Symmetric and antisymmetric states: A general feature of two-component systems

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It is shown that in both the nuclear shell model, considering explicitly proton and neutron degrees of freedom (seniority scheme), and in particle-core coupling model calculations, 2^+ levels with symmetric and antisymmetric character in the constituent unperturbed basis result. Besides the excitation energy for such 2^+ levels, the $E2$ decay properties also are studied. Constructive and destructive interference between the two distinct $E2$ matrix elements results for the 2_1^+ (symmetric) and 2_2^+ (antisymmetric) levels, respectively. Applications are shown for the Cd ($Z=48$) and the $N=84$ nuclei.

I. INTRODUCTION

Recently it was shown, within the framework of the proton-neutron interacting boson model (IBM-2), that besides the symmetric proton-neutron states (identical to IBM-1 states), a large class of states with mixed-symmetry character does arise.[1–6] This is a consequence of the fact that although the total boson wave function has to be symmetric under the interchange of *all* variables, a mixed-symmetry character can arise for both the spatial (*sd* part) and the proton-neutron charge part of it. Mathematically, this can be formulated via the reduction of the U(12) group containing both proton and neutron bosons, as $U(12)\supset U_{\pi+\nu}(6)\times SU^{(F)}(2)\supset \dots$.[7,8] In both the theoretical[1–9] and experimental[10–12] studies of such mixed-symmetry states, most interest has gone into the study of 1^+ levels. Recently, Zamick pointed out[13] that in the $1f_{7/2}$ nuclei some of the even-even Ti isotopes show 1^+ states with a character, similar to the mixed-symmetry states in IBM-2, and some of the authors[14,15] also showed that near $A\simeq 100$ (^{98}Pd) such a 1^+ level could be well described within the nuclear shell model.

It is the purpose of this paper to point out the general character of the existence of mixed-symmetry states wherever *two* distinct basic blocks are used to construct the nuclear wave functions (Sec. II). This is most easily formulated within the nuclear shell-model properly, using a single-j shell or degenerate j shell (see Sec. III), but also within the particle-core coupling model such states do arise (Sec. IV). In both Secs. III and IV we concentrate on the $J^\pi=2^+$ states, the wave functions describing these states, and their $E2$ properties.

II. GENERAL TWO-COMPONENT SYSTEMS

Before concentrating on particular nuclear model applications, we shortly describe a general nuclear two-component system. If each subsystem is characterized by only a single 0^+ and 2^+ state, i.e., described by states $|0^+(1)\rangle$, $|2^+(1)\rangle$, the combined system only contains the

$$|0^+(1)0^+(2);0^+\rangle\ ,$$

$$|2^+(1)0^+(2);2^+\rangle\,|0^+(1)2^+(2);2^+\rangle\ ,$$

and

$$|2^+(1)2^+(2);I\rangle$$

basis states. Here, one notices that for angular momentum 2^+, three basis states do result. Depending on the unperturbed energy of the 0^+ and 2^+ excitations in the separate systems and on the residual interaction, this particular two-component system can be solved numerically for its energies and wave functions. If now the $|0^+(1)2^+(2);2^+\rangle$ and $|2^+(1)0^+(2);2^+\rangle$ states are degenerate in energy and the $|2^+(1)2^+(2);2^+\rangle$ state has a much larger unperturbed energy, which is often not too far from realistic situations (see Secs. III and IV), even independent of the residual interaction V_{12} between the two subsystems, a symmetric and antisymmetric eigenstate will result, i.e., one obtains

$$|2_1^+\rangle=\frac{1}{\sqrt{2}}[\,|0^+(1)2^+(2);2^+\rangle+|2^+(1)0^+(2);2^+\rangle]\ , \tag{2.1}$$

$$|2_2^+\rangle=\frac{1}{\sqrt{2}}[\,|0^+(1)2^+(2);2^+\rangle-|2^+(1)0^+(2);2^+\rangle]\ . \tag{2.2}$$

The electric quadrupole electromagnetic decay properties ($E2$ decay) are now described by an operator which is the sum of two components, each acting in one of the subsystems, i.e.,

$$T(E2)=e_1T(E2;1)+e_2T(E2;2)\ , \tag{2.3}$$

where e_1,e_2 are the effective charges in the different subsystems and $T(E2;i)$ denotes the operator acting in the ith subsystem. The reduced transition matrix element for decay towards the 0^+ ground state $|0_1^+\rangle$, being almost purely the $|0^+(1)0^+(2);0^+\rangle$ basis state, then becomes

$$\langle 0_1^+ || T(E2) || 2_i^+ \rangle = \frac{1}{\sqrt{2}} [\langle 0^+(1) || T(E2;1) || 2^+(1) \rangle e_1 - (-1)^i \langle 0^+(2) || T(E2;2) || 2^+(2) \rangle e_2] . \quad (2.4)$$

If the separate systems have reduced matrix elements of a similar structure (both being particlelike or holelike in fermion space) and thus the same sign, coherence or incoherence results for $i=1$ and $i=2$, respectively. If $e_1 \simeq e_2$, the coherence and incoherence is optimal.

We now discuss in detail applications of two-component systems that have often been treated in describing low-energy nuclear structure:

(i) lowest seniority ($v=2$) shell-model calculations where both active protons and neutrons are present in a single j-shell approximation,

(ii) particle-core coupling where a two-particle (-hole) fermion cluster is coupled to the collective excitations of a surface quadrupole harmonic vibrator.

III. THE NUCLEAR SHELL MODEL: LOWEST SENIORITY ($v=2$) CALCULATIONS

A. Excitation energy in a single-j shell

Recently, we have pointed out how in a single-j shell starting from two valence proton particles (or holes) and two valence neutron particles (or holes) collectivity arises in an exact solvable model.[15] If we now extend the calculations to cases where one has n_π protons and n_ν neutrons, but still considering seniority $v=2$ configurations for describing the lowest 2^+ states, the basic configurations, already taking into account the pairing properties of the identical nucleon interaction in a single-j shell, are

$$|(j_\nu)^{n_\pi}_{v=2};2^+\rangle ; \; |(j_\pi)^{n_\pi}_{v=0};0^+\rangle , \quad (3.1)$$

$$|(j_\nu)^{n_\nu}_{v=2};2^+\rangle ; \; |(j_\nu)^{n_\nu}_{v=0};0^+\rangle , \quad (3.2)$$

respectively. It will now be studied how the 2_1^+ gets lowered (becomes collective) in the vibrational and transitional nuclei by taking the appropriate combinations of (3.1) and (3.2) and diagonalizing within the 2×2 model space for the 2_π^+ and 2_ν^+ levels, i.e., one obtains

$$\begin{aligned} |2_\pi^+\rangle &\equiv |[(j_\pi)^{n_\pi}_{v=2};2^+(j_\nu)^{n_\nu}_{v=0};0^+]2^+\rangle , \\ |2_\nu^+\rangle &\equiv |[(j_\pi)^{n_\pi}_{v=0};0^+(j_\nu)^{n_\nu}_{v=2};2^+]2^+\rangle , \end{aligned} \quad (3.3)$$

as unperturbed 2^+ configurations. Taking into account that these configurations describe seniority $v=2$ 2^+ states within a series of isotopes (i.e., Sn nuclei) or isotones (i.e., $N=50$, 82 nuclei), the unperturbed energy is almost constant. If we take the extra approximation to use for configurations (3.3) the same unperturbed energy, i.e., $\epsilon_2^0 \equiv \epsilon^0_{2_\pi^+} \simeq \epsilon^0_{2_\nu^+}$ the 2×2 model space is easily diagonalized when using as a residual proton-neutron interaction a quadrupole force $-\kappa Q_\pi \cdot Q_\nu$ (with $Q_\rho \equiv [\sqrt{m\omega/\hbar} r_\rho]^2 Y_2(\hat{r}_\rho)$). The coupling matrix element becomes

