{u}_f(\gamma_\mu\gamma_5F_{v1}'+q_\mu\gamma_5F_{v2}'+\sigma_{\mu\nu}Q^\nu\gamma_5F_{v3}'+\sigma_{\mu\nu}d^\nu F_{v4}'\gamma_5+\sigma_{\mu\nu}q^\nu F_{v5}'\gamma_5 +\epsilon_{\mu\nu\rho\sigma}q^\nu Q^\rho d^\sigma F_{v6}'+\epsilon_{\mu\nu\rho\sigma}\gamma^\nu Q^\rho d^\sigma F_{v7}'+\epsilon_{\mu\nu\rho\sigma}\gamma^\nu q^\rho d^\sigma F_{v8}')u_p . \tag{15}$$

If we again ignore p_α^2/M_α^2 terms we obtain

$$\langle{}^{13}C\,|\,V_\mu\,|\,p,{}^{12}C\rangle\cong\bar{u}_f(\gamma_\mu\gamma_5F_{v1}'+\sigma_{\mu\nu}Q^\nu\gamma_5F_{v3}'+\sigma_{\mu\nu}d^\nu\gamma_5F_{v4}'+\sigma_{\mu\nu}q^\nu F_{v5}'\gamma_5)u_p . \tag{16}$$

With the help of the relations

$$\begin{aligned}\bar{u}_f\gamma_\mu\gamma_5u(m_f-m)&=u_f(i\sigma_{\mu\nu}d^\nu\gamma_5+Q_\mu\gamma_5)u ,\\ \bar{u}_f\gamma_\mu\gamma_5u(m_f+m)&=u_f(i\sigma_{\mu\nu}Q^\nu\gamma_5+d_\mu\gamma_5)u ,\end{aligned} \tag{17}$$

we may write Eq. (16) as

$$\langle{}^{13}C\,|\,V_\mu\,|\,p,{}^{12}C\rangle=\bar{u}_f\{[F_{v1}'+(m+m_f)F_{v3}'+(m_f-m)F_{v4}']\gamma_\mu\gamma_5-d_\mu F_{v3}'\gamma_5-Q_\mu\gamma_5F_{v4}'+F_{v5}'\sigma_{\mu\nu}q^\nu\}u_p . \tag{18}$$

We apply the CVC condition to Eq. (18), namely,

$$\langle {}^{13}\mathrm{C} \,|\, \partial^{\mu} V_{\mu} \,|\, \mathrm{p}, {}^{12}\mathrm{C} \rangle = 0 \; . \tag{19}$$

This leads to a more complicated situation than in the nucleon case because $m_f - m$ is large and $q_\mu \neq p_{f\mu} - p\mu$. However, we obtain a set of equations[9] by using specific helicities in Eq. (19). We find that

$$\langle {}^{13}\mathrm{C} \,|\, V_\mu \,|\, \mathrm{p}, {}^{12}\mathrm{C} \rangle = \bar{u}_f (d_\mu F_{v3} \gamma_5 + Q_\mu \gamma_5 F_{v4} + \sigma_{\mu\nu} q^{\nu} \gamma_5 F_{v5}) u \; , \tag{20}$$

with

$$F_{v4} = -F_{v3} \frac{d \cdot q}{Q \cdot q} \; . \tag{21}$$

We are thus left with the necessity of determining two form factors. Ideally with sufficient data for processes such as $\mathrm{p} + {}^{12}\mathrm{C} \to {}^{13}\mathrm{N} + e^+ + e^-$, the form factors could be determined. However, no such data exist. There are data, however, for the electromagnetic process $\mathrm{p} + {}^{12}\mathrm{C} \to {}^{13}\mathrm{N} + \gamma$, which is a $q^2 = 0$ process. The matrix element $\langle {}^{13}\mathrm{C} \,|\, V_\mu \,|\, {}^{12}\mathrm{C}, \mathrm{p} \rangle$ is related to the hadronic part of the matrix element for the process $\mathrm{p} + {}^{12}\mathrm{C} \to {}^{13}\mathrm{N} + \gamma$ via the CVC hypothesis

