Optical potential approach in scattering of hadrons from light nuclei

E. Bleszynski and M. Bleszynski
Department of Physics, University of California, Los Angeles, California 90024

T. Jaroszewicz
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 6 December 1985)

A new calculational approach is described which reproduces the Watson multiple scattering series, simplified by neglecting the terms describing the repeated scatterings of a projectile on the same target nucleon. The approach involves solving sets of uncoupled wave equations (Schrödinger or Dirac) with effective optical potentials of the analogous form as the first order optical potential used in the standard impulse approximation optical models. Our approach is particularly suitable in multiple scattering calculations for elastic scattering of hadrons from light nuclei. As an example of the application of the proposed method we evaluated multiple scattering series for the elastic scattering of protons from ^{4}He with the purpose of assessing the importance of relativistic effects in this reaction.

I. INTRODUCTION

It is commonly accepted that the correct description of the hadron-nucleus interaction is given by the multiple scattering theory, and the optical potential approach is meant as an approximation scheme. It is also well known that the higher order multiple scattering terms in the multiple scattering series can be reproduced exactly only by using higher order optical potentials which are, to a great computational disadvantage, rather complicated nonlocal operators.

The objective of this paper is to describe a new, easily calculable method of approximating the multiple scattering series. The method, instead of adding higher order terms to the optical potential, consists of solving a set of uncoupled, suitably chosen wave equations with optical potentials of the same level of complexity as the first order optical potential used in standard optical models. The main advantage of our method is its simple treatment of the $1/A$ corrections (A being the target mass number) to the first order optical potential which are technically very difficult to incorporate in the framework of the standard optical model approaches. Our method may thus be particularly useful in the treatment of scattering of hadrons from light nuclei.

II. COMMON OPTICAL MODEL APPROACH

Before describing the proposed method we briefly review the basic elements of the optical potential method.[1] We are interested in the range of sufficiently high incident projectile energies at which the evaluation of the multiple scattering amplitude can be made in the fixed scatterers approximation. The collision matrix describing projectile-nucleus scattering can be written as follows:

$$T=\left\langle \sum_{k=1}^{A} t_k+\sum_{\substack{k,l=1\\k\neq l}}^{A} t_k G t_l+\sum_{\substack{k,l,m=1\\k\neq l\neq m}}^{A} t_k G t_l G t_m+\cdots \right\rangle , \quad (2.1)$$

where t_k is the projectile-target nucleon scattering operator whose matrix elements are related to the free NN scattering amplitude:

$$\langle \mathbf{p}' \mid t_i(\mathbf{r}_i) \mid \mathbf{p} \rangle = e^{i\mathbf{q}\cdot\mathbf{r}_i} f_i(\mathbf{q}) , \quad (2.2)$$

where $\mathbf{q}=\mathbf{p}'-\mathbf{p}$ and $\langle \cdots \rangle$ denotes the average with respect to the target ground state density. In Eq. (2.1), G denotes the free wave propagator which for spin zero projectile is

$$G(\mathbf{p})=\frac{1}{k^2-p^2+i\epsilon} , \quad (2.3)$$

and for the spin $\frac{1}{2}$ projectile

$$G(\mathbf{p})=\frac{\not{p}+m}{k^2-p^2+i\epsilon}=\frac{E(k)\gamma^0-\mathbf{p}\cdot\boldsymbol{\gamma}+m}{k^2-p^2+i\epsilon} . \quad (2.4)$$

Here $p=|\mathbf{p}|$ is the magnitude of the intermediate momentum, and k is defined by the energy of the incident projectile $E(k)=(k^2+m^2)^{1/2}$.

The collision matrix T of Eq. (2.1) is assumed to be composed of a finite number of terms corresponding to consecutive multiple scattering processes. [The first term, $\langle \sum_{j=1}^{A} t_j \rangle$, represents the transition in which the projectile interacts with one of the target nucleons (single scattering term), the second term, $\langle \sum_{k\neq l}^{A} t_k G t_l \rangle$, represents the process in which two target nucleons are being hit successively by the incident projectile (double scattering process).] We note that in expansion (2.1) we deliberately do not include the "reflection" terms, i.e., terms corresponding to a situation in which at least one of the target nucleons is hit more than once. The presence of such terms may be excluded on the basis of various arguments; the most convincing argument follows from the unitarity equations:[2] the reflection terms correspond to a process in which an excited mass state of a projectile propagates in between collisions. Hence the reflection terms constitute a subclass of a class of terms referred to

as inelastic shadowing terms in the multiple scattering expansion. Including such terms as only inelastic contributions, without justifying that they constitute the most important part of the inelastic shadowing, might be inconsistent.

