Siegert's theorem and nuclear electrodisintegration

W. R. Dodge and E. Hayward
National Bureau of Standards, Gaithersburg, Maryland 20899
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The connection between the electron scattering electric dipole coincidence cross section (e,e'X) and the inclusive electric dipole (e,X) cross section, differential in the angle of the outgoing X particle, is derived. Unlike the (e,e') inclusive cross section which contains contributions from only two of the four terms of the (e,e'X) cross section, the (e,X) cross section contains contributions from all four terms of the (e,e'X) cross section. Data from a previous experiment have been used to obtain the magnitude and sign of the interference term between the transverse and Coulomb reduced matrix elements (form factors) in the limit as $q \rightarrow \omega$, from the relationship commonly referred to as Siegert's theorem in the context of inclusive (e,e') scattering.

I. INTRODUCTION

The purpose of this paper is to point out that inclusive electroexcitation experiments of the type (e,X) involving excitation of an isolated, discrete $E1$ resonance are sensitive to the interference between the transverse and Coulomb form factors and to show that the excitation function for the 1^-, 16.28 MeV isobaric analog state[4] in ^{90}Zr requires the contribution from the interference term to fit the data.

In Sec. II we derive the $d^2\sigma(\text{e},X)/d\Omega_{x'}dE_f$ from the (e,e'X) coincidence cross section, $d^3\sigma(\text{e},X)/d\Omega_{ee'}d\Omega_x dE_f$, and specialize this result to a discrete, narrow level. Here $d\Omega_x$ is the X particle solid angle with respect to the momentum transfer and $d\Omega_{x'}$ is with respect to the incident electron direction. In Sec. III we relate the $d\sigma(\text{e},X)/d\Omega_{x'}$ to $E1$ virtual photon spectra differential in $d\Omega_{x'}$ and illustrate the relative magnitudes of the four terms for the proton decay of the 16.28 MeV, 1^- isobaric analog state in ^{90}Zr. Corrections for Coulomb distortions, the momentum dependence of the reduced matrix elements, and the interference between these effects are discussed.

II. THEORY

The electron scattering coincidence cross section has been the subject of several papers[1-3] and is of increasing interest because continuous wave (CW) electron accelerators with 100% duty cycles now in operation allow a broad class of these experiments to be done. The coincidence cross section describes the angular correlation between the momentum transfer, $\mathbf{q}$, of a scattered electron and some outgoing nuclear fragment, X. As written by Drechsel and Überall,[2] the (e,e'X) cross section in the plane wave Born approximation (PWBA), differential in the final electron energy, E_f, the solid angle, $\Omega_{ee'}$, of the scattered electron, and the solid angle, Ω_x, of the decay product is

$$d^3\sigma/d\Omega_x d\Omega_{ee'} dE_f = (\alpha^2/q_\mu^4)(P_f/P_0)P_x E_x[V_C(\theta_{ee'})W_C + V_T(\theta_{ee'})W_T + V_I(\theta_{ee'},\Phi_x)W_I + V_S(\theta_{ee'},\Phi_x)W_S] , \quad (1)$$

where the kinematic factors

$$V_C(\theta_{ee'}) = (q_\mu^2/q^2)^2(E_0E_f + P_0P_f\cos\theta_{ee'} + m_e^2) , \quad (2a)$$

$$V_T(\theta_{ee'}) = \kappa^2/q^2 + \tfrac{1}{2}q_\mu^2 , \quad (2b)$$

$$V_I(\theta_{ee'},\Phi_x) = (\kappa q_\mu^2/q^3)E_+\cos\Phi_x , \quad (2c)$$

and

$$V_S(\theta_{ee'},\Phi_x) = 2(\kappa^2/q^2)\cos^2\Phi_x + \tfrac{1}{2}q_\mu^2 \quad (2d)$$

contain the dependence on the angle between the incident and scattered electron $\theta_{ee'}$ and on Φ_x, the angle between the electron scattering plane defined by the momentum vectors of the incident and scattered electron, $\mathbf{P}_0$ and $\mathbf{P}_f$, and the plane defined by the momentum transfer vector, $\mathbf{q} = \mathbf{P}_0 - \mathbf{P}_f$, and the momentum, $\mathbf{P}_x$, of the disintegration product (see Fig. 1). Here $\kappa = |\mathbf{P}_0 \times \mathbf{P}_f|$, $E_+ = E_0 + E_f$, and $q_\mu^2 = q^2 - \omega^2$. The four generalized form factors are model dependent dynamic quantities, being functions of q, E_x, P_x, and θ_x, the angle between $\mathbf{q}$ and $\mathbf{P}_x$, and depend on the nature of X, the reaction mechanism, and the nuclear model used to describe the latter. Drechsel and Überall[2] evaluated the functions W using a Breit-Wigner resonance for the nuclear levels. They also assumed the level matrix was diagonal. If a multipole expansion is performed, the generalized form factors depend on the angular momentum, L, of the virtual photon absorbed, the spin and parity of the initial, excited, and final nuclear states, and the angular momentum, l, of the emitted particle, X. They are expressed in terms of the decay parameters, S, which, in turn, depend on the reduced matrix elements describing the transition involved. According to Kleppinger and Walecka,[3] the (e,e'X) cross section of Drechsel and Überall[2] corresponds to the static limit resonance approximation of their generalized (e,e'X) cross section, with their

