Coexistence models for the 0_1^+ and 0_2^+ states in Ge isotopes

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It is shown that a two-state model in which two-neutron and alpha transfer amplitude ratios vary linearly with mass leads to solutions for 0_1^+ and 0_2^+ states in Ge isotopes which give consistent agreement with experimental data. The corresponding energy gap between basis states decreases linearly with increasing neutron number, suggesting that these states differ mainly by proton excitation.

A variety of models has been used in attempts to interpret the structure of germanium isotopes and to account for the anomalous behavior of several observables as functions of neutron number. Stewart and Castel[1] proposed that low-lying states of ^{70}Ge and ^{72}Ge are mixtures of spherical vibrational and axially symmetric deformed rotational states, and showed that their model could reproduce energy levels and $B(E2)$ values if interaction matrix elements were derived from experiment. The static Hartree-Fock calculations of Ardouin *et al.*[2] suggested that a transition from prolate to oblate intrinsic states should occur as neutron number increases, and the dynamic deformation theory of Kumar,[3] which expressed Ge states as linear combinations of all possible symmetric and asymmetric deformations, was able to reproduce the trend of 0_2^+ energies. The shell-model calculations of Sakakura *et al.*[4] failed to reproduce the 0_2^+ states in ^{70}Ge and ^{72}Ge when valence nucleons were restricted to the $p_{3/2}$, $f_{5/2}$, and $p_{1/2}$ orbitals; the authors attributed this to the omission of the $g_{9/2}$ orbital, and proposed on the basis of a weak-coupling model that the 0_2^+ states of $^{68-72}$Ge are $(g_{9/2})^2$ states built on the $^{66-70}$Ge ground states. The configuration mixing IBM-2 calculation of Duval *et al.*[5] used two distinct sets of s and d bosons for protons, and chose the energy gap between configurations to reproduce the experimental 0_2^+ energies. This model, in which the 0_2^+ states are predominantly proton excitations of the ground states, was able to reproduce the general trends of (t,p) and (p,t) cross sections.

Although these models give rather different interpretations of the structure of the 0_1^+ and 0_2^+ states, a feature common to them is the coexistence of two distinct basis states characterized by particular shapes or configurations. Recently, Cardici and co-workers[6-8] have investigated the extent to which a two-state model can account for properties of these 0^+ states, while specifying little about the actual form of the basis states. In this paper we examine some of their assumptions, and show that a simple modification leads to two possible solutions which are in more consistent agreement with experimental data.

Cardici *et al.*[6] introduce basis states Φ_g^A and Φ_e^A such that the two-neutron transfer amplitudes $\Phi_g^A \rightleftarrows \Phi_g^{A+2}$, $\Phi_e^A \rightleftarrows \Phi_e^{A+2}$, $\Phi_g^A \rightleftarrows \Phi_e^{A+2}$, and $\Phi_e^A \rightleftarrows \Phi_g^{A+2}$ are in the mass-independent ratios $1:R:r:r$. They show that experimental (p,t) and (t,p) cross sections can be fitted with any R in a certain range, provided R and r are connected by the relation

$$r^2 = R + K(R+1)^2 \ , \tag{1}$$

where K is a function of the data which, to within experimental errors, is mass independent. In their paper on alpha transfer to Ge isotopes, Cardici and Fortune[8] find that the goodness of fit of calculated cross-section ratios to experimental data is independent of R. However, this result should not be surprising, because it is a consequence of the fact that *any* choice of R and r satisfying (1) gives a basis spanning the same two-dimensional space, and therefore will lead to the same observable results. Basis states $\Phi_g^A(R,r)$ and $\Phi_e^A(R,r)$ are orthogonal linear combinations of the "diagonal basis" $\Phi_g^A(R_D,0)$ and $\Phi_e^A(R_D,0)$,

$$\Phi_g^A(R,r) = a\Phi_g^A(R_D,0) + b\Phi_e^A(R_D,0) \ ,$$

$$\Phi_e^A(R,r) = b\Phi_g^A(R_D,0) - a\Phi_e^A(R_D,0) \ .$$

Amplitude ratios are then

$$R = \frac{b^2 + a^2 R_D}{a^2 + b^2 R_D} \ ,$$

and

$$r = \frac{ab(1-R_D)}{a^2 + b^2 R_D} \ .$$

Elimination of a and b gives

$$r^2 = R - \frac{R_D}{(R_D+1)^2}(R+1)^2 \ ,$$

which is the same as (1) [Eq. (9) of Ref. 6] if

$$K = -\frac{R_D}{(R_D+1)^2} \ .$$

All the bases considered by Cardici and co-workers,[6-8] consistent with $K = -0.2448$, can therefore be generated by unitary transformations of the diagonal basis with $r = 0$, $R = R_D = 1.337$.

