Deuteron-proton breakup reaction at $E_d = 7.4$ MeV

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The reaction d+p→p+p+n is investigated at $E^d_{lab} = 7.4$ MeV. Calculations of the cross section are compared with recent kinematically complete measurements. The Coulomb potential is taken fully into account in the calculations based on the strong approximation of Møller wave operator approach.

I. INTRODUCTION

The three-nucleon system has been investigated for many years in the search for a phenomenological description via NN potentials and possible effects of NNN forces. While the meson theoretical NN potentials, like the Paris or the Bonn potentials, describe NN scattering up to several hundred MeV well, the influence of NNN forces is less clear. In the ^{3}H binding energy and ^{3}He charge form factor a NNN force seems to have a sizable effect.[1-3] Slaus *et al.*[4] explain the difference in a_{nn} extracted from the reaction ^{2}H(n,p)2n and from ^{2}H(π^-,γ)2n in terms of a NNN force. On the other hand, Faddeev-type calculations of the n-d breakup process at incident neutron energies of 7.5, 14.4, and 18.5 MeV which include a NNN force give effects of maximal 10% percent in the final state region.[5] A maximum effect of 2—3 % is estimated from the p-d breakup reaction at $E_p = 14.1$ MeV reported in Ref. 6. As NNN forces are likely to produce sizable effects in the bound state region it is intriguing to look for NNN force effects also in the very low energy scattering region. For reasons of experimental accuracy one should study the charged p-d reaction. Thus recently new kinematically complete measurements of the d-p breakup cross section very close to the threshold have been performed,[7] and an upper bound of 30% for NNN effects is estimated. However, in the low energy scattering region Coulomb effects are important,[8] which from the theoretical point of view make life more complicated. While at higher energies the Coulomb effects can be taken into account by some corrections to the neutral amplitude,[9] or like in Ref. 10 for the d-α breakup, Coulomb forces have to be fully taken into account in the p-d reaction close to the threshold.

Rigorous multichannel scattering theory for Coulomb-like (i.e., Coulomb plus short range) potentials has been formulated in Refs. 11—18. However, its application to charged few-body reactions often exhibits some difficulties.[19-26]

The calculations which are presented in this paper are based on the strong approximation of Møller wave operator (SAM) approach. This is a time dependent systematic approximation scheme for the calculation of S-matrix elements. It allows one to take into account the long range Coulomb force. The technique has been introduced in Ref. 27. The applicability and accuracy of the method has been investigated in the two-body system for p-p scattering where the Coulomb plus Graz potential has been utilized.[28] Similarities of this approach to Faddeev-type integral equations, e.g., the reduction to an effective two-body system of the three-body system if the potential is separable, have been pointed out in Ref. 29. First numerical results of an application of the method to the d-p breakup reaction including the Coulomb force have been reported in Refs. 30—32. In those calculations total, relative, and subsystem angular momenta have been limited to s and p waves. Moreover, the application of the SAM approach to a field theory model has been discussed in Ref. 33.

In this paper we report new calculations of the d-p breakup reaction at 7.4 MeV incident deuteron energy for several p-p detection angles. The calculations now include also the d wave for total, relative, and subsystem angular momenta. Some improvement of the overall shape is achieved.

In Sec. II the method is briefly reviewed. Section III presents the results and a discussion.

II. FORMALISM

The approximation scheme is based on two assumptions:

(a) Corresponding to a given full Hamiltonian $H = H^0 + V$, a finite dimensional Hamiltonian H_n can be found having a finite discrete spectrum, which approximates the spectrum of H. In particular, each scattering state of H, which has evolved from an asymptotic wave packet, can be approximated as a linear combination of eigenstates of H_n, and each bound state of H can be approximated in terms of eigenstates of H_n. In mathematical terms, one demands that H_n approximates H in the sense of strong resolvent convergence.

(b) A scattering matrix element can be approximately computed from scattering states using a finite time of evolution.

