Knockout and knockout-fusion contributions to the (p,p′)-type singles cross sections

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The (p,p′)-type singles cross sections are calculated. The knockout of the target nucleons, reabsorption of the knocked-out particle, and particle-hole excitations into bound orbits are taken into account. The sum of cross sections for these three excitation modes is shown to describe nicely the observed continuum cross section data for the ^{54}Fe(p,p′) and ^{27}Al(p,p′) reactions at 62 MeV.

I. INTRODUCTION

Previously, we have analyzed continuum cross sections of the (p,p′)-type [and also (p,n)-type and (p,α)-type] reactions at a few tens of MeV. The work was based on the concept of the multistep direct reaction (MSDR),[1,2] and the results were shown to fit experiment[3,4] rather nicely.

In applying the MSDR method to the (p,p′) processes, we have used consistently the so-called collective form factor, as explained in some detail in Ref. 2. Behind the use of such a form factor lies a simplifying assumption, namely that the (p,p′) process always takes place by exciting particle-hole (ph) states, and that the target nucleons thus excited are promoted to metastable orbits. Such an assumption becomes questionable, however, when the excitation energy gets high, because then the target nucleons are promoted most of the time into unbound orbits.

The process in which the target nucleons are promoted into unbound orbits may more naturally be treated as a knockout (KO) reaction, and we intend to pursue this possibility in the present paper. It may be worth noting here that the quasifree KO mechanism is seen very clearly in the exclusive (p,2p) and (p,np) reactions at a few hundred MeV, as shown very recently by a Maryland group[5] among others. We are interested here, however, in the inclusive (p,p′) cross sections, rather than the exclusive cross sections. It is then highly conceivable that the reabsorption of the knocked-out nucleon back into the target nucleus, a process which may be called the knockout-fusion (KF) reaction, contributes significantly to the (p,p′) singles cross sections. In the present paper, we show that this is indeed the case.

Recently, we have developed a method[6] to calculate the cross sections of the so-called breakup-fusion (BF) reactions. In this reaction, the breakup of the projectile takes place first, and is then followed by fusion of one of the broken-up pair with the target. As can be naturally expected, it is easy to extend this method to evaluate the cross sections of the KF reaction, and this is what will be done in Sec. II below.

In Sec. III A we analyze the experimental data of the ^{54}Fe(p,p′) and ^{27}Al(p,p′) reactions at 62 (Ref. 3) and 65 MeV.[4] The theoretical cross sections to be compared with data are the sum of the KO, KF, and the inelastic (IE) scattering contributions. Here we mean specifically by IE the process in which inelastic excitation of the ph states takes place, i.e., the excitation mode considered earlier.[1,2] However, the particle orbits in these ph states are now restricted to the bound (but initially vacant) orbits in the target so as to avoid double counting with the KO process. Therefore the IE cross section in the present paper constitutes a (small) part of the MSDR cross section of Refs. 1 and 2. As seen in Sec. III A, the theoretical cross section thus obtained fit the data[3,4] rather well.

In Sec. III B we discuss the significance of the fit thus achieved, particularly in comparison with what we have done earlier.[1,2] We may note that, in spite of a general success in fitting the data, our MSDR analyses encountered[1] an important problem. Namely, while the experimental cross sections increased as the scattering angle decreased, the calculated cross sections flattened off below 30°. Later on, this problem was solved by considering pickup processes that form metastable dinucleon systems, which decay subsequently.[2] In the present work, this forward rise of the cross sections is accounted for by the KO + KF processes.

II. FORMULATION OF THE CALCULATIONS

The DWBA formalism for the simple KO reactions is well known.[7,8] We shall therefore concentrate on the formulation of the KF reaction. This reaction may symbolically be written as

$$A+\mathrm{p}\rightarrow B+y+\mathrm{p}'\rightarrow A^{*}+\mathrm{p}' , \qquad (1)$$

where the first step describes the simple knockout reaction, in which a nucleon y is emitted by the target A, with the residual nucleus B being left in a hole state. The p′ stands for the scattered proton. The second step describes the fusion (or reabsorption) of y and B, forming the residual nucleus A^{*}.

The process given by Eq. (1) is very similar to the breakup-fusion (BF) reaction, for which the cross section formula was already derived.[6] In the BF reaction, breakup takes place first, and is followed by fusion of one of the broken-up pair (we called it the x particle) with the target A. The channel between x and A was called the x channel, and the BF cross section was given as a constant times an expectation value of the imaginary part of the optical potential W_x of the x channel with respect to the wave function, also of the x channel. The latter wave function was obtained by projecting out the x-channel part from the total breakup channel wave function, calculated in the usual DWBA theory.