Fortune, Cardici, and Mordechai[7] examine proton occupation numbers in the ground states of Ge isotopes making use of the wave functions from Ref. 6. An assumption in their calculation is that the matrix element

$$n_{ge} = \langle \Phi_g^A | \hat{N}_f | \Phi_e^A \rangle$$

is zero, where $\hat{N}_f$ is the number operator for $f_{5/2}$-shell protons. However, this can be true for only one particular value of R. If n_{ge} is allowed to vary along with the diagonal matrix elements

$$n_g = \langle \Phi_g^A | \hat{N}_f | \Phi_g^A \rangle \ ,$$

and

$$n_e = \langle \Phi_e^A | \hat{N}_f | \Phi_e^A \rangle \ ,$$

the goodness of fit to empirical occupation numbers will be independent of R, so the plot of χ^2 vs R in Ref. 7 can have relevance only in suggesting the R value for which n_{ge} is zero. The favored range of R is not supported, however, by other considerations based on the following reasoning. If the wave functions of basis states are separable into a product of neutron and proton components, the proton overlap of $\Phi_g^A(R,r)$ and $\Phi_e^{A+2}(R,r)$ must be nonzero when $r \neq 0$ to account for the two-neutron transfer amplitude. Since it is assumed that the proton structure of Φ_e^{A+2} and Φ_e^A is the same, this implies that the proton overlap of $\Phi_g^A(R,r)$ and $\Phi_e^A(R,r)$ is also nonzero if $r \neq 0$. It is to be expected that the zero value of n_{ge} will occur for a value of R close to that giving zero proton overlap, which should be close to the diagonal basis with $R = R_D$. This is clearly not the case for the calculation of Ref. 7, because $R = 1.337$ gives large χ^2 and an unphysical negative value of n_g.

In an attempt to obtain consistent results for both two-neutron transfer and proton pickup data, we have modified the assumptions made in Ref. 6. There, R and r were taken to be mass independent, implying that R_D is also mass independent for the diagonal basis having $r = 0$. However, this is difficult to justify, because it is to be expected that the neutrons added to $\Phi_g^A(R_D,0)$ to give $\Phi_g^{A+2}(R_D,0)$, and to $\Phi_e^A(R_D,0)$ to give $\Phi_e^{A+2}(R_D,0)$, will have shell occupancies which vary somewhat with A, and the strengths for two-neutron transfer to orbitals close to the Fermi surface differ by large factors.[9,10] To allow for some variation in R_D while keeping the number of parameters to a minimum we have set

$$R_D(A) = R_D(70) + (A-70)\lambda \ ,$$

i.e., we have assumed an amplitude ratio which varies linearly with mass.

The (p,t) and (t,p) data give six cross-section ratios[6] which can be used to deduce the two-state admixtures

$$0_1^+ = \alpha_A \Phi_g^A(R_D,0) + \beta_A \Phi_e^A(R_D,0) \ ,$$

$$0_2^+ = \beta_A \Phi_g^A(R_D,0) - \alpha_A \Phi_e^A(R_D,0) \ ,$$

for even $A = 70$–76. One therefore has six data to give the four ratios $X_A = \alpha_A/\beta_A$, together with $R_D(70)$ and λ. In addition to the solution with $\lambda = 0$ found in Ref. 6, we find that the data can be fitted with $\lambda \approx -0.11$ and -0.18. The former solution gives $R_D(70) = 1.287$ and $X_A = 2.498$, 1.744, 0.686, and 0.491 for $A = 70$–76, while the second solution gives $R_D(70) = 1.435$ and $X_A = 4.891$, 2.919, 1.025, and 0.909.

Unlike the solution with $\lambda = 0$, both solutions with nonzero λ give good fits to the $f_{5/2}$ proton occupation numbers in $A = 70$–76 ground states when it is assumed that $\langle \Phi_g^A(R_D,0) | \hat{N}_f | \Phi_e^A(R_D,0) \rangle = 0$. Results are shown in Table I. Experimental values are taken from Ref. 11, renormalized so that the sum of $f_{5/2}$, $g_{9/2}$ and p-shell protons is equal to 4. For $\lambda = -0.11$ the best-fit values of n_g and n_e are 0.85 and 2.55, and the rms deviation between empirical and calculated occupation numbers is 0.03. For $\lambda = -0.18$, n_g and n_e are 1.02 and 3.15, and the rms deviation is 0.05.

