Unbroken SU(3) symmetry and the relation of interacting-boson-model parameters with the shell model

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The predictions of the exact SU(3) symmetry limit of the interacting boson model are reviewed for the Dy—Pt region. Evidence of a functional relationship of the coefficient α of the $L(L+1)$ term in the expression for energy in the SU(3) limit with the product N_pN_n is presented, with a similar relation for $B(E2,2_1^+\text{-}0_1^+)$. In contrast, the absence of a simple direct relationship of the coefficient β $(=K)$ of $C(\lambda,\mu)$ or of the SU(3) symmetry breaking term K''/K with N_p,N_n is illustrated. The possible reasons are discussed.

I. INTRODUCTION

The interacting boson model[1] (IBM) provides an algebraic description of the various symmetries involved in the collective structure of the atomic nuclei. The three dynamical symmetries of the U(6) group, SU(5), SU(3), and O(6), offer analytic solutions. The study of the deviations from these limits in actual nuclei can provide insight into the complex nuclear structure. It is of great interest[2–5] to find its links with the shell model of nuclei. An important step in this task is to find the (N,Z) dependence of the coefficients of the various interaction terms of H_{IBM}. We report in this paper a partial solution of this intermediate problem in the SU(3) limit[6] of the U(6) symmetry, and discuss the transition from the SU(3) to O(6) symmetry for the deformed nuclei in the $66\le Z\le 78$ region.

We do this by first reviewing the conditions of an exact SU(3) symmetry[6] (Sec. II A) and then test them with the experiment (Secs. II B—II E). Instead of studying a single nucleus or a few nuclei, we consider the whole deformed region of $66\le Z\le 78$ and $N\le 104$. This enables us to formulate the dependence of the coefficient of the quadrupole term and the $B(E2)$ values on the boson numbers N_p and N_n (Secs. II B and II D). In the next step we consider the effect of the symmetry breaking term $K''P\cdot P$ (Sec. II C). Our conclusions are given in Sec. III.

II. VALIDITY OF THE SU(3) SYMMETRY

A. Characteristics of SU(3) nuclei

Arima and Iachello[6] discussed the four conditions of the SU(3) symmetry. The first necessary condition of an exact SU(3) symmetry is that the energies of the g-band levels exhibit the $L(L+1)$ pattern with the energy ratio $R_4=E(4_1^+)/E(2_1^+)=\frac{10}{3}$. This condition is well satisfied for the above nuclei, which lie in the deformed region.

A second condition is that the β and γ bands be degenerate. The best examples for satisfying both conditions are in ^{156}Gd, ^{170}Er, ^{234}U (Ref. 6), ^{170}Yb, and ^{176}W. Most other nuclei exhibit a broken β- and γ-band degeneracy. We shall examine this violation of the exact SU(3) symmetry in detail below (Sec. II C).

A third stricter test is in the validity of the expression for $B(E2)$ values of intraband transitions:

$$B(E2;L+2\rightarrow L)=B_{\text{BM}}(E2;L+2\rightarrow L)\frac{(2N-L)(2N+L+3)}{(2N+\frac{3}{2})^2}\,, \tag{1}$$

which implies a cutoff factor at $L=2N$. This test of the conservation of neutron-proton bosons limited to the s-d subspace, leading to the compact U(6) group, and of the dynamical group SU(3), is difficult to verify experimentally. Only a slow fall with increasing L predicted by Eq. (1) was verified[6] in ^{164}Yb and later[7] in ^{78}Kr. There have been several reviews[8–10] of this condition vis-à-vis the results of high spin excitation experiments, and the predictions of the IBM are not borne out. In fact, the continuation of the yrast $E2$ transitions beyond $L=2N$ can be explained in terms of the core excitation,[8,9] thus changing the effective boson number N.