Some remarks seem in place. The idea to calculate scattering quantities from discrete Hamiltonians is not new in nuclear physics. It is used, e.g., in Ref. 34 for nuclear reactions in the particle-hole formalism, where the

scattering boundary conditions for the wave functions are formulated via the asymptotic behavior of the wave functions in coordinate space. In the SAM approach the boundary conditions are built in a different way, as will be pointed out below. If one wants to calculate a scattering matrix element for an N-body process in the SAM approach, only partial knowledge of the bound state spectrum is required. In general, only that part of the spectrum has to be approximated which is energetically accessible from the asymptotic states. This is in contrast to the Faddeev-Yakubovsky N-body integral equation formalism. Let us give an example. If one wants to calculate d-d elastic scattering at the total breakup threshold $E=0$ (i.e., the asymptotic state consists of a product of deuteron wave functions and a wave packet describing the d-d relative motion having a peak at $2\,|\,B_d\,|=4.45$ MeV) in the SAM approach one has to approximate the deuteron bound state. In the Faddeev-Yakubovsky formalism, where the calculation of an N-body Green's function proceeds recursively from $(N-1)$-body Green's functions (where the energy variable extends to minus infinity), in addition the triton bound state has to be calculated.

The second assumption is physically justified by a laboratory scattering measurement. Classically a finite time T can be interpreted as $R_{\rm int}/V_{\rm rel}$, where $R_{\rm int}$ is the interaction radius of a finite range potential and $V_{\rm rel}$ is the target-projectile relative velocity. Also in the case of the Coulomb potential with an infinite long range it makes sense to talk of a finite T (convergence of a T-dependent S-matrix element when T tends to infinity can be proven mathematically rigorously).

Based on the above two assumptions we obtain approximate scattering states by diagonalizing an approximate Hamiltonian and build in the scattering boundary conditions as prescribed by the Møller wave operators, using however a finite evolution time. In the case of a short range potential, where the Møller wave operators read

$$\Omega^{(\pm)}=s\text{-}\lim_{t\to\mp\infty}\exp(iHt)\exp(-iH^{as}t)\ , \tag{2.1}$$

which relates the asymptotic to the scattering states

$$|\,\psi^{(\pm)}\rangle=\Omega^{(\pm)}\,|\,\phi^{as}\rangle\ , \tag{2.2}$$

we substitute

$$\Omega^{(\pm)}\to\Omega_n(\mp T)=\exp(\mp iH_nT)\exp(\pm iH_n^{as}T)\ , \tag{2.3}$$

$$|\,\psi^{(\pm)}\rangle\to|\,\psi_n(\mp T)\rangle=\Omega_n(\mp T)\,|\,\phi^{as}\rangle\ , \tag{2.4}$$

and hence the S matrix

$$S=\Omega^{(-)^\dagger}\Omega^{(+)}\to S_n(+T,-T)=\Omega_n(+T)^\dagger\Omega_n(-T)\ . \tag{2.5}$$

The technical advantage of the substitution given by Eq. (2.3) is based on the fact that after diagonalization of H_n, H_n^{as}

$$H_n=\sum_\nu\,|\,e_\nu\rangle\epsilon_\nu\langle e_\nu\,|\ , \tag{2.6}$$

$$H_n^{as}=\sum_\nu\,|\,e_\nu^{as}\rangle\epsilon_\nu^{as}\langle e_\nu^{as}\,|\ , \tag{2.7}$$

$\Omega_n(\pm T)$ is obtained simply by

$$\Omega_n(\pm T)=\sum_{\nu,\mu}\,|\,e_\nu\rangle\exp(\pm i\epsilon_\nu T)\langle e_\nu\,|\,e_\mu^{as}\rangle\times\exp(\mp i\epsilon_\mu^{as}T)\langle e_\mu^{as}\,|\ . \tag{2.8}$$

Since the pioneering work of Dollard,[11] in cases where the long range Coulomb potential is involved the scattering boundary conditions can be formulated in terms of modified Møller wave operators.

