Compound nucleus fluctuation cross section in the intermediate coupling regime $\bar{\Gamma}/\bar{D} \approx 1$

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The compound nucleus fluctuation cross section in the intermediate absorption regime of $\bar{\Gamma}/\bar{D} \gtrsim 1$ is discussed within the optical background representation of Kawai, Kerman, and McVoy. A constraining inequality, involving $\pi(\bar{\Gamma}/\bar{D})$, a relevant parameter in the cross section formula, and other statistical parameters that appear in $\sigma_{cc'}^{\mathrm{fl}}$, is derived and analyzed.

A major problem which still confronts the statistical theory of nuclear reactions mediated by the formation of a compound nucleus (CN) is the analytical evaluation of the averaged fluctuation cross section, $\sigma_{cc'}^{\mathrm{fl}}$, in the intermediate coupling (absorption) situation of $\bar{\Gamma}/\bar{D} \approx 1$, with $\bar{D}$ the average nuclear level spacing and $\bar{\Gamma}$ the average nuclear decay width. Of course, $\sigma_{cc'}^{\mathrm{fl}}$, has been extensively discussed in the past, with analytical results usually obtained in the domains $\bar{\Gamma}/\bar{D} \gg 1$ and $\bar{\Gamma}/\bar{D} \ll 1$. Only numerical results based on Monte Carlo calculations are available for $\bar{\Gamma}/\bar{D} \approx 1$.[1]

Recently, Weidenmüller and collaborators[2] obtained an expression for $\sigma_{cc'}^{\mathrm{fl}}$ which seems to be valid for any value of $\bar{\Gamma}/\bar{D}$. The method they employed, supersymmetry averaging, enabled them to derive a triple-integral representation of $\sigma_{cc'}^{\mathrm{fl}}$. This makes a direct comparison with the experimental data rather difficult. An analytical, albeit approximate, version of the results of Ref. 2 is certainly urgently called for.

On the other hand, several years ago Kerman and Sevgen[3] derived an expression for $\sigma_{cc'}^{\mathrm{fl}}$ using the optical background representation of Kawai, Kerman, and McVoy (KKM) (Ref. 4) which is also adequate in the intermediate coupling regime. Their formula, however, contains precisely the parameter $\bar{\Gamma}/\bar{D}$, which is not manifestly directly related to the optical transmission matrix. This feature renders the Kerman-Sevgen cross section model dependent. A question naturally arises as to whether there is any way of eliminating or at least reducing this model dependence.

It is the purpose of this paper to supply a constraining relationship involving $\bar{\Gamma}/\bar{D}$ on one hand, and other parameters that appear in $\sigma_{cc'}^{\mathrm{fl}}$ on the other. This relation appears more generally in the form of an inequality,

$$\mathrm{Re\,Tr}(\mathbf{Y}\bar{\mathbf{S}}^{-1}) \leq \pi\frac{\bar{\Gamma}}{\bar{D}} \leq 2\,\mathrm{Re\,Tr}(\mathbf{Y}\bar{\mathbf{S}}^{-1}) , \tag{1}$$

where $\mathbf{Y}$ is a matrix (in channel space) that appears in $\sigma_{cc'}^{\mathrm{fl}}$ (see below) and $\bar{S}$ is the optical S matrix. The lower limit is attained under conditions of weak absorption and neutral channels, whereas the upper limit corresponds to strong absorption with strong Coulomb repulsion in the channels contained implicitly in the trace and/or slow energy variation of the elements of the Y matrix.

The starting point in our discussion is the optical background representation of the S-matrix element $S_{cc'}$ (Refs. 3 and 4),

$$S_{cc'} = \bar{S}_{cc'} - i\sum_{\mu}\frac{g_{\mu c}g_{\mu c'}}{E-\varepsilon_{\mu}} , \tag{2}$$

where the complex resonance energies, ε_{μ}, are given by

$$\varepsilon_{\mu} = E_{\mu} - i\Gamma_{\mu}/2 \tag{3}$$

and the form factors $g_{\mu c}$ are constructed in such a way as to guarantee that the energy average of the sum-over-poles term in Eq. (2), the fluctuating S matrix, S^{fl}, is identically zero.[2,3]