A fourth test was suggested[6] in the ratio

$$B(E2,2_\gamma\rightarrow 0_\beta)/B(E2,2_\gamma\rightarrow 0_1)\,.$$

In the strict SU(3) limit, the 2_γ-0_1 transition is not allowed. If the SU(3) symmetry is broken slightly, this ratio in the interacting boson approximation (IBA) would still be large. Only a few β-γ transitions which do exhibit stronger β-γ band coupling have been observed in this region.

As stated above, we apply a different criterion for the test of the zeroth approximation of SU(3). Instead of a single nucleus we test all the above SU(3) conditions simultaneously for the whole region. The F-spin symmetry in IBM-1 requires that all members of the F-spin mul-

TABLE I. Total boson number N_{calc} from Eq. (3).

$N=N_p+N_n$	Dy	Er	Yb	Hf	W	Os	Pt
12	2.0	2.7	2.8			2.3	
13	3.7	3.4	3.6			2.8	1.7
14	4.3	3.7	4.3	4.4		2.6	
15	4.3	3.8	5.2	5.2	4.4		
16	4.0	4.0	7.1	6.4			
17		4.6	8.2				

tiplet, nuclei with the same total boson number $N=N_p+N_n$ or states with $F=N/2$, have approximately the same structure for the low lying levels.[11] We apply the various tests in these F-spin multiplets.

B. The $(2N,0)$ g band

At first, we test the simplest SU(3) Hamiltonian

$$H=-K\sum_{i,j}Q_i\cdot Q_j\ , \tag{2}$$

where

$$Q=(s^\dagger\tilde{d}+d^\dagger s)^{(2)}-\frac{\sqrt{7}}{2}(d^\dagger\tilde{d})^{(2)}$$

and

$$E([N](\lambda,\mu)KLM)=\tfrac{3}{4}KL(L+1)-KC(\lambda,\mu)\ .$$

The SU(3) Casimir invariant is given by

$$C(\lambda,\mu)=(\lambda+\mu)(\lambda+\mu+3)-\lambda\mu\ .$$

In this limit, the energy difference between the levels of the lowest $(2N,0)$ and the next $(2N-4,2)$ representation with the same L value is $6(2N-1)K$. Since K is determined from $E(2_1^+)$,

$$E(2_\gamma)-E(2_1^+)=\tfrac{4}{3}(2N-1)E(2_1^+) \tag{3}$$

is a function of N only. Using this relation we derive N_{calc}. In ^{156}Gd, ^{170}Er, and ^{234}U, the good examples[6] of nearly exact SU(3) symmetry, the N_{calc} are 5.0, 4.6, and 8.2 instead of the required values of $N=12$, 17, and 13, respectively. Further, we look at all the stably deformed nuclei in the region under consideration, where the F-spin symmetry holds well[11,12] (see Table I). The calculated values of the boson number N fall short up to a factor of 6. So Eq. (3) is wholly inadequate, and using the known $N=N_p+N_n$ value for a given nucleus, one cannot predict the value of the $E(2_\gamma)$.

Arima and Iachello[6] added the $L\cdot L$ part of the two d-boson interaction to the SU(3) Hamiltonian, where

$$L=\sqrt{10}(d^\dagger\tilde{d})^{(1)}$$

is the d-boson conserving interaction. (However, see the comment in Ref. 13 on the consequence of adding this term phenomenologically and on its insufficiency.) Since it is a diagonal term in the (λ,μ) representation of SU(3), the wave functions are not affected, and

$$H=-K\sum_{i,j}Q_i\cdot Q_j-K'\sum_{i,j}L_i\cdot L_j \tag{4}$$

yields

$$E([N](\lambda\mu)KLM)=\alpha L(L+1)-\beta C(\lambda,\mu)\ ,$$

with $\alpha=(\frac{3}{4}K-K')$, $\beta=K$. Now $E(2_\gamma)$ is not related to $E(2_1^+)$ analytically, so that α and β are both parameters to be determined for each nucleus from experiment, as is done in a phenomenological calculation using the Bohr-Mottelson (BM) model.[14] Thus

