Nuclei far off stability in the N_pN_n scheme

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The phenomenological prediction of properties of nuclei far off stability is greatly simplified in the N_pN_n scheme for three reasons: The systematics for each observable is smooth, the similarity of different regions provides a paradigm for unknown cases, and, primarily, the N_pN_n scheme often transforms the process of extrapolation into one of interpolation. The same ideas find expression in the concept of N_pN_n multiplets that can link known and unknown nuclei.

It is a long-standing but key problem in nuclear physics to estimate the properties of nuclei far off stability based on those of nuclei near the valley of stability. Such predictions provide crucial tests of nuclear models as well as needed quantities for applications as diverse as astrophysics and reactor environments. Unfortunately, however, such estimates are difficult to do with reliability because nuclear systematics are complicated two-dimensional functions of *both* N and Z and are also region dependent. This may be seen, for example, by comparing the N dependence of the onset of deformation near $A=100$, 150, and 190. Moreover, one can never be sure that new values of N and Z will not disclose unexpected features in the systematics. Indeed, this possibility is one of the motivations for research in uncharted regions of the $N-Z$ plane.

It is the purpose of this note to point out that the recently proposed[1] N_pN_n scheme provides a new approach to this problem that offers much greater reliability. The essence of this scheme is to use as the underlying parameter, not N, Z, or A, but the product, N_pN_n, of the number of valence protons and neutrons counted to the nearest closed shell (that is, as holes past midshell) and with due care taken to account for significant subshell closures. There are three advantages provided by the N_pN_n scheme for nuclei far off stability. First, with few exceptions, the systematics of an observable in this scheme is a smooth, universal function of N_pN_n for a given region.[1] Second, N_pN_n plots for *different* regions are remarkably similar.[1] Thus, apparently diverse regions actually behave identically when viewed in *units* of N_pN_n. Data in one region can thus act as a guide to another. Third, the principal new point to be discussed here, the N_pN_n scheme, in effect frequently transforms the process of *extrapolation* to nuclei far off stability into one of *interpolation*, with its attendent greater reliability. In addition, since the microscopic basis of the N_pN_n scheme in terms of the p-n interaction already incorporates the dominant nonpairing residual interaction, one does not normally expect grossly new features to intervene. There is an exception possible to this. The p-n interaction is orbit dependent, and is strongest for highly overlapping proton and neutron orbits. The simple counting procedure in the N_pN_n scheme neglects this dependence, although its effects reappear implicitly in the fact that the most rapid changes in any observable occur for those N_pN_n values that correspond to the mutual filling of these crucial orbits. If such orbits are not reached in a given region for known nuclei, but can be filled for nuclei further off stability, then predictions may be risky.

Figures 1 and 2 illustrate the first two points. Figure 1 shows the systematics of the energy, $E_{2^+_1}$, of the first 2^+ state in even-even nuclei for the $A\approx 150$ region. The com-

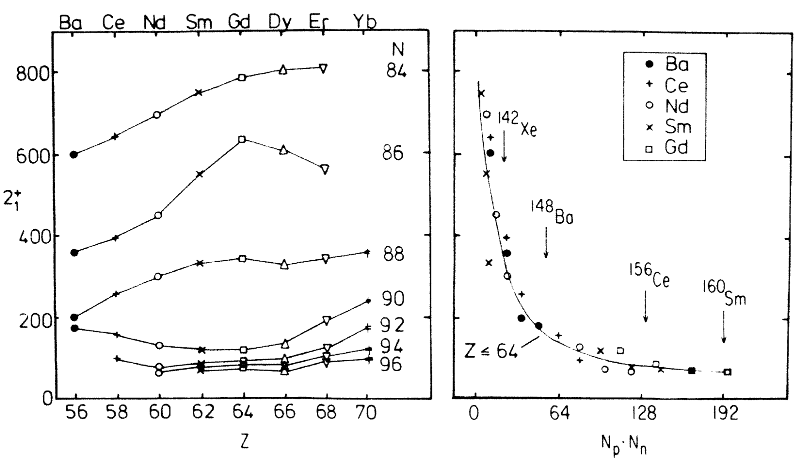

FIG. 1. Traditional and N_pN_n plots for $E_{2^+_1}$ for the rare-earth region, based on Ref. 1. As discussed there, a proton shell $Z=50$–64 has been used for $N<90$, and $Z=50$–82 for $N\geq 90$. The N_pN_n values for some typical unstudied nuclei are indicated in the figure. [The first results for ^{148}Ba have now become available (Ref. 2).]