$$\begin{aligned} \langle {}^{13}\mathrm{C} \,|\, V_\mu \,|\, {}^{12}\mathrm{C}, \mathrm{p} \rangle &= \langle {}^{13}\mathrm{C} \,|\, [I^-, J_\mu^{(3)}] \,|\, {}^{12}\mathrm{C}, \mathrm{p} \rangle \\ &= \langle {}^{13}\mathrm{N} \,|\, J_\mu^{(3)} \,|\, {}^{12}\mathrm{C}, \mathrm{p} \rangle \\ &\quad - \langle {}^{13}\mathrm{C} \,|\, J_\mu^{(3)} \,|\, {}^{12}\mathrm{C}, \mathrm{n} \rangle \; . \end{aligned} \tag{22}$$

The first and second terms on the right-hand side of Eq. (22) are simply related by charge symmetry,

$$\langle {}^{13}\mathrm{N} \,|\, J_\mu^{(3)}(0) \,|\, {}^{12}\mathrm{C}, \mathrm{p} \rangle = -\langle {}^{13}\mathrm{C} \,|\, J_\mu^{(3)}(0) \,|\, {}^{12}\mathrm{C}, \mathrm{n} \rangle \; , \tag{23}$$

so that electromagnetic data for the ${}^{12}\mathrm{C} + \mathrm{p}$ ${}^{13}\mathrm{N}$ transition should, in principle, be sufficient to determine the left-hand side of Eq. (22).

A straightforward calculation for the transition matrix element for the process ${}^{12}\mathrm{C} + \mathrm{p} \to {}^{13}\mathrm{N} + \gamma$ yields

$$\begin{aligned} |M|^2 = \frac{e^2}{m_f m} [& d^2(-p_f \cdot p + m_f m) F_{v3}^2 + 2(m_f m - p_f \cdot p) d \cdot Q F_{v3} F_{v4} \\ & + Q^2(-p_f \cdot p + m_f m) F_{v_4}^2 + 2(p_f \cdot d p \cdot q - p_f \cdot q p \cdot d) \,|\, F_{v_5} \,|\, F_{v3} \\ & + 2(p_f \cdot Q q \cdot p - p_f \cdot q p \cdot Q) \,|\, F_{v_5} \,|\, F_{v4} + (4 p_f \cdot q p \cdot q - p_f \cdot p q^2 + 3 m_f m q^2) \,|\, F_{v_5} \,|^2] \; . \end{aligned} \tag{24}$$

We have no way of determining the two independent form factors in Eq. (24). However, our experience has shown that when consistently normalized,[10] the vector form factors tend to be of the same magnitude. Because in any case we find by direct calculation that the vector current contribution to the process ${}^{12}\mathrm{C} + \mathrm{p} \to {}^{13}\mathrm{C} + \mu^+ + \nu_\mu$ is small, so we make the assumption that $F_{v3} \cong |F_{v5}|$. Then we may write Eq. (24) in terms of a single form factor F recalling from Eq. (21) that F_{v4} is related to F_{v3}. The differential cross section may be calculated as

$$\frac{d\sigma}{d\Omega} = \frac{m m_f E \,|\, M \,|^2}{4(2\pi)^2 p (E_i + E_\mathrm{p})^2} \; . \tag{25}$$

From experimental data[11] we are then able to determine

$$F^2 = 3.125 \times 10^{-5} / \{ [1 + 1.869(\theta - 0.942^2] m_f^2 \} \; . \tag{26}$$

This is only for $q^2 = 0$, but there are no additional data and as we shall see, in any case, the vector current contribution to the total matrix element is relatively small.