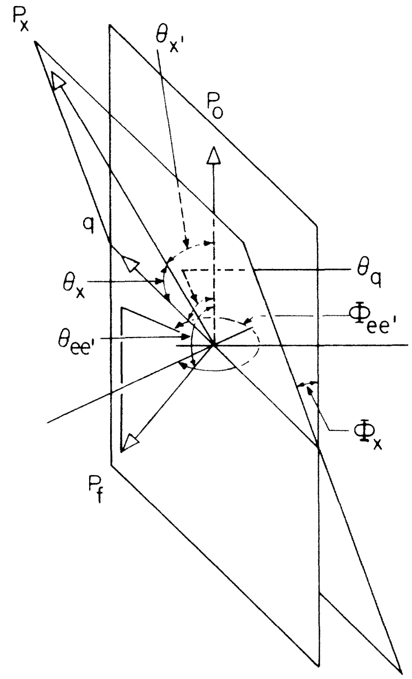

FIG. 1. Definition of the angles used in the (e,e′X) and (e,X) cross sections described in the text.

$$V_{TT}(\theta_{ee'})=[V_S(\theta_{ee'},\Phi_x)-V_I(\theta_{ee'})]/\cos2\Phi_x \ .$$

When the coincidence cross section is integrated over $d\Omega_x$, terms involving the regular and associated Legrendre polynomials, $P_L(\cos\theta_x)$ and $P_L^m(\cos\theta_x)$ with $L>0$, vanish and the cross section only contains squares of the transverse and Coulomb form factors multiplied by their respective kinematic functions $V_C(\theta_{ee'})$ and $V_T(\theta_{ee'})$. The result is the familiar Mott cross section multiplied by 4π as expected and no information about the generalized form factors W_I and W_S can be obtained from single arm (e,e′) experiments.

If, on the other hand, the coincidence cross section is integrated over $d\Omega_{ee'}$, the electrodisintegration cross section for the production of a particle X results in the following:

$$\frac{d^2\sigma_{e,x}}{d\Omega_{x'}dE_f}=\frac{d^2\sigma_C}{d\Omega_{x'}dE_f}+\frac{d^2\sigma_T}{d\Omega_{x'}dE_f}+\frac{d^2\sigma_I}{d\Omega_{x'}dE_f}+\frac{d^2\sigma_S}{d\Omega_{x'}dE_f} \quad (3)$$

which, unlike the (e,e′) cross section, contains all four terms of the coincidence cross section. The contribution of $d^2\sigma_I/d\Omega_{x'}dE_f$ depends on the relative phase of the Coulomb and transverse reduced matrix elements. This phase, in turn, depends on the definition of the Coulomb and transverse operators.

Dodge *et al.*[4] have recently reported on work which represents an experimental verification of Eq. (3). We measured the isochromat of the $E1$ virtual photon spectrum by counting the number of ground-state protons emitted at 90° by the well-known 16.28 MeV, 1^- isobaric analog state in ^{90}Zr as a function of the incident electron energy. This level has a proton width of about 77 keV, a ground state radiation width of about 100 eV, and decays by proton emission to the ground state ($\sim$63%) and second excited state ($\sim$37%) of ^{89}Y. The electroexcitation of this state followed by proton decay to the ground state of ^{89}Y is described by Eq. (45) of Drechsel and Überall.[2] For an isolated, 1^-, Breit-Wigner resonance in a nucleus of ground state spin $J_i=0$,

$$\frac{d^3\sigma}{d\Omega_x d\Omega_{ee'}dE_f}=\frac{\alpha^2}{\pi^2}\frac{P_f}{P_0}\frac{1}{q_\mu^4}\sqrt{3}P_xE_xB(\omega,\omega_0)\times\Bigg\{(\mathcal{M}_1)^2V_C(\theta_{ee'})[S^0P_0(\cos\theta_x)-\sqrt{2}S^2P_2(\cos\theta_x)]+(\mathcal{T}_1^{(e)})^2V_T(\theta_{ee'})\left[S^0P_0(\cos\theta_x)+\frac{S^2}{\sqrt{2}}P_2(\cos\theta_x)\right]$$
$$-\mathcal{T}_1^{(e)}\mathcal{M}_1V_I(\theta_{ee'},\Phi_x)S^2P_2^1(\cos\theta_x)-(\mathcal{T}_1^{(e)})^2[V_S(\theta_{ee'},\Phi_x)-V_I(\theta_{ee'})]\frac{S^2}{2\sqrt{2}}P_2^2(\cos\theta_x)\Bigg\} \ . \quad (4)$$