$$\begin{aligned} V_{\pi\nu} &\equiv \langle 2_\pi^+ | -\kappa Q_\pi \cdot Q_\nu | 2_\nu^+ \rangle \\ &= -\frac{\kappa}{5} \langle (j_\pi)^{n_\pi}_{v=2};2^+ || Q_\pi || (j_\pi)^{n_\pi}_{v=0};0^+ \rangle \langle (j_\nu)^{n_\nu}_{v=2};2^+ || Q_\nu || (j_\nu)^{n_\nu}_{v=0};0^+ \rangle \\ &= -\frac{\kappa}{5} \left[N_\pi \left[1 - \frac{N_\pi}{\Omega_\pi} \right] N_\nu \left[1 - \frac{N_\nu}{\Omega_\nu} \right] \right]^{1/2} \left[\left[\frac{2}{\Omega_\pi - 1} \right]^{1/2} \left[\frac{2}{\Omega_\nu - 1} \right]^{1/2} \langle j_\pi || Q_\pi || j_\pi \rangle \langle j_\nu || Q_\nu || j_\nu \rangle \right] , \end{aligned} \quad (3.4)$$

where we call the quantity between curly brackets $F(\Omega_\pi, \Omega_\nu)$ and N_π ($\equiv n_\pi/2$) and N_ν ($\equiv n_\nu/2$) describe the number of nucleon pairs. The matrix to be diagonalized then becomes

$$\begin{bmatrix} \epsilon_2^0 & V_{\pi\nu} \\ V_{\pi\nu} & \epsilon_2^0 \end{bmatrix} \quad (3.5)$$

with eigenvalues

$$E(2_i^+) = \epsilon_2^0 - \epsilon_i \frac{\kappa}{5} \left[N_\pi N_\nu \left[1 - \frac{N_\pi}{\Omega_\pi} \right] \left[1 - \frac{N_\nu}{\Omega_\nu} \right] \right]^{1/2} F , \quad (3.6)$$

and corresponding eigenvectors, the linear combinations (see Fig. 1),

$$|2_i^+\rangle = \frac{1}{\sqrt{2}} (|2_\pi^+\rangle + \epsilon_i |2_\nu^+\rangle) . \quad (3.7)$$

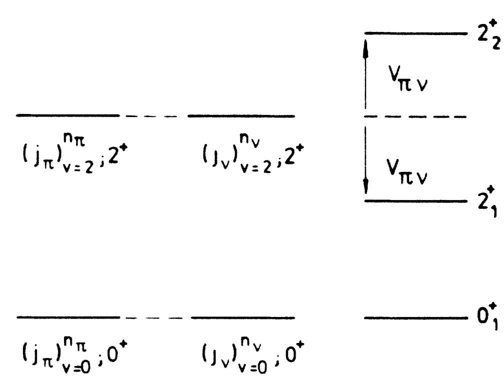

FIG. 1. The unperturbed, seniority $v=2$ proton and neutron shell-model configurations [see Eqs. (3.1) and (3.2)] and the two $J^\pi=2^+$ levels obtained after diagonalizing the quadrupole proton-neutron interaction within the two-level model space of Eqs. (3.3).

The value of $\epsilon_1=1$, $\epsilon_2=-1$ for particle-particle (or hole-hole) combinations of protons and neutrons; $\epsilon_1=-1$, $\epsilon_2=+1$ for particle-hole (or hole-particle) combinations of protons and neutrons. In the IBM-2, the lowest 2^+ state, i.e., 2_1^+ is always defined as the symmetric combination of proton and neutron boson wave functions. Thus, in a simple way, a lowest symmetric 2_1^+ level with a specific $N_\pi N_\nu$ dependence for the excitation energy $E(2_1^+)$ results. For small N_π/Ω_π, N_ν/Ω_ν the $E(2_1^+)$ energy will at first lower according to a $N_\pi N_\nu$ dependence. The correction terms $\sqrt{1-(N_\pi/\Omega_\pi)}$ and $\sqrt{1-(N_\nu/\Omega_\nu)}$ will gradually weaken the lowering and result into a minimal excitation energy at midshell occupation $N_\pi=\Omega_\pi/2$, $N_\nu=\Omega_\nu/2$. Recently, Casten[16–18] has pointed out a very striking observation that quantities such as $E(2_1^+)$, $E(4_1^+)/E(2_1^+)$, etc. lie on a smooth curve when plotted as a function not of N,Z but of the product $N_\pi N_\nu$. At least in the vibrational (transitional) region, the above simple shell-model calculations predict such a behavior for $E(2_1^+)$.

In Figs. 2 and 3, we have carried out the above analysis in a theoretical way, in order to point out that indeed all $E(2_1^+)$ energies follow a smooth behavior when plotted as a function of the product $N_\pi N_\nu$. Moreover, fits have been carried out for the Te, Xe, and Ba nuclei[19] using expression (3.6) with

(i) a constant value of κ for all nuclei (Fig. 4);

(ii) a slightly different κ for each series of isotopes (Fig. 5).

In the calculations, a value of $\epsilon_2^0=1.344$ MeV has been used, which is the average of $\epsilon_{2_\pi^+}^0$ [$=1.477$ MeV; the aver-

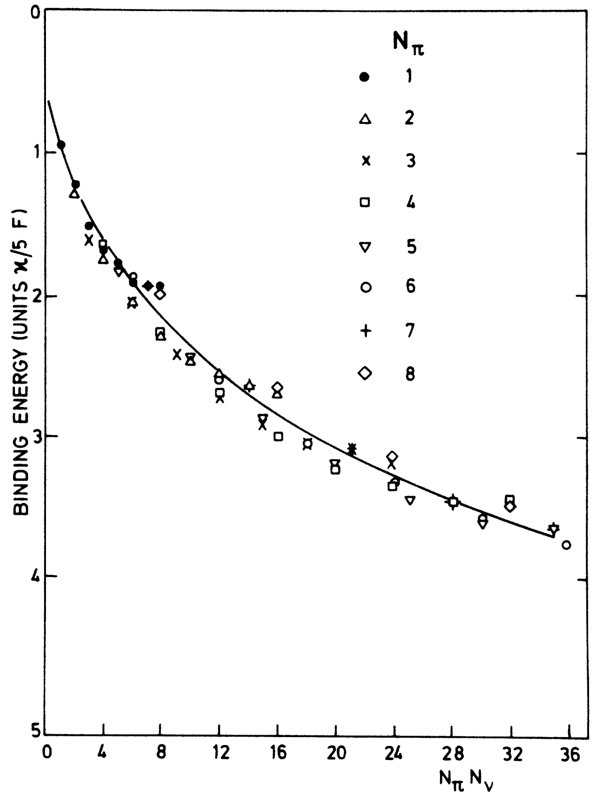

FIG. 3. The energy of the lowest 2^+ level [Eq. (3.6)], in units of $(\kappa/5)F$, as a function of the product $N_\pi N_\nu$ for nuclei in the ($N_\pi=1$) to ($N_\pi=8$) region.

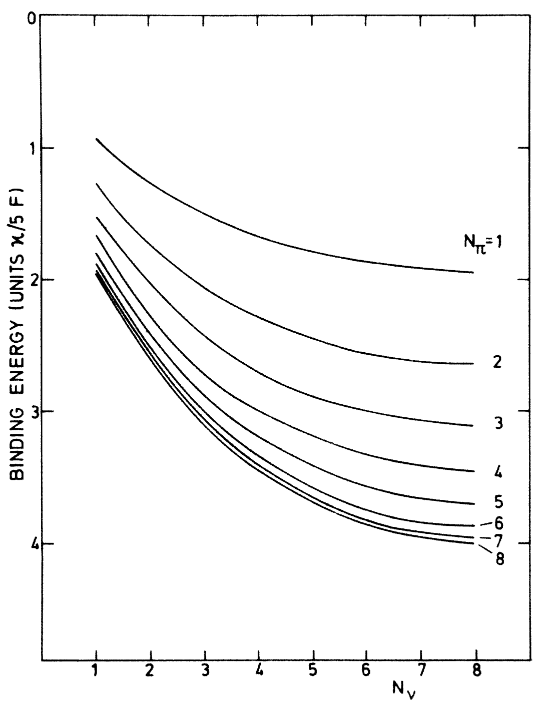

FIG. 2. The energy for the lowest 2^+ level [Eq. (3.6)], in units of $(\kappa/5)F$, as a function of the number of neutron pairs N_ν and of the number of proton pairs N_π.

