Stopping of heavy nuclei in relativistic heavy-ion collisions

Cheuk-Yin Wong
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831
(Received 19 August 1985)

We examine the space-time dynamics of the baryons in nucleus-nucleus collisions and estimate the energy density reached in the central rapidity region. It is found that with an incident laboratory energy of 15 GeV per projectile nucleon, a headon collision of Au on Au will leave a substantial spatial region with an energy density of 1.5—3 GeV/fm^3 which may exceed the energy density for a quark-gluon plasma formation.

Recently, there has been considerable interest in ultrarelativistic heavy-ion collisions and the related possibility of producing a quark-gluon plasma with these reactions.[1] The type of quark-gluon plasma which may be formed depends on the collision energies. At the energies of interest in the range of a few to a few hundred GeV per nucleon in the c.m. system, a baryon-baryon collision will lead to the production of a large number of particles in the central rapidity region. As a result, the leading baryon suffers a large loss of energy. In terms of the rapidity variable, the incident baryon loses about $\frac{1}{3}$ to 1 unit of rapidity in each baryon-baryon collision.[2–6] Thus, for collisions in the high-energy end of this energy range, the central rapidity region has only a little net baryon content. The initial energy density of the quark-gluon plasma, which may be formed in this region, was estimated previously.[7] On the other hand, for collisions at the low-energy end of this energy range, the baryon rapidity losses lead to a baryon-rich matter in the central rapidity region.[8–13] This is the "stopping" regime for which the energy density has been estimated previously from hydrodynamical considerations and nucleon stopping. Our previous work on baryon density[2,3] by following the distribution of the baryons is applicable only to the high-energy end of the energy range. We wish to examine the baryon distribution and the energy density of the baryon-rich matter in the stopping regime by following the space-time dynamics of the baryons.

In the multiple collision model, a nucleus-nucleus collision can be decomposed as a collection of tubes of projectile nucleons colliding with tubes of target nucleons.[14,15] To get a general idea of the type of energy density attained in the collision of two heavy nuclei, we study the collision of a tube of n projectile nucleons with another tube of n target nucleons, with the cross section of the tube taken to be the inelastic nucleon-nucleon cross section[4] of 29.4 mb. We call this an $n \times n$ collision. To obtain the spatial distribution, we need to follow the dynamics of the nucleons in the center-of-mass system as they make successive collisions with each other. The behavior of the transverse degree of freedom is relatively simple. The baryons will acquire a transverse momentum of about 0.35 MeV/c which does not grow much with the collision number.[16] We need to follow only the longitudinal momentum represented by the rapidity variable y_i and the spatial longitudinal coordinate z_i, assuming straight-line trajectories. The dynamics of the baryons is described by $(y_i(t),z_i(t))$ for $i=1,\ldots,2n$, as a function of time. We follow their coordinates by a Monte Carlo method. Initially, the two colliding tubes are taken to be just touching each other. The initial positions of the nucleons $z_i(0)$ are obtained from a random sampling with a uniform nuclear density. The initial rapidities $y_i(0)$ of the baryons are specified by the incident energy. When the world lines of two baryons with momenta y_1 and y_2 cross each other, there is a collision, and the new momenta of the baryons y'_1 and y'_2 are determined by a stopping law. We choose to represent the stopping law with a probability distribution for the light-cone variable x of the outgoing baryon in the form characterized by a stopping power index α:[4–6]

$$P(x)=\frac{\alpha}{1-x_L}\left[\frac{x-x_L}{1-x_L}\right]^{\alpha-1}\theta(1-x)\theta(x-x_L)\ , \qquad (1)$$

where x_L is the lower limit of the light-cone variable.[4] In terms of the rapidity variables, the stopping law can be given as a probability distribution for the rapidity variable y'_1:

$$W(y'_1;y_1,y_2)=\frac{e^{y'_1}}{e^{y_1}-e^{y_2}}\alpha\left[\frac{e^{y'_1}-e^{y_2}}{e^{y_1}-e^{y_2}}\right]^{\alpha-1}$$
$$\times\theta(y_1-y'_1)\theta(y'_1-y_2)\ . \qquad (2)$$