$$E(2_1^+)=6\alpha\ , \tag{5}$$

$$E(2_\gamma)-E(2_1^+)=6\beta(2N-1)\ . \tag{6}$$

The calculated values of the coefficient α are given in Table II. For a fixed boson number N, α is fairly constant, except in Os and Pt. Thus 2_1^+ states here form good F-spin multiplets.[11] This also leads to a constant $R_4=\frac{10}{3}$, satisfying the first condition of an $L(L+1)$ pattern of the low lying levels of the g band in all the above nuclei. As N increases, the value of α decreases on the average (see Fig. 1), but a functional relation with N is not known.

To search for a relation between α and N we plot $1/\alpha$ vs the products N_pN_n and $(N_pN_n)^{1/2}$. In each plot, the separate curves of Fig. 1 merge into a single curve with only a random small scatter (less than 10%, except for ^{180}Pt) (see Figs. 2 and 3). A linear relation seems to hold

TABLE II. The g-band energy scale parameter α (in keV) from Eq. (5).

N	Dy	Er	Yb	Hf	W	Os	Pt	Hg
11	55.8	32.0	27.7	26.5	26.1	26.5	26.5	28.5
12	23.0	20.9	20.6	20.65	20.5	22.5	25.4	61.0
13	16.5	17.0	17.1	16.8	18.65	22.0	25.8	
14	14.5	15.2	14.6	15.9	18.2	22.1		
15	13.4	13.4	14.0	15.2	17.7			
16	12.2	13.3	13.1	14.7				
17	12.8	13.1	12.75					

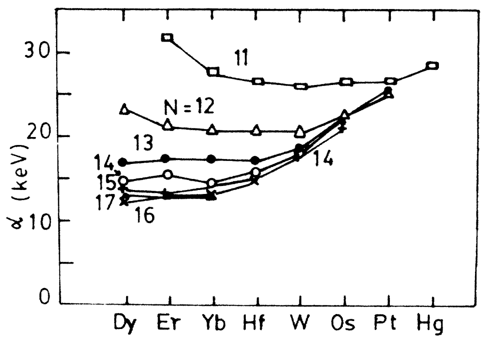

FIG. 1. Plot of the coefficient α in Eq. (5) vs boson number N.

well in both plots. So we look at the products $\alpha(N_pN_n)$ (see Table III) and $\alpha(N_pN_n)^{1/2}$ (not shown). The former product overshoots the folding with respect to N. Thus a linear relation

$$1/\alpha = a + b(N_pN_n)^{1/2} \tag{7}$$

with $a=-23$ and $b=\frac{38}{3}$ seems to be more probable from the above analysis. The uncertainty in the empirical values of (a,b) is limited to $<5\%$ only. In the geometrical picture of the BM model, $1/\alpha$ represents the moment of inertia $\mathscr{I}(2_1^+)=6/E(2_1^+)$. Compare the standard $\mathscr{I}(2_1^+)$ vs atomic mass A plots in Ref. 14 (Fig. 4.12), where for each series of isotopes a separate curve, with an initial linear rise and a final saturation value, is obtained. In the IBM, a folding of all these separate curves into a single curve is obtained in the above. For the largest value of N_pN_n, there is some indication of a saturation value. Also note that we have limited the plot to the $N\le 104$ midshell limit, $N_p+N_n=N\ge 12$, for reasons discussed at length in Ref. 12.