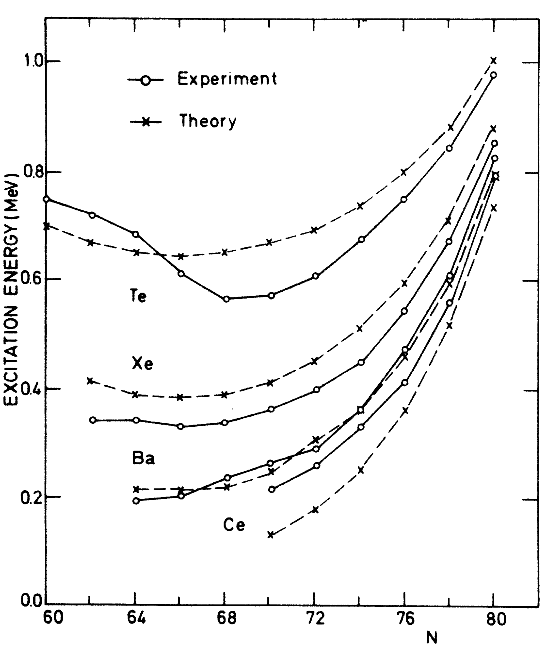

FIG. 4. Detailed fit, using Eq. (3.6), to the Te, Xe, Ba, and Ce nuclei, with the degeneracy $\Omega_\pi=16$, $\Omega_\nu=16$, using a constant value of $(\kappa/5)F=0.365$ MeV.

age of $E(2_1^+)$ in the $N=82$ nuclei with $52 \le Z \le 62$)] and $\epsilon^0_{2_\nu^+}$ [$=1.210$ MeV; the average of $E(2_1^+)$ in the Sn nuclei with $60 \le N \le 80$]. We took, moreover, degenerate single-particle states thereby using $\Omega_\pi = \Omega_\nu = 16$. The average value of the fitted parameter $(\kappa/5)F = 0.365$ MeV (see Fig. 4) can easily be converted to a value of κ, using the $j \to \infty$ estimate in calculating the factor $F(\Omega_\pi, \Omega_\nu)$ which gives a value of $F \simeq 12$ for the harmonic oscillator $N=4$ shell. One then obtains as the strength of the proton-neutron shell-model quadrupole interaction the value of $\kappa = 0.15$ MeV, which is very near the value used in calculating nuclear shell-model spectra in odd-odd Sb and I nuclei.[20]

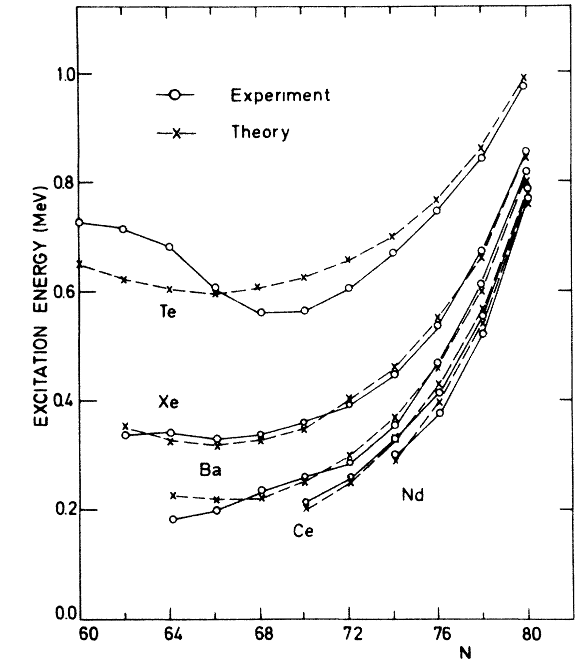

FIG. 5. See caption to Fig. 4, but now different κ values have been used for the different series of isotopes, i.e., we have used the values $\kappa' \equiv (\kappa/5)F$ Te ($\kappa'=0.386$); Xe ($\kappa'=0.388$); Ba ($\kappa'=0.362$); Ce ($\kappa'=0.340$); Nd ($\kappa'=0.328$); Sm ($\kappa'=0.312$) (κ' in MeV).

B. $E2$ decay properties

Using the wave functions from the two-level shell-model calculation, as discussed in Sec. III A, one easily calculates the $E2$ matrix elements, using the electric quadrupole operator $T(E2)$, i.e.,

$$T(E2) = e_\pi^F r_\pi^2 Y_2(\hat{r}_\pi) + e_\nu^F r_\nu^2 Y_2(\hat{r}_\nu) \ . \tag{3.8}$$

The result becomes

$$\langle 0_1^+ || T(E2) || 2_i^+ \rangle = \frac{1}{\sqrt{2}} \left\{ \left[\frac{N_\pi(\Omega_\pi - N_\pi)}{\Omega_{\pi-1}} \right]^{1/2} \langle (j_\pi)^2; 0^+ || T(E2) || (j_\pi)^2; 2^+ \rangle + \epsilon_i \left[\frac{N_\nu(\Omega_\nu - N_\nu)}{\Omega_{\nu-1}} \right]^{1/2} \langle (j_\nu)^2; 0^+ || T(E2) || (j_\nu)^2; 2^+ \rangle \right\} , \tag{3.9}$$

with

$$\langle (j_\rho)^2; 0^+ || T(E2) || (j_\rho)^2; 2^+ \rangle \equiv A_\rho e_\rho^F = -\sqrt{5/\pi} \frac{1}{4} \left[\frac{(2j_\rho - 1)(2j_\rho + 3)}{j_\rho(j_\rho + 1)} \right]^{1/2} \langle r^2 \rangle_\rho e_\rho^F \ . \tag{3.10}$$

If we assume identical $j_\pi = j_\nu = j$ (as was done in Sec. III A for the study of Te, Xe, and Ba nuclei with $60 \le N \le 80$), Eq. (3.9) reduces to

$$\langle 0_1^+ || T(E2) || 2_i^+ \rangle = \frac{1}{\sqrt{2}} A_j \left\{ \left[\frac{N_\pi(\Omega_\pi - N_\pi)}{\Omega_\pi - 1} \right]^{1/2} e_\pi^F + \epsilon_i \left[\frac{N_\nu(\Omega_\nu - N_\nu)}{\Omega_\nu - 1} \right]^{1/2} e_\nu^F \right\} , \tag{3.11}$$

which, for $N_\pi = N_\nu = 1$, reduces further to

$$\langle 0_1^+ || T(E2) || 2_i^+ \rangle = \frac{1}{\sqrt{2}} A_j (e_\pi^F + \epsilon_i e_\nu^F) \ . \tag{3.12}$$

Here, the coherence in the $E2$ matrix element for the lowest 2_1^+ level becomes clear and results in a charge $e_\pi^F + e_\nu^F$ whereas for the antisymmetric 2^+ level, the charge $e_\pi^F - e_\nu^F$ results. For the charges, normally used, this results into a $B(E2)$ ratio favoring the 2_1^+ decay over the 2_2^+ decay by almost $\simeq 10$. In the more general case $A_{j_\pi} \ne A_{j_\nu}$ we have plotted the experimental $B(E2; 0_1^+ \to 2_1^+)^{1/2}$ values versus $[N_\nu(\Omega_\nu - N_\nu)/(\Omega_\nu - 1)]^{1/2}$ to test Eq. (3.9) [or Eq. (3.11) when $A_{j_\pi} = A_{j_\nu}$], i.e., if a constant slope and identical intercept with the ordinate result, which should be the case for constant fermion charge e_π^F, e_ν^F. In Figs. 6 and 7, we give the experimental results for the $Z=50$ region (Ru, Pd, and Cd using $\Omega_\pi = 11$, and Te, Xe, and Ba using $\Omega_\pi = 16$, respectively).[19] From both figures one observes that a straight line could well be fitted to the Te, Xe, Ba and the Ru, Pd, Cd nuclei when the number of valence nucleons is not too large (then the assumption of vibrational energy spectra is a good approximation). One observes that a larger slope shows up for a larger number of valence neutrons and also that the intercept with the ordinate increases with increas-