Then, given the rapidities y_1 and y_2 of the baryons before collision, the value of y'_1 of one of the baryons after collision can be obtained from the above by a random number generator. When y'_1 is known, there is a distribution for the value of y'_2 which is likely to peak at the rapidity such that the total longitudinal momentum of the produced particles is zero in the center-of-mass system. Accordingly, we assume that y'_2 and y'_1 are correlated according to

$$y'_2=y_1+y_2-y'_1\ . \qquad (3)$$

For convenience of operation, we use the convention that when there is a collision of two baryons, the baryon with a greater rapidity before collision also has a greater rapidity after collision. With the updating of the rapidities of the colliding baryons, the dynamics of the baryons can be followed until all the projectile baryons in the tube collide with all the target nucleons in the tube.

In Eq. (2), the law of energy loss is parametrized by a stopping power index α. In our previous work,[3,4] we found that the data of $pA \to pX$, the total reaction cross section, and $dN/d\eta$ data can be well explained by using a power index of $\alpha=1$. Earlier work[17] using $pA \to pX$ data yielded $\alpha=6$. Recent studies[5,6] showed that the data of $pA \to pX$ can also be fitted with $\alpha=3$. However, the value of the stopping power index α is related intimately to the multiplicity of the produced particles. Whether or not $\alpha=3$ is consistent with the pseudorapidity density remains to be seen. The stopping power index also depends on energy. In the absence of a definitive determination of the stopping power index, we shall examine both $\alpha=1$ and $\alpha=3$ cases. A leading baryon loses on the average $1/\alpha$ units of rapidity per collision. A baryon is slowed down more if $\alpha=1$ than it is if $\alpha=3$. In Ref. 6 a distinction is made between the stopping laws for the intermediate collisions and the last collision in a nucleon-nucleus collision. However, in the collision of n projectile nucleons with n target nucleons along a tube, there is only one last collision out of about $n \times n$ collisions. We shall not make a distinction between the stopping law for this last collision and that for the other collisions.

We follow the dynamics of the baryons for an ensemble of 100 randomly chosen initial spatial configurations for an incident laboratory energy of 15 GeV per projectile nucleon. As an illustration, we show the results of the 4×4 collision in Figs. 1–3. All the projectile nucleons have collided with all the target nucleons at the time $t=3.2$ fm/c. (For the case of a 5×5 collision, all the projectile nucleons have collided with all the target nucleons at the time $t=4.5$ fm/c.) Figure 1 shows the space-time diagrams for sample cases of collisions under different stop-

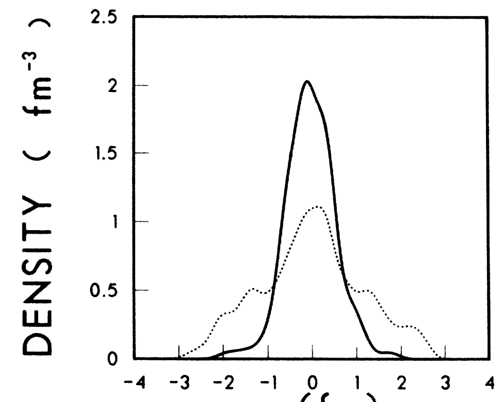

FIG. 2. Spatial density at 3.2 fm/c for the collision of four nucleons on four nucleons at a laboratory energy of 15 GeV per projectile nucleon. The solid curve represents the density for $\alpha=1$ and the dashed curve for $\alpha=3$

