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Few channel models of nuclear reactions: Proton and deuteron elastic scattering expansions

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In this paper, an asymmetric, antisymmetrized optical potential formalism is used to derive the low-order terms in multiple-scattering-type expansions for nonrelativistic proton-nucleus and deuteron-nucleus elastic scattering. In particular, the extended Faddeev theory of N-particle collisions is used to obtain the properly symmetrized, two-particle and three-particle contributions in these two cases, where it has been assumed that all potentials are pair interactions. The main purpose underlying the research reported herein has been to prove that standard results can be obtained from an asymmetric formalism. For the two-particle contributions, the usual $t^A\rho$-type expression is obtained, where t^A is an antisymmetrized, two-particle transition operator. At this level of approximation, the results for the (d,d) case are the same as for the usual folding model. The three-particle contributions in the nucleon-nucleus case are identical to those of Picklesimer and Thaler, thus further demonstrating that standard results are contained in the asymmetric approach. The antisymmetrized, three-particle contributions to the (d,d) optical potential do not seem to have been previously derived; they are the obvious analog of the nucleon-nucleus results, and are physically reasonable in form. However, at this level of approximation, the (d,d) description is no longer found to be that of a folding model.

I. INTRODUCTION

This paper extends previously published research[1] on the properties of a new class of two-fragment, antisymmetrized, elastic scattering optical potentials by deriving multiple scattering expansions for nucleon-nucleus and deuteron-nucleus collisions. It also provides an introduction to and a partial basis for the analysis of the following paper on direct nuclear reaction models.[2]

The feature distinguishing the members of the new class of optical potentials from the standard, Feshbach-type[3] is their asymmetry, i.e., they are non-Hermitian. This feature persists for energies below all inelastic thresholds, although for such energies, the new optical potentials are nonabsorptive,[1] i.e., they are real, as required by flux conservation. The fact that this asymmetry/non-Hermiticity is not a practical problem is attested to by the set of all numerical results which have been obtained by solving the three-body Faddeev equations:[4] this formalism is also asymmetric and therefore non-Hermitian.[5] Furthermore, it was noted in Ref. 1 that the members of the new class of optical potentials are as amenable as is the standard, Feshbach-type[3] to approximation by means of a potential containing adjustable parameters and which is local in coordinate space.

The ability of a new approach to elastic scattering to yield the conventional (local potential) description is clearly a necessity. A further requirement is that it should also yield other standard descriptions and/or approximations in the appropriate limits. In particular, it should lead to a "$t\rho$" type of approximation via a multiple scattering expansion.[6] Our purpose in this first paper of two is to derive such expansions for the cases of nucleon-nucleus and deuteron-nucleus scattering, using that member of the new class previously denoted the extended Faddeev (EF) theory.[1,7] The reason for choosing this particular formalism is that it leads not only to a multiple scattering representation but it and its time-reversal partner, the Bencze, Redish, and Sloan (BRS) theory,[8] also lead to a solution of the old problem of the role of Pauli principle exchange effects in the distorted wave Born approximation for particle transfer reactions.[9] Furthermore, the BRS form of the theory provides a justification of the standard three-body, deuteron-nucleus collision model.[9,10] The first of these topics is discussed in detail in the following article; the second is treated in a forthcoming paper.

The nucleon-nucleus multiple-scattering type of expansions obtained herein are not new: we find the same results as obtained earlier by Picklesimer and Thaler.[6] That is, we are able to show, despite its asymmetric character,

that the non-Hermitian EF collision formalism leads to precisely those multiple scattering expressions expected on physical grounds and previously obtained via a more standard approach. On the other hand, our deuteron-nucleus results seem to be new.

Multichannel collision formalisms are in general notationally complex and the extended Faddeev theory employed herein is no exception. The symbology is discussed in several recent publications,[1,7,9,11] but in order to make this pair of papers reasonably self-contained, a brief review of the notation and symmetrization procedure is given in Sec. II. Our main results are given in Secs. III, IV, and V.

II. NOTATION AND ANTISYMMETRIZATION

To establish notation, we first assume that the N particles forming the scattering system are distinguishable. They are labeled 1,2...N. General partitions of these particles into distinct clusters are denoted $b(j)$, where b specifies the number of clusters (n_b) as well as the number of particles in each cluster, while j, $0 \le j \le N_b$, refers to the labeling of particles in the clusters. Two-cluster partitions are denoted by lower case Greek letters, e.g., $\alpha(i)$, $\beta(j)$. The permutation operator $P_{b(j)b(k)}$ transforms partition $b(k)$ to partition $b(j)$ via $b(j)=P_{b(j)b(k)}b(k)$; $b(j)$ and $b(k)$ are members of the same equivalence class. The standard or canonical labeling for a partition is denoted by $j=0$; $\{b(0)\}_{\text{all } b}$ is the set of canonical labels,[12] and they are the only ones needed as generic labels for the equations used in the identical particle case. As a simple illustration for the four-particle ($N=4$) case, we could use $\alpha(0)=(12)(34)$, $\beta(1)=(2)(134)$, $b(3)=(1)(2)(34)$, with $n_\alpha=n_\beta=2$ and $n_b=3$.

For each $b(j)$, the Hamiltonian H of the system can be partitioned as $H=H_{b(j)}+V^{b(j)}$, where $V^{b(j)}$ is the set of intercluster interactions and $H_{b(j)}$ governs the internal as well as the relative plane wave motion of the clusters. Only those "plane wave" eigenstates of $H_{b(j)}$ are needed whose energy equals the total incident energy E; they are denoted $\Phi_{b(j)m}$, where m labels all bound state quantum numbers, $m=0$ denoting ground states. The outgoing wave Green's function in partition $b(j)$ is

$$G^{(+)}_{b(j)}(E)=(E+i0-H_{b(j)})^{-1};$$

the E dependence will usually be suppressed in the following, so that $G^{(+)}_{b(j)}(E)\to G^{(+)}_{b(j)}$, similar to the plane wave state $\Phi_{b(j)}$. Finally, the Benoist-Gueutal, L'Huillier, Redish, and Tandy distribution of $V^{d(k)}$ over all partitions $b(j)$ is[8]

$$V^{d(k)}=\sum_{b,j} C_b V^{d(k)}_{b(j)} , \tag{2.1}$$

where $C_b=(-1)^{n_b}(n_b-1)!$ and $V^{d(k)}_{b(j)}$ is the set of interactions both external to the clusters in partition $d(k)$ and internal to (binding) the clusters of partition $b(j)$. Equation (2.1) is used to derive the basic EF theory[7] and BRS precursor equations.[7,8]

Consider now a collision initiated in partition $\alpha(i)$, i.e., via $\Phi_{\alpha(i)0}$, where the relative momentum of the colliding pairs is $\mathbf{k}_\alpha$. The solution to the Schrödinger equation generated by $\Phi_{\alpha(i)0}$ is denoted $\Psi(\alpha(i))$. In the EF theory which we employ here, $\Psi(\alpha(i))$ is expanded into components $\psi_{b(j)}(\alpha(i))$, defined for all partitions $b(j)$, via

$$\Psi(\alpha(i))=\sum_{b,j}\psi_{b(j)}(\alpha(i)) , \tag{2.2}$$

where the $\psi_{b(j)}(\alpha(i))$ obey[7]

$$\psi_{b(j)}(\alpha(i))=\Phi_{\alpha(i)0}\delta_{b(j)\alpha(i)}+G^{(+)}_{b(j)} \times\sum_{d,k} V^{\mathrm{EF}}_{b(j)d(k)}\psi_{d(k)}(\alpha(i)) , \tag{2.3}$$

all $b(j)$, where

$$V^{\mathrm{EF}}_{b(j)d(k)}=C_b V^{d(k)}_{b(j)} . \tag{2.4}$$

Equation (2.3) is of the connected-kernel-type,[7] thus implying unique solutions as long as the interactions $V^{d(k)}_{b(j)}$ are suitably behaved,[13] which we assume here. Use of an integral equation form ensures that the proper boundary conditions are obeyed. It is shown in Ref. 7 that the $\psi_{b(j)}(\alpha(i))$ defined by (2.3) enjoy the property of being "true" components. That is, the amplitude describing transitions from $\alpha(i)$ to a two-body final state in two-cluster partition $\beta(j)$ is wholly contained in $\psi_{\beta(j)}(\alpha(i))$: no other $\psi_{d(k)}(\alpha(i))$, $d(k)\neq\beta(j)$, contribute (asymptotically) to such $\alpha(i)\to\beta(j)$ amplitudes. In addition, as a true component $\psi_{b(j)}(\alpha(i))$ will only contribute to final states in an arbitrary partition $d(k)$ if $d(k)=b(j)$ or if it can be obtained by breaking one or more clusters in $b(j)$ into a larger number of clusters, i.e., if $d(k)\subseteq b(j)$.