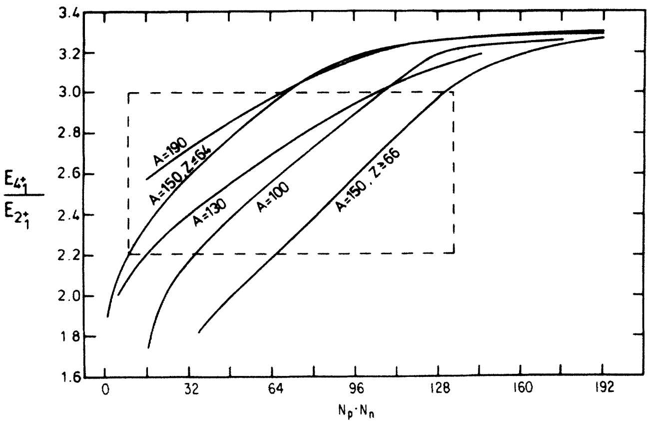

FIG. 2. Comparison of several N_pN_n curves for E_{4^+1}/E_{2^+1}.

plex behavior of the traditional plot on the left is substantially simplified in the N_pN_n plot on the right. Clearly, whether by extrapolation or interpolation, the existence of such a simple curve characterizing a region lends such greater credence to estimating the properties of unknown nuclei.

In Fig. 2, E_{4^+1}/E_{2^+1} plots are compared for five mass regions. Clearly, they are nearly identical in form and structure. Although there are differences in slopes between the $A=130$ and 190 regions and the other three, these are slight and can be easily accounted for[1] in terms of the relative strength of the p-n interaction in the crucial orbits leading to the respective transition regions. The principal regional differences lie only in varying horizontal displacements, due to the differing relative location of these crucial orbits in each region. Nevertheless, it is clear that the remarkable consistency of structure implies that even if only a few points on some curve are known, the curves for other mass regions can be used to guide further extrapolation with greater confidence.

Turning now to the third point, the extrapolation-interpolation inversion, the N_pN_n construction implies that a nucleus far off stability but with few valence particles of one type will have a lower N_pN_n product than many nuclei, nearer stability, with a more equal distribution of valence nucleons. This point is most clearly made with an example. The nucleus ${}_{56}^{148}Ba_{92}$ has $N_pN_n=6\times10=60$, whereas ${}_{62}^{154}Sm_{92}$, which is closer to stability and already well known, has $N_pN_n=12\times10=120$. Indeed, as Fig. 1 shows, the 2_1^+ systematics in the $A\approx150$ region extend out to $N_pN_n\approx200$. Whereas the data on the left in Fig. 1 would allow an E_{2^+1} for ${}^{148}Ba$ almost anywhere from 100–200 keV, the N_pN_n plot on the right predicts that $E_{2^+1}({}^{148}Ba)\approx140$ keV. The properties of a number of heretofore unknown nuclei far off stability in the neutron rich $A\approx150$ region are currently being measured at the TRISTAN isotope separator at Brookhaven National Laboratory and will provide[2] a key test of these ideas. [Indeed, a preliminary result[2] of 141.8 keV for $E_{2^+1}({}^{148}Ba)$ is very close to the above predictions.] The N_pN_n locations of some of these nuclei are indicated in Fig. 1.

The concept involved here is implicit in the isodeformation (constant N_pN_n) contours of Ref. 5 as well since the continuity of those contours from regions near stability to those far off stability implies the possibility of predictions for unknown nuclei. Indeed, an alternate phrasing of the essential point here invokes the idea of N_pN_n multiplets, analogous to F-spin multiplets.[3] An N_pN_n multiplet consists of a set of nuclei with constant (or nearly constant in a slightly relaxed definition) N_pN_n product. Such nuclei should have very similar spectra and properties and one can go from a known member of such a multiplet to an unknown one. Thus, for example, ${}^{160}Er$ and ${}^{168}Hf$ (both have $N_pN_n=140$) should be similar, and they are. Likewise, as shown in Fig. 3, the $N_pN_n\approx130$ nuclei ${}^{156}Dy$ and ${}^{180}Os$ are similar despite their wide separation in mass as are the members of the quintuplet ${}^{158}Dy$, ${}^{162}Er$, ${}^{166}Yb$, ${}^{170}Hf$, and ${}^{176}W$ whose N_pN_n values range from 160 to 168. Though the total valence nucleon numbers of the group on the left in Fig. 3 straddle that of the middle group, the former pair clearly belongs together empirically in line with their similar N_pN_n values. The latter group, with higher N_pN_n, is more collective as seen from the smaller rotational spacings, and