$$\Omega^{c(\pm)}=s\text{-}\lim_{t\to\mp\infty}\exp(iHt)\exp[-iH^{as,c}(t)]\ , \tag{2.9}$$

where the detailed form of $H^{as,c}(t)$ for the p-d breakup reaction will be given below. The same type of approximations as given in Eqs. (2.1)—(2.8) are applied also in the Coulomb case. For the particular reaction $\mathrm{p}+\mathrm{d}\to\mathrm{p}+\mathrm{p}+\mathrm{n}$ studied here the modified Møller wave operators read as follows. For the incoming channel with a deuteron in subsystem 12, and a proton being particle 3, e.g., one has

$$\Omega_{12}^{c(+)}=s\text{-}\lim_{t\to-\infty}\exp(iHt)\exp[-iH_{12}^{as,c}(t)]P_{12}^{\rm d}\ . \tag{2.10}$$

Here H denotes the full Hamiltonian

$$H=H_0+V^s+V^c\ . \tag{2.11}$$

V^s denotes the strong short range potential, which we consider here to be a sum of NN-pair potentials

$$V^s=\sum_{\alpha=12,23,31}V_\alpha^s\ , \tag{2.12}$$

and V^c denotes the p-p Coulomb potential.

$$H_{12}^{as,c}(t)=H_{12}^{as,s}t+A_{12}^c(t)\ , \tag{2.13}$$

where $H_{12}^{as,s}$ is the asymptotic Hamiltonian, which also would occur in the neutral case,

$$H_{12}^{as,s}=H_0+V_{12}^s\ , \tag{2.14}$$

and $A_{12}^c(t)$ is Dollard's anomalous term

$$A_{12}^c(t)=\mathrm{sgn}(t)e_pe_d\left[\frac{m_{\rm p,d}}{2H_0^{\rm p,d}}\right]^{1/2}\log(4H_0^{\rm p,d}\,|\,t\,|)\ , \tag{2.15}$$

where e_p and e_d are the charges of p and d, respectively, $m_{\rm p,d}$ denotes the reduced mass of p and d, and $H_0^{\rm p,d}$ is the kinetic energy of the relative motion between the proton (particle 3) and the deuteron (subsystem 12). Finally the term $P_{12}^{\rm d}$ in Eq. (2.10) denotes a projector onto the deuteron bound state in the subsystem 12.

In the outgoing channel with particle 1 being the neutron and particles 2 and 3 denoting the protons, e.g., one has

$$\Omega_0^{c(-)}=s\text{-}\lim_{t\to+\infty}\exp(iHt)\exp[-iH_0^{as,c}(t)]\ , \tag{2.16}$$

where

$$H_0^{as,c}(t)=H_0t+A_0^c(t)\ , \tag{2.17}$$

$$A_0^c(t)=\mathrm{sgn}(t)e_p^2\left[\frac{m_{p,p}}{2H_0^{p,p}}\right]^{1/2}\log(4H_0^{p,p}|t|)\,. \quad (2.18)$$

Here $m_{p,p}$ denotes the reduced mass of the two protons and $H_0^{p,p}$ is the kinetic energy of the relative motion between the two protons. The breakup S matrix

$$S_{0\alpha}^c=\Omega_0^{c(-)^\dagger}\Omega_\alpha^{c(+)} \quad (2.19)$$

is related to the breakup T matrix, which enters in the cross section, via[35]

$${}_0\langle E'|S_{0\alpha}^c|E\rangle_\alpha=-2\pi i\delta(E'-E)\,{}_0\langle E'|T_{0\alpha}^c(E+i0)|E\rangle_\alpha. \quad (2.20)$$

The differential cross section is obtained from the T-matrix element in the same way as for short range forces. The phase space factor is given in Ref. 36. Now let us describe the finite dimensional approximation of the Hamiltonians. We have used the following expansion functions. For each n we have a cutoff $q_{\mathrm{cut}}^{(n)}$ in momentum space and a partition of the interval $[0,q_{\mathrm{cut}}^{(n)}]$

$$0=q_0<q_1<\cdots<q_n<q_{n+1}=q_{\mathrm{cut}}^{(n)}\,. \quad (2.21)$$

Later the limit $n\to\infty$ will be taken under the additional condition

$$\Delta q^{(n)}=\max_{i\in 0,\ldots,n}|q_{i+1}-q_i|\to 0, q_{\mathrm{cut}}^{(n)}\to\infty\,. \quad (2.22)$$