The evaluation of the complete multiple series of Eq. (2.1) poses in general severe technical difficulties. The general objective of the optical model approach is to circumvent these difficulties by performing an approximate summation of the series (2.1). The method consists of finding an auxiliary function—"optical potential" $V_{\rm opt}$—which, when inserted into the wave equation

$$(K+V_{\rm opt})\Psi=E\Psi\ , \tag{2.5}$$

generates the collision matrix T:

$$T=V_{\rm opt}+V_{\rm opt}GT=\frac{V_{\rm opt}}{1-GV_{\rm opt}}\ . \tag{2.6}$$

In Eq. (2.5) K represents the kinetic energy operator.

The exact form of $V_{\rm opt}$, which can be obtained by formally inverting Eq. (2.6), is

$$V_{\rm opt}=T/(1+GT)=T-TGT+TGTGT+\cdots\ . \tag{2.7}$$

Equation (2.1) allows us to expand further $V_{\rm opt}$ in powers of the projectile-target nucleon scattering t matrix,

$$V_{\rm opt}=V_{\rm opt}^{(1)}+V_{\rm opt}^{(2)}+V_{\rm opt}^{(3)}+\cdots\ , \tag{2.8}$$

where

$$V_{\rm opt}^{(1)}=\left\langle\sum_{k=1}^{A}t_k\right\rangle\ ,$$

$$V_{\rm opt}^{(2)}=\left\langle\sum_{k\neq l}^{A}t_kGt_l\right\rangle-\left\langle\sum_{k=1}^{A}t_k\right\rangle G\left\langle\sum_{k=1}^{A}t_k\right\rangle\ , \tag{2.9}$$

$$\ldots\ \text{etc}\ .$$

The standard optical potential scheme allows one to approximate the multiple scattering series, to a desired accuracy, by including higher order terms in the optical potential expansion. The higher order terms, however, are in general, to a considerable computational disadvantage, quite complicated nonlocal operators. This serious technical difficulty renders the standard optical model essentially useful in cases in which only the first order approximation involving the first term in Eq. (2.9) is sufficiently accurate.

The purpose of this paper is to discuss another approximation scheme which is technically easier to deal with than the standard optical model. Before we describe our method in the next section we introduce below our notation and several elements of the optical model approach which we shall need in our further discussion.

Let us consider scattering of a projectile from a nucleus composed of A nucleons. For simplicity of the discussion, we shall assume the independent particle model density with the separable center of mass correlation

$$\rho_A(\mathbf{r}_1,\ldots,\mathbf{r}_A)=\prod_{i=1}^{A}\rho(\mathbf{r}_i)=|\,\Phi_{\rm c.m.}(\mathbf{R})\,|^2\rho_A^{\rm int}(\mathbf{r}_1-\mathbf{R},\ldots,\mathbf{r}_A-\mathbf{R})\ , \tag{2.10}$$

where

$$\mathbf{R}=\frac{1}{A}\sum_{i=1}^{A}\mathbf{r}_i\ .$$

Also in the following discussion, we shall assume the identical interactions of the projectile with protons and neutrons by setting

$$\langle\,\mathbf{p}'\,|\,t_k\,|\,\mathbf{p}\,\rangle=\langle\,\mathbf{p}'\,|\,t\,|\,\mathbf{p}\,\rangle e^{-i\mathbf{q}\cdot\mathbf{r}_k}\ . \tag{2.11}$$

Generalization to the case of the correlated wave function and nonidentical projectile-target nucleon t matrices is briefly described later on.