We now assume that the N particles are identical fermions: an isospin notation is to be employed in specifying nuclear states. The internal, bound states in $\Phi_{b(j)}$ are now taken to be antisymmetric. Hence, if $R_{b(j)}$ is the antisymmetrizer within the clusters of $b(j)$, then $R_{b(j)}\Phi_{b(j)m}=\Phi_{b(j)m}$. A properly antisymmetrized Schrödinger solution $\Psi^A(\alpha)$ is obtained from the $\Psi(\alpha(i))$ by applying to $\Psi(\alpha(0))$ the antisymmetrizer $A_{\alpha(0)}$:

$$\Psi^A(\alpha)=A_{\alpha(0)}\Psi(\alpha(0))=\hat{N}_\alpha^{-1/2}\sum_{i=0}^{N_\alpha}\hat{P}_{\alpha(i)\alpha(0)}\Psi(\alpha(0)) ,$$

where[1]

$$\hat{P}_{\alpha(i)\alpha(0)}=(-1)^{\sigma_{\alpha(i)}}P_{\alpha(i)\alpha(0)} ,$$

with $\sigma_{\alpha(i)}$ the proper fermion phase factor, and $\hat{N}_\alpha=N_\alpha+1$ (and similarly, $\hat{N}_b=N_b+1$, etc.).

Unlike $\Psi(\alpha(i))$, the components $\psi_{b(j)}(\alpha(i))$ refer to two (distinguishable-particle) partitions, and application of $A_{\alpha(0)}$ to $\psi_{b(j)}(\alpha(0))$ does not produce a fully antisymmetrized component. As discussed in Ref. 1, the procedure that leads via summation to the properly normalized, antisymmetrized components $\psi^A_b(\alpha)$ and to that canonically labeled component $\psi_{b(0)}(\alpha)$ which yields the properly normalized and antisymmetrized transition amplitudes or portions thereof, is to define $\psi_{b(j)}(\alpha)$ via

$$\psi_{b(j)}(\alpha)\equiv\hat{N}_b^{1/2}A_{\alpha(0)}\psi_{b(j)}(\alpha(0)) . \tag{2.5}$$

Then the following relations hold:[1]

$$R_{b(j)}\psi_{b(j)}(\alpha)=\psi_{b(j)}(\alpha) , \tag{2.6}$$

$$\psi_{b(j)}(\alpha)=\hat{P}_{b(j)b(0)}\psi_{b(0)}(\alpha)\ , \tag{2.7}$$

$$\psi_b^A(\alpha)=\hat{N}_b^{-1/2}\sum_{j=0}^{N_b}\psi_{b(j)}(\alpha)$$
$$=A_{b(0)}\psi_{b(0)}(\alpha)\ , \tag{2.8}$$

and

$$\Psi^A(\alpha)=\sum_b \hat{N}_b^{-1/2}\psi_b^A(a)\ . \tag{2.9}$$

The effect of relation (2.7) is that $\psi_{b(j)}(\alpha)$ and $\psi_{b(0)}(\alpha)$ asymptotically yield amplitudes differing only by the fermion phase factor $(-1)^{\sigma_{b(j)}}$ so that it is unnecessary to use $\psi_b^A(\alpha)$ to obtain (partial) amplitudes for $\alpha\to b$ processes: it is sufficient to consider only the canonically labeled components $\psi_{b(0)}(\alpha)$. These obey[1]

$$\psi_{b(0)}(\alpha)=\delta_{ba}\Phi_{\alpha(0)0}+G_{b(0)}^{(+)}\sum_d \tilde{V}_{b(0)d(0)}\psi_{d(0)}(\alpha)\ , \tag{2.10}$$

where the *exchange effect potential* $\tilde{V}_{b(0)d(0)}$ is given by

$$\tilde{V}_{b(0)d(0)}=(\hat{N}_b/\hat{N}_d)^{1/2}\sum_k R_{b(0)}V^{\rm EF}_{b(0)d(k)}\hat{P}_{d(k)d(0)}\ , \tag{2.11}$$

and $V^{\rm EF}_{b(0)d(k)}$ is defined by Eq. (2.4).

The set of equations (2.10) has N_e members, where N_e is the number of equivalence classes;[14] the size of (2.10) is much smaller than that of (2.3). Just as in the unsymmetrized case, the $\psi_{b(j)}(\alpha)$ are true components, so that, for example, only $\psi_{\beta(0)}(\alpha)$ yields amplitudes for transitions to bound states in the two-cluster partition $\beta(0)$. In the general case, these amplitudes are the plane wave matrix elements $\langle\Phi_{b(0)m}\mid T^A_{b(0)\alpha(0)}\mid\Phi_{\alpha(0)0}\rangle$ of the proper, fully antisymmetrized transition operators $T^A_{b(0)\alpha(0)}$, defined by[1]

$$T^A_{b(0)\alpha(0)}\mid\Phi_{\alpha(0)0}\rangle=\sum_d \tilde{V}_{b(0)d(0)}\psi_{d(0)}(\alpha)\ . \tag{2.12}$$

Substituting (2.10) into (2.12) leads to

$$T^A_{b(0)\alpha(0)}=\tilde{V}_{b(0)\alpha(0)}+\sum_d \tilde{V}_{b(0)d(0)}G^{(+)}_{d(0)}T^A_{d(0)\alpha(0)}\ . \tag{2.13}$$

It is shown in Ref. 1 that these $T^A_{b(0)\alpha(0)}$ are identical to the (symmetrized) class operators introduced by Bencze and Redish.[12]

Equations (2.10) and (2.13) are much easier to deal with when expressed in the following matrix form:

$$\psi_0=\Phi_0+\underline{G}_0\tilde{\underline{V}}_0\psi_0 \tag{2.14}$$

and

$$\underline{T}^A_0=\tilde{\underline{V}}_0+\tilde{\underline{V}}_0\underline{G}_0\underline{T}^A_0\ , \tag{2.15}$$

where comparison with (2.10) and (2.13) should render the notation obvious (see Ref. 1 for details). The subscript 0 serves as a reminder that only canonically-labeled partitions are used as the index set in (2.14) and (2.15).