Using the above property of the g,s together with general analytic unitarity arguments, Kerman and Sevgen were able to derive the following expression for $\sigma_{cc'}^{\mathrm{fl}} \equiv \langle |S_{cc'}^{\mathrm{fl}}|^2 \rangle$,

$$\sigma_{cc'}^{\mathrm{fl}} = X_{cc}X_{c'c'} + X_{cc'}X_{c'c} + \left[\frac{1}{\pi\dfrac{\bar{\Gamma}}{\bar{D}}} - 2\right]|Y_{cc'}|^2 , \tag{4}$$

where X and Y matrices are defined by

$$X_{cc'} = \left[\frac{2\pi}{\bar{\Gamma}\bar{D}}\right]^{1/2}\langle g_{\mu c}^{*}g_{\mu c'}\rangle_{\mu} , \tag{5}$$

$$Y_{cc'} = \frac{\pi}{\bar{D}}\langle g_{\mu c}g_{\mu c'}\rangle_{\mu} . \tag{6}$$

The optical transmission matrix, P, defined by

$$\mathbf{P} = \mathbf{1} - \bar{\mathbf{S}}^{\dagger}\bar{\mathbf{S}} , \tag{7}$$

turns out to be[3]

$$\mathbf{P} = \mathbf{X}\,\mathrm{Tr}\mathbf{X} + \mathbf{X}^2 - 2\left[1 - \frac{1}{\pi\dfrac{\bar{\Gamma}}{\bar{D}}}\right]\mathbf{Y}\mathbf{Y}^{\dagger} . \tag{8}$$

In the limit of large $\pi(\bar{\Gamma}/\bar{D})$ (strong absorption), the g's acquire rapidly oscillating phases, which renders the Y matrix small, thus enabling the neglect of the last term in (4) and (8), and accordingly only X is required for the obtainment of both P and $\sigma_{cc'}^{\mathrm{fl}}$. One can thus eliminate X, by iteration, to obtain an expression for $\sigma_{cc'}^{\mathrm{fl}}$ in terms of the elements cc' and P and its trace. The result is a series expansion of $\sigma_{cc'}^{\mathrm{fl}}$ in powers of $(\mathrm{Tr}P)^{-1}$. In the intermedi-

ate absorption case, $\pi(\bar{\Gamma}/\bar{D})\simeq 1$, which is considered here, both X and Y are important, requiring, at least, a more constraining relation besides the one supplied by unitarity. In the following, we present general arguments in favor of Eq. (1).

The expression for Γ_μ, Eq. (3), is easily obtained following KKM (Ref. 4):

$$\Gamma_\mu = -2\,\mathrm{Im}\{\langle\tilde{\mu}\,|\,V_{QP}\mathcal{G}_P^{(+)}V_{PQ}\,|\,\mu\rangle\}\ , \tag{9}$$

where $\langle\tilde{\mu}\,|\,\mu\rangle=1$, and V_{QP} and V_{PQ} are coupling matrices amply discussed in Refs. 3 and 4. The Green's function $\mathcal{G}_P^{(+)}$ operates in the open-channel subspace, P [$Q=(1-P)$ and represents the compound nucleus subspace]. Now using a spectral representation for $\mathcal{G}_P^{(+)}$, namely

$$\mathcal{G}_P^{(+)}=\sum_c\int dE'\frac{|\chi_c^{(+)}\rangle\langle\tilde{\chi}_c^{(+)}|}{E-E'+i\epsilon}\ , \tag{10a}$$

$$=\sum_{cc'}\int dE'\frac{|\chi_c^{(+)}\rangle\bar{S}^{-1}_{cc'}\langle\tilde{\chi}_{c'}^{(-)}|}{E-E'+i\epsilon}\ , \tag{10b}$$

where relation (10b) is obtained from the condition $\bar{S}_{cc'}=\langle\tilde{\chi}_c^{(-)}\,|\,\chi_c^{(+)}\rangle$ and the completeness relation satisfied by the biorthogonal states $|\chi_c^{(\pm)}\rangle$ and $\langle\tilde{\chi}_c^{(\pm)}|$, namely

$$\sum_c\int dE\,|\chi_c^{(\pm)}(E)\rangle\langle\tilde{\chi}_c^{(\pm)}(E)|=1\ .$$

We obtain for Γ_μ

$$\Gamma_\mu=-\frac{1}{\pi}\mathrm{Im}\sum_{cc'}\int dE'\frac{g_{\mu c}(E')\bar{S}^{-1}_{cc'}(E')g_{\mu c'}(E')}{E-E'+i\epsilon}\ . \tag{11}$$

In Eq. (11), we have used the KKM definition of $g_{\mu c}$,

$$g_{\mu c}=\sqrt{2\pi}\langle\tilde{\mu}\,|\,V_{QP}\,|\,\chi_c^{(+)}\rangle=\sqrt{2\pi}\langle\tilde{\chi}_c^{(-)}\,|\,V_{PQ}\,|\,\mu\rangle\ . \tag{12}$$