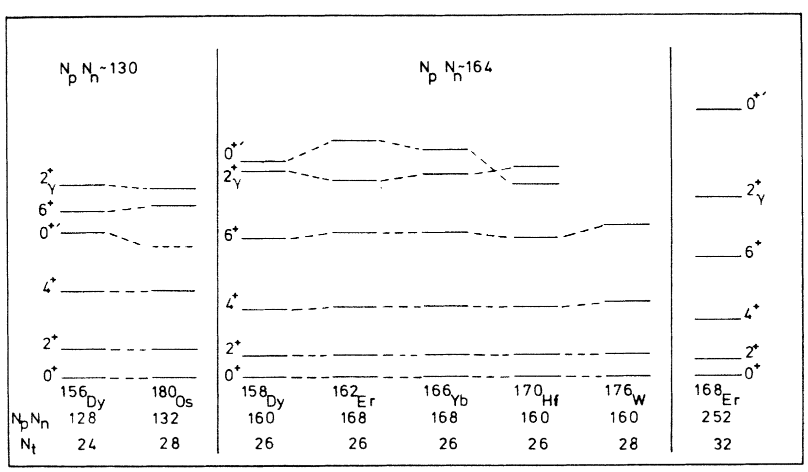

FIG. 3. N_pN_n multiplets. On the left are two widely separated nuclei with $N_pN_n\approx130$, and in the middle are five nuclei with $N_pN_n\approx164$. (The 0^+ level for ${}^{180}Os$ is located by extrapolating down from the known 4^+ and 6^+ rotational members.) At the right a nucleus, ${}^{168}Er$, not in this sequence is shown for comparison. Data from Refs. 3 and 4.

also forms a nearly ideal multiplet, even extending to the intrinsic β and γ vibrational energies. The ^{168}Er nucleus at the right shows the further lowering of ground band energies and changes of vibrational levels for a yet higher N_pN_n value. Finally, extending these arguments to an unknown nucleus, ^{152}Ba ($N_pN_n=84$) should be similar to ^{150}Ce ($N_pN_n=80$).

A more extensive and complete summary of the present ideas is given in Fig. 4 which shows the $N-Z$ plane for this region. In comparing N_pN_n plots for different regions it has become apparent[1] that differences can occur if the filling of the valence nucleons crosses midshell. Therefore, the N_pN_n scheme, in its present formulation, only links nuclei within a given quadrant of the $N-Z$ plane. The unknown nuclei whose properties can be estimated are those whose N_pN_n products are lower than the *maximum* N_pN_n value attained for *known* nuclei in the same quadrant. Since one might expect deviations from a smooth N_pN_n systematics for very low N_pN_n products where the incipient collectivity induced by the p-n interaction is still in competition with noncollective effects, it is also prudent to eliminate such nuclides from consideration. In the mass 150–200 region, these restrictions are minimal and indeed, as shown in Fig. 4, most of the unstudied nuclei in each quadrant (except for the lower right one which is completely unknown) fall within the purview of the permitted range of N_pN_n values.

