Particle-hole interacting boson approximation applied to ^{56}Ni

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The particle-hole interacting boson approximation proposed by Feshbach and Iachello is applied to the low-lying excited states of ^{56}Ni. Since the particle-hole interaction seems to be dominant in the low-lying excited states in ^{56}Ni, the particle-hole pairs are taken as the bosons. The rest of the Hamiltonian is considered as the interactions between the bosons. The first excited 2_1^+ and 4_1^+ states as well as the ground state are reasonably described without introducing any artificial treatments into the calculation. The first excited 0_1^+ state is shown to be the two-boson dominant state which is consistent with the experimental prediction. The forbidden components for a fermion system inherent in the boson wave function are investigated. As the result, the wave functions of the 0_g^+, 2_1^+, and 4_1^+ states are shown to be almost free of mixing of the components.

I. INTRODUCTION

The calculation of accurate wave functions for nuclear systems is one of the most recent matters of concern. Such wave functions are indispensable for the precise study of nuclear decay processes such as the double β decay from which valuable information concerning fundamental physics can be obtained.[1] Since these wave functions should be fully microscopic, they must be derived either from shell model calculations or from models based on the projected Hartree-Fock method. However, there is still a region where common understanding of the level structure based on such microscopic calculations is not yet obtained. In this paper we discuss the low-lying level structure of ^{56}Ni since the arguments based on microscopic investigation on this nucleus[2-9] seem not yet to be convergent.

The nucleus ^{56}Ni decays into ^{56}Fe through orbital electron capture and β^+ decay. The investigation of such a process requires an accurate wave function of the 0^+ ground state for ^{56}Ni. We do not aim, however, at the calculation of such an accurate wave function in this work. Our purpose is to give insight into the difficulties inherent in the microscopic calculations on this nucleus.

The first problem lies in the choice of the single particle energy difference between $0f_{7/2}$ and $1p_{3/2}$ orbits (hereafter we use Δ for the energy). Wong and Davies,[2] for instance, adopted a value $\Delta=1.5$ MeV which produces the best fit to the experimental data in their restricted 4p-4h calculation. With such a small value for Δ, however, it would be difficult to get a $\frac{3}{2}^-$ state as the ground state for the ^{57}Ni. Do Dang and Rabbat[6] and Motoba and Ogawa[8] (MO) used a value of about $\Delta=4.5$ MeV, which is consistent with a value obtained from the binding energies of ^{55}Co, ^{56}Ni, and ^{65}Ni nuclei. Using this Δ, they could reproduce well the positions of the lowest $\frac{3}{2}^-$, $\frac{5}{2}^-$, $\frac{1}{2}^-$, and $\frac{7}{2}^-$ states in ^{57}Ni. This choice of Δ seems very plausible, but their results on ^{56}Ni do not look as good as those of Wong and Davies.

The next problem, which seems to be more important, is the absence of the first excited 0_1^+ (3.95 MeV) state in all the microscopic calculations reported up to now. This state is excited strongly with ^{54}Fe(^{3}He,n)^{56}Ni (Refs. 10–15) (with about 50% of the ground state), whereas in the ^{58}Ni(p,t)^{56}Ni reaction,[16-18] it is weakly excited (with about 3% of the ground state transition). This 0_1^+ is understood to be a 2p-2h state following a simple shell model picture since transfer strength of a $1p_{3/2}$ pair in the (^{3}He,n) reaction is stronger than that of a $0f_{7/2}$ pair in the (p,t) reaction by a factor of about 6.[16] In the shell model calculation, the 2p-2h state is greatly affected by the existence of 4p-4h configurations. There are some calculations which include the effects of 4p-4h configurations either explicitly with much restricted form[2,3,5] or rather implicitly.[6] In the above studies many different values for Δ are adopted in each calculation, but none of them could successfully reproduce the state in the correct position. Therefore the problem seems to lie also in the truncation methods to the 4p-4h configurations. For the purpose of obtaining insight into the structure of excitations in this nucleus, we employed an approximated method to the shell model approach. The method is called the interacting boson approximation (IBA) proposed by Feshbach and Iachello.[19] This method exploits the existence of one-particle–one-hole (or particle-particle, hole-hole) excitations in nuclei to build elementary modes (bosons), and treats the residual interactions for nucleons as an effective interaction acting between the modes. This approach was already applied successfully to the low-lying states of ^{16}O.[19] It was also applied to ^{48}Ca to gain insight into the coupling scheme in the low-lying states of the nucleus.[20] But the application to ^{56}Ni must be more promising because two low-lying states (2_1^+, 2.70 MeV and 4_1^+, 3.92 MeV) in ^{56}Ni are considered to be almost 1p-1h configurations[8] and are the possible candidates for such elementary modes [d boson ($L=2$) and g boson ($L=4$)].

The present study is intended to be a breakthrough for the above-described difficulties in the microscopic calculations. We do not aim at obtaining an accurate wave function for the nucleus but rather to give a clue for resolving the difficulties inherent in the shell model calculations.

In the analysis we follow the viewpoint of MO, that is, we choose the same value for Δ as they used and we also use the Kuo-Brown matrix elements.[21] In their 2p-2h calculations they multiplied the matrix element $\langle (1f_{7/2})^2 \mid G \mid (1f_{5/2})^2 \rangle$ by 0.6 to obtain an excited state in the correct position. We neither follow such a correction for the matrix element nor use any other phenomenological interactions. In this sense, in principle, there are no free parameters in our calculation.