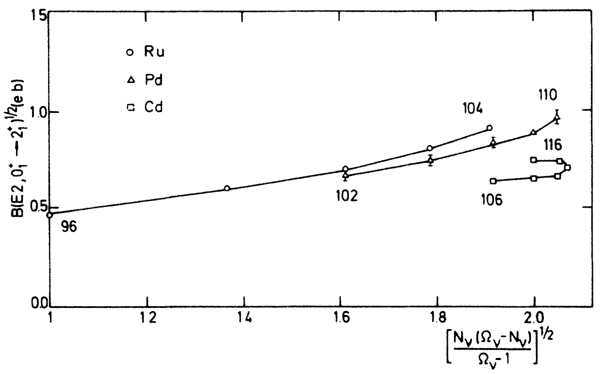

FIG. 6. Plot of the experimental values of $B(E2;0_1^+\to 2_1^+)^{1/2}$ (in units e b) as a function of $[N_\nu(\Omega_\nu-N_\nu)/(\Omega_\nu-1)]^{1/2}$ for the Ru, Pd, and Cd nuclei (nuclei with a fixed value of $[N_\pi(\Omega_\pi-N_\pi)/(\Omega_\pi-1)]^{1/2}$) in order to determine the proton fermion charge e_π^F and the neutron effective charge e_ν^F, using Eqs. (3.9) and/or (3.11). We have used the degeneracies $\Omega_\pi=11$, $\Omega_\nu=16$.

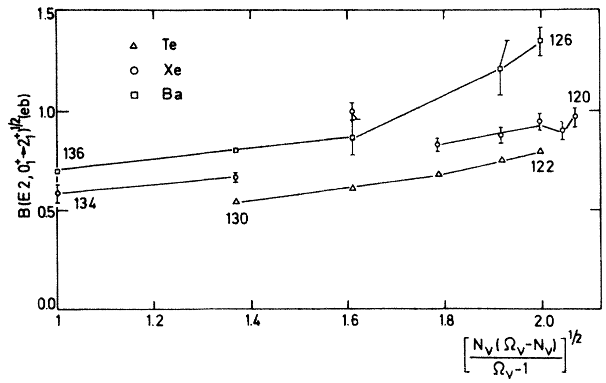

FIG. 7. See caption to Fig. 6, but for the Te, Xe, and Ba nuclei using $\Omega_\pi=16$, $\Omega_\nu=16$ as degeneracies.

ing number of valence protons. From these observations, one can conclude that a larger effective proton and neutron fermion charge e_π^F, e_ν^F is needed to accommodate the low-lying 2_1^+ level within the simple description of a two-level shell-model calculation. From the shell-model values of A_{j_π} and A_{j_ν} for the different mass regions, one obtains for the $j_\pi=\frac{31}{2}$ ($\Omega_\pi=16$) and $j_\nu=\frac{31}{2}$ ($\Omega_\nu=16$) orbitals the values $\langle\pi\rangle=0.10\ e_\pi^F$b and $\langle\nu\rangle=0.10\ e_\nu^F$b, respectively, for the coefficients multiplying the N-dependent factors in Eqs. (3.9) and (3.11). From a fit to experimental $B(E2;0_1^+\to 2_1^+)$ values in $N=82$ nuclei (^{136}Xe, ..., ^{144}Sm) and to the $Z=50$ nuclei (^{112}Sn, ..., ^{124}Sn) one determines $\langle\pi\rangle=0.202$ e b, $\langle\nu\rangle=0.161$ e b. With these values, the calculated $B(E2;0_1^+\to 2_1^+)$ values in Te, Xe, and Ba nuclei, as shown in Fig. 7, are given in Table I. Here, one observes on the average too small theoretical values, especially for nuclei with a large number of valence protons and neutrons. This again shows the limitations of the simple shell-model approach of Sec. III A for a detailed reproduction of the data.

TABLE I. Table of experimental $B(E2;0_1^+\to 2_1^+)^{1/2}$ values (Ref. 19) according to Eq. (3.9). The quantity $X_\rho\equiv[N_\rho(\Omega_\rho-N_\rho)/(\Omega_\rho-1)]^{1/2}$ ($\rho\equiv\pi,\nu$) is calculated for the 50–82 region, i.e., $\Omega_\pi=\Omega_\nu=\frac{1}{2}(82-50)=16$. The theoretical value is denoted by the relation

$$B(E2;0_1^+\to 2_1^+)_{\rm th}^{1/2}=X_\pi\langle\pi\rangle+X_\nu\langle\nu\rangle\,(\langle\pi\rangle=0.202\ e\,\text{b},\ \langle\nu\rangle=0.161\ e\,\text{b})\ .$$

Nucleus	A	N_π	N_ν	X_π	X_ν	$B(E2;0_1^+\to 2_1^+)_{\rm exp}^{1/2}$	$B(E2;0_1^+\to 2_1^+)_{\rm th}^{1/2}$
Te	122	1	6	1	2	0.816(7)	0.524
	124		5		1.915	0.755(7)	0.510
	126		4		1.789	0.691(8)	0.490
	128		3		1.612	0.614(2)	0.462
	130		2		1.366	0.539(10)	0.422
Xe	120	2	8	1.366	2.066	0.959(57)	0.609
	122		7		2.049	0.894(50)	0.606
	124		6		2	0.949(37)	0.598
	126		5		1.915	0.889(34)	0.584
	128		4		1.789	0.831(30)	0.564
	130		3		1.612	1.000(40)	0.535
	132		2		1.366	0.663(23)	0.496
	134		1		1	0.583(51)	0.437
	136		0		0	0.424(94)	0.276
Ba	126	3	6	1.612	2	1.354(73)	0.648
	128		5		1.915	1.222(132)	0.634
	130		4		1.789		0.614
	132		3		1.612	0.860(96)	0.585
	134		2		1.366	0.837(11)	0.546
	136		1		1	0.694(11)	0.487
	138		0		0	0.466(4)	0.326

Within the IBM-2, in the vibrational limit, general expressions for the $E2$ matrix elements can be derived which are valid for a nuclear Hamiltonian which is invariant under rotations in F-spin space[4–8,21,22] (F-spin symmetric Hamiltonian). For the $E2$ transitions from the lowest symmetric 2_1^+ state and from the lowest mixed-symmetry 2^+ level (which we call 2_2^+, which conforms with the restricted shell-model space) the following results were obtained:[5]

$$\langle 0_1^+ || T(E2) || 2_1^+ \rangle = \frac{(e_\pi^B N_\pi + e_\nu^B N_\nu)\sqrt{5}}{\sqrt{N}} ,$$
$$\langle 0_1^+ || T(E2) || 2_2^+ \rangle = (e_\nu^B - e_\pi^B) \left[\frac{N_\pi N_\nu 5}{N} \right]^{1/2} . \tag{3.13}$$

Identifying Eqs. (3.13) with the corresponding shell-model results of Eq. (3.11), proton and neutron boson effective charges e_π^B, e_ν^B can be obtained as a function of proton and neutron fermion charges e_π^F, e_ν^F and the number of proton and neutron valence pairs N_π, N_ν, respectively. For a single-j shell,[23] this results into

$$e_\pi^B = \frac{1}{\sqrt{10}} A_j \left[\frac{N(\Omega_\pi - N_\pi)}{N_\pi(\Omega_\pi - 1)} \right]^{1/2} e_\pi^F ,$$
$$e_\nu^B = \frac{1}{\sqrt{10}} A_j \left[\frac{N(\Omega_\nu - N_\nu)}{N_\nu(\Omega_\nu - 1)} \right]^{1/2} e_\nu^F . \tag{3.14}$$

The latter expressions are studied for the $Z=50$ and the $N=82$ mass regions (Figs. 8 and 9). In the $Z=50$ region, we carry out the calculations both for the nuclei with $Z<50$, for which both cases $\Omega_\pi=5$ ($1g_{9/2}$ orbit only) and $\Omega_\pi=11$ (the $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $1g_{9/2}$ orbitals) are considered, and for nuclei with $Z>50$, in which case we take $\Omega_\pi=16$ (the full 50–82 shell as a single degenerate $j=\frac{31}{2}$ shell). This is performed for both $e_\nu^F=0.5e$ and $e_\pi^F=1.0e$ (or $1.5e$). From Fig. 8, an important difference in the values of e_π^B, e_ν^B results, especially when the proton and neutron number of pairs are largely different. Moreover, values of the effective boson charge, as calculated here, are of the right order of magnitude as the ones used in realistic IBM-2 calculations for this mass region,[24–26] especially for the choice $e_\pi^F=1.5e$, $e_\nu^F=0.5e$. Similar conclusions are obtained for the $N=82$ region (Fig. 9) where now one observes that e_π^B is a decreasing function of N_π and e_ν^B an increasing function. Even taking into account the approximations used in obtaining expressions (3.14), a mass dependence (and thus a nuclear structure dependence) of boson charges is clearly established.

