Elastic magnetic scattering on ^{51}V

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Recent experiments have indicated that there is a large suppression of the $M3$ as well as the $M5$ elastic magnetic form factor in ^{51}V. We examine the modification of the form factors due to various configurational admixtures arising from the realistic effective interactions. It turns out that the polarization of the ^{48}Ca core plays an important role vis-á-vis the observed anomalous magnetic octupole scattering.

I. INTRODUCTION

Sometime ago, Arita *et al.*[1] reported precise measurements of the magnetization distribution of the ground state of ^{51}V by carrying out 180° scattering experiments with electrons. The experiments, involving momentum transfers ranging from 0.8 fm^{-1} to 2.5 fm^{-1}, indicated a strong suppression of the $M3$ and $M5$ part of the magnetic scattering. The experimental study of the magnetic form factor of ^{51}V has recently been extended by Platchkov *et al.*[2] to cover the momentum-transfer range $q \sim 3$–4 fm^{-1}.

An attempt was made by Arita *et al.*[1] to examine the observed quenching of the 2^L-pole magnetic form factors in ^{51}V in terms of the multishell picture involving particle-hole pairs up to $(L+1)\hbar\omega$ excitations. The calculations involved an unrenormalized effective interaction that was earlier employed by Moreira *et al.*[3] in the context of a similar calculation in ^{209}Bi. The quenching effects (for $q > 1$ fm^{-1}) due to various first-order excitations were found to be quite sensitive towards the exchange character of the interactions which were characterized by a Gaussian shape $V_0[\exp-(r/r_0^2)]$ with the range parameter $r_0 = 1.57$ fm and $V_0 = -40$ MeV; the form factor estimates resulting from the Rosenfeld mixture were about 40 percent larger than the ones resulting from the Serber mixture almost throughout the range $q = 1.8$–2.8 fm^{-1}.

Recently there has been a lot of activity centered around the calculation[4] of the contributions resulting from the mesonic exchange currents vis-á-vis the magnetic form factors for $q \geq 2$ fm^{-1}; the recent experimental study[2] of magnetic scattering in ^{51}V involving high momentum transfer was also in fact partly motivated by such considerations. An unambiguous quantitative estimate of the contributions of the various intrashell excitations towards the form factors for $1.0 < q < 4.0$ fm^{-1} is therefore extremely important from the point of view of extracting a reliable estimate of the effects due to mesonic exchange currents from the available data.

The purpose of this paper is to examine the observed anomalous magnetic scattering in ^{51}V in terms of the contributions of various excitation processes resulting from the realistic effective interactions operative in the $2p-1f$ shell that have been found quite successful in a large number of spectroscopic calculations. Specifically, we calculate the deviations from the single-particle estimates for the form factors by considering various first-order diagrams employing as an effective interaction the Kuo-Brown (KB) interaction[5] for the $2p-1f$ shell. The Kuo-Brown effective interaction has also earlier been employed in our calculations[6,7] of the renormalization of the static dipole moment operator in the second half of the $2p-1f$ shell, as well as the suppression of magnetic form factors in ^{57}Ni. It turns out that the present parameter-free calculation is quite successful in reproducing the observed large suppression of the M3 and M5 scattering covering the range $q = 0.8$–2.0 fm^{-1}. We also find that the polarization of the ^{48}Ca core is more efficacious in suppressing the form factors as compared to the excitation of only the protons. It is seen that, whereas the observed data shows quenching of the single-particle (s.p.) form factors for $q = 0.8$–1.9 fm^{-1}, it shows an enhancement with respect to s.p. estimates for $q > 1.9$ fm^{-1}. It is satisfying to note that the renormalized form factor obtained in the present calculation also displays this trend.

In Sec. II we discuss some details of the calculational framework. Section III discusses the comparison of the recent ^{51}V form factor data with the single particle as well as the renormalized values. Section IV summarizes the results.