In Sec. II we summarize the formulae of the IBA. In Sec. III we discuss the calculational results, and we give a summary of this work in Sec. IV.

II. THE CALCULATION METHOD

The IBA was proposed for the purpose of describing the complex nuclear excitations in an approximate manner. As a first step, in the IBA, the approximately independent normal modes are derived from the correlated ground state. Any excited states as well as the ground state of nuclei are then described by considering the interactions acting between the normal modes.

The nuclear Hamiltonian relative to the Fermi sea,

$$|\phi_0\rangle = \prod_{i \le N} C_i^\dagger |0\rangle \ , \tag{2.1}$$

can be written as

$$H = U_0 + H_0 + H_{\text{int}} \ , \tag{2.2}$$

where

$$\begin{aligned} U_0 &= \langle \phi_0 | H | \phi_0 \rangle \ , \\ H_0 &= \sum_\mu \epsilon_\mu a_\mu^\dagger a_\mu - \sum_i \epsilon_i b_i^\dagger b_i \ , \\ H_{\text{int}} &= \tfrac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} :C_\alpha^\dagger C_\beta^\dagger C_\delta C_\gamma: \\ &= H_{\text{p-h}} + H_{\text{p-p}} + H_{\text{h-h}} + H_{\text{cph}} + H_{\text{mph}} \ , \end{aligned} \tag{2.3}$$

and

$$a_\alpha^\dagger = C_\alpha^\dagger \ ,$$

where $\epsilon_\alpha > \epsilon_F$ (ϵ_F is the Fermi energy), and

$$b_\alpha^\dagger = (-)^{j_\alpha + m_\alpha + \frac{1}{2} + m_\alpha^t} C_{-\alpha} \ ,$$

where $\epsilon_\alpha \le \epsilon_F$. The indices $\alpha, \beta, \ldots$ label the single particle quantum number ($\alpha = n_\alpha l_\alpha j_\alpha m_\alpha t_\alpha m_\alpha^t$). States below the Fermi surface are expressed as ($-\alpha = n_\alpha l_\alpha j_\alpha - m_\alpha t_\alpha - m_\alpha^t$) and contain an additional phase factor $(-)^{j_\alpha - m_\alpha + t_\alpha - m_\alpha^t}$ to ensure the correct transformation property under rotation. The matrix elements of H_{int} are given in terms of five basic graphs and their mirror images, which are shown in Fig. 1. In the IBA $H_{\text{p-h}}$ is considered for creating the normal modes (bosons) in the 1p-1h subspace. The matrix elements of all the other parts of the H_{int} vanish exactly in this subspace. Those parts, however, are taken into account as the interactions acting between the bosons. On the other hand, the conventional random phase approximation (RPA) considers only $H_{\text{p-h}}$ and H_{cph} parts of the interactions. In this respect the IBA is beyond the RPA.

A normal mode is defined in the 1p-1h subspace as

$$B^\dagger = \sum_{\mu i} f_{\mu i} C_\mu^\dagger C_i \ , \tag{2.4}$$

where μ denotes particles and i denotes holes. By solving the equation of motion in the Tamm-Dancoff approximation,

$$[U_0 + H_0 + H_{\text{p-h}}, B^\dagger] = \hbar\omega B^\dagger \ , \tag{2.5}$$

we can obtain solutions in the 1p-1h subspace,

$$|\phi_{LT}\rangle = \sum_{\mu i} f_{\mu i}^{LT} C_\mu^\dagger C_i |\phi_0\rangle \ . \tag{2.6}$$

In the above solutions the energetically lowest one or two levels are selected as the bosons in the IBA.

In the next step, the boson-boson interaction is microscopically constructed by considering the interaction Hamiltonians $H_{\text{p-p}}$, $H_{\text{h-h}}$, H_{cph}, and H_{mph}. The two-boson state and the matrix element between the states can be written as

$$\begin{aligned} &|\phi_{L_1}\phi_{L_2}:LT\rangle \\ &\quad = \sum_{\alpha\beta\gamma\delta} f_{\alpha\beta} f_{\gamma\delta} |\alpha\beta^{-1} L_1 T_1, \gamma\delta^{-1} L_2 T_2 : LT\rangle \end{aligned} \tag{2.7}$$

and

$$\mathcal{N} \langle \phi'_{L_1}\phi'_{L_2}:LT | H_{\text{p-p}} + H_{\text{h-h}} | \phi_{L_1}\phi_{L_2}:LT \rangle \ . \tag{2.8}$$

The constant $\mathcal{N}$ is a normalization factor. This matrix element can be expanded in terms of a nucleon-nucleon interaction. The calculation of the matrix elements for the boson-boson interaction in the multiboson space is straightforward when employing the boson coefficient of fractional parentage (cfp).[22] Details of the calculation are presented in Ref. 19.

Next we consider the H_{cph} part of the interaction,

$$\langle \phi | H_{\text{cph}} | \phi_{L_1}\phi_{L_2}:L = T = 0 \rangle \ . \tag{2.9}$$

The expansion of this matrix element in terms of the nucleon-nucleon interaction is also available in the refer-

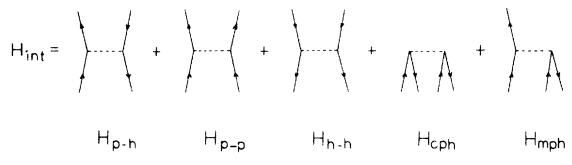

FIG. 1. The graphic representations of the five basic interactions in the H_{int}.

ence. In the matrix element, of course, only a boson pair which is coupling to $L=T=0$ can annihilate into vacuum through the $H_{\rm cph}$ interaction.