Under the above assumptions, the scattering matrix acquires the form

$$T=\sum_{n=1}^{A}[A]_n\{u\}_n\ . \tag{2.12}$$

Here we have introduced the following notation:

$$[A]_n=A(A-1)\cdots(A-n+1)\ , \tag{2.13}$$

$$\{u\}_n=u(Gu)^{n-1}\ , \tag{2.14}$$

with

$$\langle\,\mathbf{p}'\,|\,u\,|\,\mathbf{p}\,\rangle=\tilde{\rho}(\mathbf{p}'-\mathbf{p})\langle\,\mathbf{p}'\,|\,t\,|\,\mathbf{p}\,\rangle \tag{2.15}$$

and

$$\tilde{\rho}(\mathbf{q})=\int d^3r\,e^{-i\mathbf{q}\cdot\mathbf{r}}\rho(\mathbf{r})\ . \tag{2.16}$$

The first order optical potential $V_{\rm opt}^{(1)}$ can now be written in the form

$$V_{\rm opt}^{(1)}=Au\ . \tag{2.17}$$

The above potential generates the following, approximate scattering matrix $T_{\rm opt}$:

$$T_{\rm opt}=V_{\rm opt}^{(1)}+V_{\rm opt}^{(1)}GT_{\rm opt}=\sum_{n=1}^{\infty}A^n\{u\}_n\ . \tag{2.18}$$

We see that the exact multiple scattering series (2.1) and the approximate multiple scattering series (2.16) differ by the combinatorial factors multiplying $\{u\}_n$. The ratios of these factors for the single, double, triple, etc., scattering terms

$$1,\ \left[1-\frac{1}{A}\right],\ \left[1-\frac{1}{A}\right]\left[1-\frac{2}{A}\right],\ \left[1-\frac{1}{A}\right]\left[1-\frac{2}{A}\right]\left[1-\frac{3}{A}\right],\ \tag{2.19}$$

tend to differ more for light nuclei and increasing orders of scattering.

We note that the optical potential of Kerman, McManus, and Thaler (KMT) (Ref. 3) is constructed in

such a manner that its first order term reproduces the combinatorial factors in front of $\{u\}_n$ exactly up to $n=2$, because

$$V^{(1)}_{\mathrm{KMT}}=(A-1)u \tag{2.20}$$

and

$$\begin{aligned} T_{\mathrm{KMT}}&=\frac{A}{A-1}\frac{(A-1)u}{1+G(A-1)u} \\ &=A\{u\}_1+A(A-1)\{u\}_2 \\ &\quad +A(A-1)(A-1)\{u\}_3+\cdots . \end{aligned} \tag{2.21}$$

However, the combinatorial factors for the orders of scattering higher than two start to differ from those in the exact multiple scattering expansion.

The common procedure of obtaining a better approximation to the exact result (2.1) consists of including higher order terms in the expansion of the optical potential (2.8). In general, taking into account terms up to the nth order in the expansion of V_{opt} allows one to reproduce the multiple scattering series up to the nth order. Such an approach is, as we stressed above, however quite difficult to apply because of the increasing complexity of the higher order terms.

III. DERIVATION OF THE GENERALIZED OPTICAL POTENTIAL METHOD

In this section we derive a new optical potential approach which allows one to reproduce the multiple scattering series. The approach is different and significantly simpler than the standard optical potential procedure described above. Instead of including higher order corrections to the optical potential, we write the collision matrix T as a linear combination of N auxiliary collision matrices T_j

$$T=\sum_{j=1}^{N} x_j T_j , \tag{3.1}$$

where each T_j can be generated by solving the wave equation with an optical potential V_j:

$$T_j=V_j+V_jGT_j , \tag{3.2}$$

where

$$V_j=a_j u , \tag{3.3}$$

and the coefficients x_j and a_j are to be determined.

We stress here that only the linear combination of T_j's describes the physical scattering process and we do not attach any physical meaning to the auxiliary collision matrices T_j and the optical potentials V_j which generate them.

By expanding both sides of Eq. (3.1) in powers of t we obtain

$$\sum_{n=1}^{A}[A]_n\{u\}_n=\sum_{j=1}^{N}x_j\frac{a_j u}{(1-Ga_j u)} . \tag{3.4}$$

We see that in order to make both sides agree up to the *2Nth order* of scattering, it is necessary to find $2N$ coefficients

$$x_1,\ldots,x_N \text{ and } a_1,\ldots,a_N ,$$

which satisfy a set of $2N$ equations

$$\sum_{j=1}^{N}x_j a_j^n=[A]_n, \quad n=1,\ldots,2N \tag{3.5}$$

which are linear in x_j and nonlinear in a_j.

