Exchange effects in the interacting boson-fermion model

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A study of Pauli exchange effects for the odd particle in the interacting boson-fermion model is made by a comparison with a core-quasiparticle coupling model based on dynamical field theory and the BCS method. Spectra for a partly filled j shell coupled to interacting boson model cores are calculated in both models. Values of the interacting boson-fermion model exchange parameter Λ_0 are found which lead to about the same spectra as in the core-quasiparticle coupling model. Variations of Λ_0 around this value can alter the energy spectra and the Coriolis mixing between bands much more than variations of the corresponding BCS parameter Δ. However, variations of Λ_0 also effectively alter the position of the Fermi level, which should set constraints on phenomenological applications of the interacting boson-fermion model.

INTRODUCTION

The low-lying excitations of doubly even nuclei appear to have the character of bosons, and the interacting boson model (IBM) has provided a means of characterizing nuclear spectra.[1-3] Obviously, the bosons must derive in some way from fermions (e.g., Refs. 4 and 5). This must be taken into account when considering an odd fermion, for example, in connection with particle transfer or the level structure of the neighboring odd-mass nucleus. Boson-fermion models of odd-mass nuclei have therefore been formulated which include boson-fermion exchange terms.[6-9] The widely used interacting boson-fermion model (IBFM) of Iachello and Scholten[8,9] retains one such term with a phenomenological strength constant, Λ_0.

Another method to handle exchange effects between the fermion and the bosons is the core-quasiparticle coupling model (CQCM) of Ref. 10. In the CQCM, the Bardeen-Cooper-Schrieffer (BCS) technique is applied after taking into account the effect of the dynamical field of the core, in conceptual analogy with the more familiar application of BCS theory in the static deformed field of the Nilsson model. This approach provides an explicit prescription for the exchange effects, and also for relating fermion with quasifermion states. The latter leads to an exact form of the particle transfer operator. In summary, the exchange effects are uniquely prescribed in the CQCM, to within the accuracy that the BCS Fermi level, λ_F, and the BCS gap parameter, Δ, are known.

In the IBFM, there are likewise two parameters associated with the exchange term. One of these is v_j^2, which gives the degree of filling of the j shell and corresponds directly to λ_F. The other is the unspecified strength parameter Λ_0^j, which ideally should be a constant.

The present work gives numerical results of the CQCM and IBFM in model calculations which obviously differ by the treatment of the exchange effects but otherwise are completely identical.

OUTLINE OF THE MODELS

For this work, the odd-A calculations were done for a single j-shell coupling to a single core. Therefore, this outline is similarly restricted; more general discussions are available elsewhere.[9,11,12]

In the CQCM, the wave function for a collective state of total angular momentum I in an odd-A nucleus is written as

$$|I,A\rangle = \sum u_I(jR)[a_j^\dagger \,|\, R,A-I\rangle]_I + v_I(jR)[\bar{a}_j \,|\, R,A+1\rangle]_I \,, \quad (1)$$

where the first term denotes the coupling of a particle to the $A-1$ even-even core and the second term denotes the hole coupling to the $A+1$ core, with probability amplitudes u and v. The Hamiltonian for the odd-mass system, restricting to quadrupole collective modes, a monopole pair field, and taking the $A\pm1$ cores to be the same, is written as

$$H = H_{\text{core}} + H_{\text{s.p.}} + \kappa q\cdot Q + H_{\text{pair}} \,, \quad (2)$$

where H_{core} denotes the Hamiltonian of the core and $H_{\text{s.p.}}$ that of the single particle in the spherical part of the mean field. The particle and core are coupled through the third term, where q and Q are the quadrupole operators of the core and particle, respectively, and further via the pairing term

$$H_{\text{pair}}=\Delta(a_j^\dagger \bar{a}_j^\dagger+\bar{a}_j a_j)\ .$$

By composing the amplitudes u and v to a column vector $\binom{u}{v}$, the Hamiltonian (2) to be diagonalized can be written in the form of a 2×2 matrix as