We treat elastic scattering in this paper and the distorted wave Born approximation for direct nuclear reactions in the next, so that among all the components and transition operators contained in (2.14) and (2.15) only a few need be considered. We do this by means of projection operators.[3] Let $\underline{I}_0$ be the $N_e\times N_e$ unit matrix, and $\underline{P}_0$ and $\underline{Q}_0=\underline{I}_0-\underline{P}_0$ be two $N_e\times N_e$ projection operators. If $\phi_{b(0)m}$ is the product of the bound states in $\Phi_{b(0)m}$, then in the case of elastic scattering, $\underline{P}_0\to\underline{P}^{\rm el}_0$, where

$$(\underline{P}^{\rm el}_0)_{bd}=\delta_{bd}\delta_{b\alpha}P_{\alpha(0)0}\ , \tag{2.16}$$

with

$$P_{\alpha(0)0}=\mid\phi_{\alpha(0)0}\rangle\langle\phi_{\alpha(0)0}\mid\ . \tag{2.17}$$

In general, $\underline{P}_0$ will consist of 1's and/or projectors such as $P_{\alpha(0)0}$ along a few of the main diagonal entries and will be zero otherwise. It is straightforward to show that[1]

$$\underline{P}_0\psi_0=\Phi_0+\underline{P}_0\underline{G}_0\tilde{\underline{U}}_0\underline{P}_0\psi_0 \tag{2.18}$$

and

$$\underline{P}_0\underline{T}^A_0\underline{P}_0=\underline{P}_0\tilde{\underline{U}}_0\underline{P}_0+\underline{P}_0\tilde{\underline{U}}_0\underline{P}_0\underline{G}_0\underline{P}_0\underline{T}^A_0\underline{P}_0\ , \tag{2.19}$$

where

$$\tilde{\underline{U}}_0=\tilde{\underline{V}}_0+\tilde{\underline{V}}_0\underline{Q}_0(\underline{G}_0^{-1}-\underline{Q}_0\tilde{V}_0\underline{Q}_0)^{-1}\underline{Q}_0\tilde{\underline{V}}_0 \tag{2.20}$$

is the formal solution to

$$\tilde{\underline{U}}_0=\tilde{\underline{V}}_0+\tilde{\underline{V}}_0\underline{Q}_0\underline{G}_0\tilde{\underline{U}}_0\ . \tag{2.21}$$

As discussed in Ref. 1,

$$\tilde{\mathscr{V}}_\alpha=\langle\phi_{\alpha(0)0}\mid\tilde{U}_{\alpha(0)\alpha(0)}\mid\phi_{\alpha(0)0}\rangle \tag{2.22}$$

is the two-fragment, antisymmetrized, elastic scattering, asymmetric optical potential for the EF theory when $\underline{P}_0=\underline{P}^{\rm el}_0$.

III. MULTIPLE-SCATTERING TYPE OF EXPANSIONS

Our primary goal in this article is to show that an antisymmetrized $t\rho$-type approximation can be obtained for the nucleon and deuteron asymmetric optical potentials. These will form the lowest-order, two-body contributions, where t is a nucleon-nucleus transition operator and ρ is the nucleon density in the target. A secondary objective is to determine the lowest-order corrections involving three-body contributions, again taking account of the Pauli principle. Our results will be seen to be either analogous or identical to those of similar analyses based on starting points[15–18] other than the EF asymmetric collision theory. Our analysis is based on two procedures: the use of a connectivity expansion[15] or cluster decomposition[18] and the identification in such a decomposition of the two-body and three-body contributions, terms we shall define shortly. Since the results are analogous to those of a multiple-scattering series, we refer to this approach as a multiple-scattering type of expansion.

Instead of working with the incident channel optical potential $\tilde{\mathscr{V}}_\alpha$, we employ the optical potential operators $\tilde{\underline{U}}_0$ of Eq. (2.20) or equivalently, the transition operators $\underline{T}^A_0$. They are related by

$$\underline{T}^A_0=\tilde{\underline{U}}_0+\tilde{\underline{U}}_0\underline{P}_0\underline{G}_0\underline{T}^A_0\ , \tag{3.1}$$

which is abstracted from (2.19). The ground state matrix element of $T^A_{\alpha(0)\alpha(0)}$ is related to $\tilde{\mathscr{V}}_\alpha$ by Eq. (2.19).

We shall apply a cluster decomposition to (3.1). In

such a decomposition, an N-particle operator B is expressed[18] as a sum of terms $[B]_a$, each having the connectivity of partition a:

$$B=\sum_a [B]_a \ . \tag{3.2}$$

Probably the simplest realization of such a decomposition is in terms of interactions. If, for example, an interaction V is given by a sum of two-particle potentials, viz., $V=\sum_{i<j} V_{ij}$, then this sum is equivalent to Eq. (3.2) with the index a running over the partitions $(ij)(1)(2)\ldots(i-1)(i+1)\ldots(j-1)(j+1)\ldots(N)$. We note that the presence of the free, N-particle Green's function $G_0^{(+)}$ as a multiplicative factor in an (integral) equation defining a quantity does not affect the connectivity properties of that quantity.

In view of the fact that the operator product $\underline{\tilde{U}}_0\underline{P}_0=\underline{\tilde{U}}_0\underline{P}_0^{\rm el}$ in (3.1) has the connectivity of the unique one-cluster partition $a=1$, then it follows that

$$(T^A_{\alpha(0)\alpha(0)})_a=(\tilde{U}_{\alpha(0)\alpha(0)})_a, \quad a\neq 1 \ . \tag{3.3}$$

Eq. (3.3) shows that a solution of (2.19), e.g., by iteration, only changes the $a=1$, connected term. Hence, if it is reasonable to approximate $T^A_{\alpha(0)\alpha(0)}$ by one or several terms $(T^A_{\alpha(0)\alpha(0)})_a$ with $a\neq 1$, then in such an approximation, (3.3) would yield the relevant contribution to $T^A_{\alpha(0)\alpha(0)}$. Conversely, contributions to $\tilde{U}_{\alpha(0)\alpha(0)}$ from partitions $a\neq 1$ can be obtained from (3.3) by assuming $(T^A_{\alpha(0)\alpha(0)})_a$ to be known. We shall follow this latter procedure. In particular, we shall determine the two-body and three-body contributions to $\tilde{U}_{\alpha(0)\alpha(0)}$ by examining the $N-1$ and $N-2$ cluster partition [(N-1 CP) and (N-2 CP)] terms in (3.3), *viz.*,

$$\tilde{U}_{\alpha(0)\alpha(0)}\cong(T^A_{\alpha(0)\alpha(0)})_{N\text{-}1\,\mathrm{CP}}+(T^A_{\alpha(0)\alpha(0)})_{N\text{-}2\,\mathrm{CP}} \tag{3.4}$$

$$\rightarrow(T^A_{\alpha(0)\alpha(0)})_{2\text{-BC}}+(T^A_{\alpha(0)\alpha(0)})_{3\text{-BC}} \ . \tag{3.5}$$

Of course, one could extend the sum to include more complex terms, but (3.5) is sufficient for our purposes. The meaning of two- and three-body contributions [(2-BC) and (3-BC)] will be made clear in Secs. IV and V.

The use of (3.5) to obtain the two-body and three-body parts of $\tilde{U}_{\alpha(0)\alpha(0)}$ may appear cumbersome when contrasted with their seemingly simple and direct extraction using Eqs. (2.20) or (2.21) with $\underline{Q}_0^{\rm el}=\underline{I}-\underline{P}_0^{\rm el}$. In fact, however, the latter procedure is far from straightforward. First, it is necessary to use an on-shell transformation to put the terms appearing in $\tilde{U}_{\alpha(0)\alpha(0)}$ in a form more suited to a multiple-scattering type of analysis. Second, unless rather complicated manipulations on $\tilde{U}_{\alpha(0)\alpha(0)}$ are carried out, the combination of $Q^{\rm el}_{\alpha(0)}$ and the combinatoric coefficients $C_b=(-1)^{n_b}(n_b-1)!$ lead to the appearance of strange numerical factors in a Neumann expansion of $\tilde{U}_{\alpha(0)\alpha(0)}$ using Eq. (2.21). Why such factors appear and how they may be eliminated, leading to correct expressions, are discussed in Ref. 19, which may be consulted for further information.