The last part of the interaction $H_{\rm mph}$ can be considered to be the one boson annihilation interaction. Since the formulae for this interaction are not available, we present them here. Its matrix element in the boson basis can be written as

$$\mathcal{N}\langle\phi_L{:}LT\,|\,H_{\rm mph}\,|\,\phi_{L_1}\phi_{L_2}{:}LT\rangle=\mathcal{N}\sum_{\substack{\alpha\beta\\ \gamma\delta\\ \mu\nu}} f_{\alpha\beta}f_{\gamma\delta}f_{\mu\nu}\langle\gamma\delta^{-1}{:}LT\,|\,H_{\rm mph}\,|\,\alpha\beta^{-1}L_1T_1,\mu\nu^{-1}L_2T_2{:}LT\rangle\ , \tag{2.10}$$

where the matrix element in the fermion basis can be expressed as

$$\begin{aligned}&\langle\gamma\delta^{-1}{:}LT\,|\,H_{\rm mph}\,|\,\alpha\beta^{-1}L_{12}T_{12},\mu\nu^{-1}L_{34}T_{34}{:}LT\rangle\\ &\quad=\sum_{\substack{L_{13}T_{13}\\ L_{24}T_{24}}}\hat{L}_{12}\hat{L}_{34}\hat{L}_{13}^{2}\hat{L}_{24}^{2}\hat{T}_{12}\hat{T}_{34}\hat{T}_{13}^{2}\hat{T}_{24}^{2}\begin{Bmatrix}j_\alpha & j_\beta & L_{12}\\ j_\mu & j_\nu & L_{34}\\ L_{13} & L_{24} & L\end{Bmatrix}\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & T_{12}\\ \frac{1}{2} & \frac{1}{2} & T_{34}\\ T_{13} & T_{24} & T\end{Bmatrix}\begin{Bmatrix}T & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & T_{24} & T_{13}\end{Bmatrix}[A\delta_{j_\delta j_\beta}+B\delta_{j_\alpha j_\gamma}]\ ,\end{aligned} \tag{2.11a}$$

where $\hat{L}_{12}=(2L_{12}+1)^{1/2}$ and

$$A=(-)^{L+T+L_{24}+T_{24}+L_X}\begin{Bmatrix}L & j_\gamma & j_\delta\\ j_\nu & L_{24} & L_{13}\end{Bmatrix}\langle\nu\gamma\,|\,G\,|\,\alpha\mu\rangle_{L_{13}T_{13}}\ , \tag{2.11b}$$

$$B=(-)^{L+T-L_{24}-T_{24}-L_Y}\begin{Bmatrix}L & j_\delta & j_\alpha\\ j_\mu & L_{13} & L_{24}\end{Bmatrix}\langle\beta\nu\,|\,G\,|\,\delta\mu\rangle_{L_{24}T_{24}}\ , \tag{2.11c}$$

and $L_X=j_\nu+j_\beta+j_\gamma+j_\delta$, $L_Y=j_\alpha+j_\mu$. This matrix element connects the boson spaces whose boson numbers are different by one. The matrix elements of this interaction in the multiboson space can also be expressed by utilizing the boson cfp. By denoting the multiboson state $[\phi_{lt}]^n$ simply as l^n (dropping the isospin index t) we can express the matrix elements as

$$\begin{aligned}&\langle l_1^n{:}vL\,|\,H_{\rm mph}\,|\,l_1^n(v_1L_1),l_2{:}vL\rangle\\ &\qquad=n\sum_{v'L'}[l_1^{n-1}(v'L')l_1^1\}l_1^nvL][l_1^{n-1}(v'L')l_1^1\}l_1^nvL]U(L'l_1Ll_2;L_1l_1)\langle l_1\,|\,H_{\rm mph}\,|\,l_1l_2{:}l_1\rangle\end{aligned} \tag{2.12}$$

and

$$\langle l^{n_1}{:}vL\,|\,H_{\rm mph}\,|\,l^{n_2}{:}vL\rangle=\frac{n_1(n_1-1)}{2}\left[\frac{n_2!}{n_1!}\right]^{1/2}\sum_{\substack{v_1L_1\\ v_2L_2}}[l^{n_2-1}(v_2L_2)l_1^1\}l^{n_2}vL][l^{n_1-2}(v_1L_1)l^2(l)^1\}l^{n_1}vL]\langle l\,|\,H_{\rm mph}\,|\,l^2{:}l\rangle\ , \tag{2.13}$$

where $n_1=n_2+1$ and the v's are the seniority quantum numbers.

Next we consider the transition rates in the IBA approach. The reduced matrix elements for the electromagnetic operator in the boson space are given by

$$\begin{aligned}\langle\phi_{L''}||\hat{Q}_L||\phi_{L'}\rangle=\tfrac{1}{2}(e_{\rm eff}^{\rm p}+e_{\rm eff}^{\rm n})\sum_{\alpha\beta\gamma\delta}f_{\alpha\beta}^{L''}f_{\gamma\delta}^{L'}L'L''\Bigg[&\delta_{\beta\delta}\langle\gamma||\hat{C}_L||\alpha\rangle\begin{Bmatrix}L'' & L & L'\\ j_\alpha & j_\delta & j_\gamma\end{Bmatrix}(-)^{j_\gamma+j_\delta+L+L'}\\ &-\delta_{\alpha\gamma}\langle\beta||\hat{C}_L||\delta\rangle\begin{Bmatrix}L'' & L & L'\\ j_\beta & j_\gamma & j_\delta\end{Bmatrix}(-)^{j_\gamma+j_\delta+L''}\Bigg]\end{aligned} \tag{2.14}$$