$$H=\begin{bmatrix} e_j-\lambda+\Gamma+E_c & \Delta \\ \Delta & -(e_j-\lambda+\Gamma)+E_c \end{bmatrix}, \tag{3}$$

where the elements are themselves matrices:

$$\begin{aligned} &e_j-\lambda=(e_j-\lambda)\delta_{jj'}\delta_{RR'}\ , \\ &\Delta=(\Delta)\delta_{jj'}\delta_{RR'},\quad E_c=(E_R)\delta_{jj'}\delta_{RR'}\ , \end{aligned} \tag{4}$$

are all diagonal matrices containing the single particle energy in a spherical potential (e_j), the Fermi energy λ, the pairing gap energy Δ, and the core energies E_R. The matrix Γ arises from the quadrupole term and is given by

$$\begin{aligned} (\Gamma)\hat{j}\,\hat{R}\,\hat{j}'\hat{R}'=&\kappa\langle R||Q||R'\rangle\langle j||r^2Y^2||j'\rangle \\ &\times W(j'2IR;jR')\ . \end{aligned} \tag{5}$$

Therefore, in this model, the core is completely specified by the set of core energies and the reduced matrix elements of the quadrupole operator between all core states. The additional coupling parameters that must be specified are the strength of the $q\cdot Q$ interaction, κ, the single particle energy relative to the Fermi energy $e_j-\lambda$, and the pairing gap energy Δ. The eigenvalues and eigenvectors of the odd-mass system are obtained by diagonalizing the Hamiltonian; however, the above basis is overcomplete by a factor of 2 and so a method of selecting the physical subspace is required.

An approximate solution is obtained by recognizing that the adiabatic part of the Hamiltonian (omitting the core energies) has the symmetry of the Hartree-Fock-Bogolyubov (HFB) Hamiltonian and can be diagonalized in two steps. First the block matrix $e_j-\lambda+\Gamma$ is diagonalized, giving eigenvalues ϵ and eigenvectors. Next pairing is included via a Bogolyubov transformation which gives quasiparticle energies $E=\pm(\epsilon^2+\Delta^2)^{1/2}$ and occupation factors u and v for each adiabatic quasiparticle orbital. The positive energy solutions are selected as the physical subspace since they correspond to the lowest energy excitations in the odd-mass system. The nonadiabatic effects are included approximately by projecting the core Hamiltonian onto the positive energy subspace. In the case of an axial rotor core for instance, the off diagonal matrix elements resulting from this projection are the familiar Coriolis matrix elements, multiplied by the BCS pairing factors ($u_1u_2+v_1v_2$). The details of this projection technique and a general derivation of this model by the equations of motion method can be found in Ref. 13.

By comparison, in the IBFM the Hamiltonian is written as

$$H=H_{\text{core}}+H_{\text{s.p.}}+H_{\text{int}}\ , \tag{6}$$

where H_{core} is an IBM-1 description of the core and $H_{\text{s.p.}}$ is the same spherical single particle Hamiltonian as above. The interaction term consists of two parts, which can be written (for a single j-shell calculation) as

$$\Gamma_0(1-2v^2)\times\text{quadrupole coupling}$$

and (7)

$$\Lambda_0 v^2(1-v^2)\times\text{exchange}\ .$$

The first term describes quadrupole coupling between the particle and the core with a strength parameter Γ_0. The dependence on the j-shell occupancy v^2 is contained in the standard BCS pairing factor, $1-2v^2$, for a one-body operator which changes sign under particle-hole conjugation, in this case the single-particle quadrupole operator q. The IBM-1 core quadrupole operator is given by

$$Q=(d^\dagger s+s^\dagger d)^{(2)}+\chi(d^\dagger\tilde{d})^{(2)}\ , \tag{8}$$

where $s^\dagger$ ($d^\dagger$) and s ($\tilde{d}$) are the $L=0$ ($L=2$) boson creation and destruction operators, respectively. The second term is intended to account for the Pauli principle by allowing the odd fermion to exchange with one of the constituent fermions in a core boson.[8] The second term carries a parameter Λ_0 and an explicit dependence on the j-shell occupancy such that this term vanishes for a completely full or empty j shell ($v^2=1.0$ or 0.0, respectively).