II. FORMALISM

The transverse magnetic form factor is given as

$$|F_T(q)|^2 = \frac{4\pi}{Z^2}\left[\frac{1}{2J+1}\right]\sum_{\lambda} |\langle \Phi_J^{(0)} || T^{(\lambda)}(q) || \Phi_J^{(0)} \rangle|^2 , \tag{1}$$

where $T^{\lambda}(q)$ is the magnetic multipole operator defined as[8]

$$T^{(\lambda)}(q) = \int d\mathbf{r}\, j_{\lambda}(qr)\mathbf{Y}^{(\lambda\lambda l)}(\Omega)\cdot\mathbf{J}(\mathbf{r}) . \tag{2}$$

Here $j_{\lambda}(qr)$ is the spherical Bessel function and $Y_{\mu}^{(\lambda\lambda l)}$ is the vector spherical harmonic.

The first-order renormalization of the matrix element $\langle \Phi_J^{(0)} || T^{(\lambda)}(q) || \Phi_J^{(0)} \rangle$ involved in expression (1) is given by

$$\langle\langle \Phi_J^{(0)} || T^{(\lambda)}(q) || \Phi_J^{(0)} \rangle\rangle = \langle \Phi_J^{(0)} || T^{(\lambda)}(q) || \Phi_J^{(0)} \rangle + 2\sum_i \frac{\langle \Phi_J^{(0)} || T^{(\lambda)}(q) || \Phi_J^{(i)} \rangle \langle \Phi_J^{(i)} | V | \Phi_J^{(0)} \rangle}{E_0 - H_0} . \quad (3)$$

The processes involving protons and neutrons that we consider here to first order in perturbation theory are shown in Fig. 1. In Fig. 2 we present the properly antisymmetrized and normalized ground and intermediate states[9] employed here; an identical set of intermediate states have earlier been considered by Mavromatis *et al.*[10] in the context of the first-order corrections to the magnetic moments of the nuclei ^{43}Ca and ^{51}V. We obtain the following final expressions for the renormalization of the reduced matrix elements due to proton and neutron excitations (see the Appendix):

$$\begin{aligned}
&[\langle\langle \Phi^{(0)} || T^{(\lambda)}(q) || \Phi^{(0)} \rangle\rangle - \langle \Phi^{(0)} || T^{(\lambda)}(q) || \Phi^{(0)} \rangle]_{\rm p} \\
&\quad = 48 \sum_{[j',I]} \frac{1}{\Delta E} \langle (\tfrac{7}{2})^2 I, \tfrac{7}{2} \,|\}\, (\tfrac{7}{2})^3 \tfrac{7}{2} \rangle W(I \tfrac{7}{2} j' \lambda; \tfrac{7}{2} \tfrac{7}{2} \langle \tfrac{7}{2} || T^{(\lambda)}(q) || j \rangle \\
&\qquad \times \sum_i [(2i+1)(2I+1)]^{1/2} \langle (\tfrac{7}{2})^2 i \tfrac{7}{2} \,|\}\, (\tfrac{7}{2})^3 \tfrac{7}{2} \rangle W(\tfrac{7}{2} \tfrac{7}{2} \tfrac{7}{2} j'; Ii) \\
&\qquad \times [\langle \tfrac{7}{2} j' | V | \tfrac{7}{2} \tfrac{7}{2} \rangle_i^{\rm pp} - (-1)^{7/2+j'-i} \langle j' \tfrac{7}{2} | V | \tfrac{7}{2} \tfrac{7}{2} \rangle_i^{\rm pp}] , \qquad (4)
\end{aligned}$$

where $[j',I] \equiv [\frac{1}{2},4]$, $[\frac{3}{2},(2,4)]$, $[\frac{5}{2},(2,4,6)]$, and

$$\begin{aligned}
&[\langle\langle \Phi^{(0)} || T^{(\lambda)}(q) || \Phi^{(0)} \rangle\rangle - \langle \Phi^{(0)} || T^{(\lambda)}(q) || \Phi^{(0)} \rangle]_{\rm n} \\
&\quad = 2 \sum_{j'} \frac{1}{\Delta E} \langle \tfrac{7}{2} || T^{(\lambda)}(q) || j \rangle \sum_{J_3} (2J_3+1) W(j' \tfrac{7}{2} \tfrac{7}{2} \tfrac{7}{2}; \lambda J_3) \langle j' \tfrac{7}{2} | V | \tfrac{7}{2} \tfrac{7}{2} \rangle_{J_3}^{\rm pn} , \qquad (5)
\end{aligned}$$

where the explicit expressions for the single-particle reduced matrix elements of the operator $T_\mu^{(\lambda)}(q)$ appearing in Eqs. (4) and (5) are given in Ref. 8.

