Comparison of the standard Paris potential and its separable representation in the nuclear matter problem

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The nuclear matter parameters and the single-particle properties are compared for the standard (non-) local Paris potential and its separable representation, given by the Graz group. The results are discussed.

The treatment of few- and many-body models with realistic nucleon-nucleon potentials often causes, due to the local structure of the force, numerical problems of great complexity, which are sometimes intractable. In order not to sacrifice the model or to avoid an extensive numerical treatment, one can simplify the problem by using a separable representation. This method was first employed in nuclear physics by Yamaguchi[1] and was extended by numerous authors (see, for instance, Ref. 2). However, this procedure has the drawback that the off-energy behavior of the two interactions is different. It is possible to reduce this shortcoming by several methods; for instance, by utilizing the so-called EST method.[3] With this method one can reproduce, to a certain extent, both the on- and off-shell behavior of the original potential. Recently, the Graz group has obtained a separable representation of the Paris potential[4] with this procedure.[2,5] The obtained parametrization worked, it seems, reasonably well in a variety of few-body problems. Therefore, it is interesting to test the applicability of this potential for other cases. A good choice seems to be the nuclear matter problem, where one needs the effective scattering matrix in matter for an extended energy range. A successful test would open the possibility to treat more refined approximations in this field due to the tremendous cut in computer time. The test was performed for the Paris potential utilizing the so-called Λ^{00} approximation[6] of the Green's function approach, where negative energies are important, and the Brueckner approximation with the standard choice,[7] respectively. These cases, where the results for the standard Paris potential are known from the literature,[8-11] seem to be sufficient for the test, because they cover, with respect to complexity and energy domain, the basic features of the nuclear matter theory. Since the formulation of the models is outlined in detail in the literature,[7-12] we will not repeat their description in this note. We only emphasize the basic advantage of the separable potential ($\alpha = J,S,T$; g denotes the form factor; for details, see Ref. 2).

$$v^{\alpha}_{LL'}(p,p') = \sum_{i=1}^{N_L}\sum_{j=1}^{N_L} g_i^{\alpha L}(p)\lambda_{ij}^{\alpha LL'} g_j^{\alpha L'}(p') \ , \tag{1}$$

namely, the preservation of this separable structure for the T matrix

$$T^{\alpha}_{LL'}(Q;p,p';z) = \sum_{i,j=1}^{N_L} g_i^{\alpha L}(p)\tau_{ij}^{\alpha LL'}(Q;z) g_j^{\alpha L'}(p') \ . \tag{2}$$

Because of this structure the integral equations for the T matrix [Eq. (2.1) of Ref. 8 after angular decomposition] reduces—besides integration—to a linear system of equations. The gain in computer time for the treated models was approximately a factor of 40^{13}.

In the discussion we will mainly concentrate on the Λ^{00} approximation because for this case we have the most de-

TABLE I. Comparison of nuclear matter parameters.

Approximation	k_F (fm^{-1})	$E_0(A)/A$ (MeV)	ρ (fm^{-3})	κ (MeV)
$\Lambda^{(00)}$ (separable)	1.663	−13.12	0.256	114
$\Lambda^{(00)}$ (Ref. 8)	1.548	−13.17	0.213	162
A (separable)	1.548	−11.86	0.250	148
A (Ref. 4)	1.51	−11.22		

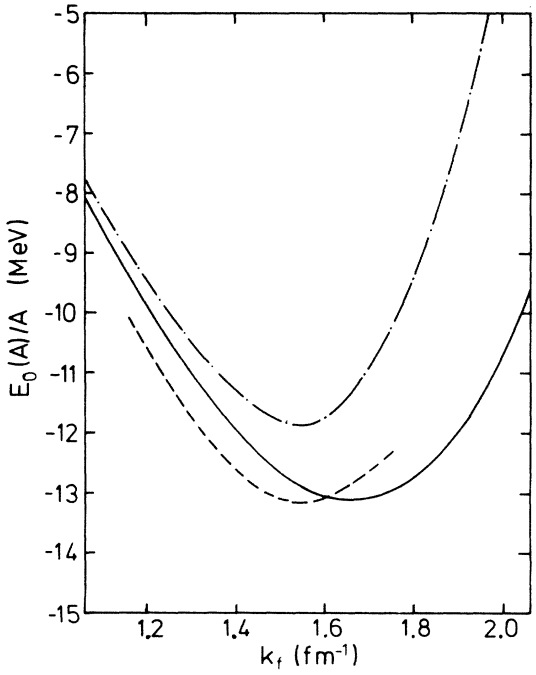

FIG. 1. Nuclear matter energy vs Fermi momentum. The dashed-dotted curve gives the outcome for the separable Paris potential in the Brueckner theory with standard gap choice. The results of the Λ^{00} approximation are given by the solid (dashed) curve for the separable (standard) Paris potential.

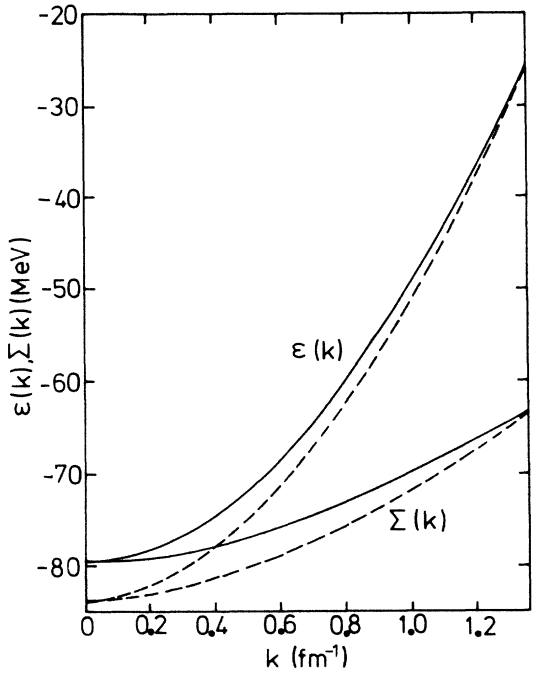

FIG. 2. Single-particle energy $\epsilon(k)$ and mass operator $\Sigma(k)$ in Λ^{00} approximation vs momentum for the separable and standard Paris potential.