Working from (3.5) not only avoids the latter problem, it also enables us to apply the on-shell transformation directly to $T^A_{\alpha(0)\alpha(0)}$. We first note that from (2.18), the elastic scattering amplitude is proportional to

$$\langle\Phi'_{\alpha(0)0}\,|\,\tilde{U}_{\alpha(0)\alpha(0)}\{\,|\,\phi_{\alpha(0)0}\rangle\langle\phi_{\alpha(0)0}\,|\,\}\,|\,\psi_{\alpha(0)}\rangle \ ,$$

where $\langle\Phi'_{\alpha(0)0}\,|=\langle\phi_{\alpha(0)0}\,|\,\langle\mathbf{k}'_\alpha\,|$, with $k'_\alpha=k_\alpha$. Substitution of (3.4) [or even (3.3)] into this expression shows that the combination $\langle\Phi'_{\alpha(0)0}\,|\,(T^A_{\alpha(0)\alpha(0)})_a$ occurs, i.e., that $T^A_{\alpha(0)\alpha(0)}$ is always left-half-on-shell when the elastic scattering amplitude is evaluated. Hence, we may evaluate the right-hand side (rhs) of (3.5) by assuming that the individual terms act to the left on $\langle\Phi'_{\alpha(0)0}\,|$. It is this step that lies at the basis of the on-shell transformations which we shall apply.

We next note that Ref. 1 implies that $T^A_{\alpha(0)\alpha(0)}$ can be expressed as

$$T^A_{\alpha(0)\alpha(0)}=\sum_{i=0}^{N_\alpha} R_{\alpha(0)}T_{\alpha(0)\alpha(i)}\hat{P}_{\alpha(i)\alpha(0)} \ , \tag{3.6}$$

where $T_{\alpha(0)\alpha(i)}$ is a transition operator of the EF theory. But, from Ref. 7, the following relation holds:

$$\langle\Phi'_{\alpha(0)0}\,|\,T_{\alpha(0)\alpha(i)}=\langle\Phi'_{\alpha(0)0}\,|\,U^{(-)}_{\alpha(0)\alpha(i)} \ , \tag{3.7}$$

where $U^{(-)}_{\alpha(0)\alpha(i)}$ is the prior form of the $\alpha(i)\rightarrow\alpha(0)$ transition operator.[20] Furthermore, $U^{(-)}_{\alpha(0)\alpha(i)}$ can be expressed in terms of the full Green's function $G^{(+)}=(E+i0-H)^{-1}$ *via*

$$U^{(-)}_{\alpha(0)\alpha(i)}=G^{(+)-1}_{\alpha(0)}G^{(+)}V^{\alpha(i)} \ . \tag{3.8}$$

Now, keeping in mind that $\tilde{U}_{\alpha(0)\alpha(0)}$ will always act to the left on $\langle\Phi'_{\alpha(0)0}\,|$, then straightforward substitution of (3.8) into (3.7) into (3.6) into (3.5) yields

$$\tilde{U}_{\alpha(0)\alpha(0)}\cong\sum_i\sum_{n=2}^{3}(G^{(+)-1}_{\alpha(0)}G^{(+)}V^{\alpha(i)})_{n\text{-BC}}\hat{P}_{\alpha(i)\alpha(0)} \ , \tag{3.9}$$

where the factor $R_{\alpha(0)}$ has been dropped due to our assuming that $R_{\alpha(0)}\,|\,\Phi'_{\alpha(0)0}\rangle=|\,\Phi'_{\alpha(0)0}\rangle$. We emphasize that the $\cong$ sign in (3.9) means that the optical potential operator is being approximated by its two- and three-body parts. [(n-BC) stands for n-body contribution.]

Use of (3.8) to obtain (3.9) [and also (3.10) below] is reminiscent of the work of Picklesimer and Thaler,[6] who employ the post form of the transition operator in place of $U^{(-)}_{\alpha(0)\alpha(i)}$ and whose antisymmetrization procedure is slightly different than ours. Given this close relationship, it will come as no surprise that our final results are identical to theirs for the nucleon-nucleon case. We intend this as a reassuring comment on the basic validity of an asymmetric approach to scattering for any readers who may still question whether an asymmetric collision theory can be either correct or practical or both.

The relation which forms the basis of our analysis results from substituting the resolvent relation

$$G^{(+)}=G^{(+)}_{\alpha(0)}(1+V^{\alpha(0)}G^{(+)})$$

into the rhs of (3.9):

$$\tilde{U}_{\alpha(0)\alpha(0)}=\sum_i\sum_{n=2}^{3}(V^{\alpha(i)}+V^{\alpha(0)}G^{(+)}V^{\alpha(i)})_{n\text{-BC}}\hat{P}_{\alpha(i)\alpha(0)} \ . \tag{3.10}$$

For each i, $0\leq i\leq N_\alpha$, the two- and three-body parts of the term in parentheses will depend on both the nature of

the clusters forming partition $\alpha(i)$ and the type of interaction. For simplicity, we shall assume that only two-particle interactions are important; the analyses of the next two sections will be for the cases of protons and then deuterons as the projectiles. Introduction of additional three-particle forces is straightforward to treat: such forces will not affect the two-body results. Similarly, n-particle forces, $n>4$, will not affect the two- or three-body results, and we do not consider them.

Since $V^{\alpha(i)}$ is now to be taken as a sum of two-particle interactions, the two-body and three-body character of the rhs of (3.10) will be determined by the connectivity structure of $G^{(+)}$. This is most simply obtained by expanding $G^{(+)}$ using the free-particle resolvent $G_0^{(+)}$:

$$G^{(+)}=G_0^{(+)}+G_0^{(+)}(V+VG_0^{(+)}V+VG_0^{(+)}VG_0^{(+)}V+\cdots)G_0^{(+)}\,, \tag{3.11}$$

where $V\equiv V^0$ is the total interaction, viz.,

$$V=\sum_{i<j}V_{ij}\,. \tag{3.12}$$

Substituting (3.11) into (3.10) gives

$$\tilde{U}_{\alpha(0)\alpha(0)}\cong\sum_i\sum_{n=2}^{3}(\{V^{\alpha(i)}+V^{\alpha(0)}[G_0^{(+)}+G_0^{(+)}(V+VG_0^{(+)}V+VG_0^{(+)}VG_0^{(+)}V+\cdots)G_0^{(+)}]V^{\alpha i}\}_{n\text{-BC}})\hat{P}_{\alpha(i)\alpha(0)}\,. \tag{3.13}$$

Evaluation of (3.13) requires knowledge of the incident channel α. We examine the nucleon-nucleus and deuteron-nucleus cases in the next two sections.

IV. NUCLEON-NUCLEUS SCATTERING

For this case, the canonical partition $\alpha(0)$ will be chosen as

$$\alpha(0)=(1)(2\ldots N)\,, \tag{4.1}$$

so that

$$\alpha(i)=P_{\alpha(i)\alpha(0)}\alpha(0)=(i)(1,2\ldots i-1,i+1\ldots N)\,. \tag{4.2}$$

The state $\phi_{\alpha(i)0}$ is given by

$$\phi_{\alpha(i)0}=\chi(i)\phi_0(1\ldots i-1,i+1\ldots N)\,, \tag{4.3}$$

where χ is a one-particle spinor and ϕ_0 is the $(N\text{-}1)$-particle, target-nucleus ground state.