In this equation the subscript x refers to the emitted proton, S^0 and S^2 are the decay parameters for the channel under consideration, in this case the proton channel, $B(\omega,\omega_0)$ is the Breit-Wigner resonance function, $1/[(\omega-\omega_0)^2+\Gamma^2/4]$, Γ is the total width of the level, and ω_0 is the resonance energy. The reduced matrix elements of the $E1$ transverse and Coulomb operators are $\mathcal{T}_1^{(e)}(q)$ and $\mathcal{M}_1(q)$, respectively. In Eq. (4) V_S-V_I appears in the fourth term; henceforth, we will represent this combination by $V_{TT}(\theta_{ee'})\cos2\Phi$ and the interference term by CT instead of I to be consistent with modern notation.[3]

Integration of Eq. (4) over $d\Omega_{ee'}$ is facilitated by the relationship

$$\int_0^{2\pi}P_L^m(\cos\theta_x)\cos m\Phi_x d\varphi_{ee'}=2\pi P_L^m(\cos\theta_q)P_L(\cos\theta_{x'}) \ . \quad (5)$$

The appearance of the term $P_L^m(\cos\theta_q)$ with $m=0,1,2$ results from the vector nature of the exchange virtual photon of spin 1 and angular momentum projections of 0 for the Coulomb, and ±1 for the transverse excitation terms. After integration over $\varphi_{ee'}$, the electrodisintegration cross section for the emission of a particle X at an angle $\theta_{x'}$ with respect to $\mathbf{P}_0$ is given by

$$\frac{d^2\sigma}{d\Omega_x dE_f} = \frac{2\alpha^2}{\pi}\frac{P_f}{P_0}\sqrt{3}P_x E_x B(\omega,\omega_0)S^0 + \int_0^\pi \frac{1}{q_\mu^4}\Bigg\{ |\mathcal{M}_1(q)|^2 V_C(\theta_{ee'})\left[1-\sqrt{2}\frac{S^2}{S^0}P_2(\cos\theta_q)P_2(\cos\theta_{x'})\right] + |\mathcal{T}_1^{(e)}(q)|^2 V_T(\theta_{ee'})\left[1+(\sqrt{2})^{-1}\frac{S^2}{S^0}P_2(\cos\theta_q)P_2(\cos\theta_{x'})\right] - \mathcal{T}_1^{(e)}(q)\mathcal{M}_1(q)(\kappa q_\mu^2 E_+/q^3)\frac{S^2}{S^0}P_2^1(\cos\theta_q)P_2(\cos\theta_{x'}) - |\mathcal{T}_1^{(e)}(q)|^2(\kappa^2/q^2)(2\sqrt{2})^{-1}\frac{S^2}{S^0}P_2^2(\cos\theta_q)P_2(\cos\theta_{x'})\Bigg\} d(\cos\theta_{ee'}) , \quad (6)$$

where

$$\cos\theta_q = (E_+\omega + q^2)/(2P_0 q) .$$

The photodisintegration ground and excited state cross sections have typical dipole angular distributions,[5]

$$\frac{d\sigma_{\gamma,x}}{d\Omega_{x'}} = \frac{\Gamma}{2\pi}B(\omega,\omega_0)a_0\left[1+\frac{a_2}{a_0}P_2(\cos\theta_{x'})\right] . \quad (7)$$

Comparison of the term multiplied by the kinematic function $V_T(\theta_{ee'})$ in Eq. (6) at $\theta_q=0$ with Eq. (7) shows that $S^2/S^0=\sqrt{2}a_2/a_0$. To integrate Eq. (6) over

$$d(\cos\theta_{ee'}) = -dq^2/(2P_0P_f) ,$$

the Coulomb and transverse $E1$ reduced matrix elements, $\mathcal{T}_1^{(e)}(q)$ and $\mathcal{M}_1(q)$, may be parametrized by model dependent expressions which reproduce experimentally measured transition radii, R_t. Alternatively, in the long wavelength limit (LWL) $\mathcal{T}_1^{(e)}(q)$ is replaced by the first-order term in the expansion of the spherical Bessel function, $j_1(qr)\simeq qR_t/3$; hence,

$$\mathcal{T}_1^{(e)}(q) = \frac{qR_t}{3} . \quad (8)$$

Siegert's theorem

$$F_T^1(q) = -\sqrt{L+1/L}\,\frac{\omega}{q}F_C^1(q) \quad (9)$$

is used to evaluate $\mathcal{M}_1(q)$:

$$\mathcal{M}_1(q) = F_C^1(q) = -\frac{q}{\omega}\frac{1}{\sqrt{2}}F_T^1(q) = -\frac{1}{\sqrt{2}}\frac{q^2R_t}{3\omega} . \quad (10)$$

There is some confusion[6] in the literature concerning the relative sign of $F_T^1(q)$ and $F_C^1(q)$. Siegert's theorem as expressed in Eq. (9) can be obtained by standard reduction techniques from the definitions of the reduced matrix elements given in Eqs. (22a) and (22b) of Dreschel and Überall.[2] In the inclusive (e,e′) cross section this phase is of no consequence; on the other hand, it is important to the magnitude inclusive (e,X) cross section as well as the $(e,e'X)$ cross section.