and

$$\langle 0||\hat{Q}_L||\phi_{L'}\rangle=\frac{1}{\sqrt{2}}(e_{\rm eff}^{\rm n}-e_{\rm eff}^{\rm p})\sum_{\alpha\beta}f_{\alpha\beta}^{L'}\delta_{LL'}(-)^{j_\beta-j_\alpha+L'}\langle\beta||\hat{C}_L||\alpha\rangle\ , \tag{2.15}$$

where $\hat{C}_L$ expresses an electroquadrupole moment operator defined by

$$\hat{C}_L=\sum_{i=1}^{z}(\tfrac{1}{2}+T_z)\gamma^LY_L^*(\theta_i,\varphi_i)\ . \tag{2.16}$$

Since in the present study we consider only $E2$ transition probabilities, only the $L=2$ boson (the d boson) can contribute to the one boson annihilation processes. Calculation of the reduced matrix elements in the multiboson

space are straightforward.

In the present study we take the $0f$-$1p$ shell as a model space. Therefore the hole state is restricted to only the $0f_{7/2}$ orbit. We consider all the configurations in the multiboson calculations. The maximum dimension for the energy matrix, in our study, is 105 in the case of the $L=4$ state of the five-boson space. As for the effective nucleon-nucleon interactions we use the Kuo-Brown matrix elements.[20] The reasons are to avoid increasing the artificial parameters in the calculation and to compare the present results with those of the previous shell model calculation[8] which used the same matrix elements.

III. CALCULATIONAL RESULTS

A. Boson creation and mutual interaction

The first step of the IBA starts with creating bosons from the fermion degrees of freedom. This is done by diagonalizing the $H_{\text{p-h}}$ part of the Hamiltonian within the 1p-1h subspace. The matrix elements for the other four types of the Hamiltonian ($H_{\text{p-p}}$, $H_{\text{h-h}}$, H_{cph}, and H_{mph}) in the 1p-1h subspace are exactly zero. In the calculation we adopt the same single particle energies as those used in the work of MO whose values are listed in Table I. These values were fixed to reproduce the low-lying $\frac{3}{2}^-$, $\frac{5}{2}^-$, $\frac{1}{2}^-$, and $\frac{7}{2}^-$ states in ^{57}Ni within (1p-0h) + (2p-1h) space. The results of the diagonalization for $L=2$, 3, and 4 ($T=0,1$) cases are listed in Table II. All the other solutions ($L=1$, 5, and 6 states) are higher in excitation energies. Among the indicated states we take the following two lowest solutions as the elementary modes:

$$E_d(L=2,T=0)=2.76 \text{ MeV } (d \text{ boson}) ,$$

$$E_g(L=4,T=0)=3.50 \text{ MeV } (g \text{ boson}) .$$

The dominant component in the d-boson state is $|p_{3/2}f_{7/2}^{-1}L=2,T=0\rangle$ (99%) and in the g-boson state is $|p_{3/2}f_{7/2}^{-1}L=4,T=0\rangle$ (72%). Parity for the boson states is everywhere positive since our model space is restricted to the $0f$-$1p$ shell. After identifying the boson spectra in the 1p-1h space we can construct any multiboson space by utilizing a properly symmetrized boson basis. The multiboson spectra can be obtained by taking into account the

TABLE I. The single particle energies for the orbits in the $0f$-$1p$ shell which is employed in the present study. The energy differences between $0f_{7/2}$ and $1p_{3/2}$ (denoted by Δ in the text) are taken as 4.62 MeV (Ref. 8) for two-boson, 5.02 MeV for four-boson, and 5.38 MeV for five-boson calculations.

Orbits	Energies (MeV)
$1p_{1/2}$	1.22
$0f_{5/2}$	0.70
$1p_{3/2}$	0.00
$0f_{7/2}$	−4.62 (for a two-boson spectrum)
	−5.02 (for a four-boson spectrum)
	−5.38 (for a five-boson spectrum)

$H_{\text{p-p}}$, $H_{\text{h-h}}$, H_{cph}, and H_{mph} parts of the total Hamiltonian as boson-boson interactions. The numerical values for the matrix elements of the Hamiltonians are indicated in Fig. 2. From the figure we can see that the average strength of the diagonal parts of the interactions is almost one-fifth of the boson self-energy (the single boson energy). This fact indicates that our bosons have good character as elementary modes within the present model space.

B. Two-boson spectrum

Before performing the multiboson space calculation we first analyze the results of the two-boson spectrum to see the characteristic futures of the present method compared to other approaches. The calculational results are shown in Fig. 3 and Table III. The ground state is shown to be 0^+ with 91% zero-boson component. This state is 0.55 MeV lower compared to the uncorrelated ground state. The value is very small compared to the 2p-2h shell model calculation by MO, whose value is about 4 MeV. The origin of this difference may be attributed to the following reasons. In the calculation we take only d- and g-boson modes and discard all the other 22 boson states which are produced in the 1p-1h calculation. Furthermore, in the IBA approach the pairing correlation is very underestimated owing to the following recoupling factors:

$$|\phi_d(p_{3/2}f_{7/2}^{-1}),\phi_d(p_{3/2}f_{7/2}^{-1}):L=0,T=0\rangle = 5\sum_{L'}\hat{L}'^2 \begin{Bmatrix} f_{7/2} & f_{7/2} & L' \\ p_{3/2} & p_{3/2} & L' \\ 2 & 2 & 0 \end{Bmatrix}_{L,T} |p_{3/2}^2 L'T',(f_{7/2}^{-1})^2 L'T':L=0,T=0\rangle . \quad (3.1)$$