We use step functions

$$h_i(q)=\theta(q_{i+1}-q)\theta(q-q_i),\quad i=0,\ldots,n \quad (2.23)$$

(i.e., the characteristic function of the subinterval $[q_{i+1},q_i]$) to describe the $|\mathbf{q}|$ dependence, and spherical harmonics to describe the $\hat{\mathbf{q}}$ dependence of our basis functions, thus $h_{ilm}(\mathbf{q})=h_i(q)Y_{lm}(\hat{\mathbf{q}})$. In order to describe the relative and subsystem momentum dependence in the three-body system we use the basis $h_{ilm}(\mathbf{p})h_{j\lambda\mu}(\mathbf{q})$. As we have used in the actual calculations potentials which do not couple spin and angular momentum, we have coupled our expansion functions to good total angular momentum and spin. The finite dimensional Hamiltonians have been obtained from the original ones via orthogonal projections on the above expansion functions.

We want to make a remark on the treatment of the orbital angular momentum in the basis $h_{ilm}h_{j\lambda\mu}$; this basis refers to a fixed choice of subsystem and third particle, e.g., 1, 2, and 3. If one wants to describe a state of orbital angular momentum λ' in subsystem 13, one has to sum over infinitely many angular momentum states from the basis $h_{ilm}h_{j\lambda\mu}$. A situation like this occurs when subsystem 13 produces a final state peak. However, at such low energy experiments as described here, the final state peaks are not very pronounced. Also Faddeev calculations for the neutral case at this energy have shown that only a few partial waves are necessary in all subsystems and relative systems.[37] The situation is different if one goes to higher energies like, e.g., 50 MeV where the final state peak is very pronounced[38] and many partial waves are needed. For such cases the use of our basis referring to a fixed subsystem could become problematic.

III. RESULTS AND DISCUSSION

The strong NN potential used in the calculations is of separable form, rank two (corresponding to the singlet and triplet channel) with s-wave form factors of Gaussian-type. The parameters were chosen to reproduce the low energy data deuteron binding energy, singlet and triplet scattering length, and singlet effective range. The potential and its numerical parameters are given in Ref. 31. We used for the expansion functions $q_{\mathrm{cut}}=p_{\mathrm{cut}}=1$ fm^{-1} and the following upper limits: Ten step functions for the $|\mathbf{p}|$ dependence as well as for the $|\mathbf{q}|$ dependence, s, p, and d partial waves for the total, relative, and subsystem angular momenta, and the full spin dependence was accounted for.

We have calculated at $E_{\mathrm{lab}}^{\mathrm{d}}=7.4$ MeV the breakup cross section for a set of p-p detection angles. However, only