By eliminating the coefficients x_j from the above $2N$ equations, we find that the coefficients a_j turn out to be the roots of the Nth order algebraic equation:

$$\begin{aligned} &(-a)^N+\begin{bmatrix}N\\1\end{bmatrix}(A-N)(-a)^{N-1} \\ &\quad +\begin{bmatrix}N\\2\end{bmatrix}(A-N)(A-N+1)(-a)^{N-2} \\ &\quad +\cdots+(A-N)(A-N+1)\cdots(A-1)=0 . \end{aligned} \tag{3.6}$$

The knowledge of the N solutions a_j allows us to determine the coefficients x_j, by solving a set of N arbitrarily chosen linear equations from the set of $2N$ equations (3.5).

In the following we shall discuss the properties of three solutions obtained with the help of the above method, corresponding to $N=1$, 2, and 3.

A. $N=1$

The simplest case of $N=1$ yields the following set of equations:

$$x_1 a_1^n=[A]_n, \quad n=1,2 \tag{3.7}$$

and, consequently, the solution

$$a_1=A-1, \quad x_1=\frac{A}{A-1} . \tag{3.8}$$

The corresponding, approximate scattering matrix has the form

$$\begin{aligned} T=x_1\frac{a_1 u}{(1-Ga_1 u)}&=A\{u\}_1+A(A-1)\{u\}_2 \\ &\quad +A(A-1)(A-1)\{u\}_3+\cdots , \end{aligned} \tag{3.9}$$

i.e., the result is equivalent to that obtained by using the first order optical potential of KMT. We note that in this approximation the terms of order higher than 2 are too large by factors

$$R_j=\frac{A(A-1)^j}{A(A-1)\cdots(A-j+1)} , \tag{3.10}$$

which, e.g., for ^{4}He nucleus yield $R_3=1.5$ and $R_4=4.5$. Also, while the multiple scattering series (1) terminates after the fourth order, the approximation (3.9) leads to the spurious fifth, sixth, etc., scattering terms. In the p-^{4}He elastic scattering around 1 GeV, differences between the multiple scattering theory and the optical model become quite large, e.g., the second diffraction maximum is overestimated by the optical model by about 100%.

B. $N=2$

The case of $N=2$ leads to the following set of equations for x_j and a_j, and to the following solutions:

$$\begin{aligned} &x_1 a_1^n + x_2 a_2^n = [A]_n, \quad n=1,\ldots,4 \\ &a_1 = a_2^* = \sqrt{A-2}(\sqrt{A-2}+i) \\ &x_1 = x_2^* = \frac{A}{2(A-2)} \frac{\sqrt{A-2}-i}{\sqrt{A-2}+i} . \end{aligned} \tag{3.11}$$

The resultant scattering matrix is

$$\begin{aligned} T &= x_1 \frac{a_1 u}{(1-G a_1 u)} + x_2 \frac{a_2 u}{(1-G a_2 u)} \\ &= A\{u\}_1 + A(A-1)\{u\}_2 + A(A-1)(A-2)\{u\}_3 \\ &\quad + A(A-1)(A-2)(A-3)\{u\}_4 \\ &\quad + A(A-1)(A-2)^2(A-5)\{u\}_5 + \cdots . \end{aligned} \tag{3.12}$$

The above expansion reproduces, as expected, the multiple scattering series up to the fourth order. We also note that, in the case of p-^{4}He elastic scattering, the scattering terms of order higher than 4 are much smaller in the above approximation than in the case of the KMT optical potential.

C. $N=3$

Here we find

$$\begin{aligned} &x_1 a_1^n + x_2 a_2^n + x_3 a_3^n = [A]_n, \quad n=1,\ldots,6 , \\ &a_1 = x+y+A-3 , \\ &a_2 = a_3^* = -\tfrac{1}{2}(x+y)+A-3+i\sqrt{3/4}(x-y) , \\ &x_j = A \frac{(A-1)(A-2)-(A-1)(a_k+a_l)+a_k a_l}{a_j(a_j-a_k)(a_j-a_l)} , \\ &\qquad\qquad \text{for } j,k,l = 1,2,3 , \end{aligned} \tag{3.13}$$