This transformation is necessary in calculating the boson-boson interaction microscopically. The study by MO has made clear that the pairing correlation inherent in the Hamiltonian plays a dominant role in the low-lying excited states of ^{57}Ni. The inclusion of the f boson which appeared next to the g boson (see Table II) is expected to increase the correlation, which will lead to a greater depression of the ground state energy. Therefore the f boson may also be important in our calculation. But f bosons are expected to play a minor role compared to the d and g bosons in ^{56}Ni because only the 1p-1h nature of 2_1^+ and 4_1^+ is observed in the low-lying excited region. For this reason we neglect f bosons in the present approximated calculation.

Because of the small depression of the ground state energy, however, we can obtain the 2_1^+ and 4_1^+ states in the correct positions. In our calculation the excitation energies of the 2_1^+ and 4_1^+ states are 3.18 MeV and 3.93 MeV, respectively. The 2_1^+ level is almost a one-boson (d boson 94%) state with 99% $|p_{3/2}f_{7/2}^{-1}L=2,T=0\rangle$ component.

TABLE II. The eigenvalues and the components of wave functions for boson states obtained from the diagonalization of $H_{\text{p-h}}$ in the 1p-1h subspace.

L	T	Eigenvalue (MeV)	$(1p_{3/2},0f_{7/2}^{-1})$	$(0f_{5/2},0f_{7/2}^{-1})$	$(1p_{1/2},0f_{7/2}^{-1})$
2	0	2.76	0.996	−0.084	
2	0	6.09	−0.084	0.996	
3	0	3.96	0.937	−0.012	−0.349
3	0	5.05	0.050	0.994	0.101
3	0	6.28	0.346	−0.112	0.932
4	0	3.50	0.846	−0.366	−0.388
4	0	5.02	0.359	0.929	−0.093
4	0	5.45	0.395	−0.060	0.917
2	1	5.67	0.735	0.678	
2	1	6.36	−0.678	0.735	
3	1	4.97	0.957	0.046	−0.286
3	1	5.82	−0.172	0.885	−0.432
3	1	6.60	0.233	0.463	0.855
4	1	4.67	0.962	0.081	0.262
4	1	5.71	0.019	0.934	−0.357
4	1	6.83	−0.273	0.349	0.896

In shell model terminology this state is a 1p-1h state. The 4_1^+ state is also a one-boson (g boson 93%) state. In the work of MO, a large depression of the ground state results in the high excitations of the 2_1^+ and 4_1^+ states. They attributed the reason to the omission of 3p-3h configurations in their model space since the configurations significantly affect the 1p-1h states, whereas 2p-2h configurations act upon the 0p-0h components. To see an effect from the three-boson components on the one-boson states in the IBA, we extend the calculation up to the three-boson space. As a result, the 2_1^+ and 4_1^+ states reduce their energies about 0.4 MeV with respect to the uncorrelated ground state. However, we cannot find any particular change or improvements in both energy levels and the wave functions. Therefore in our approach the three-boson components seem not to play an important role in the 2_1^+ and 4_1^+ states. Following the maximum symmetry principle,[3] Jaffrin selected certain 2p-2h and 4p-4h configurations which involve the maximum binding energies. With such configurations, without 3p-3h components, he successfully reproduced the low-lying spectra of ^{56}Ni.

C. Multiboson spectrum

One of the main purposes of this work is to reproduce the 0_1^+ (3.95 MeV) state in the correct position and to predict its structure. Up to now no theoretical discussions were made on this level.

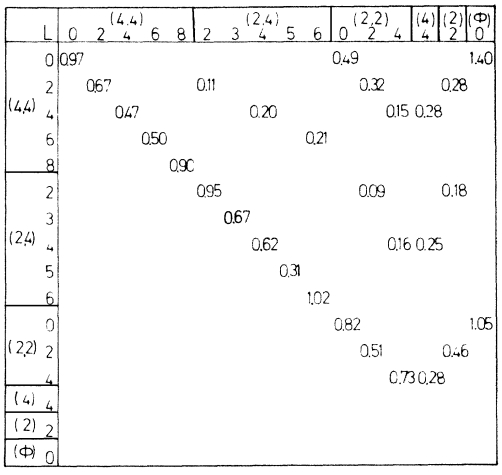

FIG. 2. The boson-boson interaction matrix elements. (2,4) means a coupling state of a d boson and a g boson and (4) is one g-boson state. The symbol ϕ represents the zero-boson state. All boson states are $T=0$. All the energy values are negative and the unit is MeV.

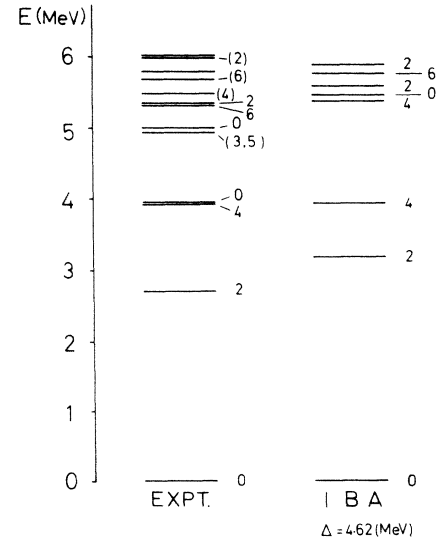

FIG. 3. The results of the diagonalization of the $H_{\text{p-p}}$, $H_{\text{h-h}}$, H_{cph}, and H_{mph} in the two-boson space.