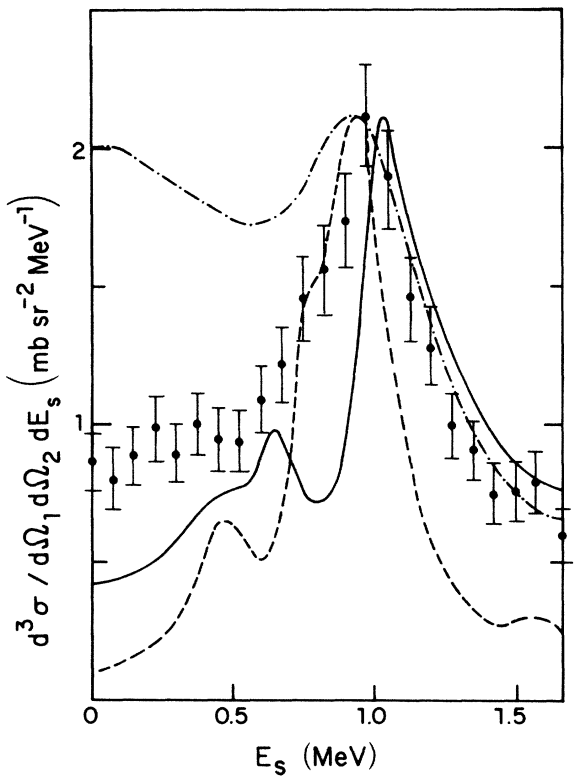

FIG. 1. Kinematically complete d-p breakup cross section at $E_{\mathrm{lab}}^{\mathrm{d}}=7.4$ MeV. E_s is measured along the kinematical curve of the two outgoing protons. $E_s=0$ corresponds to the point where the outgoing protons have the same energy (close to the origin). The p-p detection angles are 13°—13°. The dashed and the full curve display the results of the SAM calculation. In the dashed curve s and p waves have been used for total, relative, and subsystem angular momenta. The full curve includes also the d wave. Both curves are normalized to the height of the experimental peak. For comparison the dashed dotted line shows the results of a d-n Faddeev calculation (Ref. 37) also normalized to the height of the experimental peak of the d-p reaction.

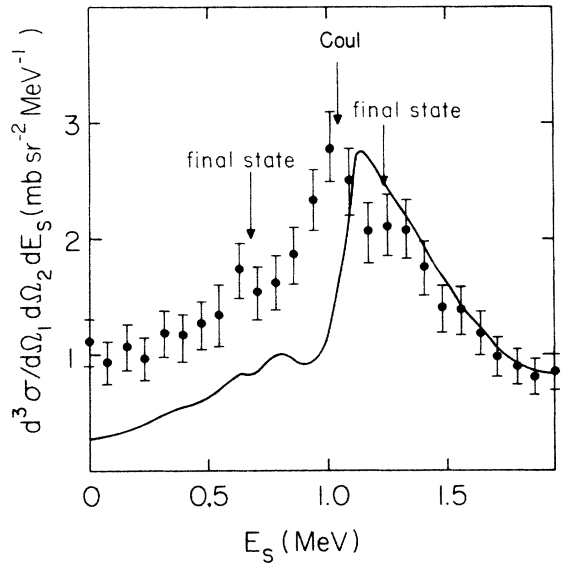

FIG. 2. Same as Fig. 1, but at 12°—12°.

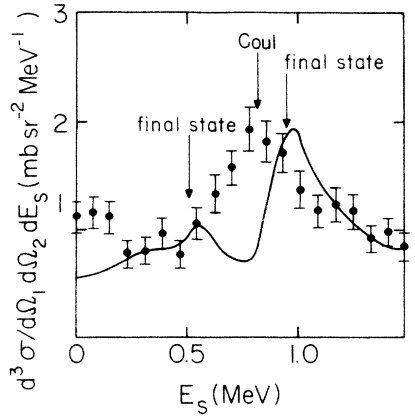

FIG. 4. Same as Fig. 1, but at 13.5°—13.5°. Same comment as in Fig. 3 concerning the region around $E_s=0$.

the relative shape has been calculated, the absolute normalization was adjusted to the experimental peak height. The results are displayed in Figs. 1—5. In Fig. 1 the dashed curve shows the results of the SAM calculation including only s and p waves for total, subsystem, and relative angular momenta. The results of the calculation including s, p, and d waves are displayed in Figs. 1—5 by the full curve. It is known from Faddeev calculations[37] at similar energies in the case of the neutral n-d breakup reaction that the inclusion of d waves gives a smaller contribution. The rather strong effect of d waves in the d-p breakup could result from the Coulomb field interference or from a numerical inaccuracy; this point has not been resolved yet.