In Eq. (3.13), the sum on i contains N terms. We consider the $i=0$ case first, *viz.*,

$$I_0=\sum_{n=2}^{3}\left\{\sum_{j\neq1}V_{1j}+\sum_{j\neq1}V_{1j}\left[G_0^{(+)}+G_0^{(+)}\left[\sum_{i<k}V_{ik}+\sum_{i<k}V_{ik}G_0^{(+)}\sum_{l<m}V_{lm}+\cdots\right]G_0^{(+)}\right]\sum_{p=1}V_{1p}\right\}_{n\text{-BC}}$$

$$=\sum_{j\neq1}\left[I_{0,2}(j)+\sum_p I_{0,3}(j,p)\right]\,, \tag{4.4}$$

where

$$I_{0,2}(j)=\left\{V_{1j}+V_{1j}\left[G_0^{(+)}+G_0^{(+)}\left[\sum_{i<k}V_{ik}+\sum_{i<k}V_{ik}G_0^{(+)}\sum_{l<m}V_{lm}+\cdots\right]G_0^{(+)}\right]\sum_{p\neq1}V_{1p}\right\}_{2\text{-BC}} \tag{4.5}$$

and

$$I_{0,3}(j,p)=\left\{V_{1j}+V_{1j}\left[G_0^{(+)}+G_0^{(+)}\left[\sum_{i<k}V_{ik}+\sum_{i<k}V_{ik}G_0^{(+)}\sum_{l<m}V_{lm}+\cdots\right]G_0^{(+)}\right]V_{1p}+V_{1j}(\cdots)V_{1j}\right\}_{3\text{-BC}}$$

$$+(\text{same expression with } j \text{ and } p \text{ interchanged})_{3\text{-BC}}\,. \tag{4.6}$$

The usage "two-body contribution" and "three-body contribution" can now be defined in terms of the cluster decomposition, Eq. (3.2). The two-body contributions are obtained by truncating the sum in (3.2) to partitions of the form

$$a_{N-1}=(1j)(2)\ldots(j-1)(j+1)\ldots(N)\,. \tag{4.7}$$

In other words, the term denoted by $I_{0,2}(j)$ in Eq. (4.4) is the single term in the cluster decomposition of I_0 which has the connectivity of the partition a_{N-1}. In a similar fashion, the three-body contributions to I_0 are obtained by picking out the terms in its cluster decomposition having the connectivity of the partition

$$a_{N-2}=(1jp)(2)\ldots(j-1)(j+1)\ldots(p-1)(p+1)\ldots(N)\,. \tag{4.8}$$

Thus the term $I_{0,3}(j,p)$ in Eq. (4.4) is the part of I_0 with connectivity a_{N-2}. The preceding definition of two-body and three-body contributions to the transition amplitude were introduced by Kowalski[15] and their use was referred to him as forming a connectivity expansion. Of course, this is not the only possible procedure. An alternative is to adopt the inclusive connectivity expansion introduced by Picklesimer, Tandy, and Thaler[16] and later expounded on by Kowalski.[17] We choose to concentrate on the former method since it is simpler, and our major goal in this paper is to establish that standard approximation schemes to the standard optical potential can be developed within an asymmetric formalism.

The two-body contributions can now be extracted from the expression in (4.5). It is clear that the only terms in the various sums which can contribute to $I_{0,2}(j)$ are those which contain the interaction V_{1j}. Therefore,

$$I_{0,2}(j)=V_{1j}+V_{1j}G_0^{(+)}V_{1j}+V_{1j}G_0^{(+)}V_{1j}G_0^{(+)}V_{1j}+\cdots \equiv t_{1j}\ , \tag{4.9}$$

where t_{1j} is the two-particle transition operator for particles 1 and j, embedded in the N-particle space (*via* $G_0^{(+)}$).

Summing the result (4.9) on j as indicated by (4.4) gives

$$\sum_{j\neq 1} I_{0,2}(j)=\sum_{j\neq 1} t_{1j}\ . \tag{4.10}$$

This is the familiar impulse approximation, an expected result, since restricting the sum in (3.13) is equivalent to making a distinguishable particle approximation. It is the first indication that reasonable approximations can be recovered from an asymmetric formalism.

The energy dependence of t_{1j} in (4.10) has been suppressed, but since the *free* N-particle Green's function appears in (4.9), then this implies that t_{1j} should be evaluated at the shifted "energy" E^+-H_0. This feature has led to a major criticism of the connectivity expansion,[16] *viz.*, the binding effects in the target nucleus have been ignored. If, however, one were to use the inclusive connectivity expansion, then $(E^+-H_0)^{-1}$ would be replaced by $(E^+-H_\beta)^{-1}$, where

$$\beta=(1j)(2\ldots j-1,j+1\ldots N)\ ,$$

thereby including binding effects. Similar comments hold for the other expressions derived in this paper. However, for reasons stated before, we continue with our present procedure.

The three-body contributions can be determined from $I_{0,3}(j,p)$ in (4.6). Only the first expression in curly braces will be considered in detail, as the expression in parentheses in Eq. (4.6) can be gotten from it by a simple interchange of indices. It is immediately clear that the first term containing V_{1j} alone has already been counted as a two-body contribution and must therefore be ignored.

The second term contains both V_{1j} and V_{1p}. This ensures that it will at least have the connectivity of the partition a_{N-2} defined by (4.8). It is thus only necessary to eliminate the pieces with higher connectivity. This is easily done by restricting the potentials which appear in the expansion in parentheses to V_{1j} and V_{1p}. The three-body contributions to the second term are then seen to be

$$V_{1j}(G_0^{(+)}+G_0^{(+)}V_{a_{N-2}}G_0^{(+)}+G_0^{(+)}V_{a_{N-2}}G_0^{(+)}V_{a_{N-2}}G_0^{(+)}+\cdots)V_{1p}\ . \tag{4.11}$$

The series in parentheses is the expansion for the channel Green's function $G_{a_{N-2}}^{(+)}$. (It is not necessary that the series converges as the expansion is only used as a tool to study the connectivity properties of the operator I_0.) The above expression can therefore be rewritten as

$$V_{1j}G_{a_{N-2}}^{(+)}V_{1p}\ . \tag{4.12}$$

The third term must now be examined for possible three-body contributions. In order for this term to produce a true three-body contribution, the expression sandwiched between the V_{1j}'s must contain at least one interaction involving nucleon k, $1\neq k\neq j$. All such three-body contributions must be summed over k. Since k is a dummy variable, it can be set equal to p, which is already summed over as indicated by (4.4). Thus, all three-body contributions to the third term in (4.7) are contained in the expression

$$V_{1j}(G_0^{(+)}V_{a_{N-2}}G_0^{(+)}+G_0^{(+)}V_{a_{N-2}}G_0^{(+)}V_{a_{N-2}}G_0^{(+)}+\cdots)V_{1j}-V_{1j}(G_0^{(+)}V_{1j}G_0^{(+)}+G_0^{(+)}V_{1j}G_0^{(+)}V_{1j}G_0^{(+)}+\cdots)V_{1j}\ , \tag{4.13}$$

where the part that has been subtracted off has already been counted as a two-body contribution. The above expression can be rewritten in terms of channel Green's functions as

$$V_{1j}(G_{a_{N-2}}^{(+)}-G_{1j}^{(+)})V_{1j}\ , \tag{4.14}$$

where $G_{1j}^{(+)}$ is the channel Green's function for the partition a_{N-1} defined by (4.7).