III. CALCULATION OF THE CROSS SECTION

We are now ready to obtain the cross section for the process $\mathrm{p} + {}^{12}\mathrm{C} \to {}^{13}\mathrm{C} + \mu^+ + \nu_\mu$. The matrix element for the process is given by

$$\langle \mu^+ \nu {}^{13}\mathrm{C} \,|\, H_w \,|\, \mathrm{p}, {}^{12}\mathrm{C} \rangle = \frac{G}{\sqrt{2}} \cos\theta \bar{u}_\nu \gamma^\lambda (1 - \gamma_5) v_\mu \langle {}^{13}\mathrm{C} \,|\, J_\lambda \,|\, {}^{12}\mathrm{C}, \mathrm{p} \rangle \; , \tag{27}$$

where

$$J_\mu = V_\mu - A_\mu \tag{28}$$

and the matrix elements for V_μ and A_μ are determined by Eqs. (3), (10), and (14) and by Eqs. (20), (21), and (26), respectively. With these choices we obtain

$$\begin{aligned} | \langle \mu^+ \nu {}^{13}\mathrm{C} \,|\, H_w \,|\, \mathrm{p}, {}^{12}\mathrm{C} \rangle |^2 = \frac{G^2 \cos^2\theta_c}{2} \frac{1}{2 m m_f m_\mu} & (2F_1^2 (p_f \cdot \mu p \cdot \nu + p_f \cdot \nu p \cdot \mu - m_f m \mu \cdot \nu) + F_2{}^2 m_\nu^2 \nu \cdot \mu (p_f \cdot p + m_f m) \\ & + F_{v3}^2 (2 d \cdot \nu d \cdot \mu - \nu \cdot \mu d^2)(p_f \cdot p - m_f m) + F_{v4}^2 (2 Q \cdot \nu Q \cdot \mu - \nu \cdot \mu Q^2)(p_f \cdot p - m_f m) + |F_{v5}|^2 \\ & \times \{ -4(p_f \cdot \nu p \cdot \nu m_\mu^2 + p_f \cdot \nu \nu \cdot p \nu \cdot \mu + p_f \cdot \mu p \cdot \mu \nu \cdot \mu) + p_f \cdot p \nu \cdot \mu m_\mu^2 \\ & \quad - m_f m [4(\nu \cdot \mu)^2 + 3 m_\mu^2 \nu \cdot \mu] \} - 2 F_1 F_2 m_\mu^2 (m p_f \cdot \nu + m_f p \cdot nu) + 2 F_{v3} F_{v4} (p_f \cdot p - m_f m) \\ & \times [2 p_f \cdot \mu p_f \cdot \nu - 2 p \cdot \mu p \cdot \nu - (m_f^2 - m^2) \nu \cdot \mu] \end{aligned}$$

$$-2F_{v3}|F_{v5}|\{(d\cdot\nu-d\cdot\mu)(p_f\cdot\nu p\cdot\mu-p_f\cdot\mu p\cdot\nu)$$

$$+\mu\cdot\nu[(m_f^2-p_f\cdot p)(p\cdot\mu+p\cdot\nu)-(p_f\cdot\mu+p_f\cdot\nu)(p_f\cdot p-m^2)]\}$$

$$-2F_{v4}|F_{v5}|\{(p_f\cdot\nu+p\cdot\nu-p_f\cdot\mu-p\cdot\mu)(p_f\cdot\nu p\cdot\mu-p_f\cdot\mu p\cdot\nu)$$

$$-\mu\cdot\nu[(p_f\cdot\nu+p_f\cdot\mu)(p_f\cdot p+m^2)-(p_f\cdot p+m_f^2)(p\cdot\mu+p\cdot\nu)]\}$$

$$-4F_1F_{v5}[(m_\mu^2+\mu\cdot\nu)(mp_f\cdot\nu-m_fp\cdot\nu)+\mu\cdot\nu(m_fp\cdot\mu-mp_f\cdot\mu)]\}\ , \qquad (29)$$

where we have summed and averaged over appropriate spins.