We may similarly express the KF cross section in terms of the expectation value of W_y with respect to the y-channel wave function, where the y channel is the system consisting of y and B. The y-channel wave function is then calculated by taking an appropriate projection of the total knockout wave function.

The KF cross section is given (in the partial wave expanded form) as

$$d^2\sigma^{\mathrm{KF}}/dE_{\mathrm{p'}}d\Omega_{\mathrm{p'}} = \sum_{j_1 l_1 l_y l m} |C^{(\mathrm{KF})}_{BAj_1l_1}|^2 d^2\sigma^{\mathrm{KF}}_{j_1l_1l_yl}/dE_{\mathrm{p'}}d\Omega_{\mathrm{p'}} , \tag{2a}$$

where $C^{(\mathrm{KF})}_{BAj_1l_1}$ is the spectroscopic amplitude of the knockout of a nucleon from the shell model orbit $(j_1 l_1)$, leaving the residual nucleus in its state B. Also, $d^2\sigma^{\mathrm{KF}}_{j_1l_1l_yl}/dE_{\mathrm{p'}}d\Omega_{\mathrm{p'}}$ is the partial wave cross section specified by a set of quantum numbers j_1, l_1, l_y, and l. Here l_y is the orbital angular momentum which y carries, while l is the total orbital angular momentum transferred, i.e., $l = l_y + l_1$. The above partial cross section can explicitly be written as,

$$d^2\sigma^{\mathrm{KF}}_{j_1l_1l_yl}/dE_{\mathrm{p'}}d\Omega_{\mathrm{p'}} = (2\pi/\hbar v_a)\rho(E_{\mathrm{p'}})(\langle u_{j_1l_1l_ylm} | W_y | u_{j_1l_1l_ylm}\rangle/\pi) . \tag{2b}$$

In Eq. (2b), $\rho(E_{\mathrm{p'}})$ is the phase space volume of the outgoing proton, while $u_{j_1l_1l_ylm}$ is the y-channel partial wave function and satisfies the following inhomogeneous differential equation:

$$\{(\hbar^2/2\mu_y)[d^2/dr_1^2 - l_y(l_y+1)/r_1^2] + E_y - U_y\} u_{j_1l_1l_ylm} = z_{j_1l_1l_ylm} , \tag{3a}$$

with

$$z_{j_1l_1l_ylm} = \sum_{m_1 m_y} \langle l_1 m_1 l_y - m_y | lm\rangle (-)^{l_y - m_y} r_1 (i^{l_y} Y_{l_y m_y}(\hat{\mathbf{r}}_1)\chi^{(-)}_{\mathrm{p'}}(\mathbf{r}_a) | V(\mathbf{r}_a - \mathbf{r}_1) | \phi_{j_1l_1m_1}(\mathbf{r}_1)\chi^{(+)}_{\mathrm{p}}(\mathbf{r}_a)\rangle . \tag{3b}$$

The symbol $(| \; | \rangle$ in Eq. (3b) denotes integration over all coordinates, excepting the y-channel radial coordinate r_1. $\chi^{(+)}_{\mathrm{p}}$ and $\chi^{(-)}_{\mathrm{p'}}$ are the incident and outgoing proton distorted wave functions, respectively, and $\phi_{j_1l_1m_1}$ is the spacial part of the single particle wave function. E_y and U_y are the kinetic energy and the optical potential in the y channel, respectively. W_y in Eq. (2b) is defined as $W_y \equiv -\mathrm{Im}U_y$. E_y is related to the incident and outgoing energies, E_{p} and $E_{\mathrm{p'}}$, respectively, and also to the binding energy B_1 of the state $(j_1 l_1)$. The relation is given by conservation of energy,

$$E_y = E_{\mathrm{p}} - E_{\mathrm{p'}} + B_1 . \tag{4a}$$

The Q value of the reaction is then given in terms of B_1 and E_y as,

$$Q = B_1 - E_y . \tag{4b}$$

The interaction V that appears in Eq. (3b) is the two-body nucleon-nucleon interaction potential that is responsible for the knockout reaction. Note that z defined by Eq. (3b) includes the projection of the knockout channel wave function onto the p′ channel. This projection is taken care of by the scalar multiplication of $(\chi^{(-)}_{\mathrm{p'}}|$.