We are now ready to evaluate the three-body contributions $I_{0,3}(j,p)$ of Eq. (4.6). Combining (4.12) and (4.14) plus the same expressions with j and p interchanged, yields

$$I_{0,3}(j,p)=(V_{1j}+V_{1p})G_{a_{N-2}}^{(+)}(V_{1j}+V_{1p})-V_{1j}G_{1j}^{(+)}V_{1j}-V_{1p}G_{1p}^{(+)}V_{1p}\ . \tag{4.15}$$

The above equation for $I_{0,3}(j,p)$ can be put into simpler form by adding and subtracting $V_{1j}+V_{1p}$:

$$I_{0,3}(j,p)=[(V_{1j}+V_{1p})+(V_{1j}+V_{1p})G_{a_{N-2}}^{(+)}(V_{1j}+V_{1p})]-(V_{1j}+V_{1j}G_{1j}^{(+)}V_{1j})-(V_{1p}+V_{1p}G_{1p}^{(+)}V_{1p})\ . \tag{4.16}$$

The first term in square brackets is seen to be the three-body elastic transition operator describing nucleon 1

scattering off nucleons j and p; this term will be denoted $T^{1(jp),1(jp)}_{a_{N-2}}$. Like the t_{1j} operator given by Eq. (4.9), it is a few-body operator embedded in the full N-particle space. The second and third terms in parentheses on line two of Eq. (4.16) are simply two-body t matrices, again embedded in the N-particle Hilbert space. Hence, $I_{0,3}(j,p)$ can be written in the compact form

$$I_{0,3}(j,p)=T^{1(jp),1(jp)}_{a_{N-2}}-t_{1j}-t_{1p} . \tag{4.17}$$

The $I_{0,3}(j,p)$ term can now be summed on all pairs j and p to yield

$$\sum_{\substack{j\neq p\\ j,p\neq 1}} (T^{1(jp),1(jp)}_{a_{N-2}}-t_{1j}-t_{1p}) . \tag{4.18}$$

This is the three-body correction to the impulse approximation (4.10) which one would expect on physical grounds. The first term yields the amplitude for the projectile to scatter off the pair of target nucleons j and p. The second and third are subtracted terms which prevent the overcounting of processes already accounted for by the two-body impulse approximation.

We have so far only considered terms in Eq.(3.13) for which $i=0$. Since this ignores exchange terms, what we have done so far amounts to a distinguishable particle approximation for the nucleon-nucleus amplitude. However, one of the advantages to working with a connected-kernel formalism is that exchange effects can be included in a straightforward and consistent manner. We will evaluate exchange effects starting from (3.13) by selecting a term in the sum on i with $i\neq 0$, and then summing our result over all i.

In analogy with (4.4), we write

$$I_i=\sum_{j\neq i}\left[I_{i,2}(j)+\sum_p I_{i,3}(j,p)\right]\hat{P}_{i1} , \tag{4.19}$$

where $\hat{P}_{i1}$ is an explicit representation of the operator that takes $\alpha(i)$ into $\alpha(0)$ and is taken to mean an interchange of particles 1 and i, multiplied by the appropriate phase factor.

In an analogy to the $i=0$ case, $I_{i,2}(j)$ can be expressed as

$$I_{i,2}(j)=\left[\left\{V_{ij}+V_{ij}\left[G_0^{(+)}+G_0^{(+)}\left[\sum_{n<k}V_{nk}+\sum_{n<k}V_{nk}G_0^{(+)}\sum_{l<m}V_{lm}+\cdots\right]G_0^{(+)}\right]\sum_{p=i}V_{pi}\right\}_{2\text{-BC}}\right](\hat{P}_{ij}) . \tag{4.20}$$

In similar fashion, $I_{i,3}(j,p)$ can be written as

$$I_{i,3}(j,p)=\left[\left\{V_{ij}+V_{ij}\left[G_0^{(+)}+G_0^{(+)}\left[\sum_{n<k}V_{nk}+\sum_{n<k}V_{nk}G_0^{(+)}\sum_{l<m}V_{lm}G_0^{(+)}+\cdots\right]\right]V_{1p}+V_{ij}(\cdots)V_{ij}\right\}_{3\text{-BC}}\right]\hat{P}_{j1}$$
$$+[(\text{same expression with } j \text{ and } p \text{ interchanged})_{3\text{-BC}}]\hat{P}_{p1} . \tag{4.21}$$

We will now extract the two-body contributions from the curly-bracketed term in (4.20). Because of the presence of the $\hat{P}_{ij}$ operator, it is clear that the only two-body contributions can come from the interactions V_{1i}. Therefore, the only two-body contributions occur when $j=1$. Once this observation has been made, the two-body contributions can be evaluated just as in the distinguishable particle analysis ($i=0$). The result is

$$I_{i,2}(j)=t_{1i}\delta_{j1}\hat{P}_{i1} . \tag{4.22}$$

Next, summing on all $j\neq i$ as indicated by (4.19), and then summing on all $i\neq 0$ as indicated by (3.13), yields

$$\sum_{\substack{i\neq 0\\ j\neq i}} I_{i,2}(j)=\sum_{i\neq 1} t_{i1}\hat{P}_{i1} . \tag{4.23}$$

Combining this with the distinguishable particle result given by (4.10) gives

$$(\tilde{U}_{\alpha(0)\alpha(0)})_{2\text{-BC}}=\sum_{i\neq 1} t_{ij}(I+\hat{P}_{i1})$$
$$\equiv\sum_{i\neq 1} t^S_{1i} , \text{ nucleon-nucleus case} , \tag{4.24}$$

for the two-body contributions to the optical potential, where t^S_{1i} is the physical two-body t matrix describing nucleon-nucleon scattering *with* particle exchange taken into account. This "antisymmetrized" $t\rho$ expression is a familiar approximation for the nucleon-nucleus optical potential and is a result that is expected on physical grounds as the natural extension of Eq. (4.10). Again, we have shown that physically reasonable and well-known approximations can be recovered from an asymmetric optical potential formalism.

The three-body exchange contributions to the optical potential are obtained from (4.21). Similar manipulations as in the $i=0$ case yield the expression

$$I_{i,3}(j,p)=(T^{j(1p),1(jp)}_{a_{N-2}}-t_{1j})\hat{P}_{1j}+(T^{p(1j),1(jp)}_{a_{N-2}}-t_{1p})\hat{P}_{1p} . \tag{4.25}$$

The operator $T^{j(1p),1(jp)}_{a_{N-2}}$, for example, is the prior-form three-body transition operator (embedded in the N-particle space) for scattering from the channel $1(jp)$ to the channel $j(1p)$. As in Eq. (4.17), the two-body t matrices are subtracted to prevent overcounting of two-body contributions. In addition, these subtractions also ensure that $I_{i,3}(j,p)$ has the connectivity of the partition a_{N-2}.

To obtain the full three-body contributions to the optical potential, $I_{i,3}(j,p)$ must be properly summed over j

and p and the result added to that obtained in (4.18). This yields

$$\sum_{\substack{j\neq p\\ j\neq 1\neq p}} [T^{1(jp),1(jp)}_{a_{N-2}} + T^{j(1p),1(jp)}_{a_{N-2}}\hat{P}_{1j} + T^{p(1j),1(jp)}_{a_{N-2}}\hat{P}_{1p} - t_{1j}(I+\hat{P}_{1j}) - t_{1p}(I+\hat{P}_{1p})] \ . \tag{4.26}$$

Although this expression looks unwieldy, it may be put into simpler form. It can be seen that the first three terms are equivalent to the correctly symmetrized three-body transition operator[21] describing identical particle scattering of the particle 1 off of the pair j and p. This will be denoted by $T^{1(jp)^S}_{a_{N-2}}$, where the a_{N-2} subscript is to remind us that it is a three-body operator embedded in the N-particle space. Furthermore, the last two terms are symmetrized two-body matrices that have been defined by (4.24). Thus (4.26) can now be rewritten as

$$(\tilde{U}_{\alpha(0)\alpha(0)})_{3\text{-BC}} = \sum_{\alpha=\{j,p\}} (T^{1(jp)^S}_{a_{N-2}} - t^S_{1j} - t^S_{1p}) \ , \qquad \text{nucleon-nucleus case} \ ; \tag{4.27}$$

where α runs over all pairs of nucleons that do not include 1. As was the case with the two-body contributions, the result (4.27) is a natural generalization of Eq. (4.18) to the case of identical particles in which all of the operators are replaced by their symmetrized counterparts.

