# Coupled cluster description of pion-nucleon systems

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The (nonperturbative) coupled cluster method is applied to the pion-nucleon system. After solving the vacuum and one-nucleon problem, a reasonable description of the deuteron is obtained.

## I. INTRODUCTION

The assumption that the forces between nucleons are mediated mainly by mesons has survived all more sophisticated theories, such as quantum chromodynamics (QCD). This is why there still are many papers based on this model.[1,2] However, even within this picture approximations have to be made which cannot easily be controlled. The validity of perturbation theory certainly is questionable in view of the large coupling constant. Therefore nonperturbative methods have a certain appeal. One of the methods which has been considered is the coupled cluster method (CCM).[3,4] It has been successfully used in various many body systems. More recently it has been formulated specifically for the meson-nucleon system.[5] In this case the vacuum corresponds to the closed shell many body problem, the wave function being written in exponential form; the one, two, etc., nucleon problems correspond to the one, two, etc., valence problems. The mesonic degrees of freedom can be systematically eliminated such that one ends up with a many nucleon theory with effective interactions. In Ref. 5—thereafter quoted as I—these ideas have been presented explicitly. We refer the reader to this reference for further details. Here we present the results for the vacuum, and the calculations of the one and two nucleon masses and the deuteron binding energy. The meson mass enters on a higher level of CCM approximation such that it is justified to put the bare meson mass equal to the physical one. The vacuum is not merely a luxury: the contributions to the wave functions have a definite influence on the other quantities. The vacuum energy by itself, of course, is of no interest.

The main problem we encountered is the necessity to regularize. Unfortunately there is still no method to reconcile the (perturbative) renormalization schemes with the (nonperturbative) CCM. So we were forced to introduce a (standard) form factor into the pion-nucleon Hamiltonian.[6] We note in passing that in $1+1$ or $2+1$ dimensional field theories these difficulties either do not occur[7,8] or can be removed.[9]

## II. THE MODEL

The Hamiltonian density used in this paper is of the standard isospin invariant form with free (relativistic) mesons, free (relativistic) nucleons, and $\gamma_5$ pion-nucleon interaction with form factors:

$$\mathscr{H}=\mathscr{H}_0+\mathscr{H}_{\text{int}} , \tag{2.1}$$

$$\mathscr{H}_0=\tfrac{1}{2}[\dot{\phi}_t^\dagger(x)\dot{\phi}_t(x)+\nabla\phi_t^\dagger(x)\cdot\nabla\phi_t(x)+m^2\phi_t^\dagger(x)\phi_t(x)] +\bar{\psi}_t(x)(-i\gamma\cdot\nabla+M)\psi_t(x) , \tag{2.2}$$

$$\mathscr{H}_{\text{int}}(x)=-ig\int d^3x'F(x-x') \times\bar{\psi}_t(x)\gamma_5\tau_{t'}\psi_t(x)\phi_{t'}(x') . \tag{2.3}$$

We follow the usual summation convention; $t,t'$ label the isospin, $\tau_t$ are the isospin matrices, and $F(x-x')$ is the form factor for the meson momentum of the nucleon-pion vertex; in momentum space we use the form[6]

$$F(|q|)=\frac{\lambda^2-m^2}{\lambda^2+q^2} \tag{2.4}$$

with the cutoff parameter $\lambda$; $m/M$ is the (bare) pion-nucleon mass ratio and $g$ the (bare) coupling constant. We shall fix the parameters later.

## III. CCM FOR THE PION-NUCLEON SYSTEM

As details of the general scheme can be found in I, we note here only those facts specific to the model under discussion. The first step is the calculation of the ground state (=vacuum). It is given in the form

$$|\psi_{\text{vac}}\rangle=\exp S\,|\phi_0\rangle . \tag{3.1}$$

Here $\phi_0$=bare vacuum and

$$S=\sum_{n,m}^{\infty}S_{n,m} . \tag{3.2}$$

$n$ labels the number of nucleon-antinucleon pairs created out of the bare vacuum; $m$ labels the number of mesons; explicitly

$$S_{n,m}=\frac{1}{n!m!}\sum_{p_1\cdots p_n}\sum_{\bar{p}_1\cdots\bar{p}_n}\sum_{k_1\cdots k_m}S_{n,m}(\bar{p}_1\cdots\bar{p}_n,p_1\cdots p_n,k_1\cdots k_m) \times b^\dagger(k_1)\cdots b^\dagger(k_m)a^\dagger(p_1)\cdots a^\dagger(p_n)\bar{a}^\dagger(\bar{p}_1)\cdots\bar{a}^\dagger(\bar{p}_n) . \tag{3.3}$$