TABLE III. The wave functions for 0_g^+, 2_1^+, 4_1^+, and 0_1^+ states resulting from the two-boson calculations. The numbers express the percentage of the components in the wave functions.

States \ Number of bosons	0	1	2
0_g^+	91		9
2_1^+		94	6
4_1^+		93	7
0_1^+	6		94

TABLE IV. The wave functions for 0_g^+, 2_1^+, 4_1^+, and 0_1^+ states resulting from the four-boson calculations.

States \ Number of bosons	0	1	2	3	4
0_g^+	88		10	0	2
2_1^+		71	9	16	4
4_1^+		64	13	16	7
0_1^+	8		49	15	28

In the shell model investigation the choice of the single particle energies is very crucial in determining the level structure. Δ especially plays an essential role in ^{56}Ni. In the multiboson calculations we also follow MO's value for Δ. In the present case, however, the model space is extended up to four- and five-boson configurations, then we try to change the value from $\Delta=4.62$ to 5.02 MeV for the four-boson system and to 5.38 MeV for the five-boson system as the optimal values. These values still seem reasonable since they are also close to the value ($\sim$6 MeV) obtained from the binding energy calculations of ^{55}Co, ^{56}Ni, and ^{57}Ni. In the four-boson case the single boson energies become $E(d$ boson$)=3.16$ MeV and $E(g$ boson$)=3.90$ MeV, and all the other states are higher in excitation energies. Calculated results with up to four-boson components are shown in Fig. 4 and Table IV.

Following the experimental analysis of Nann and Benenson,[16] the dominant component for the 0_1^+ state is expected to be 2p-2h configurations as discussed in the Introduction of the present study. In the IBA approach a 0^+ state is obtained at 4.07 MeV with 49% two-boson and 28% four-boson components. The two-boson part is composed of 91% $[d$ boson$]^2_{L=T=0}$ component. Thus the dominant structure of this state can be described as a two-particle-hole-pairs state with each pair forming $L=2^+$ and totally coupled to $L=T=0$.

Although the four-boson components are not so large in the above result, we consider these components to be significant in reproducing this state, because we could not find the 0^+ state in the two-boson spectrum discussed in the preceding subsection.

Above 5 MeV we cannot make any correspondence between calculated and experimental states. The present approach especially fails to reproduce the 0_2^+ (5.00 MeV) state observed in the experiments. This state is weakly excited in both (p,t) and (^{3}He,n) reactions and is pointed out as a 4p($T=0$)$-$4h($T=0$) state.[23,24] In the experiment we can find the 2_2^+ (5.35 MeV) and 4_2^+ (5.49 MeV) states just above the 0_2^+ state. Following the quadrupole pairing model by Fucks *et al.*,[11] these three levels can be understood as 4p-4h states provided the 2_1^+ (2.70 MeV) state is a quadrupole pairing (2p-2h) state. The differential cross sections of these 2_2^+ and 4_2^+ states show almost one-tenth (at $\theta_{lab}=0$) of that of the 2_1^+ and 4_1^+ states in (p,t) reactions. Since they are also weakly excited in (^{3}He,n) reactions, there is a possibility that they are the 4p-4h dominant states. This idea, however, contradicts both the present IBA result and the work of MO; thus it is still not clear about the structure of the 0_2^+ state.

We can find further three 0^+ states at 6.66, 7.91, and 9.92 MeV in the data. These three levels are nicely understood as $T=0$, 1, and 2 states by utilizing the Zamick formula,[18,25]

$$E=E_0+bT(T+1)/2$$

with $E_0=6.66$ MeV provided we assume the following coupling scheme for the above 0^+ states:

$$[^{58}\mathrm{Cu};J=0^+,T=1]\otimes[^{54}\mathrm{Co};J=0^+,T=1]\ .$$

This fact implies that above 5 MeV, the $H_{\text{p-p}}$ and $H_{\text{h-h}}$ parts of the Hamiltonian play dominant roles compared to the $H_{\text{p-h}}$ part. Thus our boson modes (particle-hole pairs) must be already broken in the energy region. Therefore, in the application of the IBA to ^{56}Ni, we should restrict ourselves to the discussions for the energy region less than 5 MeV.

In the four-boson spectrum, 0_g^+, 2_1^+, and 4_1^+ levels are

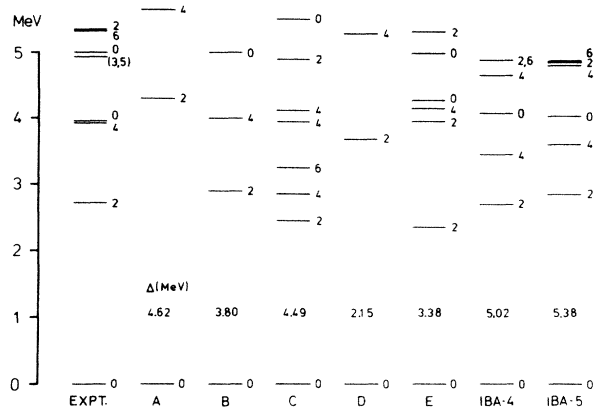

FIG. 4. The comparison of the previous and the present works with the observed data. The experimental data are taken from Ref. 16. The letters $A-E$ correspond to the following works: *A*, Motoba and Ogawa; *B*, Jaffrin; *C*, Do Dang and Rabbat; *D*, Boiti *et al.*; and *E*, Oberlechner and Richert. IBA-4 means the calculational result in the four-boson space. The symbol Δ represents the single particle energy difference between $0f_{7/2}$ and $1p_{3/2}$. The parity of the states is everywhere positive.