Using the hybrid rotational model of Moshinsky[15] which takes advantage of the extended BM model and of the SU(3) limit of the IBM for the deformed nuclei, Partensky and Quesne[16] showed that $\langle\beta^2\rangle$ is proportional to N. Since $1/\alpha$ itself is approximately proportional to $\langle\beta^2\rangle$, it should be proportional to N in the exact SU(3) limit. In the "two-fluid model" (TFM) of a rotating nucleus, Vallieres *et al.*[17] calculated $\mathscr{I}(2^+)$ and $N_{\rm wing}$, the number of nucleons in the rotating wing, using the experimental $B(E2)$ values for extracting the deformation parameter β. Their $N_{\rm wing}$ show greater correspondence with

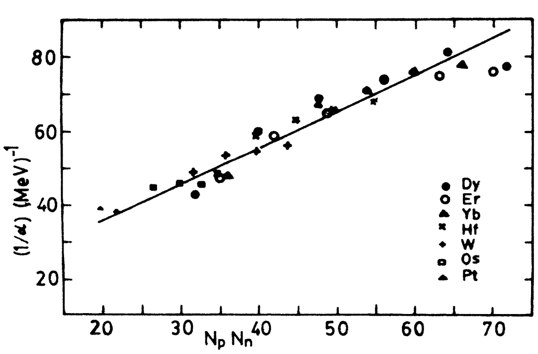

FIG. 2. Linear relation of $1/\alpha$ to N_pN_n.

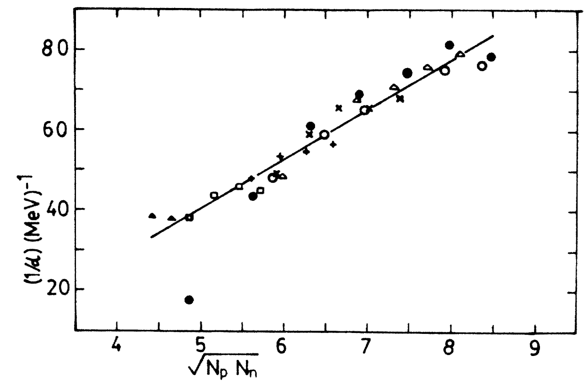

FIG. 3. Linear relation of $1/\alpha$ to $\sqrt{N_pN_n}$. The symbols for the data are the same as in Fig. 2. The single data point far below corresponds to $N=11$ for Dy.

the $\mathscr{I}(2^+)$ vs A curves than to the actual $N=N_p+N_n$ values (see Figs. 1 and 2 in Ref. 17). Thus our present analysis shows that the $N_{\rm wing}$ values derived from such a naive model as the TFM are a function of the N_pN_n product rather than of the total boson number N. This is logical, since the collectivity in this region depends on the dominant attractive proton-neutron boson interaction rather than on the total number of bosons N. While N represents the static effect, the product N_pN_n represents the dynamics of the deformation. Casten[18] used the N_pN_n product for unifying the R_4 data in the $A\sim 150$ region.

C. The $(2N-4,2)$ β and γ bands

Next we look at the systematics of the coefficient β of the Casimir invariant $C(\lambda,\mu)$, derived from $E(2_\gamma)$ and $E(2_1^+)$ (see Table IV). Over the stably deformed nuclei in the Dy—Os region, β varies through a factor of 2, even for the two neighboring nuclei with the same boson number N and approximately the same N_pN_n product, e.g., in ^{170}Er and ^{174}Yb, the good SU(3) nuclei.

Further, in most nuclei, the β- and γ-band degeneracy is broken (the second condition).[6] So a given value of the β coefficient (and of α) will not fit both the β- and γ-band levels simultaneously. In fact, a measure of this symmetry breaking can be the degree of splitting in the 2_β and 2_γ levels. A more general Hamiltonian for the SU(3)→O(6) transitional nuclei is obtained by adding the O(6) Casimir invariant term $K''P\cdot P$.

$$H=-KQ\cdot Q-K'L\cdot L+K''P\cdot P\ , \tag{8}$$

where $P=\frac{1}{2}((\tilde{d}\cdot\tilde{d})-s\cdot s)$. The pairing term raises the $K^\pi=0^+$ band upwards, the $K^\pi=2^+$ remaining unaffected. Casten and Warner[19] suggested a one-parameter measure of the SU(3) symmetry breaking in the ratio K''/K, since K represents only the energy scaling factor and $L\cdot L$ is a diagonal term not contributing to the relative energy of two levels of same L. The ratio K''/K was related to the expression

$$R_B=\frac{E(0_\beta)}{E(2_\gamma^+)-E(2_1^+)}-1\ , \tag{9}$$

TABLE III. The product $\alpha(N_p N_n)$ in keV.