Rose[7–9] defines the transverse current operator with a sign opposite to that of Drechsel and Überall,[2] as well as most other authors. For this reason, Rose[7–9] writes Siegert's theorem with a positive sign. However, when the reduced transverse matrix elements defined by Rose[7–9] are inserted in the $(e,e'X)$ cross section defined by Rose,[7–9] the results are identical with the $(e,e'X)$ cross sections of other authors. Explicitly, in the LWL the four terms in $d^2\sigma/d\Omega_{x'}dE_f = -d^2\sigma/d\Omega_{x'}d\omega$ are

$$\frac{d^2\sigma_T}{d\Omega_{x'}d\omega} = D\left[N_T^{E1}(E_0,\omega) + \left\{\left[1+\frac{3m_e^2}{2P_0^2}\left[\frac{E_+^2}{E_0^2+E_f^2}\right]\right]N_T^{E1}(E_0,\omega) - \frac{3}{2}\frac{m_e^2}{P_0^2}\left[\frac{\omega^2}{E_0^2+E_f^2}\right]N_C^{E1}(E_0,\omega) - \frac{3\alpha}{2\pi}\frac{P_f}{P_0^3}\left[E_0E_f + m_e^2\left[1-\frac{4E_0E_f}{E_0^2+E_f^2}\right]\right]\right\}\frac{a_2}{a_0}P_2(\cos\theta_{x'})\right] , \quad (11a)$$

$$\frac{d^2\sigma_C}{d\Omega_{x'}d\omega} = D\left[N_C^{E1}(E_0,\omega) + \left\{\left[1-\frac{3}{4}\frac{\omega(E_0+3E_f)}{P_0^2}\right]N_C^{E1}(E_0,\omega) - \frac{3\alpha}{2\pi}\frac{P_f}{P_0}\left[\frac{3E_0E_f+m_e^2}{P_0^2}\right]\right\}\frac{a_2}{a_0}P_2(\cos\theta_{x'})\right] , \quad (11b)$$

$$\frac{d^2\sigma_{CT}}{d\Omega_{x'}d\omega} = 3D\left\{\left[\frac{\omega E_f + 2m_e^2}{P_0^2}\right]N_C^{E1}(E_0,\omega) + 2\frac{\alpha}{\pi}\left[\frac{P_f}{P_0}\left[\frac{E_0E_f+m_e^2}{P_0^2}\right] - \frac{m_e^2}{P_0^2}\frac{E_0E_+}{P_0^2}\ln\lambda\right]\right\}\frac{a_2}{a_0}P_2(\cos\theta_{x'}) , \quad (11c)$$

$$\frac{d^2\sigma_{TT}}{d\Omega_{x'}d\omega}=\tfrac{3}{4}D\left\{\left[\frac{\omega^2-4m_e^2}{P_0^2}\right]N_C^{E1}(E_0,\omega)-2\frac{\alpha}{\pi}\left[\frac{P_f}{P_0}\left[\frac{E_0E_f+3m_e^2}{P_0^2}\right]-4\frac{m_e^2}{P_0^2}\frac{E_0E_f}{P_0^2}\ln\lambda\right]\right\}\frac{a_2}{a_0}P_2(\cos\theta_{x'})\ , \tag{11d}$$

where

$$N_T^{E1}(E_0,\omega)=\frac{\alpha}{\pi}\left[\frac{E_0^2+E_f^2}{P_0^2}\ln\lambda-\frac{E_+^2}{2P_0^2}\ln\xi-\frac{P_f}{P_0}\right]\ , \tag{11e}$$

$$N_C^{E1}(E_0,\omega)=\frac{\alpha}{\pi}\left[\frac{E_+^2}{2P_0^2}\ln\xi-\frac{P_f}{P_0}\right]\ , \tag{11f}$$

and

$$D=\sqrt{3}\alpha P_xE_x(R_t/3)^2B(\omega,\omega_0)S^0\ , \tag{11g}$$

$$\lambda=(E_0E_f+P_0P_f-m_e^2)/m_e\omega\ , \tag{11h}$$

$$\xi=(P_0+P_f)/(P_0-P_f)\ , \tag{11i}$$

and

$$\frac{S^2}{S^0}=\sqrt{2}\frac{a_2}{a_0}\ . \tag{11j}$$

The ratio of the total width to the excitation energy of the 16.28 MeV, 1^- isobaric analog state in ^{90}Zr is sufficiently small so that the only function of ω which varies rapidly in the integration of $\sigma_\gamma(\omega)$ or $d^2\sigma_{e,x'}/(d\Omega_{x'}d\omega)$ over this state is the Breit-Wigner function. With this caveat, the transverse $E1$ reduced matrix element, $\mathcal{T}_1^{(e)}(q)$, is related to the photon absorption cross section into this level by[10]

$$\int\sigma_\gamma(\omega)d\omega=(2\pi)^3\frac{\alpha}{\omega}\,|\mathcal{T}_1^{(e)}(\omega)|^2\ . \tag{12}$$