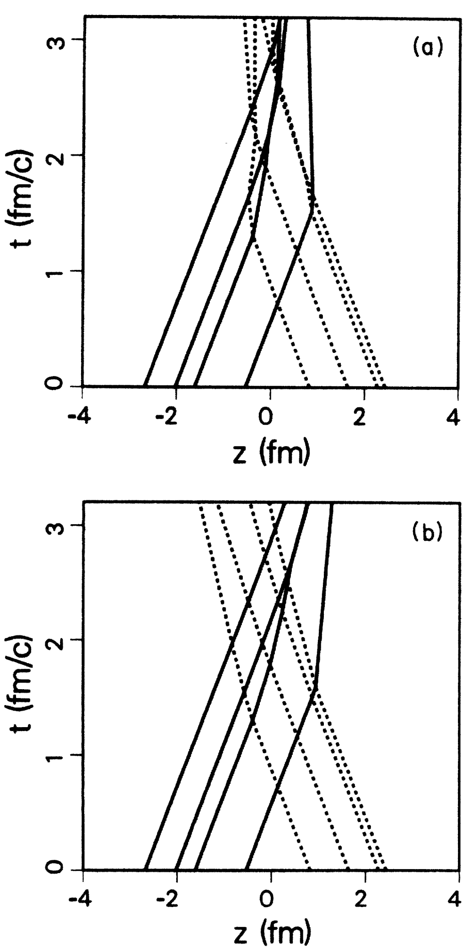

FIG. 1. Sample cases of baryon space-times dynamic for the collision of a tube of four nucleons on another tube of four nucleons at a laboratory energy of 15 GeV per projectile nucleon. The dynamics is described in the center-of-mass frame. Solid lines represents the world lines of the projectile nucleons, and the dashed lines represent the target nucleons. Part (a) is for the case $\alpha=1$ and part (b) for $\alpha=3$.

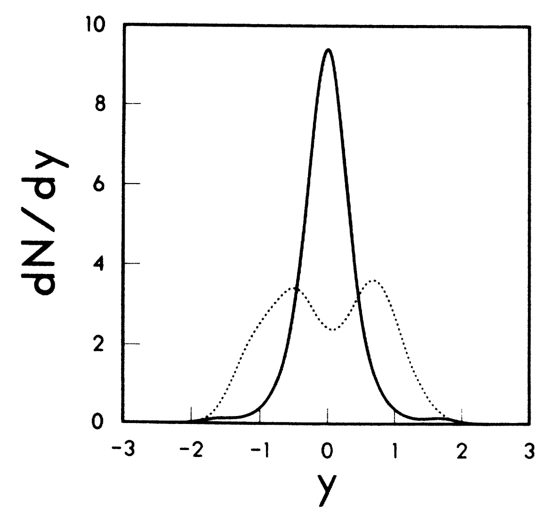

FIG. 3. Rapidity density dN/dy at 3.2 fm/c for the collision of four nucleons on four nucleons at a laboratory energy of 15 GeV per projectile nucleon. The solid curve represents the rapidity density for $\alpha=1$ and the dashed curve for $\alpha=3$.

ping laws. In Fig. 1(a), where the stopping power index is $\alpha=1$, one finds that the slopes of the world lines of the particles at $t=3.2$ fm/c are very large, indicating that these baryons have small longitudinal momenta. One can roughly describe the baryons as being "stopped" in the center-of-mass system. In Fig. 1(b), for the case of $\alpha=3$, the baryons, after collision, still have substantial speeds in the z direction, and the two systems continue to separate from each other. There is apparently no complete stopping of baryons if the stopping power index is 3.

We can examine the spatial density ρ and the momentum density (rapidity distribution) dn/dy for the baryons at the time $t=3.2$ fm/c. To present the data in a continuous curve instead of in histograms, we obtain the spatial density of a baryon by smearing the point distribution by a Gaussian distribution with a standard deviation of 0.5 fm. We obtain the rapidity density dn/dy by a Gaussian distribution with a standard deviation of 0.2 units of rapidity. The results for the spatial density are shown in Fig. 2. As one observes, for $\alpha=1$ the central density reaches about two baryons/fm^3, which is very high indeed. The full width at half-maximum of the density distribution is about 1.5 fm. For $\alpha=3$, the maximum density attained is still quite large, and the width of the density distribution is about 3 fm. Another view of the dynamics can be obtained from the rapidity distribution dn/dy in Fig. 3. For $\alpha=1$, the final rapidity density peaks at $y=0$ with a full-width at half maximum of about 1 unit of rapidity. The rapidities of the baryon at $t=3.2$ fm/c is small, and there appears to be a complete stopping of baryons. On the other hand, for the case of $\alpha=3$, the rapidity distribution has two peaks and the distribution is much broader than that for $\alpha=1$. There is no complete stopping of the baryons in such a case.