The single-particle energies (in MeV), relative to the $f_{7/2}$ level, appearing in expressions (4) and (5) have been taken as $\epsilon^{\pi}_{p_{3/2}}$ $(\epsilon^{\nu}_{p_{3/2}}) = 4.40$ (2.17), $\epsilon^{\pi}_{f_{5/2}}$ $(\epsilon^{\nu}_{f_{5/2}}) = 5.90$ (6.12), and $\epsilon^{\pi}_{p_{1/2}}$ $(\epsilon^{\nu}_{p_{1/2}}) = 6.90$ (4.19). These single-particle energies have been taken from the observed spectra[5] of the nuclei ^{49}Sc and ^{49}Ca. The energy of the $f_{7/2}$ orbit relative to the ground state in ^{49}Ca is obtained by carrying out the spherical Hartree-Fock (HF) calculation for the nucleus ^{48}Ca employing the Kuo-Brown effective interaction. For the evaluation of the single-particle matrix elements we use the harmonic oscillator constant $b = 1.94$ fm. The center-of-mass correction and the proton finite size effect are taken into account by multiplying the reduced matrix elements appearing in Eq. (1) by the factors $\exp(-b^2q^2/4A)$ and $(1 + q^2/7.5)^{-1}$, respectively.[7,11]

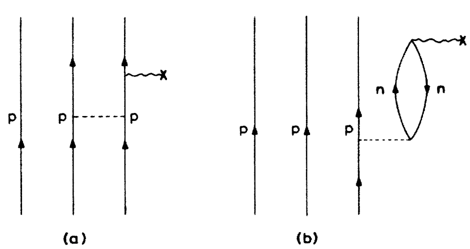

FIG. 1. First-order graphs involving (a) proton excitations and (b) neutron excitations contributing to the renormalization of the single-particle magnetic form factors in ^{51}V.

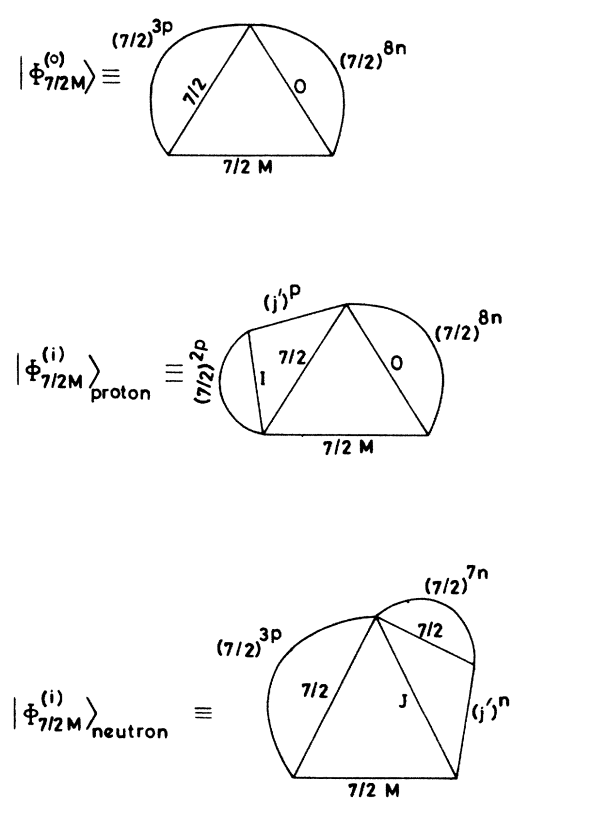

FIG. 2. Angular-momentum coupling schemes for the ground state as well as the intermediate states involving proton and neutron excitations.

III. RESULTS AND DISCUSSION

In Fig. 3 we compare the recent data[1] on the ^{51}V form factors with the single-particle as well as the renormalized estimates. The experimental form factors reported by de Vries[1,12] as well as by de Witt Huberts *et al.*,[13] for $q<1$ fm^{-1} and $q>1.8$ fm^{-1}, respectively, have also been presented. The calculated form factor is in excellent agreement with the observed one for $0.9<q<2.4$ fm^{-1}. The single-particle prediction for the form factors reproduces the gross qualitative features—a large $M1$ diffraction maximum and a plateau covering $q=1.0$–1.7 fm^{-1} followed by a rather abrupt drop. Quantitatively, however, the observed form factors show large quenching for $q=1.3\pm0.3$ fm^{-1}; the suppression factors are as large as 2. It is seen that the present parameter-free calculation employing the KB interaction reproduces the observed dramatic suppression of the form factors around the first maxima of $M3$ as well as $M5$ form factors.