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Here the summation over spins and isospins is implied; $b^\dagger$ creates mesons, $a^\dagger$ nucleons, and $\bar{a}^\dagger$ antinucleons.

The equation for the vacuum problem is then obtained via standard procedures: write the Schrödinger equation with (3.1) in the form

$$\exp(-S)H\exp(S)\,|\,\phi_0\rangle = E_{\text{vac}}\,|\,\phi_0\rangle \tag{3.4}$$

and project onto a complete orthonormal set of Fock states: $|\,\phi_0\rangle$, $a^\dagger|\,\phi_0\rangle$, $a^\dagger\bar{a}^\dagger|\,\phi_0\rangle$, $a^\dagger\bar{a}^\dagger b^\dagger|\,\phi_0\rangle$, etc. [see also I Eq. (3.2)]. The one nucleon problem is solved with the ansatz equation

$$|\,\psi_1(p)\rangle = \exp(S)(1+F^{(1)})a^\dagger(p)\,|\,\phi_0\rangle\ , \tag{3.5}$$

[see I, Eq. (3.3)] where $F^{(1)}=\sum F^{(1)}_{n,m}$ is of a structure similar to $S_{n,m}$. It changes the nucleon with momentum $p$ and creates nucleon-antinucleon pairs and/or mesons [see I, equations after (3.3)]. Similarly, the two nucleon wave function is written as

$$|\,\psi_2(\alpha)\rangle = \int dp_1 \int dp_2 \exp(S)(1+F^{(1)}+\tfrac{1}{2}{:}F^{(1)2}{:}+F^{(2)})a^\dagger(p_1)a^\dagger(p_2)\,|\,\phi_0\rangle\tau(p_1p_2)\ , \tag{3.6}$$

[see I, Eq. (3.9)]. $F^{(2)}$ now changes two nucleons. The tricks to be used to eliminate the vacuum from both equations and the one body part from the two body equation are described in I (and in former papers on many body theory[4,10]). All these techniques are straightforward.

## IV. TRUNCATION AND TECHNICALITIES

The solution of the coupled cluster equations is a major task. Not only are the explicit equations rather lengthy (containing, however, an enormous amount of partially summed perturbative terms), they also are nonlinear and coupled to higher amplitudes. The nonlinearity turned out to be a minor nuisance: iterative procedures always worked. The technical difficulties mainly had their origin in the many dimensional integrals, which could only be overcome in part by partial wave expansions, a great deal of Clebsch-Gordan algebra, etc. So, we had to truncate on a rather low level. But we hasten to emphasize that still many different terms of arbitrary high order were included. We list here the amplitudes which were included: $S_{1,0}$, $S_{1,1}$, $F^{(1)}_{0,1}$ ($F^{(1)}_{0,2}$, $F^{(1)}_{1,0}$, $F^{(1)}_{1,1}$ in low order). For the two nucleon problem only those terms containing amplitudes from the vacuum and one particle problem were included. This includes the (most important) one Boson exchange terms, of course. On this level of approximation the meson self-energy is a higher order term. Therefore its bare mass was put equal to the physical one. For more technical details of this work see Ref. 11.

## V. NUMERICAL RESULTS

The solution of the coupled cluster equations for the vacuum could be done consistently using the form factor (2.4). In a higher order calculation it turns out that a further cutoff for the nucleon momenta is needed. However, as long as one is not interested in the vacuum energy itself (only in its wave function) it is sufficient to cut off only the meson momenta according to (2.3) and (2.4). This is what we did here. The results depend on the parameters $M$, $g$, and $\lambda$. The bare meson mass $m$ is chosen to be equal to 139 MeV. The one body problem fixes the bare mass $M$, and the deuteron problem the coupling constant $g$. Although the convergence of the CCM procedure turned out to be excellent due to the softening effects of the form factor, the $g$ dependence was still rather strong. Nevertheless, it was no problem to find a $g$ not far from the experimental $g^2=4\pi\times 14.4$. In contrast, the $\lambda$ dependence is rather weak in accordance with similar results obtained by other authors.[6] Details are shown in Table I.