The integrated partial cross section, resulting in the emission of particle X, may be obtained by integrating Eq. (7) over energy and angle:

$$\int\sigma(\gamma,X)d\omega=4\pi a_0=\frac{\Gamma_x}{\Gamma}\int\sigma_\gamma(\gamma)d\omega=\frac{\Gamma_x}{\Gamma}(2\pi)^3\frac{\alpha}{\omega}\,|\mathcal{T}_1^{(e)}(\omega)|^2 \tag{13}$$

Using Eq. (8) and $S^0=\pi\Gamma_x/(\sqrt{3}P_xE_x)$ from Eq. (29c) of Drechsel and Überall,[2] $D=(\Gamma/2\pi)B(\omega,\omega_0)(a_0/\omega)$. Integrating Eqs. (11) over ω, we have in the LWL:

$$\frac{d\sigma_T}{d\Omega_{x'}}=\frac{1}{\omega}\left[N_T^{E1}(E_0,\omega)a_0+\left\{\left[1+\frac{3m_e^2}{2P_0^2}\left[\frac{E_+^2}{E_0^2+E_f^2}\right]\right]N_T^{E1}(E_0,\omega)-\frac{3}{2}\frac{m_e^2}{P_0^2}\left[\frac{\omega^2}{E_0^2+E_f^2}\right]N_C^{E1}(E_0,\omega)\right.\right.$$
$$\left.\left.-\frac{3\alpha}{2\pi}\frac{P_f}{P_0^3}\left[E_0E_f+m_e^2\left[1-\frac{4E_0E_f}{E_0^2+E_f^2}\right]\right]\right\}a_2P_2(\cos\theta_{x'})\right]\ , \tag{14a}$$

$$\frac{d\sigma_C}{d\Omega_{x'}}=\frac{1}{\omega}\left[N_C^{E1}(E_0,\omega)a_0+\left\{\left[1-\frac{3}{4}\frac{\omega(E_0+3E_f)}{P_0^2}\right]N_C^{E1}(E_0,\omega)-\frac{3\alpha}{2\pi}\frac{P_f}{P_0}\left[\frac{3E_0E_f+m_e^2}{P_0^2}\right]\right\}a_2P_2(\cos\theta_{x'})\right]\ , \tag{14b}$$

$$\frac{d\sigma_{CT}}{d\Omega_{x'}}=\frac{3}{\omega}\left\{\left[\frac{\omega E_f+2m_e^2}{P_0^2}\right]N_C^{E1}(E_0,\omega)+2\frac{\alpha}{\pi}\left[\frac{P_f}{P_0}\left[\frac{E_0E_f+m_e^2}{P_0^2}\right]-\frac{m_e^2}{P_0^2}\frac{E_0E_+}{P_0^2}\ln\lambda\right]\right\}a_2P_2(\cos\theta_{x'})\ , \tag{14c}$$

$$\frac{d\sigma_{TT}}{d\Omega_{x'}}=\frac{3}{4\omega}\left\{\left[\frac{\omega^2-4m_e^2}{P_0^2}\right]N_C^{E1}(E_0,\omega)-2\frac{\alpha}{\pi}\left[\frac{P_f}{P_0}\left[\frac{E_0E_f+3m_e^2}{P_0^2}\right]-4\frac{m_e^2}{P_0^2}\frac{E_0E_f}{P_0^2}\ln\lambda\right]\right\}a_2P_2(\cos\theta_{x'})\ . \tag{14d}$$

Combining terms,

$$\frac{d\sigma_{e,x}}{d\Omega_{x'}}=\frac{1}{\omega}\left\{a_0P_0(\cos\theta_{x'})+\left[1+\frac{3}{2}\frac{m_e^2}{P_0^2}\left[\frac{E_+^2}{E_0^2+E_f^2}\right]\right]a_2P_2(\cos\theta_{x'})\right\}N^{E1}(E_0,\omega)$$
$$-\frac{3}{2}\frac{\alpha}{\pi}\frac{1}{\omega}\left\{\frac{P_f}{P_0}\left[\frac{E_0E_f}{P_0^2}\right]+\frac{m_e^2}{P_0^2}\left[\frac{4E_0^2}{P_0^2}\ln\lambda+\frac{P_f}{P_0}\left[1-\frac{4E_0E_f}{E_0^2+E_f^2}\right]\right]\right\}a_2P_2(\cos\theta_{x'})\ , \tag{15}$$

where

$$N^{E1}(E_0,\omega)=N_T^{E1}(E_0,\omega)+N_C^{E1}(E_0,\omega)\ .$$

For $P_f \gg m_e$, this is the result of Dodge and Barber.[11]