where

$$x = [(A-3)(1+\sqrt{A-2})]^{1/3}$$

and

$$y = [(A-3)(1-\sqrt{A-2})]^{1/3} .$$

The $N=3$ approximation reproduces the multiple scattering series (2.1) exactly up to the sixth order, i.e.,

$$\begin{aligned} T &= x_1 \frac{a_1 u}{(1-G a_1 u)} + x_2 \frac{a_2 u}{(1-G a_2 u)} + x_3 \frac{a_3 u}{(1-G a_3 u)} \\ &= A\{u\}_1 + [A]_2\{u\}_2 + [A]_3\{u\}_3 + [A]_4\{u\}_4 \\ &\quad + [A]_5\{u\}_5 + [A]_6\{u\}_6 + \cdots . \end{aligned} \tag{3.14}$$

In the case of p-^{4}He elastic scattering, the approximation we propose has an additional advantage over the KMT approximation: the dominant spurious terms, the fifth and sixth order scattering terms, vanish.

The above examples demonstrate the usefulness of our approach. By using our method it is possible to reproduce the combinatorial factors in front of the first $2N$ terms in the multiple scattering expansion (1) exactly by solving N uncoupled wave equations with the optical potentials proportional to the first order optical potential of KMT:

$$V^j = a_j u, \quad j=1,\ldots,N . \tag{3.15}$$

In order to achieve the same level of accuracy in the framework of the standard KMT approach, it would be necessary to solve one wave equation with the $2N$th order optical potential; this is in practice totally unfeasible.

Our method can be generalized in order to include correlations as well as the nonidentical t matrices in the analogous way as in the standard optical model approaches, by replacing the potentials $a_j u$ with the expansions

$$V_j = V_j^{(1)} + V_j^{(2)} + V_j^{(3)} + \cdots , \tag{3.16}$$

where

$$\begin{aligned} V_j^{(1)} &= a_j \left\langle \sum_{k=1}^{A}{}' t_k \right\rangle , \\ V_j^{(2)} &= a_j^2 \left[\left\langle \sum_{\substack{k,l=1 \\ k\neq l}}^{A}{}' t_k G t_l \right\rangle - \left\langle \sum_{k=1}^{A}{}' t_k \right\rangle G \left\langle \sum_{l=1}^{A}{}' t_l \right\rangle \right] , \\ V_j^{(3)} &= a_j^3 \left[\left\langle \sum_{\substack{k,l,m=1 \\ k\neq l\neq m}}^{A}{}' t_k G t_l G t_m \right\rangle - \left\langle \sum_{\substack{k,l=1 \\ k\neq l}}^{A}{}' t_k G t_l \right\rangle G \left\langle \sum_{m=1}^{A}{}' t_m \right\rangle - \left\langle \sum_{k=1}^{A}{}' t_k \right\rangle G \left\langle \sum_{\substack{l,m=1 \\ l\neq m}}^{A}{}' t_l G t_m \right\rangle - \left\langle \sum_{\substack{k,m=1 \\ k\neq m}}^{A}{}' t_k G \left\langle \sum_{l=1}^{A}{}' t_l \right\rangle G t_m \right\rangle \right. \\ &\quad \left. + \left\langle \sum_{k=1}^{A}{}' t_k \right\rangle G \left\langle \sum_{l=1}^{A}{}' t_l \right\rangle G \left\langle \sum_{m=1}^{A}{}' t_m \right\rangle \right] . \end{aligned} \tag{3.17}$$

Here the summation runs over $i,k,l=1,\ldots,A$ such that i,k,l are all different, and the prime indicates that a given sum has to be divided by $[A]_n$, n being the number of summation indices. The above expression is essentially an expansion in terms of the correlation functions in the target density, i.e., the first term $V_j^{(1)}$ is proportional to the single particle density, the second term $V_j^{(2)}$ to the two-particle correlation function, etc. (cf. Ref. 4). We

stress, however, that in the case of independent particle model target density only the first order term $V_j^{(1)}$ contributes to the expansion (39). This is not the case of the KMT potential, in which, in general higher order terms in the expansion do not vanish even if there are no correlations between the target nucleons.