As a conclusion of the shell-model calculations of Sec. III, it is clear from constructing proton-neutron coupled wave functions, through the proton-neutron quadrupole interaction, that symmetric and antisymmetric 2^+ states arise in a very natural way. The dependence of the excitation energy $E(2_1^+)$ in the vibrational and transitional region can be estimated to be a smooth function of the product $N_\pi N_\nu$. Moreover, calculating $E2$ reduced transition probabilities a quenching for the antisymmetric $2_2^+ \to$symmetric 0_1^+ transition compared to the symmetric

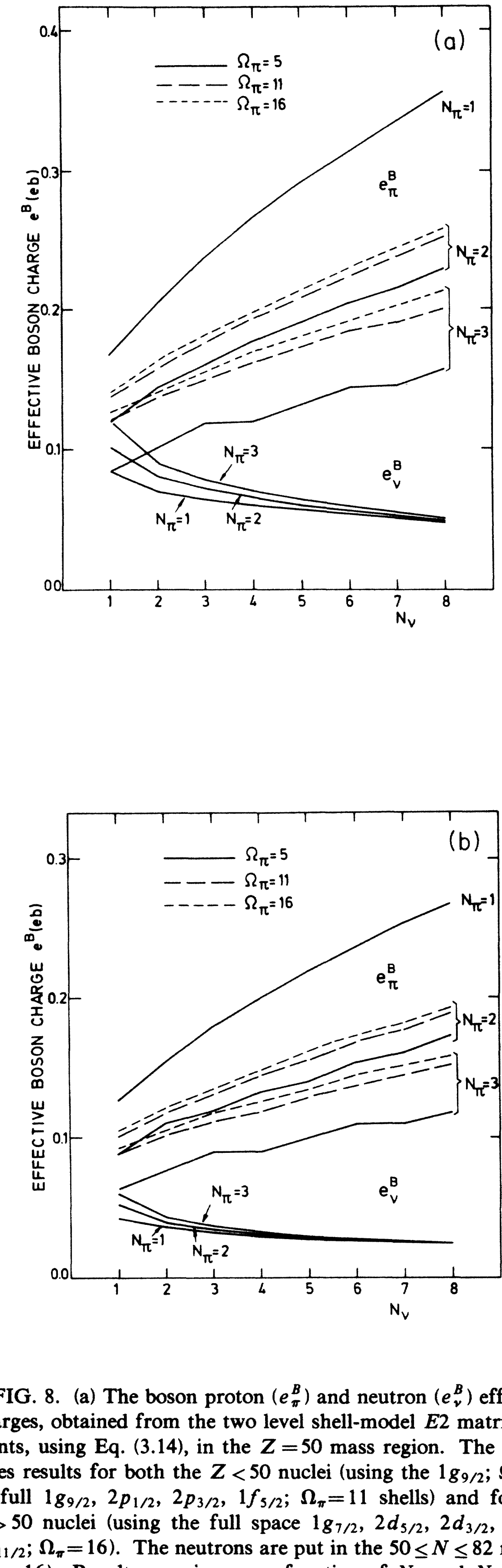

FIG. 8. (a) The boson proton (e_π^B) and neutron (e_ν^B) effective charges, obtained from the two level shell-model $E2$ matrix elements, using Eq. (3.14), in the $Z=50$ mass region. The figure gives results for both the $Z<50$ nuclei (using the $1g_{9/2}$; $\Omega_\pi=5$ or full $1g_{9/2}$, $2p_{1/2}$, $2p_{3/2}$, $1f_{5/2}$; $\Omega_\pi=11$ shells) and for the $Z>50$ nuclei (using the full space $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, $1h_{11/2}$; $\Omega_\pi=16$). The neutrons are put in the $50 \le N \le 82$ region ($\Omega_\nu=16$). Results are given as a function of N_π and N_ν using the values $e_\pi^F=1.5e$, $e_\nu^F=0.5e$. (b) See caption to (a), but using the fermion charges $e_\pi^F=1.0e$, $e_\nu^F=0.5e$.

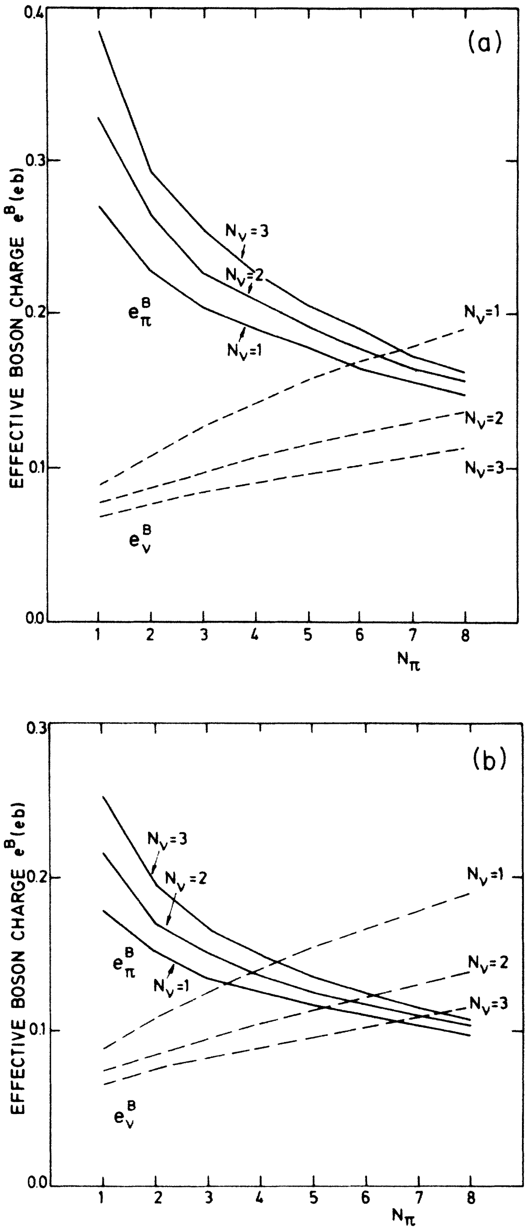

FIG. 9. (a) The same caption as for Fig. 8(a), but now for the $N=84$ mass region. For the proton degeneracy we use the full $50 \le Z \le 82$ shell ($\Omega_\pi = 16$) and for the neutrons, we consider the $2f_{7/2}$, $3p_{3/2}$, $1h_{9/2}$, and $1i_{13/2}$ orbitals ($\Omega_\nu = 18$). Results are given as a function of N_π and N_ν using the values $e^F_\pi = 1.5e$, $e^F_\nu = 0.5e$. (b) See caption to (a), but using the fermion charges $e^F_\pi = 1.0e$, $e^F_\nu = 0.5e$.

$2^+_1 \rightarrow$symmetric 0^+_1 transition results, making it clearly difficult to observe such antisymmetric 2^+ levels in vibrational nuclei. Moreover, by equating the shell-model and IBM-2 results, a $N_\pi(N_\nu)$ dependence of the boson effective charges on the fermion charges occurs.

C. Comparison with a more realistic calculation

In the analysis carried out before, a number of approximations have been imposed:

(i) the assumption of degeneracy in energy for the proton and neutron 2^+_1 excited states;

(ii) the use of a separable proton-neutron quadrupole force;

(iii) the use of a single-j shell for both protons and neutrons;

(iv) the neglect of higher seniority ($v=4,6,\ldots$) and higher spin states ($J^\pi = 4^+, 6^+, 8^+, \ldots$).