described as 88% zero-boson components, 71% one-boson components, and 64% one-boson components, respectively. The main trends of these structures are not changed compared to the two-boson spectrum, and they are also conserved when we extend the space up to five-boson configurations. The calculational results for the five-boson spectrum are shown in Fig. 4. We can find 2^+ and 4^+ states near the 2_2^+ and 4_2^+ states, but they are not the four-boson dominant states. As discussed already, $H_{\text{p-p}}$ and $H_{\text{h-h}}$ act strongly in this energy region. Therefore we cannot have any discussion about the states within the present approach.

Before closing this subsection, we give a brief discussion on $E2$ transition rates. We calculate $B(E2;2_1^+\rightarrow 0_g^+)$ and $B(E2;4_1^+\rightarrow 2_1^+)$ in five boson space. In the experiment only $B(E2;2_1^+\rightarrow 0_g^+)=77\pm 32$ ($e^2\text{fm}^4$) is reported.[15] In the calculation we use effective charges for protons and neutrons as $e_{\text{eff}}^{\text{p}}=1.19e$ and $e_{\text{eff}}^{\text{n}}=0.49e$, respectively. These values were adopted in the 4p-4h shell model calculation.[5] The results are $B(E2;2_1^+\rightarrow 0_g^+)=64$ ($e^2\text{fm}^4$) and $B(E2;4_1^+\rightarrow 2_1^+)=49$ ($e^2\text{fm}^4$), and the former is very close to the observed value. If we set the additional charges as $\Delta e^{\text{p}}=\Delta e^{\text{n}}=0.42e$, then we can reproduce the experimental value. These additional charges are also close to the value $\Delta e_{\text{eff}}=(0.53\pm 0.17)e$ which is obtained from a simple shell model estimation.[26]

From the above results we can say that the 0_g^+ and 2_1^+ states are reasonably described within the IBA approach and their dominant components are zero-boson and one-boson states, respectively. Sharma investigated the ground state of ^{56}Ni with the Hartree-Fock method[7,9] and came to the conclusion that a $0f_{7/2}$ orbit is occupied by 13 particles, that is, the 0_g^+ state is an 81.3% closed shell state.

D. Comparison with other studies

In this subsection we compare the present results for 0_g^+, 2_1^+, 4_1^+, and 0_1^+ states with several other investigations. We use the following indices for the references: *A*, Motoba and Ogawa (Ref. 8); *B*, Jaffrin (Ref. 3); *C*, Do Dang and Rabbat (Ref. 6); *D*, Boiti *et al.* (Ref. 4); and *E*, Oberlechner and Richert (Ref. 5). The energy levels obtained in these works as well as the IBA results are compared with the experiments in Fig. 4. Among these works *A*, *C*, *D*, and *E* are the shell model studies, whereas *B* follows the projected Hartree-Fock method. The space is restricted up to 2p-2h configurations in *A* and *D*, while in the works of *B*, *C*, and *E*, 4p-4h configurations are taken into account in a restricted form. Relatively large values for Δ are adopted in *A*, *B*, and *C*, whereas in *D* and *E* the employed values are somewhat smaller.

1. 0_g^+

In *A* the energy depression of the ground state is about 4 MeV. In order to reproduce the 0_3^+ (6.66 MeV, $T=0$) state in the correct position they multiplied the matrix element $\langle (f_{7/2})^2 \mid G \mid (f_{5/2})^2\rangle_{J,T}$ by 0.6 and reduced the depression to about 2 MeV. In their resultant wave function 0p-0h components are included to about 80%. The energy depression is also large in *D*, but only 54% 0p-0h components exist in the 0_g^+ state. The reason is that their value for Δ is very small ($\Delta=2.2$ MeV). In *B* and *C*, 0_g^+ is almost a closed shell state.

2. 2_1^+ and 4_1^+

In *A* both levels consist of more than 90% 1p-1h configurations. The 2_1^+ state is almost a $\mid p_{3/2}f_{7/2}^{-1}L=2,T=0\rangle$ state. In *B* both levels are pure 1p-1h states. In *C* the two levels include more than 85% 1p-1h states. In *D* the 2_1^+ state consists of 87% 2p-2h configurations and the 4_1^+ is 40% 1p-1h configurations because of the small value for Δ.

3. 0_1^+

There are neither any discussions nor calculational results concerning this state in works *A*, *B*, *C*, and *D*. In work *E*, the 0^+ (4.23 MeV) state is found which includes 47.4% 4p-4h configurations. However, considering their small value for Δ (3.38 MeV), this level seems to be a possible candidate for the 0_2^+ (5.00 MeV) state since the level is expected to be a 4p-4h state as discussed in subsection C. They also reproduced the 0^+ (4.26 MeV) state in a separate calculation with $\delta\equiv\epsilon(f_{5/2})-\epsilon(f_{7/2}^{-1})=2.5$ MeV. With such a value for δ, however, it will be difficult to reproduce the ground state of ^{57}Ni.

Following the above discussions, we can see that the IBA approach can reasonably describe the 0_g^+, 2_1^+, and 4_1^+ states without relying on any artificial adjustments of the parameters.