tailed information available.[8] Furthermore, the behavior of the T matrix for negative energies, needed in more sophisticated approximations,[7,10-12] is significant in this model. The comparison of the nuclear matter parameters is given in Table I and Fig. 1. A first glance might suggest a satisfactory agreement, at least, with respect to binding energies. However, a closer inspection reveals greater discrepancies and the agreement of the binding energies may be accidental. For this purpose we show in Figs. 2 and 3 the effective single-particle potential and the momentum distribution, respectively. This comparison exhibits a weaker single-particle potential and a steeper decrease of the momentum distribution for the separable potential. Since the Λ^{00} approximation is completely determined by the free off-shell S matrix for (large) negative energies [see, for instance, Eqs.

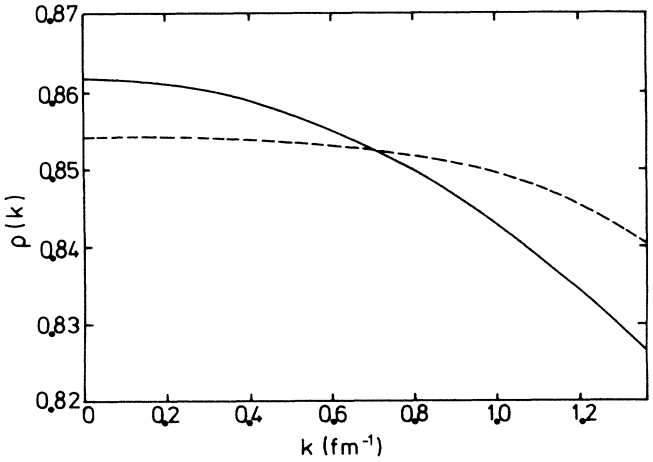

FIG. 3. Momentum distribution in Λ^{00} approximation for the separable and standard Paris potential at $k_F = 1.36$ fm^{-1}.

TABLE II. Partial-wave contributions to the mass operator in MeV (Λ^{00} approximation).

Channel	$k=0$ Separable	$k=0$ Paris	$k=k_F=1.36$ fm^{-1} Separable	$k=k_F=1.36$ fm^{-1} Paris
$^1S_0$	−36.9	−38.0	−24.4	−25.2
$^1P_1$	6.2	6.7	7.5	7.6
$^1D_2$	−2.8	−3.0	−5.7	−5.9
$^3D_2$	−4.0	−4.6	−8.1	−8.6
$^3S_1$	−43.0	−46.9	−31.9	−34.2
$^3D_1$	0.5	1.5	1.5	2.1
$^3D_3$	0.0	0.3	0.1	0.7

(55), (75), and (76) of Ref. 6], this result can only be explained by a different off-shell behavior of the S matrices in this negative energy domain. In most nuclear applications this difference seems to be insignificant and it might be caused by favoring the (physical) positive energy range in the fitting procedure of the separable potential. The shown structure of Σ and ρ implies, by taking the weighting factor p^2 into account, a higher saturation density. It is quite possible that a combination of these features causes a compensation effect for the energy but obviously leads to higher saturation densities and (in-) compressibilities.[13] Therefore, the approximate agreement of the binding energies alone does not prove the equivalence of both potentials within these models. For completeness the differences for the dif-

TABLE III. Partial-wave contributions to the potential energy in MeV at $k_F = 1.36$ fm^{-1} using the Brueckner approximation with standard gap choice.

Channel	Separable	Paris (Ref. 9)
$^1S_0$	−15.88	−16.35
$^1D_2$	−2.71	−2.82
$^3S_1$	−17.09	−17.25
$^3D_1$	1.06	1.49
$^3D_2$	−3.80	−5.76[a]
$^3P_0$	−3.17	−3.55
$^3P_1$	10.49	10.52
$^3P_2$	−7.18	−7.25
$^3F_2$	−0.27	−0.60
$^1P_1$	4.37	4.40
$^3D_3$	0.09	0.11
$^3G_3$	0.10	0.22
Total	−33.99	−36.85 (−35.39)[a]

[a]Lejeune *et al.* (Ref. 10) recalculated the channel contributions and obtained for the $^3D_2$ case −4.3 MeV, which seems more in accordance with our result. A further comparison with Ref. 11, where −4.62 MeV was obtained for $k_F = 1.4$ fm^{-1}, suggests, that the result of Ref. 10 is more reliable.

ferent channels are exhibited, in more detail, in Tables II and III. In the Brueckner approximation, where the involved domain of negative energies is smaller,[7] one expects and obtains a better agreement (see Tables I and III). Unfortunately, a direct test of the single-particle properties was not possible because the mass operator is not given in Ref. 9.

From the given comparison one can conclude that the separable representation of the Paris potential is sufficient for nuclear matter calculations within the Brueckner scheme (see Fig. 1, Tables I and III), since here the T matrix in matter is needed in an energy domain which is similar to the free nucleon-nucleon problem. For the Λ^{00} approximation (see Figs. 1–3, Tables I and II) the agreement, it seems, is not as good as in the Brueckner theory. A probable cause is the different behavior of the T matrixes for larger negative energies (see discussion of the mass operator).

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