An important feature of the data for the high momentum-transfer part ($q\geq1.5$ fm^{-1}) is the following. It is seen that although the experimental values of the form factors are lower than the single-particle estimates for $q<1.8$ fm^{-1}, the latter are distinctly smaller than the observed values for $q>2.0$ fm^{-1}. The results obtained in the present calculation are quite consistent with this observed trend; the renormalized form factor curve goes through a turning point vis-à-vis its magnitude relative to the s.p. estimates at $q=1.9$ fm^{-1}.

Considering next the low momentum-transfer part of the form factors, it is found that the inclusion of the configuration mixing worsens the agreement between the single-particle values and the experiments. The present calculation suggests significant quenching for the value of the form factors around its first maximum. Further, the static $M1$ moment of the ground state in ^{51}V—and this is just the limit of

$$\left[-i\sqrt{6\pi}(2M/q^{-1})\langle\langle f_{\frac{7}{2}}||T^{(1)}(q)||f_{\frac{7}{2}}\rangle\rangle\right]_p$$

as q tends to zero—resulting from the present calculation is 4.32 μ_N which is significantly smaller than the Schmidt value $\mu_{\text{Schmidt}}=5.79\ \mu_N$. It is seen that although the measured value[14] of the $M1$ static moment ($\mu_{\text{expt}}=5.15\ \mu_N$) is qualitatively consistent with the results obtained here, the experimental values for the form factors at $q\sim0.6$ fm^{-1} are slightly higher than the single-particle estimates. In view of the possible uncertainties due to the large contribution of the charge scattering[15] at low momentum transfer, the apparent inconsistency between the measured static moment and the low-q form factors seems to warrant a careful redetermination of the latter.

In Fig. 4 we have given the contribution to the renormalized form factors for various multipoles arising separately from the proton as well as neutron excitations. We find that the renormalization of the form factors due to the excitations from the ^{48}Ca core is considerably more for various multipoles compared to that due to the proton excitations. This is primarily due to the fact that the overall strength of the proton-neutron interaction is larger—by roughly a factor of 1.5—compared to that of the interaction in the proton-proton channel.[16]

As pointed out earlier, the observed data gives unambiguous indications concerning dequenching of the form factors for $2.0<q<2.5$ fm^{-1}. The results presented in Fig. 4 reveal that the observed dequenching owes its

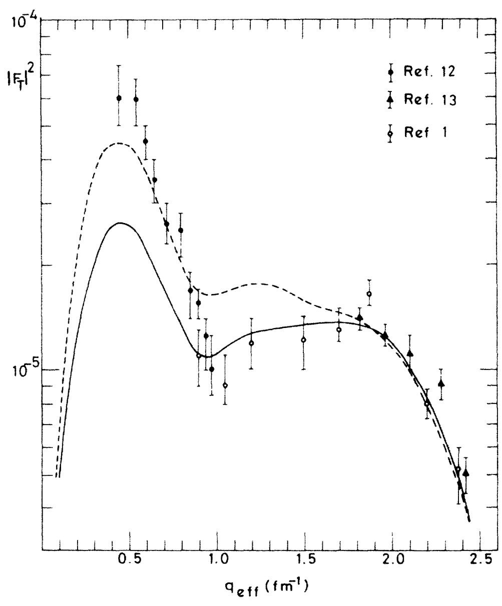

FIG. 3. Experimental as well as theoretical magnetic form factors for ^{51}V. The dashed and the solid curves represent the single particle and the renormalized squared form factors, respectively.