In Fig. 1 the inclusion of the d wave gives a relative better shape at both ends of the kinematical curve. It would be important to study the influence of higher partial waves and also the number of step functions. The

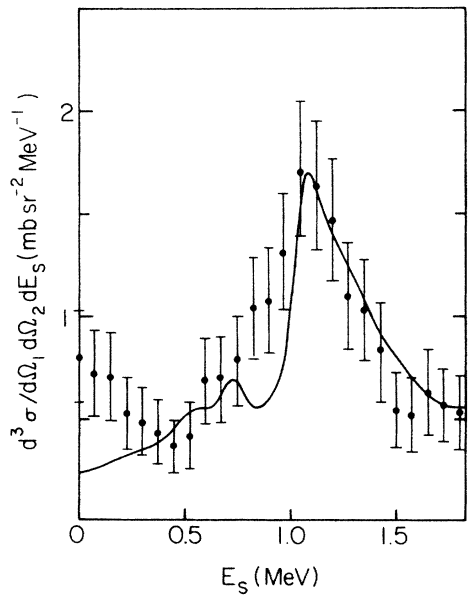

FIG. 3. Same as Fig. 1, but at 12.5°—12.5°. There is a noticeable discrepancy near $E_s=0$ which could be ascribed to the two-pion exchange three-body force (Ref. 7).

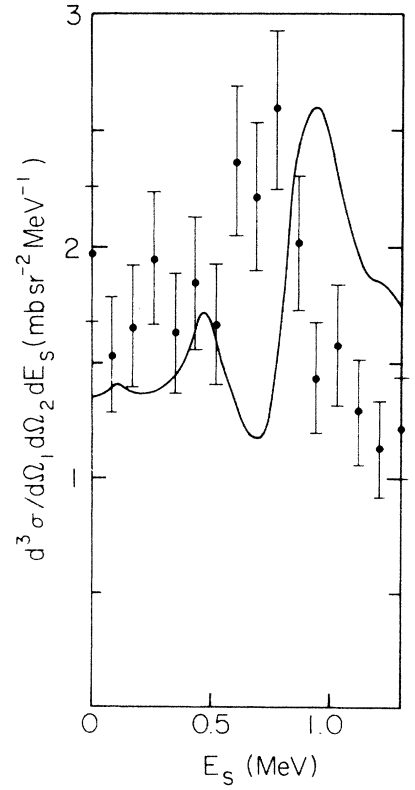

FIG. 5. Same as Fig. 1, but at 14°—14°.

present restrictions are due to the limited available computer storage.

In order to demonstrate the effect of the Coulomb force at the low energy considered here, we have displayed in Fig. 1 the result of a Faddeev calculation[37] for the neutral d-n breakup reaction. Also in this case the calculated curve has been adjusted to the height of the experimental d-p peak (the calculated d-n peak is about a factor 47 larger than the measured d-p peak). The relative shape of the d-n curve differs also largely from the measured and calculated d-p curve. In Ref. 32 it has been pointed out that in all cases, with different p-p angles, the position of the experimental peak lies very close to the point where the relative energy of the outgoing protons has a maximum (indicated in Figs. 2 and 4 by "Coul"). Thus the general shape of the cross sections having a pronounced peak and a sharp fall off behavior could be explained in terms of the repulsive p-p Coulomb force in the outgoing channel which tends to suppress configurations with a small relative momentum (or energy) of the protons and enhances configurations with a large relative momentum. It should be noted that in the final state the center of mass energy of the three nucleons is only 0.241 MeV, and at very low energies the Coulomb potential tends to dominate over the strong short range potential. On the other hand, the well pronounced peak with a sharp fall off behavior is not found in the neutral d-n case. However, a weak maximum is found there at nearly the same position. Thus it is possible that in the d-p and at the d-n spectra a peak reminiscent of the quasifree scattering peak is present, although strictly speaking quasifree scattering can only occur above the threshold $E_d > 4 \mid B^d \mid$.

IV. CONCLUSION

Kinematically complete measurements of the d-p breakup cross section at $E_{lab}^{d}=7.4$ MeV are compared with calculations which fully take into account the Coulomb potential. The calculations roughly reproduce the overall shape. Some improvement is obtained by inclusion of d waves. The importance of the Coulomb force is clearly demonstrated because a major difference is obtained between the neutral d-n breakup cross section obtained from a Faddeev calculation and the cross section of the charged d-p reaction obtained from experiment as well as from the calculations. Some clear discrepancies remain in the region corresponding to symmetric energy partitions between the two protons in the final state.

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