The simple knockout partial wave cross section can be given[7,8] as,

$$d^2\sigma^{K}_{j_1l_1l_ylm}/dE_{\mathrm{p'}}d\Omega_{\mathrm{p'}} = (2\pi/\hbar v_a)\rho(E_{\mathrm{p'}})\rho(E_y)|\beta_{j_1l_1l_ylm}|^2 , \tag{5a}$$

$$\beta_{j_1l_1l_ylm} = (4\pi/k_y)\int z_{j_1l_1l_ylm}(r_1)\chi_{l_y}(r_1)dr_1 , \tag{5b}$$

where $\chi_{l_y}(r_1)$ is the usual distorted wave in the y channel. The total cross section is then given as a simple sum of Eqs. (2a) and (5a), as well as the IE cross section, as remarked in the Introduction.

III. RESULTS OF NUMERICAL CALCULATIONS AND DISCUSSIONS

A. Comparison with experiments

In this section, we present the results of the numerical calculations, and compare them with experiments. The calculations were performed for the ^{54}Fe(p,p′) and ^{27}Al(p,p′) reaction at $E_{\mathrm{p}} = 62$ MeV.

In order to compare with experimental data, we first calculated contributions coming from the IE process. We followed the same procedure as that in Ref. 2, but took only the particle states up to 8.85 and 8.27 MeV above the Fermi sea, respectively, in ^{54}Fe and ^{27}Al. The average single particle strength parameter β is chosen as 0.046, a reasonable choice as discussed in Ref. 2.

In both IE and the KO and KF calculations, the optical model parameters for the incident and exit protons were taken from Menet *et al.*[9] Those for the knocked-out nucleons were taken from Perey and Perey.[10] The effective proton-proton interaction ($M3Y$ potential) used in the KO and KF calculations was taken from Bertsch *et al.*[11] The

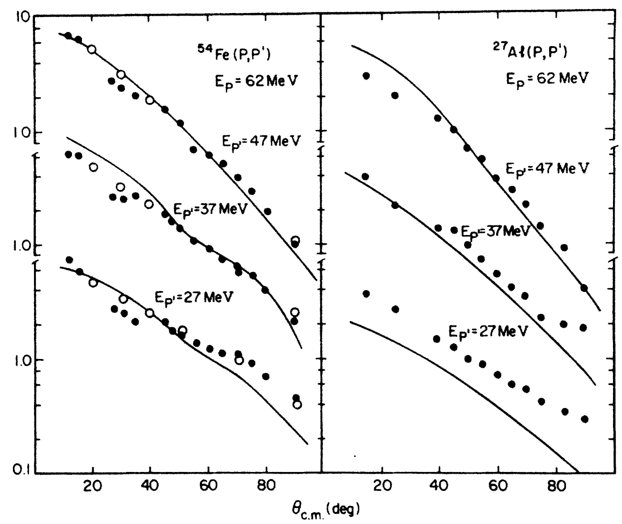

FIG. 1. Comparison of the calculated ^{54}Fe(p,p′) and ^{27}Al(p,p′) cross sections at $E_p = 62$ MeV with the corresponding experimental data. The data (with solid circles) were taken from Ref. 3. We also included the data (with open circles) from Ref. 4 for the ^{58}Ni(p,p′) reaction at $E_p = 65$ MeV. [The data from Ref. 4 were multiplied by 1.2 to normalize to ^{54}Fe(p,p′) data at 60°.]

proton-neutron interaction was simply assumed to be the same as the proton-proton interaction. We took into account the knockout of the proton and neutron from the $f_{7/2}$, $d_{3/2}$, $s_{1/2}$, $d_{5/2}$, $p_{1/2}$, and $p_{3/2}$ orbits for the ^{54}Fe(p,p′) reaction, and from the $d_{5/2}$, $p_{1/2}$, $p_{3/2}$, and $s_{1/2}$ orbits for the ^{27}Al(p,p′) reaction. We assumed that the spin orbit partners of the above orbits were degenerate, and also that the proton (neutron) binding energies of the f, d, s, and p orbits in ^{54}Fe were, respectively, 8.5 (10.5), 18.5 (20.5), 18.5 (20.5), and 28.5 (30.5) MeV. Those of the d, p, and s orbits in ^{27}Al were taken, respectively, as 8.3 (13.1), 18.3 (23.1), and 28.3 (33.1) MeV.