TABLE I. Deuteron binding energies.

| $g$ | $\lambda$ (MeV) | bare mass $M$ (MeV) | Deuteron binding energy (MeV) |
|---|---|---|---|
| | 1000 | 726 | $-9.4$ |
| $\sqrt{4\pi\times 14.4}$ | 1300 | 664 | $-10.5$ |
| | 1500 | 651 | $-10.8$ |
| $\dfrac{\sqrt{4\pi\times 14.4}}{1.28}$ | 1000 | 771 | $-2.22$ |

One may ask how reliable these numbers are. All indications are that, indeed, the terms not included can be neglected: already in the approximation scheme we have used relatively few terms and all terms of higher order turned out to be numerically small. This is in line with the numerical experience in many body theory. Roughly speaking it is due to the fact that chains (ladders) of infinite length have been summed. Only the number of particles excited "at the same time" is very much restricted.

## VI. DISCUSSION AND SUMMARY

The pion-nucleon Hamiltonian used in this paper has a long history. It has been described in standard text books a long time ago.[11,12] Nowadays we know that actually one has to deal with a many quark system. Lacking a tractable theory one again is relying on relatively simple (bag, Skyrme, etc.) models.[13–15] Usually, it is no problem to adjust their parameters such that the properties of nucleons and the deuteron can be reproduced. One cannot expect the same accuracy with the present rather simple—and in a sense old fashioned—model. But it was not the intention of this paper to obtain an *accurate* description of the one- and two-nucleon physics. Rather we wanted to show that the CCM is both a feasible as well as a reasonable approach to rather complex systems. In view of this the fact that we for instance have recovered the "experimental coupling constant" has to be put into the proper perspective: indeed in a theory with a cutoff it is not at all clear whether or not this is already the renormalized (experimental) coupling constant. The cutoff function certainly simulates a large set of high order renormalized diagrams. It is hard to say which diagrams are included in a given approximation. The experimental value referred to above traditionally is based on a low order (renormalized) pion-nucleon vertex calculation and would have to be modified in higher orders. Therefore one cannot expect more than qualitative agreement. We note in passing that actually a double cutoff function is needed, which connects the pion field $\phi$ separately with $\psi$ and $\bar{\psi}$.[16] We could ignore this in our approximation for the simple reason that in the one- and two-nucleon problems there are one or two more nucleons than antinucleons and in our approximation the number of antinucleons at the same time is so restricted that they do not produce infinities. This was not so for the vacuum state: It either becomes infinite if one uses the form (2.3) with only one cutoff function (as we did) or it depends strongly on the cutoff parameter. But this has no influence on the masses of one- or two-nucleon systems representing energy differences to the vacuum energy. Lorentz invariance anyway requires subtracting the vacuum energy from the Hamiltonian. These problems are inherent in all the papers based on the same model[6,17,18] although they typically have not been mentioned. We note in passing that Ref. 19 not only uses the same model, it also is related to our technique: the folded diagram expansion is known to be the perturbative solution of the CCM equations for open shell systems.[10]

We feel that the main objection against this approach is the enormous amount of numerics. This did not show up explicitly in the present short paper, see, however, Ref. 19. It is the inclusion of so many high order terms which makes the equations so complex and lengthy. But one should keep in mind that the same amount of numerics is quite common in quantum chemistry. There, however, the CCM has developed into a standard technique and standard CCM procedures and computer programs exist, which can be applied to a wide range of objects. This also could be done in nuclear physics, and the drawback of the CCM mentioned above may be removed. But one certainly should then use better models typically based on or closely related to QCD: Here again CCM could be useful, certainly in the lattice forms of whichever theory. Work in this direction is under way.

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