Thus, the two models both have three parameters. The Fermi energy λ in the CQCM and the orbital occupancy v^2 in the IBFM both control the number of particles on the j shell so equivalent values of these parameters can be chosen accordingly. The quadrupole strength parameters, κ for CQCM and Γ_0 for IBFM, are the same within a constant factor such that, for a full or empty j shell, identical energy spectra result from the two models. This leaves the parameters Δ and Λ_0 free for comparison in the partially filled j shell. Finally, the $E2$ operator can be given the same form in the two models and so differences in calculated $E2$ transition rates can be traced to differences in the odd-A wave functions.

NUMERICAL CALCULATIONS

Numerical results for the CQCM and the IBFM are given below for coupling a $j=\frac{9}{2}$ particle to SU(5), SU(3), and O(6) cores. For a given core and j-shell occupancy, the energies of the first few states in the odd-A system are shown as a function of Δ or Λ_0. The strength of the quadrupole interaction ($\Gamma_0=0.567$ MeV, which is equivalent to $\kappa=6.34$ MeV) and the parametrization of the IBM cores are taken directly from Ref. 2 with the minor exception of taking χ in the IBM core quadrupole operator [Eq. (8)] equal to zero for the SU(5) core. This gives vanishing diagonal quadrupole matrix elements for the SU(5) core.[1]

In Fig. 1 coupling to an SU(5) core is shown for $v^2=0.2$. Clearly the resulting spectra are quite similar as Δ ranges from 1.5 to 0.5 MeV and Λ_0 ranges from 0.5 to 1.5 MeV. However, it is also clear that no values of Δ and Λ_0 in this range give identical spectra. In both the CQCM and the IBFM, the $\frac{7}{2}$ state $(j-1)$ is the most sensitive to the parameters Δ and Λ_0. However, as v^2 gets closer to 0.5, the $\frac{7}{2}$ state becomes much more sensitive to Λ_0 than to Δ. This sensitivity of the $(j-1)$ state to the IBFM exchange term for a vibrational core has been not-

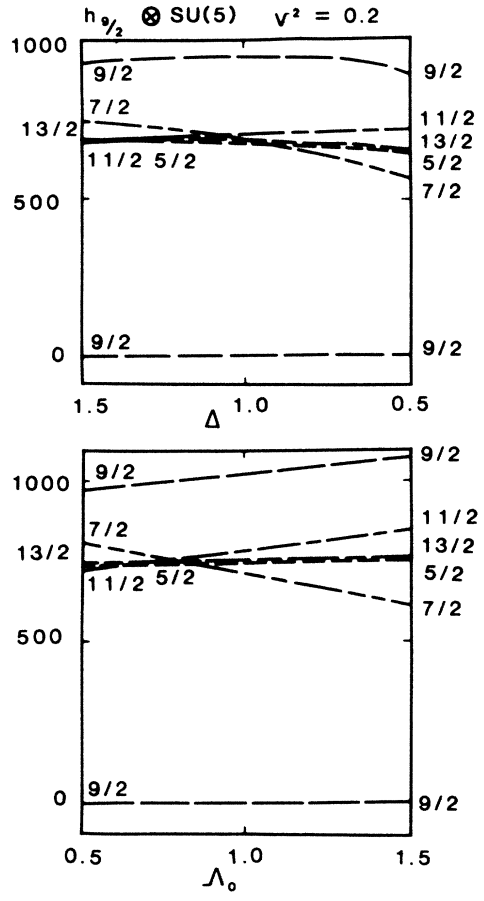

FIG. 1. Energy levels for a $j=\frac{9}{2}$ particle with a j-shell occupancy of 0.2 coupling to an SU(5) core. Energies relative to the ground state are plotted as a function of the pairing gap parameter Δ for the CQCM, and as a function of the exchange term parameter for the IBFM.