IV. APPLICATIONS

As an interesting example demonstrating the usefulness of the method proposed here we compare the nonrelativistic and relativistic calculations of the amplitude for p-^{4}He elastic scattering. Our objective is to estimate the quantitative importance of the relativistic effects in this reaction. Such a calculation, in principle, consists of comparing the multiple scattering amplitudes composed of the single, double, triple, and quadruple collision terms:

$$F(\mathbf{Q})=F_1(\mathbf{Q})+F_2(\mathbf{Q})+F_3(\mathbf{Q})+F_4(\mathbf{Q}) \tag{4.1}$$

evaluated with the Schrödinger and the Dirac propagators, respectively.

In either Schrödinger or Dirac case the amplitudes F_j, $j=1,4$ can be written schematically as follows:

$$\begin{aligned}
F_1&=A\langle\Psi\,|\,h_1(\mathbf{Q})\,|\,\Psi\rangle\ ,\\
F_2&=A(A-1)\int d^3p\langle\Psi\,|\,h_2(\mathbf{p}-\mathbf{k}_f)G(p)h_1(\mathbf{k}_i-\mathbf{p})\,|\,\Psi\rangle\ ,\\
F_3&=A(A-1)(A-2)\int d^3p_1d^3p_2\langle\Psi\,|\,h_3(\mathbf{p}_2-\mathbf{k}_f)G(p_2)h_2(\mathbf{p}_1-\mathbf{p}_2)G(p_1)h_3(\mathbf{k}_i-\mathbf{p}_1)\,|\,\Psi\rangle\ ,\\
F_4&=A(A-1)(A-2)(A-3)\int d^3p_1d^3p_2d^3p_3\langle\Psi\,|\,h_3(\mathbf{p}_3-\mathbf{k}_f)G(p_3)h_3(\mathbf{p}_2-\mathbf{p}_3)G(p_2)h_2(\mathbf{p}_1-\mathbf{p}_2)G(p_1)h_3(\mathbf{k}_i-\mathbf{p}_1)\,|\,\Psi\rangle\ .
\end{aligned} \tag{4.2}$$

In the above formulas, $A=4$ is the target mass number, Ψ is the target wave function,

$$h_n(\mathbf{q})=e^{i\mathbf{q}\cdot\mathbf{r}_n}f(\mathbf{q}),\quad n=1,\ldots,4 \tag{4.3}$$

are the isospin averaged amplitudes for scattering off the nth nucleon.

In the nonrelativistic approach we adopt the elementary projectile-target nucleon amplitude in the following form:

$$f(\mathbf{q})=A(q)+C(q)\sigma\cdot\hat{\mathbf{y}}\ , \tag{4.4}$$

i.e., we neglect the projectile interaction with the spins of the target nucleons and retain only the two relevant spin components of the NN amplitude. The amplitudes A and C are obtained from the phase shift compilation of Arndt and Roper.[5] This allows us to consider the relativistic representation of a NN amplitude in a simple form

$$f(q)=f_s(q)+f_v(q)\gamma_0\ . \tag{4.5}$$

(This choice is essentially the same as that made in Ref. 6. It differs only in our neglect of terms associated with the target nucleons spin degrees of freedom which give negligible contributions for scattering off large, spin zero nuclei.) The scalar $f_s(q)$ and vector $f_v(q)$ components are constructed by equating the matrix elements of the amplitudes between the four component positive energy Dirac spinors and two component Pauli spinors, respectively:[7]

$$\begin{aligned}
&\bar{u}(\mathbf{k}_0-\tfrac{1}{2}\mathbf{q},s_f)[f_s(q)+f_v(q)\gamma_0]u(\mathbf{k}_0+\tfrac{1}{2}\mathbf{q},s_i)\\
&\qquad=\chi(s_f)^\dagger[A(q)+C(q)\sigma\cdot\hat{\mathbf{y}}]\chi(s_i)\ .
\end{aligned} \tag{4.6}$$

Here $\mathbf{k}_0$ is the Breit frame average momentum determined by the "optimal" energy, $\chi(s)$ is a Pauli spinor, and $u(\mathbf{p},s)$ is a positive energy Dirac spinor. The following transformation formulas between the amplitudes $A(q)$, $C(q)$ and $f_v(q)$, $f_s(q)$ can be obtained in a straightforward manner from the last equation,

$$A(q)=\frac{E+m}{2m}[f_s(q)+f_v(q)]+\frac{k^2-q^2/2}{2m(E+m)}[f_v(q)-f_s(q)]\ , \tag{4.7a}$$

and

$$C(q)=\frac{1}{i}\frac{(k^2-q^2/4)^{1/2}}{2m(E+m)}[f_v(q)-f_s(q)]\ . \tag{4.7b}$$