N	Dy	Er	Yb	Hf	W	Os	Pt	Hg
12	735	733	743	723	655	608	507	671
13	660	714	716	672	671	660	568	
14	694	746	702	715	726	728		
15	753	752	758	759	778			
16	783	838	788	810				
17	922	917	842					

which is zero in the exact SU(3) limit. In the general case, for the SU(3) nuclei ($R_4=\frac{10}{3}$) a mathematical relation between K''/K and the term R_B is not known. For $N=16$, Casten and Warner[19] presented a correspondence between the two quantities. But in general, one has to find the value of K''/K by a least square fit to the 2_1^+, 2_γ, and 0_β levels. Then in the IBM-1, other $(E(2_i)-E(2_1^+))/K$ are a function of K''/K only.[19] In order to search for a shell model dependence of the symmetry breaking parameter, we limit our study to the exactly knowable quantity R_B itself. However, instead of the above expression for R_B, one can also define it as

$$R_B'=\frac{E(2_\beta)-E(2_\gamma)}{E(0_\beta)} . \tag{10}$$

This should be more reliable, since all three numbers in the above involve the levels of the same $(2N-4,2)$ representation. Look at the values from both expressions in Tables V and VI. More reasonable and consistent values are obtained from our modified expression for R_B'. For example, the large value of $R_B\sim 1$ for ^{168}Er (which leads to $K''/4K$ to ~ 5; see Fig. 1, Ref. 19), a good SU(3) nucleus with 2_γ well above 4_1^+, is now reduced to $R_B'=\frac{1}{2}$. All other values are generally reduced in the second definition of $R_{B'}$ and vary more slowly with N.

Even with the improved values of R_B' (see Table VI) there seems to be no correlation of R_B' (much less for K''/K) with the boson number N or N_pN_n. Negative values of R_B' ($E_{2_\beta}<E_{2_\gamma}$) in Yb and Hf are difficult to interpret with O(6) symmetry breaking. Ronnigen *et al.*[20] had noted the similarity of the β and γ bands in the Gd and Hf isotopes, the 2_β being the lowest and $B(E2,\,0_1^+\text{-}2_\beta)$ the highest in both ^{154}Gd and ^{174}Hf and higher (lower) in heavier isotopes. But unlike the Gd isotopes, we do not expect a SU(5) to SU(3) transition here; all these Hf isotopes have $R_4\sim 3.3$. Draayer and Weeks[10] examined the SU(3) symmetry in the pseudo-coupling scheme in the fermion space and showed the need for third and fourth order Q terms, to obtain the K splitting in the SU(3) itself, but they used a different representation, and the simplicity of the IBM-1 is not available. The intruder orbit $i_{13/2}$ in the $N=5$ shell of neutrons implies a breakdown of the harmonic oscillator structure which implies that real SU(3) symmetry may not be good here.[10] Casten *et al.*[21] have suggested the use of the adjustable parameter χ instead of $\sqrt{7}/2$ in the second term of the expression for Q,

$$Q=(d^\dagger s+s^\dagger\tilde{d})^{(2)}+\frac{\chi}{\sqrt{5}}(d^\dagger\tilde{d})^{(2)} . \tag{11}$$

Instead of the SU(3) value of $\chi\simeq -3$, they obtained $-0.5\le\chi\le -1.2$ for Dy and Er. The values for Yb and Hf for $N\le 104$ are not given where we have found major differences.

In the task of deriving the IBA parameters from a shell model, this lack of regularity with N (or N_p,N_n) appears to be the first hurdle to be crossed for the lowest two excited bands.