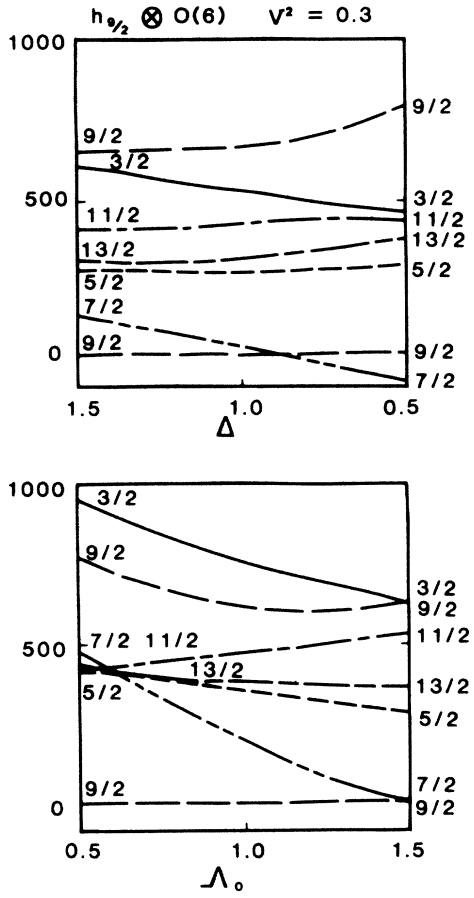

FIG. 2. Same as Fig. 1, but for $v^2=0.3$ and an O(6) core.

ed before,[9,11] and is the only significant difference between IBFM and CQCM.

Examination of Figs. 2 and 3 [$v^2=0.3$, O(6) core and $v^2=0.4$, SU(3) core, respectively] leads to similar conclusions: the two sets of spectra are similar but not identical. In addition, one sees here that the exchange term in the IBFM can have a larger effect on spectra than the BCS gap parameter Δ within the physically reasonable range of 0.5–1.5 MeV.

The greatest differences observed between the IBFM and the CQCM are shown in Fig. 4 for $v^2=0.8$ and an SU(3) core. In the geometrical picture of a hole coupled to a prolate rotor, these states correspond to rotational bands built upon $K=\frac{9}{2}$ and $K=\frac{7}{2}$ bandheads. A striking difference between the two models is the rapid lowering of the $K=\frac{7}{2}$ band as Λ_0 increases, while only small changes result from varying Δ. This effect with Λ_0 is quite similar to a changing Fermi energy and is readily understood since the exchange term is multiplied by $\Lambda_0 v^2(1-v^2)$. Thus, changes in Λ_0 can also simulate changes in v^2 (i.e., for $v^2>0.5$, increasing Λ_0 has the same effect as decreasing v^2 in the exchange term, or equivalently, as lowering the Fermi energy). The small changes with Δ are due to the BCS compression effect (changing the adiabatic quasiparticle energy).

Another difference between the two sets of spectra is that the spacing between band members is quite different, i.e., the moments of inertia are different in the IBFM and CQCM calculations. For example, in the CQCM the $K=\frac{9}{2}$ band has a moment of inertia 1.36 times that of the core, which is understood as due to Coriolis coupling with the $K=\frac{7}{2}$ band. By comparison, the $K=\frac{9}{2}$ band in the IBFM has a moment of inertia slightly smaller than that of the core. This suggests that the IBFM may be able to attenuate the Coriolis interaction, i.e., that the mixing of the K bands which arises from the core Hamiltonian may be attenuated by the exchange term. The most direct way to study this mixing is to examine interband collective $E2$ transition rates, since these vanish between pure K bands in the geometric picture.[14] Interband transitions occur due to the Coriolis mixing, and thus the transition rates are directly related to these interaction matrix elements. Furthermore, the $E2$ operator has the same form in both the CQCM and the IBFM, namely