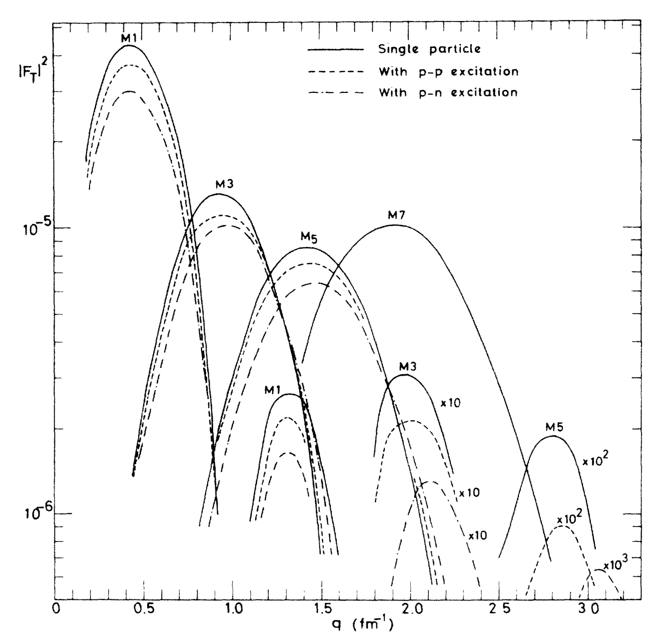

FIG. 4. Renormalization of the form factors for various multipoles due to proton and neutron excitations separately.

origin to the enhancement of the single-particle estimates of $M5$ form factors arising from both proton and neutron excitations. However, for the momentum-transfer range $2.5 < q < 3.2$ fm^{-1} our results indicate appreciable quenching of the $M5$ form factor due to the intrashell $(0\hbar\omega)$ particle-hole excitations. An assessment of the mesonic exchange contributions—and these are known to enhance the form factors—usually ignores the role of the intrashell excitations. The results obtained here suggest that the role of mesonic exchange currents—as well as other subnucleonic degrees of freedom—may be of greater quantitative significance than heretofore expected vis-à-vis the large observed dequenching for $q > 3.0$ fm^{-1}, as reported very recently by Platchkov *et al.*[2]

IV. SUMMARY

Summarizing, a reasonably successful interpretation of the observed large suppression of $M3$ and $M5$ scattering in ^{51}V can be provided by invoking various first-order perturbative processes in conjunction with the realistic interactions. The results for the large momentum transfers ($q \gtrsim 2$ fm^{-1}) are expected to provide the basis for an unambiguous discussion of the effects due to mesonic degree of freedom. In view of the existing inconsistency between the available static moments and the low momentum-transfer form factors, our results further underline the necessity of a careful reexamination of the latter.

APPENDIX: DERIVATION OF THE MATRIX ELEMENTS OF TWO- AND ONE-BODY OPERATORS IN THE CASE OF NEUTRON EXCITATIONS

To facilitate the evaluation of the matrix elements of V between the ground state and the intermediate states involving the excitation of neutrons, we first rewrite the ground state as follows:

$$\begin{aligned}
|\Phi^{(0)}_{\frac{7}{2},M}\rangle &= |[(\tfrac{7}{2})^{3\mathrm{p}}_{7/2}\times(\tfrac{7}{2})^{8\mathrm{n}}_{0}]\tfrac{7}{2}M\rangle \\
&= \sum_{J_1} \langle(\tfrac{7}{2})^2J_1,\tfrac{7}{2}\,|\}(\tfrac{7}{2})^3\tfrac{7}{2}\rangle\langle(\tfrac{7}{2})^7\tfrac{7}{2},\tfrac{7}{2}\,|\}(\tfrac{7}{2})^8 0\rangle\,|[\{(\tfrac{7}{2})^{2\mathrm{p}}J_1\times(\tfrac{7}{2})^{\mathrm{p}}\}\tfrac{7}{2}\times\{(\tfrac{7}{2})^{7\mathrm{n}}\tfrac{7}{2}\times(\tfrac{7}{2})^{\mathrm{n}}\}0]\tfrac{7}{2}M\rangle \\
&= -\sqrt{8}\sum_{J_1J_2J_3}[(2J_2+1)(2J_3+1)]^{1/2}\langle(\tfrac{7}{2})^2J_1,\tfrac{7}{2}\,|\}(\tfrac{7}{2})^3\tfrac{7}{2}\rangle \\
&\qquad\times\langle(\tfrac{7}{2})^7\tfrac{7}{2},\tfrac{7}{2}\,|\}(\tfrac{7}{2})^8 0\rangle W(\tfrac{7}{2}\tfrac{7}{2}\tfrac{7}{2}\tfrac{7}{2};J_2 0)W(J_1\tfrac{7}{2}J_2\tfrac{7}{2};\tfrac{7}{2}J_3) \\
&\qquad\times|[\{(\tfrac{7}{2})^{2\mathrm{p}}J_1\times(\tfrac{7}{2}^{\mathrm{p}}\tfrac{7}{2}^{\mathrm{n}})J_3\}J_2\times(\tfrac{7}{2})^{7\mathrm{n}}\tfrac{7}{2}]\tfrac{7}{2}M\rangle\ .
\end{aligned} \tag{A1}$$