III. RESULTS

The integral of the photonuclear absorption cross section over the 16.28 MeV, 1^- isobaric analog state in ^{90}Zr that results in protons populating the ground state of ^{89}Y is related to the photon width, Γ_γ, the ground-state proton width, Γ_{p_0}, and the total width, Γ, of this level by

$$\int \sigma_{\gamma,p_0}(\omega)d\omega=(\pi\lambda\!\!\!^{-}{}^2)\frac{2J_0+1}{2J_i+1}\frac{\Gamma_\gamma\Gamma_{p_0}}{\Gamma} =4\pi a_0\ , \qquad (16)$$

where J_0 and J_i are the spins of the excited and ground states. Dodge *et al.*[4] obtained $(d\sigma_{e,p_0}/d\Omega_{p_0})_{\rm exp}$ from their measurements. If a factor of (a_0/ω) is extracted from $d\sigma_i/d\Omega_{x'}$, Eqs. (14a)–(14d), the remaining factor is the $E1$ virtual photon spectrum differential in $d\Omega_{x'}$, i.e., $d\sigma_i/d\Omega_{x'}=(a_0/\omega)dN_i/d\Omega_{p_0}$, for $i=T$, C, CT, and TT. The ratio[12] of the coefficients, a_2/a_0, for the ^{90}Zr, 1^-, 16.28 MeV isobaric analog state, contained in Eqs. (14), is -0.61.

Therefore

$$a_0=\frac{\omega\left[\dfrac{d\sigma_{e,p_0}}{d\Omega_{p_0}}\right]_{\rm exp}}{\left[\dfrac{dN_T}{d\Omega_{p_0}}+\dfrac{dN_C}{d\Omega_{p_0}}+\dfrac{dN_{CT}}{d\Omega_{p_0}}+\dfrac{dN_{TT}}{d\Omega_{p_0}}\right]} \qquad (17)$$

and

$$\frac{\Gamma_\gamma\Gamma_{p_0}}{\Gamma}=4\frac{\omega^3}{\pi(\hbar c)^2}\frac{2J_i+1}{2J_0+1}\times\frac{\left[\dfrac{d\sigma_{e,p_0}}{d\Omega_{p_0}}\right]_{\rm exp}}{\left[\dfrac{dN_T}{d\Omega_{p_0}}+\dfrac{dN_C}{d\Omega_{p_0}}+\dfrac{dN_{CT}}{d\Omega_{p_0}}+\dfrac{dN_{TT}}{d\Omega_{p_0}}\right]} \qquad (18a)$$

Numerically,

$$\frac{\Gamma_\gamma\Gamma_{p_0}}{\Gamma}=(4.72\ {\rm eV}/\mu{\rm b})\times\frac{\left[\dfrac{d\sigma_{e,p_0}}{d\Omega_{p_0}}\right]_{\rm exp}}{\left[\dfrac{dN_T}{d\Omega_{p_0}}+\dfrac{dN_C}{d\Omega_{p_0}}+\dfrac{dN_{CT}}{d\Omega_{p_0}}+\dfrac{dN_{TT}}{d\Omega_{p_0}}\right]}\ . \qquad (18b)$$

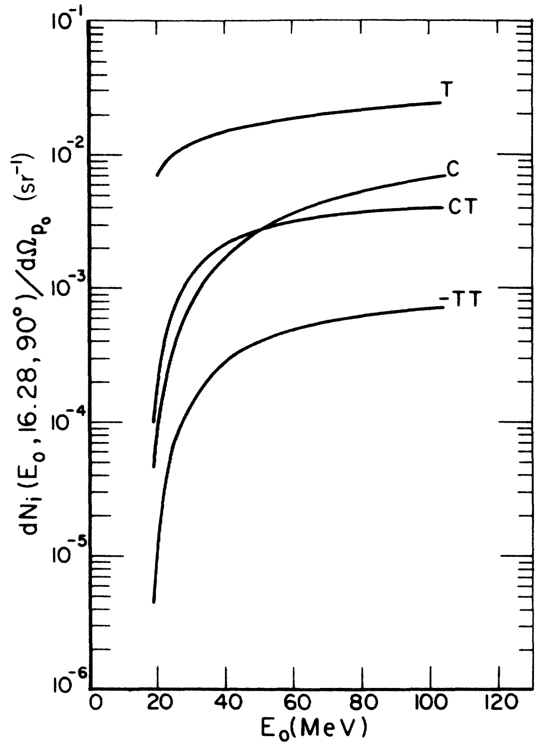

FIG. 2. A comparison of the four terms, $dN_i/d\Omega_{p_0}$ (90°), Eq. (14). They represent the number of 16.28 MeV virtual photons absorbed to produce ground-state protons at 90° as a function of incident electron energy, E_0. The interference term, $dN_{TC}/d\Omega_{p_0}$, is comparable to the Coulomb term, $dN_C/d\Omega_{p_0}$, and amounts to about 10% of the virtual photon spectrum. The $dN_{TT}/d\Omega_{p_0}$ term is an order of magnitude smaller and negative at this angle.