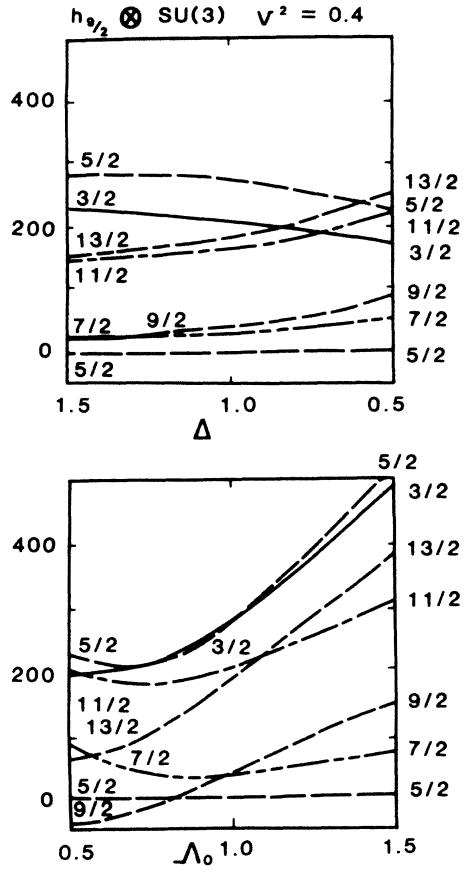

FIG. 3. Same as Fig. 1, but for $v^2=0.4$ and an SU(3) core.

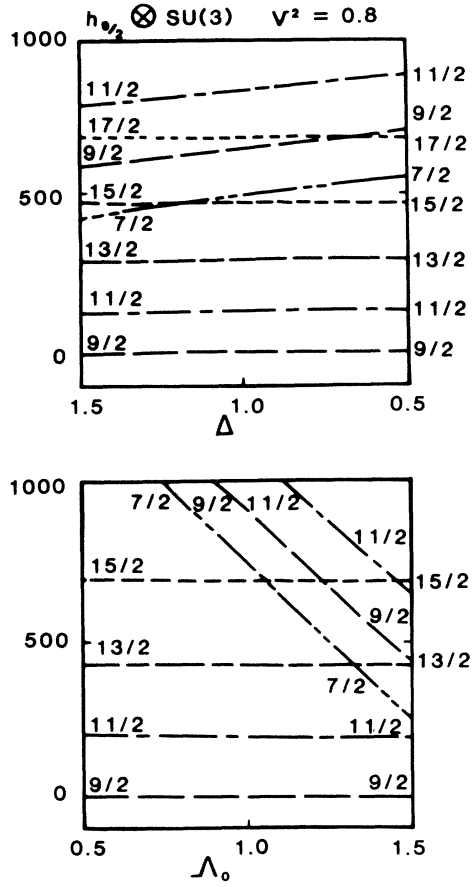

FIG. 4. Same as Fig. 1, but for $v^2=0.8$ and an SU(3) core.

$$T^{(E2)}=e_c Q+e_F q\ , \tag{9}$$

where e_c and e_F are the effective charges for the core and the odd fermion and Q and q are the core and particle quadrupole operators. By choosing the fermion effective charge equal to zero and using the same effective core charge in the two models, the differences in calculated $E2$ transition rates can be related to the effective Coriolis mixing matrix elements. The simplest way to extract the mixing matrix elements is to consider two band mixing only and to approximate the SU(3) core quadrupole matrix elements with those of a rigid axial rotor, i.e.,

$$\langle R'||Q||R\rangle \simeq Q_0\sqrt{2R+1}\langle 2R\,020\,|\,R'0\rangle\ , \tag{10}$$

where Q_0 is chosen to reproduce the diagonal SU(3) matrix elements. This results in collective $E2$ transition rates between the mixed bands given by

$$B(E2;I\rightarrow I')=e_c^2Q_0^2\left[\sum A_K B_K\langle IK20\,|\,I'^2K\rangle\right]\ , \tag{11}$$

where e_c is the effective charge in Eq. (8) and the amplitudes A_K and B_K are determined from the two band mixing calculation. For $K=\frac{9}{2}, \frac{7}{2}$ mixing, the $I=\frac{9}{2}\rightarrow I'=\frac{9}{2}$ transition is quite simple because