For a collision at a laboratory energy of 15 GeV per projectile nucleon, the energy per nucleon in the nucleon-nucleon center-of-mass system is 2.73 GeV. Thus, the initial baryon energy for the collision of n projectile nucleons with n target nucleons is $E_i=2n\times2.73$ GeV. From the rapidity distribution, we can obtain the final baryon energy E_f after collision by assuming a transverse momentum of $p_T=0.35$ GeV/c:

$$E_f=\int \frac{dn}{dy} m_T \cosh y \, dy \;, \tag{4}$$

where

$$m_T=(m_N^2+p_T^2)^{1/2} \tag{5}$$

and m_N is the nucleon rest mass. With the Monte Carlo program, we can record the locations of all the collisions for each event. These are the longitudinal coordinates with respect to which the produced energy (which manifests itself as produced particles) will emerge about 1 fm/c after a collision takes place.[18] The average separation Δz between the leftmost and the rightmost collisions then defines the length of the sources of the produced energy, each of which will be distributed over a longitudinal length. At the time of 1 fm/c after a collision the produced particles begin to emerge and the flow of energy from the collision point reaches at most a longitudinal distance of 1 fm. To allow approximately for the additional distribution in space, we add a length of 1 fm to both ends of Δz as the length of the tube over which the produced energy will distribute itself. Accordingly, the energy density of the produced "matter" ϵ_{pr} can be estimated as

$$\epsilon_{pr}=\frac{(E_i-E_f)}{\sigma_{in}(\Delta z+2\text{ fm})} \;. \tag{6}$$

Besides the produced matter, there are also baryons in the central rapidity region which contribute to the total energy density. The number of baryons N_b in the interval of $\Delta z+2$ fm in the central rapidity region can be obtained by integrating the baryon density. We can get a conservative estimate of the total energy density ϵ_{tot} by including the rest masses of these baryons.

We show in Table I the relevant quantities and the energy densities for collisions at a laboratory energy of 15 GeV per projectile nucleon after all the projectile nucleons have collided with all the target nucleons. In the 4×4 collision, if $\alpha=1$, the final baryon energy at $t=3.2$ fm/c is 8.7 GeV and the total energy density ϵ_{tot} is 2.2 GeV/fm^3. For $\alpha=3$, the total energy density ϵ_{tot} is 1.6 GeV/fm^3. The stopping is more effective when the number of nucleons in the tube is greater, and the total energy

TABLE I. We list here the relevant quantities in collisions at a laboratory energy of 15 GeV per projectile nucleon. The cases considered involve the collision of four projectile nucleons on four target nucleons and five projectile nucleons on five target nucleons. The quantity α is the stopping power index [Eq. (1)], E_f is the final baryon energy after collision, Δz is the average separation between the leftmost and the rightmost locations of baryon-baryon collisions, N_b is the number of baryons in the interval of $\Delta z+2$ fm, ϵ_{pr} is the energy density of the produced matter in the interval $\Delta z+2$ fm, and ϵ_{tot} is the total energy density including the baryon rest masses.

Case	α	E_f (GeV)	Δz (fm)	N_b (baryons)	ϵ_{pr} (GeV/fm^3)	ϵ_{tot} (GeV/fm^3)
4×4	1	8.7	1.1	8.0	1.4	2.2
4×4	3	10.7	1.8	6.7	0.99	1.6
5×5	1	10.2	1.3	9.8	1.74	2.69
5×5	3	12.4	2.4	8.6	1.2	1.8

density of the baryon-rich matter is also higher (Table I). For the collision of five projectile nucleons on five target nucleons, the total energy density at $t=4.5$ fm/c is 2.7 GeV/fm^3 for $\alpha=1$ and 1.8 GeV/fm^3 for $\alpha=3$.

It should be pointed out that there are additional effects which need to be included in future, more refined calculations. As the baryons pile up together, the equation of state will lead to a repulsive mean field which will lead to an outward explosion of baryons. The number of baryons in the central rapidity region decreases but the total energy density may get a compensation from the baryon energy which resides in the equation of state. About 1 fm/c after the occurrence of the first nucleon-nucleon collision, particles such as pions will materialize and participate in secondary collision with nucleons which are in the central rapidity region. These collisions will slow down the baryons and increase the energy density. Furthermore, if a quark-gluon plasma is formed during the course of the collision, the dynamics will follow a course quite different from the description of a collection of baryons. The present calculation without those effects gives an estimate of the energy density. We intend to examine in the future how these effects may modify the energy density estimated here.