The intermediate state can be written as

$$\begin{aligned}
|\Phi^{(i)}_{7/2,M}\rangle &= |[(\tfrac{7}{2})^{3\mathrm{p}}\tfrac{7}{2}\times\{(\tfrac{7}{2})^{7\mathrm{n}}\tfrac{7}{2}j'^{\mathrm{n}}\}K]\tfrac{7}{2}M\rangle \\
&= \sqrt{8}\sum_{K'LL'}[(2K+1)(2L+1)(2L'+1)]^{1/2}\langle(\tfrac{7}{2})^2K',\tfrac{7}{2}\,|\}(\tfrac{7}{2})^3\tfrac{7}{2}\rangle W(\tfrac{7}{2}j'\tfrac{7}{2}\tfrac{7}{2};LK)W(K'\tfrac{7}{2}Lj';\tfrac{7}{2}L') \\
&\qquad\times|[\{(\tfrac{7}{2})^{2\mathrm{p}}K'\times(\tfrac{7}{2}^{\mathrm{p}}j'^{\mathrm{n}})L'\}L\times(\tfrac{7}{2})^{7\mathrm{n}}\tfrac{7}{2}]\tfrac{7}{2}M\rangle\ .
\end{aligned} \tag{A2}$$

Using Eqs. (A1) and (A2) the matrix element of $\langle\Phi^{(i)}|V|\Phi^{(0)}\rangle$ is easily obtained as

$$(-1)^K[(2K+1)/8]^{1/2}\sum_{J_3}(2J+1)W(j'\tfrac{7}{2}\tfrac{7}{2}\tfrac{7}{2};KJ_3)\langle j'\tfrac{7}{2}|V|\tfrac{7}{2}\tfrac{7}{2}\rangle^{\mathrm{pn}}_{J_3}\ . \tag{A3}$$

Furthermore,

$$\langle\Phi^{(0)}||T^{(\lambda)}(q)||\Phi^{(i)}\rangle = 8(-1)^K\langle(\tfrac{7}{2})^{8\mathrm{n}}0||T^{(\lambda)}(q)||[(\tfrac{7}{2})^{7\mathrm{n}}\tfrac{7}{2}\times j'^{\mathrm{n}}]K\rangle W(0\tfrac{7}{2}\lambda\tfrac{7}{2};\tfrac{7}{2}K)\delta_{K\lambda}\,, \tag{A4}$$

where we have

$$\langle(\tfrac{7}{2})^{8\mathrm{n}}0||T^{(\lambda)}(q)||[(\tfrac{7}{2})^{7\mathrm{n}}\tfrac{7}{2}\times j'^{\mathrm{n}}]K\rangle = 8\sqrt{2K+1}\,W(\tfrac{7}{2}j'0\lambda;K\tfrac{7}{2})\langle(\tfrac{7}{2})^7\tfrac{7}{2},\tfrac{7}{2}\,|\}(\tfrac{7}{2})^8 0\rangle\langle\tfrac{7}{2}||T^{(\lambda)}(q)||j'\rangle\ . \tag{A5}$$

Finally we get

$$\langle\Phi^{(0)}||T^{(\lambda)}(q)||\Phi^{(i)}\rangle = (-1)^{\lambda}[8/(2\lambda+1)]^{1/2}\langle\tfrac{7}{2}||T^{(\lambda)}(q)||j'\rangle\ . \tag{A6}$$

The definition of the reduced matrix elements employed here is the same as in Ref. 17.

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