If more realistic calculations are carried out, however, the error in assumption (i) is negligible and is borne out by the approximate equality of 2^+ excitation energies in a series of single-closed proton and neutron shell nuclei for a given mass region, i.e., Sn and $N=82$ 2^+_1, energies in the $A \simeq 130$ mass region (see also Sec. III A). Also, the use of a separable quadrupole proton-neutron interaction compared to more realistic interactions in the $Z=50$, $N=50$ (Refs. 14 and 15) and the $Z=50$, $N=82$ (Refs. 27 and 28) regions does not modify the formation of a symmetric and antisymmetric 2^+ level in an important way. Of course, the use of restricted model spaces and simplifications of it as described under (iii) and (iv) have a rather large influence on the detailed energy spectra and electromagnetic decay properties.[14,27,28] However, the general conclusions discussed above are not drastically changed.

When describing the $B(E2;2^+_1 \rightarrow 0^+_1)$ values, using Eq. (3.11), in Figs. 6 and 7, it is clear that even the single-j shell results give a good overall description of the available data. This of course reflects the fact that the charges e^F_π and e^F_ν become "renormalized" as a function of Z (and N) in order to accomplish good agreement over a large number of nuclei (see Sec. III B).

We also point out that, concerning the $N_\pi N_\nu$ systematics, the good behavior obtained here by using the two-level model may not be overestimated. After all, we have used lowest order ($v=2$) seniority shell-model wave functions and a realistic description of the lowest 2^+ level in vibrational and transitional nuclei gives of course more complex results. For deformed nuclei, the above nuclear coupling scheme where the pairing force is the dominant component of the nucleon-nucleon interaction is clearly not the best starting point. Here, one will rather have to start from a deformed potential.[29] However, for the 2^+_1 level, the basic content of the energy lowering and $B(E2;2^+_1 \rightarrow 0^+_1)$ values are clearly contained within the simple two-component proton-neutron system.

IV. PARTICLE-CORE COUPLING CALCULATIONS

As was pointed out in the Introduction, any model description where two distinct building blocks determine, when coupled, the low-lying nuclear levels, symmetric and antisymmetric states will result. In the present section we use the particle-core coupling model[29-33] where a few (one, two, three, ...) fermions are explicitly treated and coupled to a set of surface quadrupole vibrational excitations of the underlying core nucleus. Since the particle-core coupling model has been discussed at length, especially by Paar and co-workers (see Refs. 30–32, and references therein), we quote here some of the necessary expressions for the further discussion to be self-contained.

A. Excitation energy in particle-vibration coupling

The standard particle-core coupling Hamiltonian reads

$$H_{\rm int}=-\left[\frac{\pi}{5}\right]^{1/2}\xi_2\hbar\omega_2\sum_{i,\mu}[b_{2\mu}+(-1)^{\mu}b^{\dagger}_{2,-\mu}]Y_{2\mu}(\hat{r}_i)\ , \tag{4.1}$$

where ξ_2 denotes the particle-core coupling strength, $\hbar\omega_2$ the quadrupole phonon energy [$\simeq E(2_1^+)$ of the core nucleus], $b^{\dagger}_{2\mu}$ is the creation operator for the quadrupole surface oscillations, and $Y_{2\mu}(\hat{r}_i)$ describes the particle (fermion) coordinates. With the present Hamiltonian, the coupling strength ξ_2 is related to the quadrupole core collectivity via the relation

$$\xi_2=\left\langle r\frac{dV}{dr}\right\rangle_{\alpha_{\lambda\mu}=0}\frac{4\sqrt{5\pi}}{\hbar\omega_2}\frac{B(E2;2_1^+\rightarrow 0_1^+)^{1/2}}{3ZeR_0^2}\ , \tag{4.2}$$

(or to the notation frequently used by Paar,[30] according to $\xi_2=a_2 2\sqrt{5}/\hbar\omega_2$). In Eq. (4.2), V denotes the vibrating single-particle potential taken at the spherical equilibrium shape. A typical estimate of the strength

$$\left\langle r\frac{dV}{dr}\right\rangle_{\alpha_{\lambda\mu}=0}\simeq 40-50\ \text{MeV}$$

results in heavy and medium-heavy nuclei.[33]

Correspondingly, the $E2$ transition operator will contain, besides the single particle term, also a collective term and is written as

$$T(E2;\mu)=\sum_{i=1}^{A}e_i r_i^2 Y_{2\mu}(\hat{r}_i)+B(E2;2_1^+\rightarrow 0_1^+)^{1/2}[b^{\dagger}_{2\mu}+(-1)^{\mu}b_{2,-\mu}]\ , \tag{4.3}$$

where e_i gives the fermion charge and $B(E2)^{1/2}$ the collective vibrational charge, characterizing the collective part of the $E2$ transition. Furthermore, one has the relation

$$B(E2;0_1^+\rightarrow 2_1^+)=\beta_2^2\left[\frac{3}{4\pi}ZeR_0^2\right]^2\ , \tag{4.4}$$

with

$$\beta_2=\left[\frac{5\hbar\omega_2}{2C_2}\right]^{1/2}\ . \tag{4.5}$$

Within the particle-core coupling model, basis functions contain a particle cluster wave function $|(j)^n\alpha JM\rangle$ (n is the number of extra fermions) coupled to a collective wave function, the latter denoted by the number of quadrupole phonons N and the collective quadrupole angular momentum R, resulting in the basis wave functions

$$|(j)^n\alpha J,NR;IM\rangle\ . \tag{4.6}$$

In particle-core coupling, near closed shells, in many cases a single-j shell (or a linear combination) determines the proton (or neutron) fermion cluster wave function. In the Cd nuclei, i.e., the two proton holes, coupled to the Sn core states, move in the $1g_{9/2}$ orbital whereas for the $N=84$ nuclei, the two neutrons move in the $2f_{7/2}$, $3p_{3/2}$, $1h_{9/2}, \ldots$ orbitals, being close in energy. In both cases, the separate subsystems are described by (i) fermion part

$$|(j)^2;0^+\rangle;\ |(j)^2;2^+\rangle\ ,$$

(ii) collective part

$$|00;0F;0^+\rangle;\ |12;2^+\rangle\ , \tag{4.7}$$

where $|(j)^2;0^+\rangle$ and $|(j)^2;2^+\rangle$ describe the paired and the one-broken pair fermion state[34] and $|00;0F;0^+\rangle$, $|12;2^+\rangle$ the zero- and one-phonon quadrupole phonon states, respectively. If we consider the approximate case that the pair matrix element

$$|\langle(j)^2;0^+|V|(j)^2;0^+\rangle|\simeq\hbar\omega_2\ ,$$

which is indeed the case for both the $Z=50$ and $N=82$ regions, the 2^+ levels will result, when considering only the basis states (4.7), from a two-level model diagonalization with the basis states,

$$\begin{aligned}&|(j)^2 0^+,12;2^+\rangle\ ,\\&|(j)^2 2^+,00;2^+\rangle\ .\end{aligned} \tag{4.8}$$

So, one obtains the linear combinations

$$|2_i^+\rangle=\frac{1}{\sqrt{2}}[\,|(j)^2 0^+,12;2^+\rangle+\epsilon_i\,|(j)^2 2^+,00;2^+\rangle]\ . \tag{4.9}$$

In the case of two particle-core coupling, the lowest 2^+ state has $\epsilon_1=-1$, $\epsilon_2=+1$ for the 2_2^+ level. As will be discussed in Sec. IV A 2, in the 2_1^+ decay, both the collective and two-particle contributions add up coherently and therefore the 2_1^+ state can be called the "symmetric" state. For two hole-core coupling, the opposite situation results, i.e., $\epsilon_1=+1$, $\epsilon_2=-1$. The 0^+ ground state remains basically the

$$|0_1^+\rangle=|(j)^2 0^+,00;0^+\rangle \tag{4.10}$$

configuration. Before studying the $E2$ transition probabilities from the states (4.9) to the 0^+ ground state, we shortly discuss the results for both the $Z=50$ (Cd nuclei) and the $N=82$ ($N=84$) regions.