The main results of this section are Eqs. (4.24) and (4.27) which, when folded over the nuclear density, give the two- and three-body contributions, respectively, to the antisymmetrized nucleon-nucleus optical potential. In the next section, we show how the same techniques can be used to construct low-order approximations to the deuteron-nucleus optical potential.

V. DEUTERON-NUCLEUS SCATTERING

To examine the two- and three-body contributions to deuteron-nucleus elastic scattering, where the projectile is now a composite particle, we again use Eq. (3.13) as a starting point. We will choose the canonical partition to be

$$\alpha(0)=(01)(2\ldots N) \ , \tag{5.1}$$

so that $N+1$ particles comprise the system. This enables us to make use of results derived in the preceding section and also facilitates the comparison of composite particle scattering with that of nucleon-nucleus scattering. We note that the operator $\tilde{U}_{\alpha(0)\alpha(0)}$ must be folded over both the target density and the deuteron density in order to recover the optical potential, rather than just the target density as was the case for nucleon-nucleus scattering.

Before we proceed, it should be noted that a popular method to relate the deuteron-nucleus optical potential to the nucleon-nucleus optical potential is to take the deuteron potential to be the sum of the nucleon potentials each evaluated at half of the incident deuteron energy and folded over the deuteron density. This prescription is known as the folding model,[22] and we will bear it in mind when comparing our results for the deuteron-nucleus case to those from the preceding section.

We now work with Eq. (3.13) to determine the two- and three-body contributions to deuteron-nucleus scattering. We first examine the equivalence class of partitions that contains $\alpha(0)$. There are four types: first, $\alpha(0)$, as given by (5.1); next, there are the two types of partitions

$$\alpha^0(i)=(1i)(0,2\ldots i-1,i+1\ldots N) \tag{5.2}$$

and

$$\alpha^1(i)=(0i)(1,2\ldots i-1,i+1\ldots N) \ , \tag{5.3}$$

and finally there are partitions of the form

$$\alpha^{01}(i)=(ij)(0,1,2\ldots i-1,i+1\ldots j-1,j+1\ldots N) \ . \tag{5.4}$$

It can be seen that while $\alpha^0(i)$ and $\alpha^1(i)$ can be obtained from $\alpha(0)$ by the interchange of only two nucleons, the simplest permutation that takes $\alpha(0)$ into $\alpha^{01}(i)$ is $\hat{P}_{0i}\hat{P}_{1j}$, which involves *four* nucleons. Since we are only considering two- and three-body effects, contributions from the $\alpha^{01}(i)$ partitions will be ignored. Physically, this simply means that the double exchange processes, in which the incident deuteron trades places with two of the target nucleons, do not make any two- or three-body contributions to deuteron-nucleus scattering.

In addition to the various partitions occurring in Eq. (3.13), there also appears the infinite series $G_0^{(+)}VG_0^{(+)}+G_0^{(+)}VG_0^{(+)}VG_0^{(+)}+\ldots$. Since the operator V is the sum of all of the pairwise potentials, substitution of this sum for V leads to an expression for (3.13) similar to that of (4.4). In order to apply our earlier results most easily, it is helpful to decompose the preceding series into the sum of four operators:

$$G_0^{(+)}VG_0^{(+)}+G_0^{(+)}VG_0^{(+)}VG_0^{(+)}+\cdots \equiv A^{(01)}+A^{(0i)}+A^{(1i)}+A^{(ij)} \ , \tag{5.5}$$

where $A^{(01)}$ contains terms involving only the potential V_{01}, $A^{(0i)}$ contains all terms involving potentials with particle 0 but not 1, $A^{(1i)}$ contains all terms involving potentials with particle 1 but not 0, and $A^{(ij)}$ contains all terms involving potentials with neither particles 0 nor 1.

We begin by considering the term in Eq. (3.13) in which $\alpha(i)$ is equal to $\alpha(0)$. This is analogous to the distinguishable particle analysis of Sec. IV, and we shall be able to carry over a number of results from that section. The operator $V^{\alpha(0)}$ can be expressed as a sum of two contributions:

$$V^{\alpha(0)}=\sum_{j>2} V_{0j}+\sum_{j>2} V_{1j} \ . \tag{5.6}$$

Using this decomposition and the definition (5.5), we introduce the operator I_0, in analogy with the notation of Sec. IV:

$$I_0=\sum_{n=2}^{3}\Bigg[\sum_{j>2}V_{0j}+\sum_{j>2}V_{0j}(G_0^{(+)}+A^{(0i)}+A^{(ij)})\sum_{p>2}V_{0p}+\sum_{j>2}V_{1j}+\sum_{j>2}V_{1j}(G_0^{(+)}+A^{(0i)}+A^{(ij)})\sum_{p>2}V_{1p}$$
$$+\sum_{j>2}V_{0j}(A^{(01)}+A^{(1i)})\sum_{p>2}V_{0p}+\sum_{j>2}V_{1j}(A^{(01)}+A^{(0i)})\sum_{p>2}V_{1p}$$
$$+\sum_{j>2}V_{0j}(G_0^{(+)}+G_0^{(+)}VG_0^{(+)}+\cdots)\sum_{p>2}V_{1p}+\sum_{j>2}V_{1j}(G_0^{(+)}+G_0^{(+)}VG_0^{(+)}+\cdots)\sum_{p>2}V_{0p}\Bigg]_{n\text{-BC}}. \quad (5.7)$$

The reason for dividing the series (5.5) into four terms now becomes clear: It can be seen from a comparison with (4.4) that the first four terms correspond, with one slight difference, exactly to the separate scattering of particles 0 and 1 off the target nucleus. All of the work needed for extracting the two- and three-body contributions from these terms has already been carried out in Sec. IV. The one difference is that while the operators in (4.4) are operators in an N-body Hilbert space, the operators in (5.7) are operators in an $(N+1)$-body Hilbert space. Thus, for example, although particle 1 does not appear in any of the interactions in the first two terms of (5.7), it still appears in the kinetic energy terms of $G_0^{(+)}$. This difference does not affect the form of the operators: only their "energy dependence" is changed.

We are now ready to evaluate the two-body, "distinguishable particle" contributions to deuteron-nucleus scattering. By inspection of (5.7), it is seen that all but the first four j sums involve at least three particles. The two-body contributions from the first four j sums have been evaluated already and are given by

$$\sum_{j>2}(t_{0j}+t_{1j}). \quad (5.8)$$

The above result can be compared with the distinguishable particle two-body contribution to the nucleon-nucleus optical potential, which will be denoted by $[U^0(E)]_2$. If the two-body contributions to the deuteron-nucleus potential are labeled by $[U^{01}(E)]_2$, then upon comparison of (4.10) with (5.8) it is seen that

$$[U^{01}(E)]_2=\langle\phi_{01}|[U^0(E^+-K_1)]_2+[U^1(E^+-K_0)]_2|\phi_{01}\rangle, \quad (5.9)$$

where ϕ_{01} is the (deuteron) bound state wave function for particles 0 and 1, and U^0 and U^1 are evaluated at shifted energies because they are operators embedded in the $(N+1)$-particle space. If it is assumed that K_0 and K_1 can be replaced by an energy equal to half of the deuteron incident energy, then the folding model[22] is reproduced at the level of two-body contributions. It is only in the three-body contributions that deviations from the folding model start to appear. This remark also holds for the exchange contributions.