The calculated KO + KF + IE cross sections are presented as solid curves in Fig. 1, and are compared with experiments for the Q values of -15, -25, and -35 MeV. The individual contributions (for the case of ^{54}Fe) of the three processes are displayed in Fig. 2. It is clearly

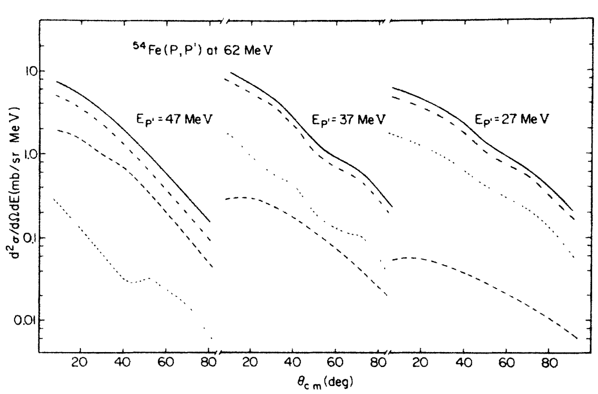

FIG. 2. Contributions from KO (dotted curves), KF (dot-dashed), and IE (dashed) processes in the ^{54}Fe(p,p′) reaction. The solid lines represent the sum of these three cross sections and are the same as the solid lines given in Fig. 1.

seen that the cross sections due to the IE process are significant only when the $|Q|$ value is low, a result which is not unexpected. The IE contributions are responsible for 30%, 3%, and 1% of the measured cross sections of ^{54}Fe(p,p′), respectively, at $Q = -15$, -25, and -35 MeV, and for 40%, 4%, and 2% of those of ^{27}Al(p,p′), respectively. Between KO and KF, the latter dominates for most of the Q values.

In presenting the theoretical KF cross sections in Figs. 1 and 2, we have introduced arbitrary normalization factors of $N = 0.5$ for the ^{54}Fe(p,p′) reaction and $N = 0.25$ for ^{27}Al(p,p′). This shows that our calculations somewhat overestimated the observed cross sections. As remarked, the singles cross sections come mostly from the KO and KF contributions (particularly for the larger $|Q|$ values), and the latter depends strongly on the magnitude of the imaginary part of the y-channel optical potential. In other words, the uncertainty and ambiguity of the optical potential is the major cause of $N \neq 1$.

B. Behaviors of the KO, KF, and IE cross sections

It is interesting to remark that the magnitude of the experimental cross sections are more or less independent of the $|Q|$ values, as seen in Fig. 1. Note that an elementary KO cross section decreases as $|Q|$ increases. However, as $|Q|$ is increased, the number of (occupied) orbits in the target that participate in the KO process increases, explaining the increase of the KO cross section as $|Q|$ is increased; see Fig. 2. When $|Q|$ is small, however, the y particle is strongly absorptive, because its energy is low. This makes KF dominate over KO at the lower $|Q|$ values. At very low $|Q|$, the IE process also gives nonnegligible contributions. When all these effects are combined, the near $|Q|$ independence of the experimental cross section is nicely accounted for, as shown in Fig. 1.

As also seen in Fig. 1, the calculated angular distributions are consistent in shape with the data, which in particular means that the present calculation explains the observed rise of the cross sections at forward angles, less than 30°. As remarked in the Introduction, our previous MSDR calculations, which used the collective form factor, failed to explain this. Note that the collective form factor has a rather short tail. In the KO (+ KF) calculations, on the other hand, the y particles are unbound, and because of this, the tail of the form factor gets much longer than it was in the collective form factor, and this is the reason why the forward cross sections get larger. [We actually cut off the KO form factor for $r > 7$ fm, and redid the KO + KF calculations. As expected, the resultant angular distributions became very much the same as they were with the collective form factor.[1,2]]

As we also remarked in the Introduction, we had previously considered[2] a nucleon pickup process to form metastable dinucleon systems and found that forward angle peaking of the cross section resulted. The KO process considered here is to be regarded as an alternate way to describe the same process. It should, nevertheless, be remarked that the KO process emphasizes the correlation of the two nucleons [p′ and y in Eq. (1)] individually with the target [or the nucleus B in Eq. (1)] over that between