1. The Cd nuclei

Although many detailed calculations have been carried out[31,32] using the two hole-core coupling model, we simplify the present discussion in order to find out the basic mechanisms at work. Taking into account a single-j shell (the $1g_{9/2}$ orbital) and up to three quadrupole phonon excitations, we illustrate some numerical results for a case where we take $\hbar\omega_2=1$ MeV, $\xi_2=2.5$. The excited states 2_1^+, 2_2^+, and 2_3^+ are shown in Fig. 10, and very much resemble a typical even-even Cd nucleus. The wave functions become

$$
\begin{aligned}
|0_1^+\rangle &= -0.77\,|(1g_{9/2})^2 0^+,00;0^+\rangle \\
&\quad -0.54\,|(1g_{9/2})^2 2^+,12;0^+\rangle\ , \\
|2_1^+\rangle &= -0.55\,|(1g_{9/2})^2 2^+,00;2^+\rangle \\
&\quad -0.56\,|(1g_{9/2})^2 0^+,12;2^+\rangle\ , \\
|2_2^+\rangle &= -0.23\,|(1g_{9/2})^2 2^+,00;2^+\rangle \\
&\quad +0.50\,|(1g_{9/2})^2 0^+,12;2^+\rangle \\
&\quad +0.48\,|(1g_{9/2})^2 2^+,12;2^+\rangle, \\
|2_3^+\rangle &= +0.54\,|(1g_{9/2})^2 2^+,00;2^+\rangle \\
&\quad -0.26\,|(1g_{9/2})^2 0^+,12;2^+\rangle \\
&\quad +\text{higher components}\ .
\end{aligned} \tag{4.11}
$$

Compared to the $|2_1^+\rangle$ wave function, both the $|2_2^+\rangle$ and $|2_3^+\rangle$ wave functions have an antisymmetric character, although distributed over the two levels (the 2_3^+ being the better antisymmetric state).

2. The $N=84$ nuclei

The $N=84$ nuclei have been studied in detail (realistic situation for both the neutron single-particle energies, coupling strength, and phonon energies) before.[33,35] Here, we basically have redone the calculations of Ref. 35. Since here, not a single-j shell but the $2f_{7/2}$, $3p_{3/2}, \ldots$ orbitals are considered, some slight complications arise. Thus, we call the neutron 2^+ excitation, the linear combination,

$$
|2_\nu^+\rangle \equiv a\,|(2f_{7/2})^2;2^+\rangle + b\,|(3p_{3/2})^2;2^+\rangle + \cdots\ . \tag{4.12}
$$

Diagonalizing the particle-core coupling Hamiltonian of Eq. (4.1), with the parameters as discussed in Ref. 35, one obtains the following wave functions for the 0^+ ground state and the low-lying 2^+ levels:

$$
\begin{aligned}
|0_1^+\rangle &= 0.27\,|(3p_{3/2})^2;0^+\rangle + 0.67\,|(2f_{7/2})^2;0^+\rangle + 0.24\,|(1h_{9/2})^2;0^+\rangle + \cdots \\
&\quad -[0.31\,|(3p_{3/2}2f_{7/2})2^+,12;0^+\rangle + 0.35\,|(2f_{7/2})^2 2^+,12;0^+\rangle + \cdots]\ , \\
|2_1^+\rangle &= -0.24\,|(3p_{3/2}2f_{7/2})2^+,00;2^+\rangle - 0.33\,|(2f_{7/2})^2 2^+,00;2^+\rangle + \cdots \\
&\quad +[0.23\,|(3p_{3/2})^2 0^+,12;2^+\rangle + 0.60\,|(2f_{7/2})^2 0^+,12;2^+\rangle + \cdots]\ .
\end{aligned} \tag{4.13}
$$

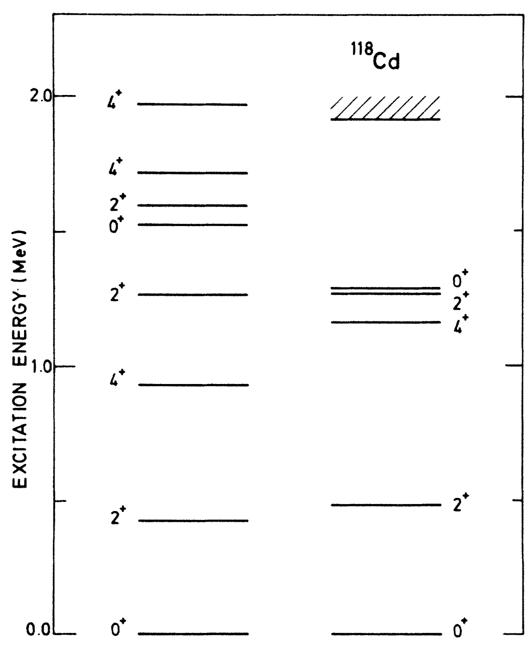

FIG. 10. The low-lying excited $J^\pi=0^+$, 2^+, and 4^+ levels obtained from a schematic two-hole core coupling calculation [using a single j shell (the $1g_{9/2}$ orbital) and the values $\hbar\omega_2=1$ MeV, $\xi_2=2.5$ and up to three quadrupole phonons] for the Cd region. A comparison is made with the lower part of the experimental spectrum in ^{118}Cd.

Here, one recognizes the symmetric structure

$$
a\left[\sum_{j,j'}\beta_{j,j'}\,|(jj')2^+,00;2^+\rangle\right] + b\left[\sum_j |(j)^2 0^+,12,2^+\rangle\right]\ . \tag{4.14}
$$

For the other $2_2^+,2_3^+$ levels (see Table I of Ref. 35), more complicated admixtures occur, but for the 2_3^+ level a large component

$$
0.41\,|(2f_{7/2})^2 2^+,00;2^+\rangle + 0.38\,|(2f_{7/2})^2 0^+,12;2^+\rangle \tag{4.15}
$$

does result. Thus, we find two major components of the $|2_1^+\rangle$ wave function, but with the opposite sign. Therefore, the analysis for 2_3^+ levels in some $N=84$ nuclei, as carried out by Hamilton *et al.*,[36] although using a purely collective approach for both proton and neutron constituents of the nuclear wave functions (IBM-2) is not too far from the results obtained from a semimicroscopic calculation as indicated here. Thus, even for more realistic particle-core coupling calculations (Cd, $N=84$), besides the lowest-lying strongly collective symmetric combination of particle and collective excitations, an antisymmetric 2^+ level results albeit with a more strongly mixed wave function to other configurations outside the purely collective IBM-2—type subspace.

B. $E2$ decay properties

Starting from the above particle-core coupled wave functions describing the different 2^+ levels (see Sec. IV A) and using the $E2$ operator, discussed in Eq. (4.3), one easily calculates the reduced $E2$ matrix elements

$$A' \equiv \langle 0_1^+ || T(E2) || 2_1^+ \rangle$$
$$= \langle (j)^2 0^+, 00; 0^+ || \sum_{i=1}^{2} e_i r_i^2 Y_2(\hat{r}_i) + B(E2; 2_1^+ \to 0_1^+)^{1/2} (b_2^\dagger + \tilde{b}_2) || [a \mid (j)^2 0^+, 12; 2^+ \rangle + b \mid (j)^2 2^+, 00; 2^+ \rangle]$$
$$= aB(E2; 2_1^+ \to 0_1^+)^{1/2} \sqrt{5} + b\, 2 \langle r^2 \rangle_{j_i} \left[\frac{5}{4\pi} \right]^{1/2} e_i \frac{\frac{3}{4} - j_i(j_i+1)}{[j_i(j_i+1)(2j_i-1)(2j_i+3)]^{1/2}} . \quad (4.16)$$

The latter term reduces (in a single-particle limit; $j_i \to \infty$) to $-b\frac{3}{5}(r_0)^2 A^{2/3} \sqrt{(5/4\pi)} e_i$, which for $r_0 = 1.3$ fm reduces to the expression $-b\, 0.35\sqrt{5} A^{2/3} e_i$. So one obtains for A' and B' $[B' \equiv \langle 0_1^+ || T(E2) || 2_2^+ \rangle]$ the approximate expressions

$$A' = aB(E2; 2_1^+ \to 0_1^+)^{1/2} \sqrt{5} - b\, 0.35\sqrt{5} A^{2/3} e_i ,$$
$$B' = aB(E2; 2_1^+ \to 0_1^+)^{1/2} \sqrt{5} + b\, 0.35\sqrt{5} A^{2/3} e_i . \quad (4.17)$$

Coherence and destructive interference thus results for the symmetric 2^+ and the antisymmetric 2^+ particle-core coupled states, respectively. In the particular situation of the Cd ($Z = 50$ region) nuclei and the $N = 84$ ($N = 82$ region) nuclei, the following results are obtained.