D. The intraband $B(E2)$ values

Next we test the third condition on $B(E2,\,L+2\to L)$ in Eq. (1). First we study the $2_1^+\text{-}0_1^+$ transition, taking data from Ref. 22, and look for a functional dependence of $B(E2,\,2_1^+\to 0_1^+)$ on N_p,N_n (see Figs. 4 and 5). For a given series of isotopes, at first the $B(E2)$ value increases

TABLE IV. The coefficient β of the Casimir invariant of SU(3), in keV.

N	Dy	Er	Yb	Hf	W	Os	Pt
12	5.45	5.28	5.36				3.8
13	5.65	5.32	5.53			4.25	3.42
14	5.43	4.75	5.53	6.05		4.56	
15	4.64	4.05	6.10	6.53	5.81		
16	3.70	3.99	7.46	6.74			
17		4.31	7.87				

TABLE V. The SU(3) symmetry breaking parameter

$$R_B = \left| \frac{E_{0_\beta}}{E_{2_\gamma} - E_{2_1}} - 1 \right|$$

as a measure of the K''/K ratio.

N	Dy	Er	Yb	Hf	W
12	−0.103	0.226	0.319		
13	0.169	0.361	0.257		
14	0.450	0.620	0.290	−0.111	
15	0.401	1.070	0.007	−0.272	−0.010
16		0.642	−0.248	−0.082	
17		0.044	−0.045		

linearly with neutron number and then almost saturates to a maximum value [Fig. 4(a)]. On the other hand, if one links the data points of the same total boson number N, the $B(E2)$ value seems to saturate at $N \geq 13$, irrespective of N_p, N_n [Fig. 4(b)]. Figures 5(a) and (b) exhibit the folding of the distribution when $B(E2)$ is plotted vs N_pN_n or $(N_pN_n)^{1/2}$. The choice between the two plots is not unambiguous. Note the initial faster rise [transition to SU(3)] and a later slow rise corresponding to the saturation.

The predicted variation with L of the $B(E2, L+2 \to L)$ values is not supported by experiment in general, the actual fall being too slow. In the pseudo-SU(3) approach of Draayer *et al.*[10] referred to above, using (LQL) and (LQQL) interaction terms, a slower fall with L is predicted. Wu Hua-Chuan[23] showed that in the SU(15)⊃SU(3) chain, a slower fall of $B(E2)$ with L is predicted, in agreement with experiment, e.g., in ^{164}Yb, ^{168,170}Hf, and ^{232}Th. Thus SU(3) symmetry may be good, but an appropriate representation is required.

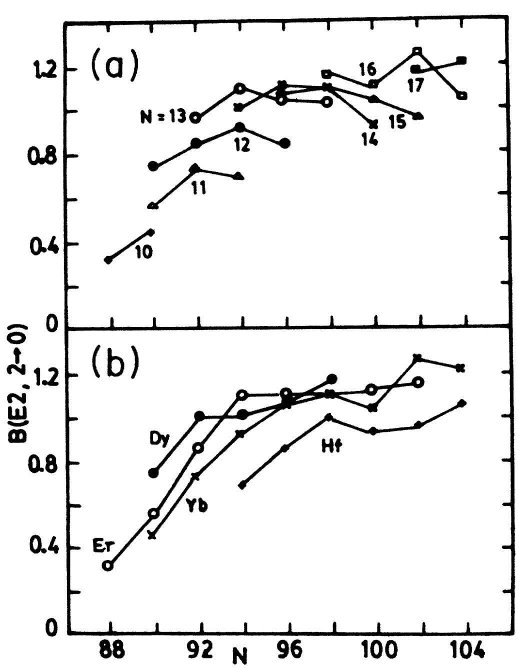

FIG. 4. (a) Variation of $B(E2;\ 2_1^+\text{-}0_1^+)$ in e^2b^2 with neutron number N. Broken lines link data for the same Z. (b) The broken lines link data for the same boson number.