To illustrate the relative magnitudes of the four terms, $dN_i/d\Omega_{p_0}$ (90°), in Fig. 2 we plot the number of 16.28 MeV virtual photons absorbed to produce ground-state protons at 90° as a function of incident electron energy, E_0. It may be seen that the interference term, $dN_{CT}/d\Omega_{p_0}$, is comparable to the Coulomb term

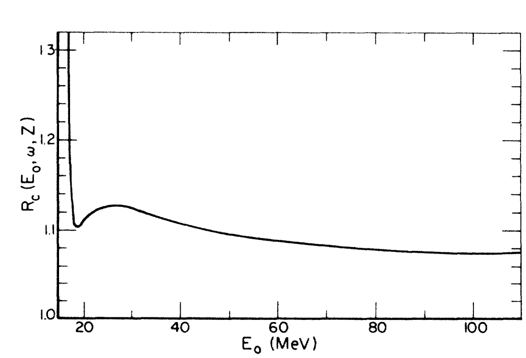

FIG. 3. The ratio, $R_C(E_0,16.28,40)$, of the $E1$ DWBA virtual photon spectrum of Ref. 10 to the $E1$ PWBA virtual photon spectrum $N^{E1}(E_0,\omega)$, defined in the text, as a function of E_0. $R_C(E_0,16.28,40)$ shows the effect of Coulomb distortion in ^{90}Zr on the $E1$ virtual photon spectrum at an excitation energy of 16.28 MeV.

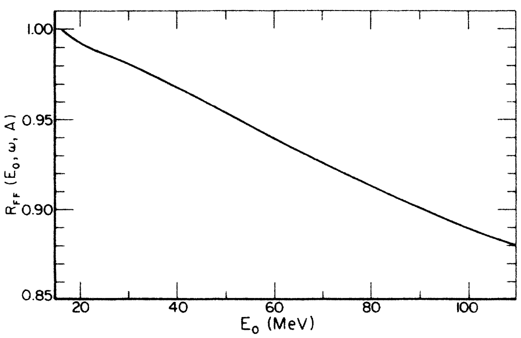

FIG. 4. The ratio, $R_{FF}(E_0, 16.28, 90)$, of $E1$ PWBA virtual photon spectrum obtained with the Helm model form factors to $E1$ PWBA virtual photon spectrum for a point nucleus as a function of E_0. $R_{FF}(E_0, 16.28, 90)$ shows the effect of the q dependence of the generalized Helm model form factors on the ^{90}Zr $E1$ virtual photon spectrum at an excitation energy of 16.28 MeV.

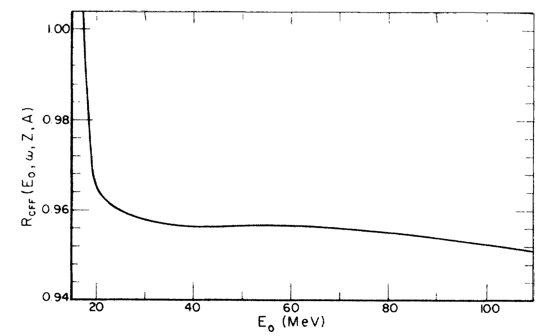

FIG. 5. The ratio, $R_{CFF}(E_0, 16.28, 40, 90)$, of the $E1$ SOBA virtual photon spectrum of Ref. 11 to the $E1$ PWBA virtual photon spectrum with multiplicative Coulomb and form factor corrections as a function of E_0.

$dN_C/d\Omega_{p_0}$, and amounts to about 10% of the virtual photon spectrum while the $dN_{TT}/d\Omega_{p_0}$ contribution is negative at this angle but negligible in any case.

Equations (14) do not include the Coulomb distortion of the incoming and outgoing electron wave. The distorted wave Born approximation (DWBA)[13] and second-order Born approximation (SOBA)[14] calculations of the Coulomb distortions have only been carried out for total (e,X) cross sections. However, Soto Vargas *et al.*[13] have produced a computer program which calculates the ratio of

$$\frac{N^{E1}_{\rm DWBA}(E_0,\omega,Z)}{N^{E1}_{\rm PWBA}(E_0,\omega)} = R_C(E_0,\omega,Z) ,$$

where

$$N^{E1}_{\rm PWBA}(E_0,\omega) = \int \left[\frac{dN^{E1}_T}{d\Omega_{p_0}} + \frac{dN^{E1}_C}{d\Omega_{p_0}} + \frac{dN^{E1}_{CT}}{d\Omega_{p_0}} + \frac{dN^{E1}_{TT}}{d\Omega_{p_0}} \right] d\Omega_{p_0} . \qquad (19)$$

This ratio is plotted as a function of E_0 for $\omega = 16.28$ MeV in Fig. 3. The Coulomb correction for these kinematic conditions is about 10% and nearly independent of E_0. For these reasons we have corrected for Coulomb effects by dividing Eq. (18) by $R_C(E_0,\omega,Z)$. The effects of using model dependent expressions for $F^1_T(q)$ and $F^1_C(q)$ were investigated by numerically integrating Eq. (4) assuming Helm model form factors.[4] The ratio of

$$N_{FF}(E_0,\omega,A)/N_P(E_0,\omega) = R_{FF}(E_0,\omega,A)$$

is plotted in Fig. 4; here A is the mass number of the nucleus, FF denotes form factor, and P a point nucleus. Because of the forward peaking of the transverse part of the cross section, the model dependence of $N_{FF}(E_0,\omega,A)$ is slight as long as the model reproduces the same electron scattering charge and transition charge radii.