1. The Cd nuclei

In Table II, we give the $B(E2; 0_1^+ \to 2_1^+)$ values as taken from Ref. 37. An averaged value of $\simeq 2000\ e^2\mathrm{fm}^4$ results, giving rise to a collective charge, to be used in Eq. (4.17) of

$$e_{\mathrm{coll}} = B(E2; 2_1^+ \to 0_1^+)_{\mathrm{Sn}}^{1/2} \sqrt{5} \simeq 45e . \quad (4.18)$$

For the even-even Cd nuclei, one obtains the approximate single-particle charge

$$e_{\mathrm{sp}} = 0.35\sqrt{5} A^{2/3} e_\pi \simeq 20 e_\pi . \quad (4.19)$$

In the particle-core coupling calculations,[30–32] effective proton charges $1.5e \le e_\pi \le 2.0e$ have been used. Restricting to a single j shell ($1g_{9/2}$ orbital) [see Eq. (4.16)], renormalization will even imply a larger effective proton charge. This means that for an equal mixing (for degenerate 2^+ levels in the unperturbed two-level model) $a = -b = 1/\sqrt{2}$ and since the collective and single-particle charges are almost equal, a value for the matrix element

$$\langle 0_1^+ || T(E2) || 2_1^+ \rangle \simeq \sqrt{2} B(E2; 2_1^+ \to 0_1^+)_{\mathrm{Sn}}^{1/2} \sqrt{5} \quad (4.20)$$

or

$$B(E2; 2_1^+ \to 0_1^+)_{\mathrm{Cd}} \simeq 2B(E2; 2_1^+ \to 0_1^+)_{\mathrm{Sn}} \quad (4.21)$$

results. This means that the coherence between the two-proton cluster and the collective vibration is optimal and almost doubles the $B(E2)$ values in going from Sn to the Cd nucleus with the same number of neutrons. Detailed calculations for Cd nuclei, using a particle-core coupling model description, give the same results. Similar results were also obtained and discussed by Paar, starting from perturbation theory.[31,32] For the antisymmetric 2_2^+ state, on the contrary, a very small reduced matrix element $\langle 0_1^+ || T(E2) || 2_2^+ \rangle$ will result.

2. The N = 84 nuclei

In Table III, $B(E2; 0_1^+ \to 2_1^+)$ values, as taken from Ref. 37, for the $N = 82$ nuclei, are given. An averaged value of $\simeq 3200\ e^2\mathrm{fm}^4$ is representative for this mass region. If we apply Eq. (4.17), but now for a typical value of $A = 140$, one obtains the effective charges

$$e_{\mathrm{coll}} = B(E2; 2_1^+ \to 0_1^+)^{1/2} \sqrt{5} \simeq 55e ,$$
$$e_{\mathrm{sp}} = 0.35\sqrt{5} A^{2/3} e_\nu \simeq 22 e_\nu . \quad (4.22)$$

In particle-core coupling calculations[30–32] effective neutron charges $0.5e \le e_\nu \le 1.0e$ have been used. Restricting now even more to a single j shell [see Eq. (4.16)], renormalization will imply an even larger effective neutron charge. For optimal mixing, $a = -b = 1/\sqrt{2}$ one approximately gets

$$\langle 0_1^+ || T(E2) || 2_1^+ \rangle = \left[\frac{1}{\sqrt{2}} 55e + \frac{1}{\sqrt{2}} 22e \right] \simeq 55e$$

or (4.23)

$$B(E2; 2_1^+ \to 0_1^+)_{N=84} \simeq B(E2; 2_1^+ \to 0_1^+)_{N=82} .$$

One notices that the $B(E2)$ value remains almost con-

TABLE II. Reduced $E2$ transition probabilities $B(E2; 0_1^+ \to 2_1^+)$ for the even-even Sn nuclei (Ref. 37).

A	$B(E2; 0_1^+ \to 2_1^+)$ ($e^2\mathrm{fm}^4$)
112	2570
114	2300
116	2160
118	2180
120	2060
122	2020
124	1690

TABLE III. Reduced $E2$ transition probabilities $B(E2; 0_1^+ \to 2_1^+)$ for the even-even $N = 82$ nuclei (Ref. 37).

A	$B(E2; 0_1^+ \to 2_1^+)$ ($e^2\mathrm{fm}^4$)
138	3800
140	2900
142	3900
144	2500

stant, which is almost the case since for the $N=84$ 2_1^+ levels compared with the $N=82$ 2_1^+ levels for the same neutron number only about a 10% increase in the $B(E2)$ value results.

C. Comparison with more realistic calculations

As was discussed in Sec. III C, a number of approximations underlie the above discussion, especially when compared with the more realistic cases such as the Cd and the $N=84$ nuclei. Here, the approximations are the following:

(i) the equality between the quadrupole phonon energy $\hbar\omega_2$ and the pairing matrix element in a given j shell for medium-heavy and heavy nuclei;

(ii) the truncation of the shell-model space to a single j shell (using only seniority $v=0$ and $v=2$ states) and of the quadrupole phonon model space to zero- and one-phonon states.

The more detailed calculations on Cd nuclei[31,32] and on $N=84$ nuclei[33,35] give of course more levels and at the same time a better description of the actual experimental data (the ^{118}Cd data in Fig. 10 can then be reproduced in a much better way). The basic idea, however, of obtaining a low-lying 2^+ level where both the single particle and the collective component of the $E2$ decay operator act coherently thereby producing a large $B(E2;2_1^+\rightarrow 0_1^+)$ value is retained.

V. CONCLUSION

In the present paper we have shown that symmetric and antisymmetric couplings of two separate building blocks occur naturally from the most simple two-level models, as obtained within the proton-neutron shell model and in particle-core coupling model calculations. Such two-level models can be obtained as a good approximation, at least for the lowest-lying 2^+ level as discussed here, to the actual shell-model situation in nuclei. It is the pronounced pairing property within the identical nucleon systems and the subsequent strong proton-neutron quadrupole interaction which causes the symmetric and antisymmetric 2^+ states to result. For the lowest 2^+ level, moreover (for vibrational-like nuclei), a specific dependence for the excitation energy $E(2_1^+)$ on the number of valence protons and neutrons is obtained and is shown to correlate very well with the experimental data in the Te, Xe, Ba, and Ce regions with $60\leq N\leq 80$.

In calculating the $E2$ decay properties, both in the shell model and in the particle-core coupling model, constructive (for the 2_1^+ level) and destructive (for the 2_2^+ level) interference is obtained between the proton and neutron fermion charges or between the fermion cluster and the quadrupole core collective charges, respectively. Within the shell model, when identifying the shell-model $E2$ matrix elements for the lowest 2_1^+ level to the 0^+ ground state with the corresponding IBM-2 expression, a relation between the proton and neutron effective boson charges on the fermion proton and neutron charges as well as on the number of valence protons and neutrons is obtained.

The above observations point out that a clear observation of such antisymmetric 2^+ levels will be very difficult to be carried out experimentally. Mixing with other, nearby 2^+ levels will moreover spread out the $E2$ decay strength. We expect that, from the present study (see Sec. III), the best cases for observing 2^+ antisymmetric states will be the ones where a large difference between the number of valence protons and neutrons occurs. So, as we pointed out in Sec. IV, in the $N=84$ nuclei, the 2_3^+ level could indeed be a good candidate as well as the analogous 2_3^+ level in the even-even Cd nuclei.

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with

$$\sum_i \alpha_i^2 = 1 \ ,$$

$$|(j)^2;2^+\rangle = \sum_{i\le k} \beta_{ik}\,|(j_i j_k);2^+\rangle$$

with

$$\sum_{i\le k} \beta_{ik}^2 = 1 \ .$$

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