Cardici *et al.*[6] deduce the effective interaction matrix element connecting Φ_g^A and Φ_e^A, which in a two-state model is given by

$$V_A = -E_A \alpha_A \beta_A \ ,$$

where E_A is the actual energy gap between the 0_1^+ and 0_2^+ states in ^{A}Ge. Although the authors state that it might be hoped that V_A would be independent of A, this does not appear to be required by the model. Of more interest is Δ_A, the energy gap between $\Phi_e^A(R_D,0)$ and $\Phi_g^A(R_D,0)$ before mixing, because if the two basis states differ only in their proton structure, and if this is independent of A, then Δ_A will arise from the monopole part of the neutron-proton interaction and should decrease linearly with neutron number. Figure 1 shows the gap, deduced from

$$\Delta_A = E_A(1 - 2\beta_A^2) \ .$$

Deviations from linearity are small, especially for $\lambda = -0.11$, lending support to the proton-excitation hypothesis. The fact that R_D is not one requires that there be *some* difference between the neutron structure of Φ_e^A and Φ_g^A, but the deduced magnitude of R_D is quite close to unity and can be explained with only minor configurational differences because of the extreme sensitivity of the two-neutron transfer cross section to the orbitals which are occupied.[9,10]

Cardici and Fortune[8] investigate alpha stripping and pickup to the 0_1^+ and 0_2^+ states. They find that the two-state

TABLE I. $f_{5/2}$ proton occupation numbers.

	Expt.	Calc. $\lambda = -0.11$	Calc. $\lambda = -0.18$
^{70}Ge	1.12	1.08	1.10
^{72}Ge	1.23	1.27	1.24
^{74}Ge	1.98	2.01	2.06
^{76}Ge	2.25	2.22	2.18

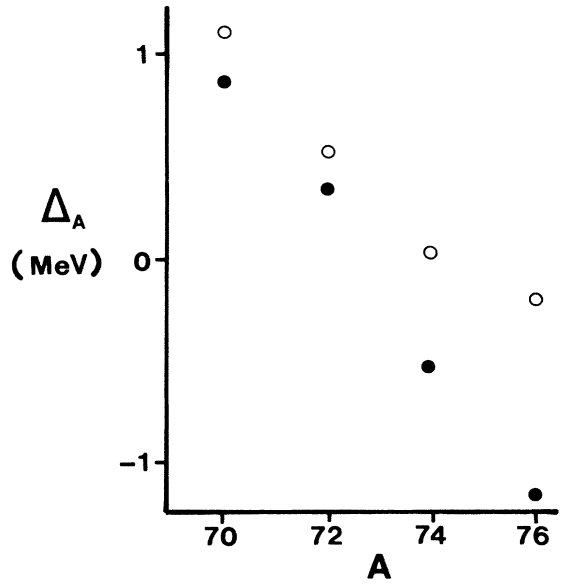

FIG. 1. Energy gap between $\Phi_e{}^A(R_D,0)$ and $\phi_g^A(R_D,0)$ in ^{A}Ge. Closed circles are for $\lambda = -0.11$, open circles are for $\lambda = -0.18$.

model with $\lambda = 0$ cannot account for empirical cross-section ratios if u_A and v_A, defined as

$$u_A = \frac{\langle {}^{A+4}\mathrm{Se}|\Phi_e^A + \alpha\rangle}{\langle {}^{A+4}\mathrm{Se}|\Phi_g^A + \alpha\rangle}$$

and

$$v_A = \frac{\langle {}^{A-4}\mathrm{Zn} + \alpha|\Phi_e^A\rangle}{\langle {}^{A-4}\mathrm{Zn} + \alpha|\Phi_g^A\rangle}$$

are taken to be mass independent. We find that the same is true for the solutions with nonzero λ. However, a simple linear dependence of u_A and v_A on neturon number is all that is required to give calculated cross-section ratios in agreement with experiment. With the values of X_A derived from $\lambda = -0.11$, the (d, ^{6}Li) ^{A}Ge data for even $A = 70$–76 is reproduced within experimental errors by

$$u_A = 1.2 + 0.2(A - 70) \ ,$$

and the (^{6}Li, d) ^{A}Ge data for even $A = 70$–74 is reproduced within errors by

$$v_A = 0.17 + 0.025(A - 70) \ .$$

In summary, we have shown that a modification of the two-state model of Cardici and co-workers can, with a few plausible assumptions, account for properties of the 0_1^+ and 0_2^+ states in even mass Ge isotopes of even $A = 70$–76. It remains to be seen whether the numerical values of the parameters of the model can be justified by detailed calculations using more microscopic models; in particular, the shell model with inclusion of the $g_{9/2}$ orbital for at least the neutrons.

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