$$(A_K B_K)^2=V^2/(\Delta E)^2\ , \tag{12}$$

where ΔE is the energy difference of the two $\frac{9}{2}$ states after mixing and V is the mixing matrix element. Therefore, from the calculated $E2$ rate for the $I=\frac{9}{2}\rightarrow I'=\frac{9}{2}$ transition, the magnitude of the effective mixing matrix element V can be extracted using Eqs. (10)–(12). Using the rigid rotor approximation to obtain Eq. (10) is well justified since, for $I\simeq j$ the odd particle couples to the lowest members of the core ground state band, where the difference between the SU(3) and rigid rotor quadrupole matrix elements is a few percent or less.[2] Furthermore, the additional implicit assumptions of the rigid rotor, namely that the core ground state band has an $R(R+1)$ energy dependence and that the odd particle couples only to the ground state band, are exactly satisfied for the SU(3) core. The validity of this procedure for extracting the mixing matrix element can be checked directly for the CQCM due to the two step diagonalization procedure used there. For high-K bands, the extracted Coriolis matrix elements agreed within 2% with the values used by the CQCM computer code from the projection of the core energies.

The effective Coriolis matrix elements obtained from the IBFM $(\frac{9}{2})_2\rightarrow(\frac{9}{2})_1$ $E2$ transition rate for an SU(3) core and $v^2=0.8$ are shown in Fig. 5 as a function of Λ_0. As Λ_0 varies from 1.25 to 2.00 MeV, the energy difference

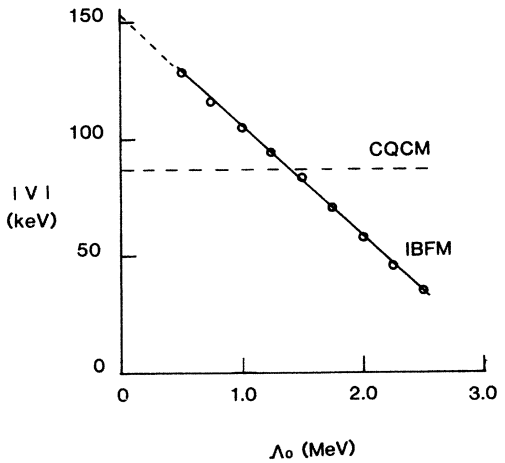

FIG. 5. The effective Coriolis matrix element extracted from the IBFM $E2$ transition rate between the first and second $\frac{9}{2}$ states, for $v^2=0.8$ and an SU(3) core. The IBFM results are shown as a function of the exchange term parameter Λ_0, and the dashed line shows the CQCM result for $\Delta=1$ MeV.

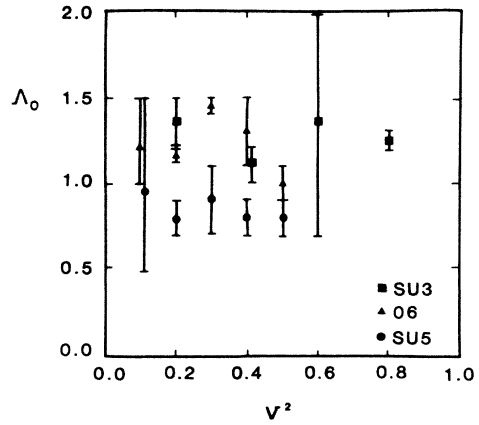

FIG. 6. The values of Λ_0 for which the IBFM spectrum most closely resembles the CQCM spectrum with $\Delta=1$ MeV, shown as a function of the j-shell occupancy v^2 for the three different cores. The two models give identical spectra at $v^2=0.0$ and 1.0 for all values of Λ_0, and the spectra are symmetric about $v^2=0.5$ for the O(6) and SU(5) cores.