E. Forbidden components in the IBA wave function

In this subsection we refer to the existence of the forbidden components which can be easily recognized in the expansion of the boson wave function. The two-boson state can be expanded as

$$\begin{aligned}\mid \phi_{L_{12}}\phi_{L_{34}}:LT\rangle &= \sum f_{L_{12}}f_{L_{34}} \mid j_\alpha j_\beta^{-1}L_{12}T_{12},j_\gamma j_\delta^{-1}L_{34}T_{34}:LT\rangle \\ &= \sum f_{L_{12}}f_{L_{34}}\{L-9j\}\{T-9J\} \mid j_\alpha j_\gamma L_{13}T_{13},j_\beta^{-1}j_\delta^{-1}L_{24}T_{24}:LT\rangle \ . \end{aligned} \tag{3.2}$$

In the above expansion we can find a forbidden component for a fermion system such as

$$\mid p_{3/2}^2(L_{13}=2,T_{13}=0),f_{7/2}^{-2}(L_{24}=2,T_{24}=0):L=2,T=0\rangle \ . \tag{3.3}$$

These components, however, are necessary for fulfilling the orthonormality relations among the boson states. An admixture of such components into the boson wave function is inevitable when one describes a nuclear system with the IBA approach. In the derivation of the matrix

elements for the $H_{\text{p-p}}$, $H_{\text{h-h}}$, H_{cph}, and H_{mph} between the boson states, however, these forbidden components do not come into play in the calculations [see Eq. (3.15) in Ref. 19]. The existence ratios of these components in every boson basis ($|$one-boson$\rangle$,$|$two-boson$\rangle$,...) are already fixed when one creates bosons from the diagonalization of the $H_{\text{p-h}}$ part of the Hamiltonian. The ratio is unaltered during the multiboson calculational process.

Clearly the forbidden components are not included in the one-boson states. Therefore one-boson dominant states, such as 2_1^+ and 4_1^+ in ^{56}Ni, are almost free from these components. In the realistic calculation of the two-boson spectrum, 0_g^+, 2_1^+, and 4_1^+ states are shown to include the "permitted" components of 95.5%, 97.1%, and 95.3%, respectively. Since the nature of these three levels is conserved also in the multiboson spectrum, our calculational results with the IBA approach might be acceptable.

On the other hand, the dominant components in the 0_1^+ state are two-boson and four-boson configurations. In the $[(d \text{ boson})^2]_{L=T=0}$ state 49.6% forbidden components are included. Therefore the IBA description for multiboson states, such as the 0_1^+ in ^{56}Ni, suffers from such admixture of the components into the boson wave function no matter what its simplicity for the interpretation of the level structure.

IV. SUMMARY

The low-lying level structures of ^{56}Ni are discussed within the framework of the IBA. Although several shell model studies on this nucleus already exist, their results seem to be still confusing. The main discrepancies may be attributed to the different choice for the single particle energy differences between $0f_{7/2}$ and $1p_{3/2}$ and to the treatments of the 4p-4h configuration in their model spaces. In the present work we followed the viewpoint of Motoba and Ogawa as to the choice of the energy difference.

It requires huge effort to carry out the whole 4p-4h calculation. Then we employ the IBA as an effective truncation method to the shell model approach. The IBA is expected to work well when $H_{\text{p-h}}$ dominates over the other part of the Hamiltonian. The nucleus ^{56}Ni seems to satisfy this condition.

Our investigations are summarized as follows:

(1) After the diagonalization of the $H_{\text{p-h}}$ part of the Hamiltonian, d and g bosons are taken as the elementary modes in the 1p-1h subspace of ^{56}Ni. The strength of the interaction acting between these bosons is about $\frac{1}{5}$ of the boson self-energies.

(2) 0_g^+, 2_1^+, and 4_1^+ states are reasonably reproduced. The 0_g^+ is a 91% closed shell state, 2_1^+ is a 61% one-boson state, and 4_1^+ is also a 53% one-boson state within the five-boson calculation. The above structures are compatible with the result of the shell model study of Motoba and Ogawa. In our study, however, the positions of the levels are well reproduced without relying on any artificial adjustments of a matrix element which was introduced in their work. The IBA also works well in reproducing the $B(E2;2_1^+ \rightarrow 0_g^+)$ transition rate within the permissible additional charges.

(3) The excitation energy of the 0_1^+ state is well reproduced in our multiboson calculation. This level cannot be identified in the two- and three-boson spectra. The state consists of 29% two-boson and also 29% four-boson components. In shell model terminology 4p-4h excitations may play a crucial role in reproducing the level in the correct position.

(4) The pairing correlation seems to be much reduced within the present truncated space. Introduction of the f boson (3.96 MeV, see Table II) into our calculation serves to increase the correlation, which results in a greater depression of the ground state energy. Since the 2_1^+ and 4_1^+ states are almost 1p-1h states, the above effect may shift up these levels. Therefore the study of an interacting three-type-boson system (d, g, and f bosons) as well as a large space 4p-4h shell model calculation seems to be indispensable for making the more definite comparison between the two approaches.

(5) Some forbidden components for the fermion systems are included in the IBA wave functions. The ratio of the components is large for the state in which more than two-boson components are dominant. Since zero- and one-boson states do not include such components, 0_g^+, 2_1^+, and 4_1^+ levels are almost free from the problem. On the other hand, the 0_1^+ state suffers from the mixing of such components in the boson wave function. This problem, however, is unavoidable in our approximate method.

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