The inverse relations are

$$f_v(q)=\frac{m}{E+m}A(q)-\frac{(-2mi)}{(k^2-q^2/4)^{1/2}}\left[E-\frac{q^2}{4(E+m)}\right]C(q) \tag{4.8a}$$

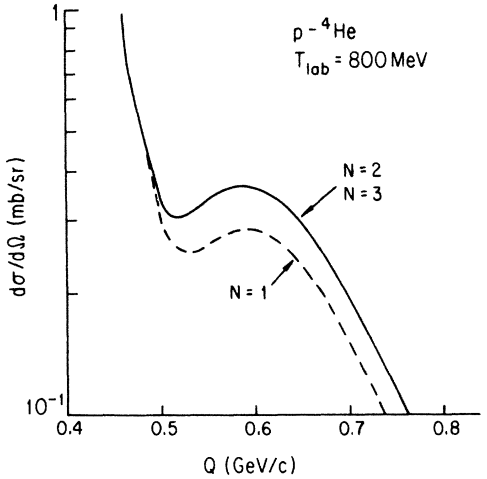

FIG. 1. Comparison between the successive approximations $N=1,N=2,N=3$ for the differential cross section for p-^{4}He elastic scattering at 800 MeV. The ^{4}He wave function Ψ was parametrized as the product of the single particle s-wave functions with the harmonic oscillator parameter $b=1.37$ fm.

and

$$f_s(q)=\frac{m}{E+m}A(q) - \frac{(-2mi)}{(k^2-q^2/4)^{1/2}}\left[m+\frac{q^2}{4(E+m)}\right]C(q)\,. \quad (4.8b)$$