The cross section is obtained from

$$\sigma=\frac{mm_fm_\mu}{8\pi^3p(E_i+E)}\int_{m_\mu}^{E_{\mu f}}\int_{E_{f\,\min}}^{E_{f\,\max}}|M|^2dE_fdE_\mu \qquad (30)$$

by computer numerical integration routines. The quantity $|M|^2$ is given by Eq. (24). The results are given in Fig. 1. We find by setting the vector form factors equal to zero that the vector contribution to Eq. (30) is of the order of 3% or 4% which is not significant at the order of approximation used here.

IV. DISCUSSION OF RESULTS

As has been mentioned before, a calculation[12] of this process has been previously undertaken. The authors of that calculation used an impulse approximation based method and PCAC correction to obtain an order of magnitude result. They noted certain features of the reaction, particularly the large role played by the time component of the axial current and the necessity of using a variable in addition to q^2 to describe the process. As we have noted, these features appear in a straightforward way in the treatment given here. They also obtained results for $d\sigma/d\Omega_\mu$ for several other states besides the results for the transition considered here.

When their results

$$\frac{d\sigma}{d\Omega}(p+{}^{12}C\rightarrow{}^{13}C_{g.s.}+\mu^++\nu)$$

are integrated over angle one obtains an approximate $\sigma\cong1.16\times10^{-46}$ cm^2 for 148 MeV protons (laboratory). When we compare this with our results after transforming the protons to the c.m. frame we obtain $\sigma=3.3\times10^{-46}$ cm^2, so that our σ is approximately three times as large. The authors of Ref. 2 speculated that the inclusion of PCAC relations between the nuclear axial current and nuclear pion field might enhance their results by an order of magnitude. Our result, which in fact does relate the nuclear pion field to the nuclear axial current, has been calculated conservatively and agrees generally with these speculations. We are unable to make calculations for the other states considered by Weiss and Walker at this time due to the absence of necessary experimental data. However, it may be possible to make an inclusive calculation of the form $p+{}^{12}C\rightarrow\mu^++\nu_\mu+X$ where X is limited to excited nuclear states. We shall consider this in a future calculation.

The observation of the reaction $p+{}^{12}C\rightarrow{}^{13}C+\mu^++\nu_\mu$ would provide a way of testing several ideas which require a timelike four-momentum. In the reaction considered here because $q=-(p_\mu+p_\nu)$,

$$q^2=m_\mu^2+2E_\mu E_\nu-2\mathbf{p}_\mu\cdot\mathbf{p}_\nu\ , \qquad (31)$$

which is strictly timelike. Some recent work[13] has implied that the axial current form factor $F_A(q^2)$ may be mirrorlike in the near timelike region, $q^2\cong m_\pi^2$, so that $F_A(m_\pi^2)\cong F_A(-m_\pi^2)$. In order to check this result, a timelike process is needed and so a breakup process of the type described here may be useful.

Another long-standing problem which should be mentioned in connection with the reaction considered here is that of anomalous threshold contributions to the nuclear matrix elements $\langle N_f|\partial_\mu A^\mu(0)|N_i\rangle$. In the nucleon case one obtains a pion-pole contribution and a cut contribution starting at $q^2=(3m_\pi)^2$. For the nuclear case the cut contribution begins at a q^2 less than $(3m_\pi)^2$ because virtual breakup of the nucleus is possible. For example, in the case of ${}^3H\leftrightarrow{}^3He$ the anomalous contribution starts at approximately $(1.7m_\pi)^2$ because virtual breakup of the

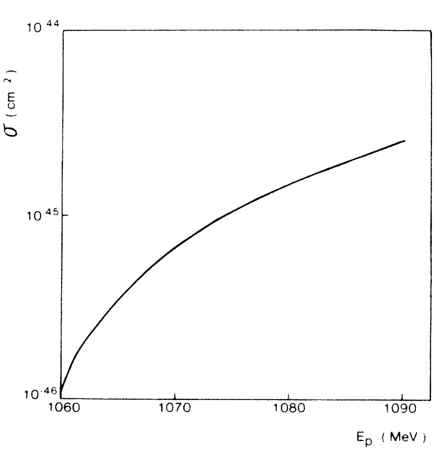

FIG. 1. Plot of the total cross section $\sigma(p+{}^{12}C\rightarrow{}^{13}C+\mu^++\nu_\mu)$ as a function of center of mass proton total energy.