The question naturally arises as to whether there may be unknown subshell effects far off stability, such as those already known near $Z=38$ and 64, which would affect the N_pN_n counting and, therefore, the predictive power of the N_pN_n scheme. Since the highly overlapping $1h_{11/2\pi}$ and $1h_{9/2\nu}$ orbits are already filling for known nuclei in Fig. 4, such effects are unlikely in the lower left quadrant. However, they are possible near the upper ends of the N,Z ranges in the figure since intruder orbits from above $Z=82$ or $N=126$ may be involved. This would suggest caution for the extreme neutron rich nuclei ($N\approx 122,124$) for $Z>66$ (e.g., the heaviest Er-W nuclei) and for the extremely neutron deficient nuclei with Z close to 82 (e.g., the lightest Hg, Pt, and Os nuclei). Indeed, in the Pt and Hg nuclei of this region, intruder states are already well known.[6] Moreover, if the collectivity of known nuclei of a given Z passes a maximum and begins to decrease (e.g., lightest Os isotopes), then an effective midshell may have been crossed, and predictions further from stability would be unwise. It should be emphasized, though, that deviations from the N_pN_n predictions are *not* unwelcome since, as already shown[1] (e.g., $N=60$ and 90 nuclei, ^{184}Hg), such deviations usually lead to interesting physical insights: The N_pN_n scheme in a sense removes or accounts for the smooth behavior of various observables and therefore highlights deviant features, and their associated physics.

To summarize, the N_pN_n scheme offers the promise of a much simplified and more reliable technique to estimate the properties of unknown nuclei far from stability. These predictions certainly encompass ground band energies and $B(E2)$ values and, most likely, the lowest collective vibrational energies and isotope shifts. Whether they can be extended to other properties such as binding (or separation) energies is still under investigation. Likewise the present analysis applies to even nuclei. Extensions to odd mass nuclei are also being considered and look promising. Experimental tests of the N_pN_n predictions in all these areas would be extremely useful and are encouraged.

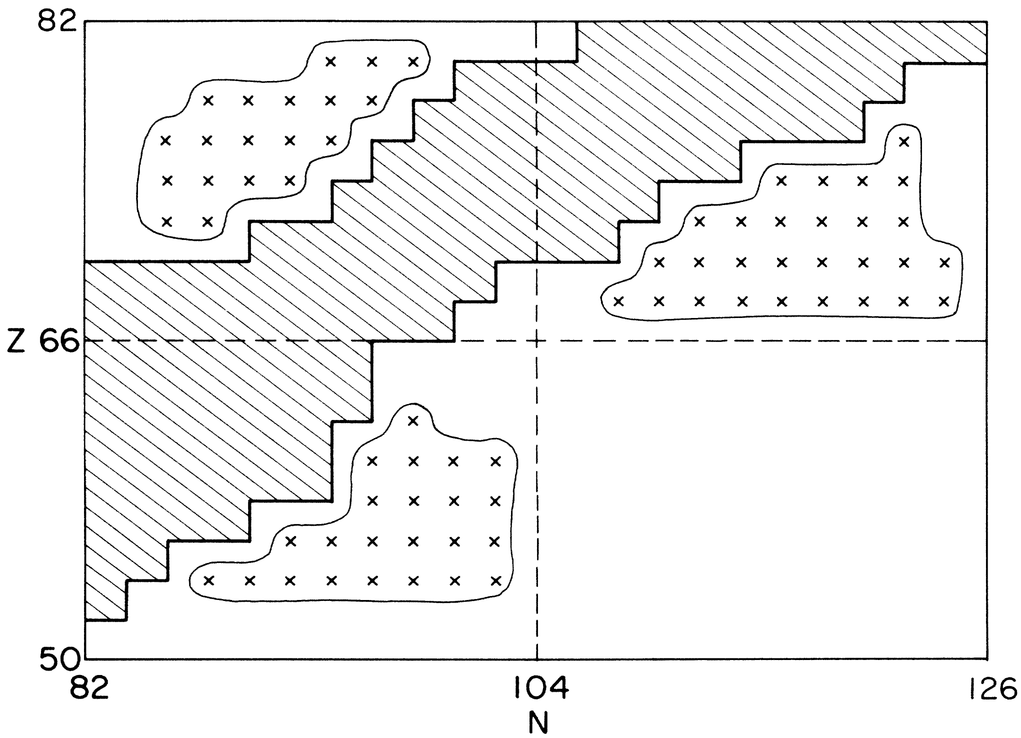

FIG. 4. The $N-Z$ plane for $50\leq Z\leq 82$ and $82\leq N\leq 126$. The hatched area between thick solid lines indicates known nuclei. The thin dashed lines divide the plot into four quadrants. Encircled ×'s indicate those unstudied nuclei for which the N_pN_n scheme should be a useful predictor.

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