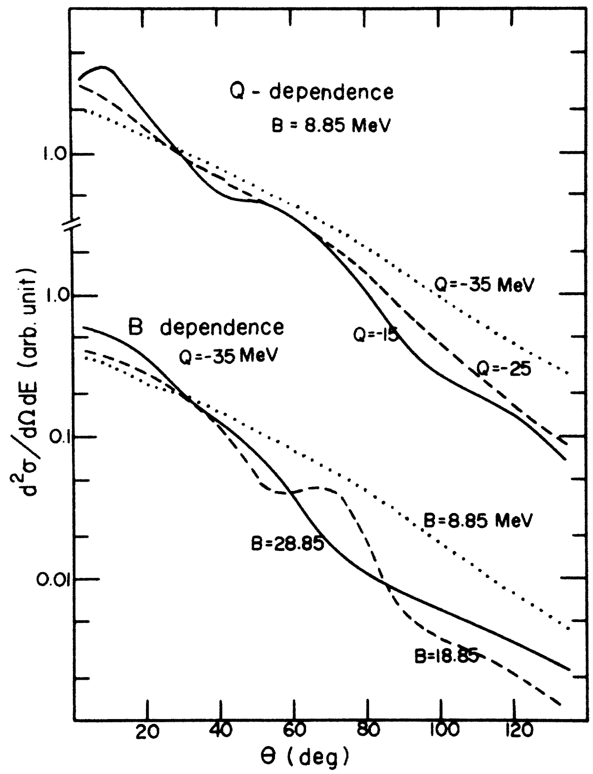

FIG. 3. The $|Q|$ value and binding energy dependence of the calculated KO + KF cross sections for the case of proton knockout in the ^{54}Fe(p,p′) reaction.

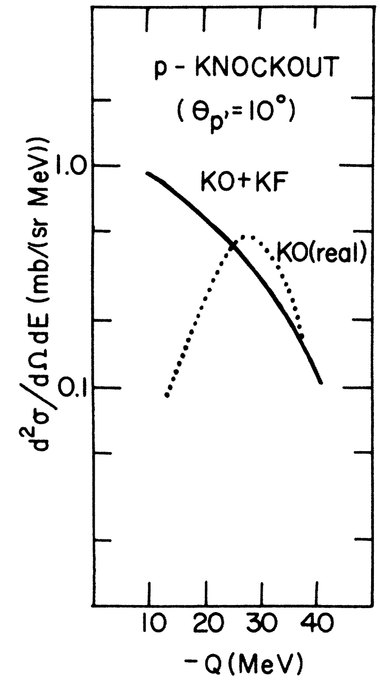

FIG. 4. The calculated KO + KF and KO(real) cross sections as functions of Q value in the ^{54}Fe(p,p′) reaction.

the two nucleons. On the other hand, the emphasis in the pickup description is reversed. To describe the matter more accurately, both correlations of course should be taken into account on an equal footing, and for this purpose the best method known to date would be the coupled-channel method of Kamimura *et al.*[12]

The observed angular distributions tend to flatten off as the $|Q|$ value increases. To see this somewhat more closely, we plotted in Fig. 3 the calculated angular distributions for the various values of Q and B. It is interesting to note that the calculated angular distributions remain forward peaked, when $|Q|$ and B are small. In such a case, the quasielastic nature of the knockout mechanism will remain intact.

C. Use of real U_y

As shown above, we have calculated the KO and KF contributions separately. In doing this, we used the optical model potential, which is complex, for the particle y, as well as for x. Since the KF calculation is much more involved than is the KO calculation, one may seek a simpler way. And in particular one may hope[13] that, if a real potential is used for y, a pure KO calculation might suffice. It is not clear, however, whether such a calculation, which we shall refer to as a KO(real) calculation, is indeed equivalent to our KO + KF calculation.

In order to see whether this equivalence holds or not, we have performed a KO(real) calculation, and the result is shown in Fig. 4. As seen, the KO(real) cross section can have a magnitude that is comparable to our KO + KF cross section. However, the two cross sections have quite different Q dependences. In particular, the KO(real) cross section is very small at small $|Q|$, and thus cannot account for the data for such Q values.

IV. CONCLUDING REMARKS

Calculations of the ^{54}Fe(p,p′) and ^{27}Al(p,p′) reactions at $E_p = 62$ MeV were performed by assuming that the target nucleon knockout (KO), knockout fusion (KF), and the inelastic (IE) ph excitation into bound orbits are the major mechanisms that are responsible. The singles cross sections were then obtained as a sum of the contributions of these processes.

The present work may be regarded as a replacement of our previous works,[1,2] in which we treated the continuum (p,p′) process as entirely due to ph mode excitation, i.e., to the IE processes in the terminology of the present paper. However, while the particles in the ph pairs in the IE process of the present paper were restricted to stay in the bound orbits, this restriction was not made in previous work. In other words, we used[1,2] in place of the actual continuum orbits a set of metastable states, the nature of which had to be assumed. Therefore, the previous treatment was of a more phenomenological nature, compared with the present work. And it is thus very pleasing to find that the present work, of a more microscopic nature, does account for the data rather nicely.

We had to use, in the present calculation, an overall normalization constant N (<1), though it did not differ very much from 1. This constitutes an unsatisfactory feature in the present work. The other conceivable source of uncertainty for the present work is the fact that there could also be contributions from processes other than those we have considered, particularly when $|Q|$ is large. For example, we have not considered the contribution from a process in which the projectile is absorbed, while the knocked-out proton is observed. Removing these possible uncertainties as much as possible needs to be done in the future.

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