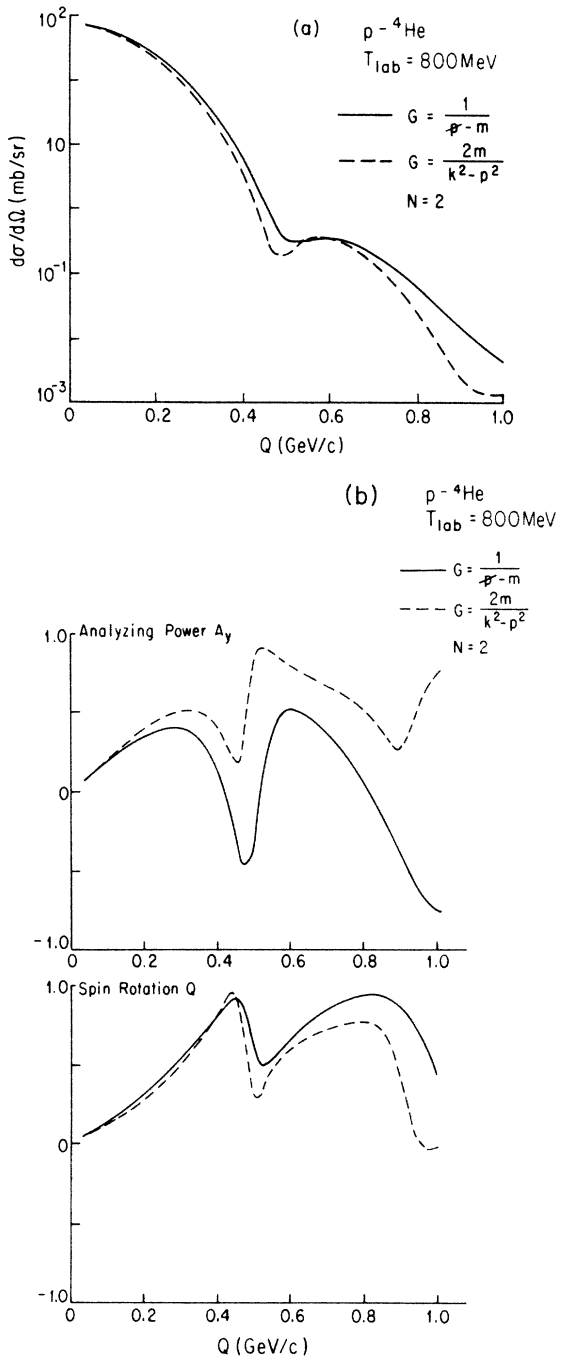

FIG. 2. (a) Differential cross section for p-^{4}He scattering at $T_{\rm lab}=800$ MeV. Solid and dashed lines represent the relativistic and nonrelativistic calculations, respectively. (b) Analyzing power and spin rotation parameter for p-^{4}He scattering at $T_{\rm lab}=800$ MeV. Solid and dashed lines represent the relativistic and nonrelativistic calculations, respectively.

Similarly as in the nonrelativistic approach, the accepted form of the relativistic NN amplitude determines the form of the relativistic optical potential in the impulse approximation,[6]

$$V(q)=V_s(q)+V_v(q)\gamma_0\,, \quad (4.9a)$$

where

$$V_s(r)=-\frac{ik}{m}\frac{1}{(2\pi)^3}\int d^3q\, e^{-iq\cdot r}f_s(q)S(q) \quad (4.9b)$$

and

$$V_v(r)=-\frac{ik}{m}\frac{1}{(2\pi)^3}\int d^3q\, e^{-iq\cdot r}f_v(q)S(q) \quad (4.9c)$$

are the scalar and vector optical potentials.

The relativistic nuclear scattering amplitude and the observables can be found by solving numerically the Dirac equation with the relativistic optical potential of Eq. (4.9). An alternative approach is to cast the Dirac equation into the mathematically equivalent form of a Schrödinger equation containing the central and spin-orbit potential components:

$$V(\mathbf{r})=V^c(r)+V^{\rm so}(r)\boldsymbol{\sigma}\cdot\mathbf{l}\,, \quad (4.10a)$$

which are related to the Dirac scalar and vector potentials

$$\tilde{V}^c(r)=\frac{E}{m}V_v(r)+V_s(r)+\frac{1}{2m}[V_s^2(r)-V_v^2(r)]+V_D(r)\,, \quad (4.10b)$$

$$\tilde{V}^{\rm so}(r)=\frac{1}{2mr}\frac{\frac{d}{dr}[V_v(r)-V_s(r)]}{E+m+V_s(r)-V_v(r)}\,, \quad (4.10c)$$

where V_D is the Darwin term which, because of its negligible contribution, was omitted in our calculations.

In Figs. 1 and 2 we present the elastic differential cross sections and spin observables for p-^{4}He scattering at 800 MeV obtained using the method described above. Figure 1 represents the elastic differential cross section calculated in the $N=1$ (i.e., the first order optical potential), $N=2$, and $N=3$ approximations. While the $N=2$ approximation differs dramatically from the $N=1$, the difference between the $N=2$ and $N=3$ approximations is, for the range of momentum transfers covered by Fig. 1, practically negligible.

In Fig. 2 we compare predictions of the relativistic and nonrelativistic approaches for the differential cross section and the spin observables. On the basis of the results of our preliminary calculations (the ^{4}He wave function Ψ was parametrized as the product of the single particle s-wave functions with the harmonic oscillator parameter $b=1.37$ fm and the isospin degrees of freedom were taken into account through the isospin averaged NN amplitudes), we expect the relativistic effects to play a very important role in p-^{4}He scattering observables.

This work was supported in part by the U.S. Department of Energy.

[1]K. M. Watson, Phys. Rev. **105**, 1388 (1957).
[2]R. J. Eden, P. V. Landshoff, D. I. Olive, and J. C. Polkinghorne, *The Analytic S-Matrix* (Cambridge University, Cambridge, 1966); E. Bleszynski, M. Bleszynski, and T. Jaroszewicz (unpublished).
[3]A. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N.Y.) **8**, 551 (1959).
[4]H. Feshbach and J. Hüfner, Ann. Phys. (N.Y.) **56**, 268 (1970).
[5]R. A. Arndt and L. D. Roper, Phys. Rev. D **25**, 2011 (1982).
[6]J. Shepard, J. A. McNeil, and S. J. Wallace, Phys. Rev. Lett. **50**, 1443 (1983).
[7]D. L. Adams and M. Bleszynski, Phys. Lett. **150B**, 405 (1985).