We now proceed to extract the three-body contributions from I_0. The first four terms are of the same form as those that have appeared in Sec. IV, and their three-body contributions are given by (4.18). Adapting that result to the present case yields

$$\sum_{\substack{j\neq p\\ j,p>2}}(T^{1(jp),1(jp)}_{a^1_{N-2}}-t_{1j}-t_{1p}+T^{0(jp),0(jp)}_{a^0_{N-2}}-t_{0j}-t_{0p}), \quad (5.10)$$

where

$$a^1_{N-2}=(1jp)(0)(2)\ldots(j-1)(j+1)\ldots(p-1)(p+1)\ldots(N) \quad (5.11)$$

and

$$a^0_{N-2}=(0jp)(1)(2)\ldots(j-1)(j+1)\ldots(p-1)(p+1)\ldots(N). \quad (5.12)$$

The suppressed energy dependence of the operators in (5.10) refers to the fact that they are embedded in an $(N+1)$-particle Hilbert space.

Thus far, there are still no deviations from the folding model. However, the remaining terms in I_0 will give different results than the folding model. In the fifth j sum, j and p must be set equal to each other, or else this term will involve four nucleons. For the same reason, the terms appearing in $A^{(1i)}$ must be restricted to those containing only the potentials V_{1j}. The three-body contributions from this term are thus found to be

$$\sum_{j>2}[V_{0j}(G_0^{(+)}V_{01}G_0^{(+)}+G_0^{(+)}V_{01}G_0^{(+)}V_{01}G_0^{(+)}+\cdots)V_{0j}+V_{0j}(G_0^{(+)}V_{1j}G_0^{(+)}+G_0^{(+)}V_{1j}G_0^{(+)}V_{1j}G_0^{(+)}+\cdots)V_{0j}]. \quad (5.13)$$

This expression can be rewritten in terms of channel Green's functions as

$$\sum_{j>2}V_{0j}(G_{01}^{(+)}+G_{1j}^{(+)}-2G_0^{(+)})V_{0j}. \quad (5.14)$$

A similar analysis yields

$$\sum_{j>2}V_{1j}(G_{01}^{(+)}+G_{0j}^{(+)}-2G_0^{(+)})V_{1j} \quad (5.15)$$

for the three-body contributions from the sixth j sum.

Now examining the seventh and eighth j sums in I_0, we see that the three-nucleon contributions occur only when p is equal to j. As these terms are very much like those in Eqs. (4.11) and (4.12), we easily find the three-body contributions from the seventh and eighth j sums in I_0 to be

$$\sum_{j>2}(V_{0j}G^{(+)}_{b_{N-2}}V_{1j}+V_{1j}G^{(+)}_{b_{N-2}}V_{0j}), \quad (5.16)$$

where

$$b_{N-2}=(01j)(2)\ldots(j-1)(j+1)\ldots(N)\ . \tag{5.17}$$

Adding together Eqs. (5.14), (5.15), and (5.16) yields, after the use of some Green's function identities,

$$\sum_{j>2}[(V_{0j}+V_{1j})G^{(+)}_{b_{N-2}}(V_{0j}+V_{1j}) - V_{0j}G_{0j}V_{0j}-V_{1j}G_{1j}V_{1j}]\ . \tag{5.18}$$

Adding and subtracting two-body potentials as was done in Eq. (4.16) gives for (5.18) the result

$$\sum_{j>2}(T^{j(01),j(01)}_{b_{N-2}}-t_{0j}-t_{1j})\ . \tag{5.19}$$

Finally, adding Eq. (5.10) to (5.19) produces the full, dinstinguishable-particle, three-body contributions to deuteron-nucleus scattering, *viz.*,

$$\sum_{\substack{j\neq p\\ j,p>2}}(T^{1(jp),1(jp)}_{a^1_{N-2}}-t_{1j}-t_{1p}+T^{0(jp),0(jp)}_{a^0_{N-2}}-t_{0j}-t_{0p}) + \sum_{j>2}(T^{j(01),j(01)}_{b_{N-2}}-t_{0j}-t_{1j})\ . \tag{5.20}$$

As noted earlier, it is the term given by (5.19) which provides the first correction to the folding model. It physically corresponds to the scattering of the bound deuteron by each of the target nucleons, which is necessarily a three-body contribution. This is another example of a physically reasonable result that has been recovered, despite the fact that an asymmetric optical potential formalism has been used as the starting point.

To now determine the low-order exchange contributions to deuteron-nucleus scattering, the sum in Eq. (3.13) must be evaluated for partitions of the form $\alpha^0(i)$ and $\alpha^1(i)$. Using the preceding analyses, this calculation is straightforward but lengthy, and details will not be reproduced here. [An apparent problem involves expressions of the form $t_{0i}\hat{P}_{1i}$ that arise from terms analogous to the first four terms of (5.8). However, these expressions are exactly canceled by contributions from other terms.]

As might be expected, the only modification of the results already obtained in this section when exchange effects are fully included is that the unsymmetrized operators are replaced by their fully symmetrized counterparts. Thus, from Eq. (5.8), the two-body contributions to deuteron-nucleus scattering are the following:

$$(\tilde{U}_{\alpha(0)\alpha(0)})_{2\text{-BC}}=\sum_{j\geq 2}(t^S_{0j}+t^S_{1j}),\ \text{deuteron-nucleus case}\ , \tag{5.21}$$

while the three-body contributions are given by

$$(\tilde{U}_{\alpha(0)\alpha(0)})_{3\text{-BC}}=\sum_{\alpha=\{j,p\}}(T^{0(jp)^S}_{a_{N-2}}-t^S_{0j}-t^S_{0p}+T^{1(jp)^S}_{a_{N-2}}-t^S_{1j}-t^S_{1p})+\sum_{j\geq 2}(T^{j(01)^S}_{b_{N-2}}-t^S_{0j}-t^S_{1j}),\ \text{deuteron-nucleus case}\ . \tag{5.22}$$

Both of these equations fully account for all exchange effects within the approximations that only the two- and three-body contributions to the scattering process are included.

The major results of this section are Eqs. (5.21) and (5.22), which give, when folded over the product of the deuteron and target densities, the two- and three-body contributions to the antisymmetrized deuteron-nucleus optical potential. What we have analyzed here is the optical model, which reduces deuteron-nucleus scattering to an effective two-body problem. However, due to the ease with which the deuteron can be broken up, it is often useful to treat deuteron scattering via an effective three-body model, so that the composite nature of the deuteron can be treated explicitly. In fact, the folding model can be regarded as an approximate solution to the full three-body model. It is possible to use the formalism developed in Secs. II and III, specialized to the BRS theory,[8] to analyze the three-body deuteron-nucleus model with exchange effects included and to show what approximations are necessary to arrive at the standard form of the three-body model. This has been done and will be presented in the third paper of this series.[23]

VI. SUMMARY

In this paper, we have used the antisymmetric optical potential formalism of Adhikari, Kozack, and Levin[1] to generate low-order approximations to both the nucleon-nucleus and the deuteron-nucleus optical potentials. We have assumed that elastic scattering is caused by the interactions of only a few nucleons and have evaluated contributions to the scattering from both two- and three-nucleon interactions. Since we started with optical potentials that include exchange effects *ab initio,* our low-order approximations contain exchange effects in a correct and consistent fashion.

Although some of our final results have been obtained by other workers using different approaches, one of our major points was to show that well-established and physically reasonable results can be obtained from an optical potential which was asymmetric. (The asymmetry here refers to the fact that the optical potential is nonlocal and not equal to itself when its two arguments are interchanged.) For the case of nucleon-nucleus scattering, we have recovered both the standard impulse approximation and corrections from three-body effects.

We have also obtained some new results. To our knowledge, the calculations in Sec. V are the first concerning deuteron-nucleus scattering which consistently include the effects of exchange. As stated earlier, this will be examined in more detail in a subsequent paper, where the question of deuteron breakup will be addressed.

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