E. The interband E2 strength

Finally we test the fourth condition of larger γ-β $E2$ strength compared to γ-g. There are almost no 2_γ-0_β observed transitions in Dy—Hf,[24] since the β band is expected to lie higher in the SU(3) nuclei considered here. Only two 2_β-2_γ transitions are available in ^{156}Er and ^{164}Dy. The $B(E2,\ 2_\beta\text{-}2_\gamma)/B(E2, 2_\beta\text{-}2_1)$ ratios (assuming the same quadrupole moment for the excited bands) are 38 and 15, respectively, showing stronger β-γ band coupling than the β-g coupling. This is expected in both the IBM-SU(3) and the BM models.

III. DISCUSSION

Our study of the four SU(3) conditions shows that the SU(3) symmetry is good where one is concerned with the rotational symmetry O(3), e.g., the moment of inertia $\mathscr{I}$ and the $B(E2,\ 2\text{-}0)$ values show a direct dependence on

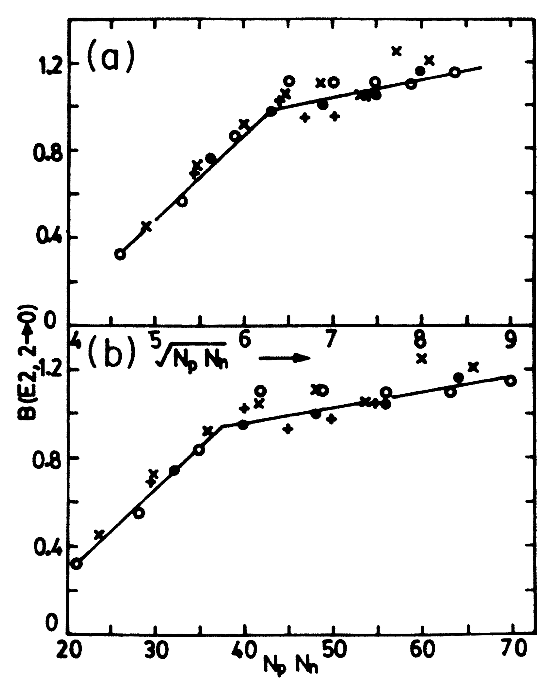

FIG. 5. (a) The $B(E2;\ 2_1^+\text{-}0_1^+)$ plotted versus the product N_pN_n. The data points ● correspond to Dy, ○ to Er, × to Yb, and + to Hf. (b) The same data as in (a) plotted versus $\sqrt{N_pN_n}$. The data points on the fast rising portion belong to $N = 10$—12.

TABLE VI. SU(3) symmetry breaking parameter $R'_B=(E2_\beta-E2_\gamma)/E0_\beta$ as a measure of K''/K.

N	Dy	Er	Yb	Hf
12	−0.092	0.160	0.215	
13	0.140	0.249		
14	0.301	0.365	0.216	−0.141
15	0.281	0.508	−0.007	−0.395
16		0.374	−0.334	−0.100
17		0.031	−0.049	

the valence nucleon (or hole) boson numbers N_p,N_n, which in turn are mainly responsible for the stable deformation of the nuclear core. Note that the main contribution in α comes from K', since $\beta=K$ is small (see Tables II and IV). It involves the d-boson preserving interaction, coupled to angular momentum one. The significant role of the N_pN_n product is made more transparent here, which reflects the dynamics of the nuclear deformation expressed in the H_{pn} term of $H_{\text{IBM-2}}$.

On the other hand, the intrinsic β and γ vibrations show irregular deviations from the SU(3) symmetry. Neither the coefficient of the $C(\lambda,\mu)$ term nor that of the O(6) term $P\cdot P$ is a simple function of N_p,N_n. The local shell effects are important here, and the effect of the neglected nuclear core will be maximum for the intrinsic vibrations. The complexity of the nuclear structure remains hidden in the parameters of the model determined by fitting to the data. Our simple analysis differs from the usual application of the IBM for deriving the L-dependent properties in a given nucleus. We have presented the quantities which often serve only for normalization of the L-dependent calculated quantities.

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