We obtain $\bar{\Gamma}$ by averaging Eq. (12) over the compound states μ, getting, with the defining equation of the Y matrix, Eq. (6), the following:

$$\frac{\pi\bar{\Gamma}}{\bar{D}}=-\frac{1}{\pi}\mathrm{Im}\int dE'\frac{\mathrm{Tr}[\mathbf{Y}(E')\bar{\mathbf{S}}^{-1}(E')]}{E-E'+i\epsilon}\ . \tag{13}$$

Equation (13) can also be written as a delta-part contribution plus a principal integral, viz.,

$$\frac{\pi\bar{\Gamma}}{\bar{D}}=\mathrm{Re}\,\mathrm{Tr}(\mathbf{Y}\bar{\mathbf{S}}^{-1})-\frac{1}{\pi}\mathrm{Im}\left\{\mathrm{P}\int dE'\frac{\mathrm{Tr}[\mathbf{Y}(E')\bar{\mathbf{S}}^{-1}(E')]}{E-E'}\right\}. \tag{14}$$

To proceed further, we have to have a picture of the energy variation of $\mathrm{Tr}(Y\bar{S}^{-1})$. Let us now consider one element of the trace and take $\bar{S}^{-1}$ to be diagonal. At this point, I remind the reader that all the formulae that have been discussed so far refer to one particular partial wave, l. I take

$$\bar{S}_l(E')=(1+\exp\{[-l+\bar{l}(E')]/\Delta(E')\})^{-1}\exp(i\delta_l(E'))\ .$$

Therefore

$$\bar{S}_l^{-1}(E')=(1+\exp\{[-l+\bar{l}(E')]/\Delta(E')\})\times\exp(-i\delta_l(E'))\ ,$$

which I write in the following form:

$$\bar{S}_l^{-1}(E')=e^{-i\delta_l(E')}+e^{-i\xi_l(E')-i\delta_l(E')}\ . \tag{15}$$

I further write a similar representation for the diagonal elements of the Y matrix.

$$Y_{cc}(E')=e^{-y_l(E')+i\eta_l(E')}\ . \tag{16}$$

Expanding the δ_l and η_l to first order in the off-shell energy difference $E'-E\equiv Z$ and the $\xi_l(E')$ and $y_l(E')$ to second order, namely

$$\begin{aligned}\delta_l&\simeq\delta_l^{(0)}+\delta_l^{(1)}Z\ ,\\ \eta_l&\simeq\eta_l^{(0)}+\eta_l^{(1)}Z\ ,\\ \xi_l&\simeq\xi_l^{(0)}+\xi_l^{(1)}Z+\xi_l^{(2)}Z^2\ ,\\ y_l&\simeq y_l^{(0)}+y_l^{(1)}Z+y_l^{(2)}Z^2\ ,\end{aligned} \tag{17}$$

enables us to calculate the principal value integral for each of the diagonal terms in $\mathrm{Tr}\sqrt{S^{-1}}$ straightforwardly. We feel justified in using (17), since both Y and $\bar{S}^{-1}$ are energy-averaged (or ensemble-averaged) matrices, and therefore their elements are expected to vary smoothly with E. Since $\bar{S}_l^{-1}(E)$ is composed of two distinct terms, Eq. (15), it is convenient to write

$$Y\bar{S}^{-1}=(Y\bar{S}^{-1})_1+(Y\bar{S}^{-1})_2\ ,$$

with the first term associated with $\bar{e}^{\,i\delta}$ and the second with $\bar{e}^{\,i\delta-\xi}$.

The analysis of the contributions of the terms in $\mathrm{Tr}\,Y\bar{S}^{-1}$ arising from the nondiagonal elements of $\bar{S}^{-1}$ can be carried out along the same lines as above. It is, however, expected that these terms are smaller than the diagonal $Y_{\alpha\alpha}\bar{S}^{-1}_{\alpha\alpha}$ ones, and we accordingly drop them.