The matter in the central rapidity region is baryon-rich and any quark-gluon plasma evolved therefrom is a baryon-rich plasma. The critical energy density for a transition from hadronic matter to this type of quark-gluon plasma has been found to be rather insensitive to the net baryon content.[19] For a pure quark-gluon plasma, the critical energy density has been estimated[20] to be about 1–2 GeV/fm^3. The results of Table I show that for either $\alpha=1$ or $\alpha=3$ the collision of a tube of four or more nucleons on four or more other nucleons may lead to an energy density in the range of 1.5 GeV to 3 GeV/fm^3. This energy density may exceed the energy density for a transition to a baryon-rich quark-gluon plasma. In the head-on collision of Au on Au, an area up to an impact parameter of 5 fm has tube-tube collisions with more than four nucleons in each tube. Therefore, within this area of π (5 fm)^2, the total energy density is at least in the range of 1.5–3 GeV/fm^3 and may be energetically capable of forming a quark-gluon plasma. The results here are in rough agreement with those of Gylassy[9] who obtained, for a bombarding energy of 15 GeV per projectile nucleon, an energy density in the range of 0.9–2.2 GeV/fm^3, depending on the method of estimation and other attributes of the equation of state.

In conclusion, we have examined the baryon energy loss in nucleus-nucleus collisions and estimate that for a head-on collision of Au on Au at a laboratory energy of 15 GeV per projectile nucleon the total energy density in a substantial spatial region is in the range of 1.5–3 GeV/fm^3 and may allow the formation of a baryon-rich quark-gluon plasma.

The author wishes to thank Prof. C. M. Ko, Dr. A. Shor, and Dr. U. Heinz for helpful discussions. This research was sponsored by the Division of Nuclear Physics, U. S. Department of Energy, under Contract No. DE-AC05-840R21400 with Martin Marietta Energy Systems, Inc.

[1]See, e.g., *Quark Matter '84,* edited by K. Kajantie (Springer-Verlag, Berlin, 1985); Quark Matter '83, edited by T. Ludlam and H. E. Wagner, Nucl. Phys. A418, 1 (1984); and Proceedings of the 7th High Energy Heavy-Ion Study, Darmstadt, 1984, Gesellschaft für Schwerionenforschung Report 85-10, 1985.
[2]C. Y. Wong, Phys. Rev. Lett. **52**, 1393 (1984).
[3]C. Y. Wong, Phys. Rev. D **30**, 972 (1984).
[4]C. Y. Wong, Phys. Rev. D **32**, 94 (1985).
[5]J. Hüfner and A. Klar, Phys. Lett. **145B**, 167 (1984).
[6]S. Date, M. Gyulassy, and H. Sumiyoshi, Phys. Rev. D **32**, 619 (1985).
[7]C. Y. Wong, Phys. Rev. D **30**, 961 (1984).
[8]M. Gyulassy, Nucl. Phys. **A418**, 59c (1984).
[9]M. Gyulassy, Nucl. Phys. **A400**, 31c (1983).
[10]J. Kapusta, Phys. Rev. C **27**, 2037 (1983).
[11]W. Busza and A. S. Goldhaber, Phys. Lett. **139B**, 235 (1984); W. Busza, Nucl. Phys. **A418**, 635C (1984).
[12]T. S. Biro and J. Zimanyi, Nucl. Phys. **A395**, 525 (1983).
[13]H. Stöcker *et al.*, Phys. Lett. **95B**, 192 (1980).
[14]R. J. Glauber and G. Matthiae, Nucl. Phys. **B21** 135 (1970).
[15]J. Hüfner and J. Knoll, Nucl. Phys. **A290**, 460 (1977).
[16]Sa Ban-Hao and C. Y. Wong, Phys. Rev. D **32**, 1706 (1985).
[17]K. Kinoshita, A. Minaka, and N. Sumiyhoshi, Prog. Theor. Phys. **61**, 165 (1979).
[18]J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).
[19]U. Heinz (private communication).
[20]M. Jacobs, Nucl. Phys. **A418**, 7c (1984).