nucleus N_i, into $N_i'+n$ is considered. There is in fact some speculation that for more exotic breakup processes, lower anomalous thresholds might be possible. Most weak processes which are observed are processes for spacelike q^2. The anomalous contributions for these processes affect F_P and are relatively small[14] and hard to observe. Experiments in the timelike region should enhance the effects of the anomalous thresholds. In particular if the cut region is reached, virtual processes in the spacelike region become allowed breakups in the timelike region and should be observable in cross-section measurements.

For all of these reasons additional theoretical estimates for this and related processes are very desirable and, if feasible, experimental searches for these processes might yield interesting results.

---

[1]We note that the time components for the axial currents are the large components because of the spin and parity assignments for ^{12}C, p, and ^{13}C. This effect is accentuated because all of the momentum vectors Q_μ, d_μ, and q_μ in the hadronic matrix element are timelike. This is true for Q_μ and d_μ by their construction and q because $q_\mu=-(P_\mu+P_\nu)$ which near threshold is $(m_\mu, \sim 0)$ and is always timelike.

[2]D. L. Weiss and G. E. Walker, Phys. Rev. C **25**, 991 (1982).

[3]H. Lehmann, K. Symanzik, and W. Zimmerman, Nuovo Cimento **1**, 1425 (1955).

[4]On the basis of helicities there are $2\times4\times2=16$ independent amplitudes. Parity removes half of them leaving eight. For a discussion of this see S. L. Mintz, Phys. Rev. D **8**, 2946 (1973).

[5]Again using helicity arguments there are $2\times1\times2=4$ independent amplitudes but parity removes half leaving two amplitudes.

[6]This follows immediately from a standard dispersion analysis

$$\langle {}^{13}\mathrm{C}\bar{\mathrm{p}}\,{}^{12}\bar{\mathrm{C}} \mid A_\mu \mid 0\rangle_{q^2+i\epsilon} - \langle {}^{13}\mathrm{C}\bar{\mathrm{p}}\,{}^{12}\bar{\mathrm{C}} \mid A_\mu \mid 0\rangle_{q^2-i\epsilon} = \sum_i \delta(p_f+\bar{p}+\bar{p}_C-p_i)\langle {}^{13}\mathrm{C}\bar{\mathrm{p}}\,{}^{12}\bar{\mathrm{C}} \mid i\rangle\langle i \mid A_\mu \mid 0\rangle$$

and choosing $|\, i > = |\, \pi\rangle$. For detailed descriptions see Ref. 4.

[7]C. W. Kim and H. Primakoff, in *Mesons in Nuclei*, edited by M. Rho and D. H. Wilkinson (North-Holland, Amsterdam, 1979), p. 69.

[8]F. Soga *et al.*, Phys. Rev. C **24**, 570 (1981).

[9]See Ref. 4 for methods of extraction of the form factors for matrix elements of this type.

[10]See for example, C. W. Kim and S. L. Mintz, Phys. Lett. **31B**, 503 (1970), J. Frazier and C. W. Kim, Phys. Rev. **177**, 2560 (1969), Ref. 4, and S. L. Mintz, Phys. Rev. C **19**, 476 (1979).

[11]S. L. Blatt *et al.*, Phys. Rev. C **30**, 423 (1984).

[12]See Ref. 2. Also seminar at the Summer Workshop, Queens University, 1983.

[13]B. Bosco, C. W. Kim, and S. L. Mintz, Phys. Rev. C **25**, 1986 (1982).

[14]See Ref. 7, and also, for example, S. L. Mintz, submitted to Nuovo Cimento. For a discussion of anomalous thresholds see for example, C. Jarlskog and F. J. Yndurain, Nuovo Cimento **12A**, 801 (1972).