The final expression we obtain for the principal value integral of Eq. (14) is the following:

$$\mathrm{P}\int dE'\frac{\mathrm{Tr}(\mathbf{Y}(E')\bar{\mathbf{S}}^{-1}(E'))}{E-E'}=-i\pi\,\mathrm{Tr}[\mathrm{erf}(Z_1)(\mathbf{Y}\bar{\mathbf{S}}^{-1})_1+\mathrm{erf}(Z_2)(\mathbf{Y}\bar{\mathbf{S}}^{-1})_2]\ , \tag{18}$$

where

$$Z_i=\frac{\eta^{(1)}-\delta^{(1)}+2i(y^{(2)}+\xi^{(2)})t_i}{2(y^{(2)}+\xi^{(2)})^{1/2}}\ , \tag{19}$$

$$t_1=\frac{y^{(1)}}{2(y^{(2)}+\xi^{(2)})},\quad t_2=\frac{y^{(1)}+\xi^{(1)}}{2(y^{(2)}+\xi^{(2)})}\ , \tag{20}$$

and $\mathrm{erf}(Z_i)$ is the error function[5]

$$\mathrm{erf}(Z_i)=\frac{2}{\sqrt{\pi}}\int_0^{Z_i}e^{-\chi^2}d\chi\ . \tag{21}$$

Of course, all quantities appearing inside the square

brackets in Eq. (18) refer to a given partial wave and a given diagonal element of $Y\bar{S}^{-1}$.

Clearly, when both Z_1 and Z_2 become very large, $\mathrm{erf}(Z_i)$ tends to unity and the two terms in Eq. (18) combine to give for the right-hand side (rhs) $-i\pi\,\mathrm{Tr}(Y\bar{S}^{-1})$, exactly the same as the delta function contribution. Under these conditions, Eq. (14) gives $\pi(\bar{\Gamma}/\bar{D}) = 2\,\mathrm{Re\,Tr}(Y\bar{S}^{-1})$. In the opposite limit, Z_1 and $Z_2 \to 0$, we obtain $\pi(\bar{\Gamma}/\bar{D}) = \mathrm{Re\,Tr}(Y\bar{S}^{-1})$. For intermediate (and necessarily realistic) values of Z_1 and Z_2 for the different channels, $\pi(\bar{\Gamma}/\bar{D})$ could take any value within the above two extremes. Thus the inequality, Eq. (1).

The critical parameter that determines the values of the Z_i is $\delta^{(1)}/(\xi^{(2)}+y^{(2)})^{1/2}$ [Eq. (19)]. When this quantity becomes large and negative, we obtain large positive values for the Z_i's. This happens if the elastic scattering, described by $\bar{S}$, in a given channel is dominated by strong Coulomb repulsion, which renders $\delta^{(1)} = d\delta/dE$ negative, and the diagonal elements of Y have very slow variations with energy [within our parametrization, Eq. (17), such a variation is described by a very wide Gaussian] exemplified by small $y^{(2)}+\xi^{(2)}$. In general it is expected that, owing to the Coulomb barrier inhibition of charged particle decay channels with strong Coulomb repulsion (heavy fragments channels), the upper limit is more likely to be attained through the smallness of $y^{(2)}+\xi^{(2)}$. This latter condition, namely very slow energy variation of the elements of the Y matrix, is guaranteed to be met, since, by definition, all the parameters that specify the average fluctuation cross section are energy-averaged quantities.

In a recent article, Dagdeviren and Kerman[6] found, within a schematic reaction model (with no Coulomb interaction), that $\pi(\bar{\Gamma}/\bar{D})$ is close to the value $\mathrm{Re\,Tr}(Y\bar{S}^{-1})$ in a wide range of values of the transmission coefficient (up to 0.8). We feel that in more realistic situations $\pi(\bar{\Gamma}/\bar{D})$ becomes larger than $\mathrm{Re\,Tr}\,Y\bar{S}^{-1}$ (limited by the upper value $2\,\mathrm{Re\,Tr}\,Y\bar{S}^{-1}$), even at intermediate values of the transmission (~ 0.5). Thus a more careful application of Eq. (1) is necessary in order to pin down the required constraining relation among the parameters of $\sigma^{\mathrm{fl}}_{cc'}$.

In conclusion, we have discussed in this paper the compound nucleus fluctuation cross section in the intermediate absorption regime of $\bar{\Gamma}/\bar{D} \sim 1$ using the optical background representation of KKM. A constraining inequality involving $\pi(\bar{\Gamma}/\bar{D})$ on one hand and $\mathrm{Re\,Tr}\,Y\bar{S}^{-1}$ on the other hand is obtained. Since both $\pi(\bar{\Gamma}/\bar{D})$ and the elements of Y are important parameters of the cross section in the $\bar{\Gamma}/\bar{D} \sim 1$ case, the derived inequality should be useful in eliminating some of the model dependence, and in supplying a useful criterion for the realistic model calculation and subsequent comparison with the data.

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