between the two $\frac{9}{2}$ states ranges from 650 to 140 keV, and the extracted matrix element varies from 94 to 60 keV. A linear extrapolation gives an intercept of 153 keV for the unattenuated Coriolis matrix element, in very good agreement with the rigid rotor value of 150 keV. The dashed line on the figure shows the result of the CQCM calculation for $\Delta=1.0$ MeV and a j-shell occupancy of 0.8. This result was not very sensitive to the placement of the Fermi energy. Moving the Fermi energy so that the energy difference between the two $\frac{9}{2}$ states ranges from 600 to 180 keV changes the effective Coriolis matrix element by only 2 keV. This is because the Fermi energy is approximately midway between the two Nilsson states and so small changes in the Fermi level affect the BCS pairing factor $u_1u_2+v_1v_2$ very little.

The effective Coriolis matrix element also varies with v^2 in the IBFM due to the $v^2(1-v^2)$ factor in the exchange term. For example, allowing v^2 to vary from 0.80 to 0.74 with Λ_0 kept fixed at 1.25 MeV produces a range in energy differences between the two $\frac{9}{2}$ states of 650 to 170 keV. The resulting variation in the extracted matrix element is from 94 to 83 keV, a significantly smaller variation than that obtained with Λ_0.

Finally, we investigate if there exists any global value of Λ_0 that gives energy spectra corresponding to the physically reasonable value of $\Delta=1$ MeV. In Fig. 6 the approximate value of Λ_0 corresponding to the CQCM calculations for each of the cores considered is shown as a function of v^2. For each core, there is a range of Λ_0 values for which the IBFM gives a level spectrum similar to the CQCM; Figs. 1 and 2 exemplify this. Figure 6 shows that for a given core and with the quadrupole interaction strength fixed, keeping $\Delta=1$ MeV in the CQCM and varying the Fermi level through the j shell is roughly equivalent to fixing Λ_0 in the IBFM and varying v^2 through the shell. For the SU(5) core, $\Lambda_0=0.8$ MeV approximately corresponds to $\Delta=1$ MeV, while for the SU(3) and O(6) cores, the appropriate value for Λ_0 is about 1.2 MeV. The value of Λ_0 is expected to scale with the $q\cdot Q$ strength parameter Γ_0, but since the same value of Γ_0 was used for all three cores,[9] it is not clear why a different value of Λ_0 should be required for the SU(5) core.

SUMMARY AND CONCLUSIONS

Calculations have been made, in which a partly filled $j=\frac{9}{2}$ shell is coupled to IBM cores with dynamical symmetries SU(5), SU(3), and O(6), respectively. Both the CQCM (Ref. 10) and IBFM (Refs. 8 and 9) models were used with the same cores, the same core-particle quadrupole-quadrupole coupling strength, and the same filling of the j shell. Variations of the remaining CQCM parameter, the gap parameter Δ, in the physical range 0.5—1.5 MeV, produce rather small variations of the odd-A level spectrum. The remaining IBFM parameter, Λ_0, could be chosen to give a similar level spectrum. It was borne out that a single value of Λ_0 will approximately reproduce the CQCM results for any degree of filling of the shell. In practical applications of IBFM phenomenology, however, Λ_0 should be expected to vary as follows. Firstly, Λ_0 obviously scales with Γ_0, the scaling parameter of the core quadrupole field. Secondly, the present CQCM results suggest that Λ_0 should be reduced by a factor of about 0.6 for nuclei near the SU(5) limit relative to nuclei nearer to SU(3) and O(6) limits (Fig. 6).

Variations of the IBFM parameter Λ_0 produce relatively large variations in the odd-A level spectrum (Figs. 1—4) and also in the Coriolis mixing of K bands (Fig. 5). At first sight this would seem to give the IBFM more flexibility than the CQCM on the phenomenological level. However, for a given nucleus, the variation of Λ_0 is effec-

tively similar to a variation of v^2, or the position of the Fermi level in the shell. Thus, if many Nilsson bandheads are known in one nucleus, or if a whole sequence of nuclei which successively exhibit many bandheads is to be described with a smooth and plausible variation of v^2, the relative position of the bandheads can be expected to constrain Λ_0.

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