Durgapal and Onley[14] have pointed out that Coulomb distortion and form factor corrections to virtual photon spectra can interfere, and hence factorized Coulomb and form factor corrections cannot always be used. However, for the 16.28 MeV, 1^- isobaric analog state in ^{90}Zr, a direct comparison of the SOBA calculation of Durgapal and Onley with the PWBA calculation with multiplicative Coulomb and form factor corrections shows that interference effects are small. The ratio

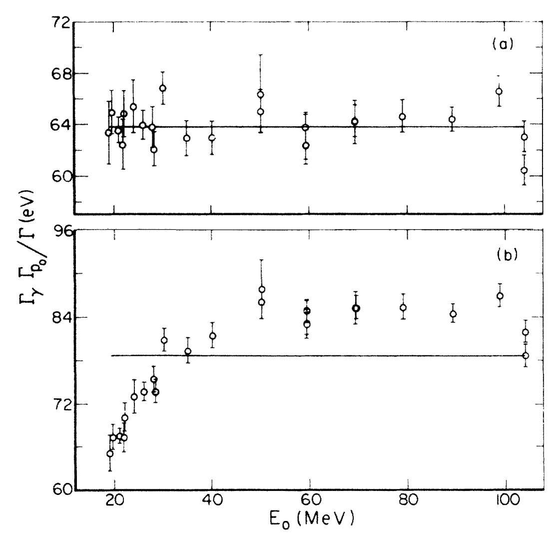

FIG. 6. (a) The invariant quantity $\Gamma_\gamma \Gamma_{p_0}/\Gamma$ plotted as a function of the incident electron energy, E_0, obtained with the choice of relative sign of the Coulomb and transverse form factors given in Eq. (1) with Coulomb and form factor corrections described in the text. (b) The invariant quantity $\Gamma_\gamma \Gamma_{p_0}/\Gamma$ plotted as a function of the incident electron energy, E_0, obtained as in (a) except the choice of relative sign of the Coulomb and transverse form factors was taken to opposite that given in Eq. (1).

TABLE I. Summary of χ^2 fits to the weighted mean of $\Gamma_\gamma\Gamma_{p_0}/\Gamma$ as a result of applying various corrections to $E1$ PWBA differential virtual photon spectra. For definitions of $R_C(E_0,16.28,40)$, $R_{FF}(E_0,16.28,90)$, and $R_{CFF}(E_0,16.28,40,90)$ see the text.

Coulomb correction $R_C(E_0,16.28,40)$	Form factor correction $R_{FF}(E_0,16.28,90)$	Coulomb-form factor interference correction $R_{CFF}(E_0,16.28,40,90)$	Weighted mean of $\Gamma_\gamma\Gamma_{p_0}/\Gamma$ (eV)	Reduced χ^2 a
No	Yes	No	66.06±0.27	9.13
Yes	No	No	60.21±0.36	5.33
No	No	No	70.14±0.29	1.99
Yes	Yes	No	63.82±0.26	1.20
Yes	Yes	Yes	66.14±0.28	1.20

[a]For 24 degrees of freedom.

$$R_{CFF}(E_0,16.28,40)=\frac{N^{E1}_{\rm SOBA}(E_0,16.28,40,90)}{[N^{E1}_{\rm PWBA}(E_0,16.28)R_C(E_0,16.28,40)R_{FF}(E_0,16.28,90)]}$$

is plotted in Fig. 5.

Figure 6(a) shows the invariant quantity $\Gamma_\gamma\Gamma_{p_0}/\Gamma$ plotted as a function of E_0 for the relative sign relationship between the transverse and Coulomb form factors given by Eq. (9). Figure 6(b) shows the same quantity with the sign of Eq. (9) taken to be positive. Table I summarizes the results of making various combinations of corrections described above in terms of a χ^2 test for goodness of fit to the weighted mean of $\Gamma_\gamma\Gamma_{p_0}/\Gamma$. This comparison shows that the interference term of the (e,e′p) coincidence cross section is clearly observed in our (e,p) experiment and must be included in the $d\sigma_{e',p}/d\Omega_p$.

IV. CONCLUSION

Equations (14a)–(14d) are the PWBA expressions for the electric dipole virtual photon spectra including the angular dependence of an outgoing nucleon. They contain all four terms in the electron scattering coincidence cross section. In the experiment[4] we measured the invariant quantity, $\Gamma_\gamma\Gamma_{p_0}/\Gamma$, using electrons in the energy range, 17–105 MeV, as well as bremsstrahlung having end-point energies 60–105 MeV. The results of these two measurements were consistent when the correction shown in Fig. 5 was applied to the (e,p) data and if the Davies-Bethe-Maximon bremsstrahlung cross section[15] was used in the interpretation of the real photon data. This experiment represents a clear observation of the interference term in the coincidence cross section.

The minus sign of Eq. (9) is compatible with the conventions of Drechsel and Überall.[2] Other definitions of the operators could lead to a positive sign, but with these definitions of the operators the sign of the Coulomb-transverse interference term in the coincidence cross section would change, thus resulting in the same coincidence cross section and virtual photon spectra. This result is related to the ^{12}C(e,e′γ) experiment of Papanicolas *et al.*[16] where an $E2$ excitation was studied.

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