Noncoplanar geometry for detecting nuclear renormalization effects in exclusive $(\pi,\pi N)$ reactions

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We consider a simple model of the π-N scattering amplitude in nuclear matter, in which the intermediate pion propagators appearing in π-N diagrams are dressed through the scattering of the pion from the background nucleons. We use this model to calculate the differential cross section for the exclusive $(\pi,\pi N)$ knockout reaction with the geometry of the experiment especially chosen to maximize the possibility of observing renormalization of the Δ resonance in the nuclear interior.

The Δ isobar dominates both low-energy π-N scattering and π-nucleus scattering.[1,2] It is generally believed the nuclear medium renormalizes the π-N interaction, so that the mass and width of the Δ need not be the same as found in scattering from a free nucleon.[3] Previously we have used a simple model of this renormalization to calculate exclusive $(\pi,\pi N)$ cross sections[4] in distorted-wave impulse approximation (DWIA), and compared the results with recent data of Ziock *et al.* on $^{12}C(\pi^+,\pi^+p)^{11}B$.[5] The data were taken with the beam momentum $\mathbf{k}$, the outgoing pion momentum $\mathbf{k}'$, and the outgoing proton momentum $\mathbf{p}'$ *coplanar.* The outgoing pion and proton energies, the beam energy, and the magnitude of the momentum of the recoiling nucleus, $|\mathbf{k}-\mathbf{k}'-\mathbf{p}'|$, were kept fixed. Of the five kinematic variables characterizing a three-body final state, two angles then remain free to be varied in the experiment. The coplanar experimental geometry fixes one of these angles (because it represents the angle between the planes whose normals are $\mathbf{k}\times\mathbf{p}'$ and $\mathbf{k}\times\mathbf{k}'$). The remaining angle was varied in order to sweep the barycentric energy $\sqrt{s'}$—in the outgoing π-N subsystem—through the mass of the Δ, and thereby produce the resonance. The experimentalists had hoped in this way to see a bump identifiable with the Δ—perhaps at $\sqrt{s'}\neq 1232$ MeV.

In I (see Ref. 4) we used a deliberately schematic model of nuclear Δ renormalization—based on pure p-wave, spin-independent π-N scattering—to inquire whether an experiment like that of Ziock *et al.*[5] could hope to detect nuclear renormalization of the Δ. We felt that if no observable effect was predicted with a schematic model, the added complexities of a realistic theory were hardly likely to improve matters. In fact, our simple model[4] showed that the experiment[5] suffered from two defects originating from its coplanar geometry.

(1) The range of variation of $\sqrt{s'}$ obtained by varying the angles between the outgoing particles (in coplanar geometry) is too small to define the resonance, that is, even the free resonance varies too little over the available range to produce a clear resonance shape from which the position and width may be deduced.

(2) In the DWIA, a pure p-wave π-N t matrix contains the factor

$$H=(\mathbf{k}'-k^0{}'\mathbf{p}'/M)\cdot[\mathbf{k}-k^0(\mathbf{k}'+\mathbf{p}'-\mathbf{k})/M] . \tag{1}$$

Unfortunately, $\underline{H}$ varies rapidly with the angle that is used to sweep $\sqrt{s'}$ (and, in fact, it is this variation which dominates the shapes of the theoretical and experimental curves shown in Fig. 3 of I and repeated in Fig. 1 herein). Thus the data[5] reflect the p-wave nature of the π-N scattering near the Δ resonance rather than the energy dependence of the resonance *per se.*

While concluding that the experiment[5] contained no information on Δ renormalization in nuclei, we wondered whether these defects could be remedied using *noncoplanar* geometry. We found that by varying the dihedral angle Φ (between the vectors $\mathbf{k}\times\mathbf{k}'$ and $\mathbf{k}\times\mathbf{p}'$) as well as the polar angles θ_π,θ_p, defined by

$$\begin{aligned}\hat{\mathbf{k}}'&=\hat{\mathbf{k}}\cos\theta_\pi+\hat{\mathbf{x}}\sin\theta_\pi ,\\ \hat{\mathbf{p}}'&=\hat{\mathbf{k}}\cos\theta_p+\hat{\mathbf{x}}\sin\theta_p\cos\Phi+\hat{\mathbf{y}}\sin\theta_p\sin\Phi ,\end{aligned} \tag{2}$$

one might define a regime in which both the recoil momentum and the π-N p-wave factor H could be held constant while $\sqrt{s'}$ varies sufficiently to be of interest. As we shall report in this paper, we have found such a regime; we have recalculated our simple theory and present the results in Figs. 2 and 3.

We now recapitulate our simple model of Δ renormalization in nuclear matter which we used to test the $(\pi,\pi p)$ data[5] for shifts of the energy and/or width of the Δ in finite nuclei. This is a deliberately oversimplified model in which we have incorporated both "multiple scattering" (MS) and "pure renormalization" (PR). By varying the model's parameters we explore the dependence of nuclear Δ renormalization on various aspects of the problem that we cannot as yet confidently calculate from first principles. An important oversimplification is our neglect of the not-inconsiderable s-wave π-nucleus interaction which is expected to contribute to $(\pi,\pi N)$ knockout at energies below the Δ resonance. Our philosophy in neglecting s waves is that if one cannot observe renormalization of the Δ under ideal conditions, the situation will not be improved by interfering processes. We shall next describe the main ideas of our model, and then we will sketch how we use it in a form of distorted-wave impulse approxima-

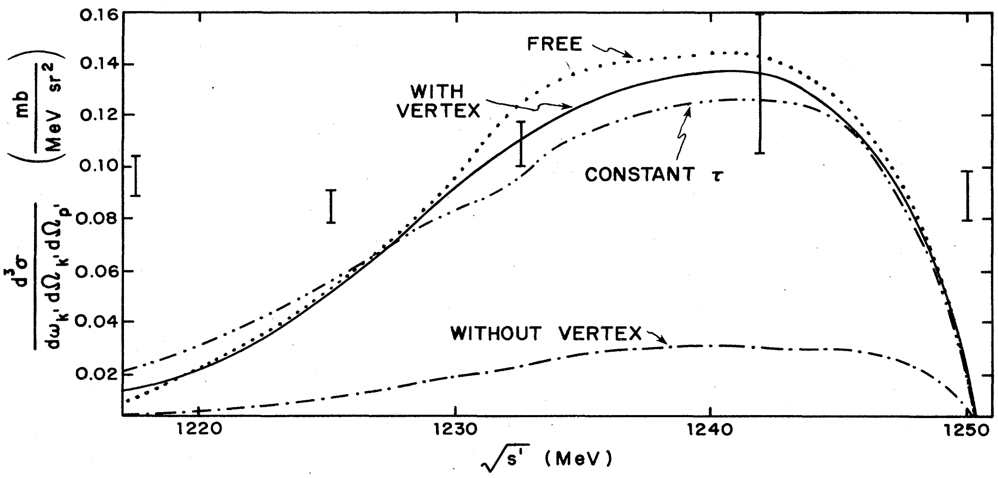

FIG. 1. Reprise of Fig. 3 of Ref. 4: The fivefold differential cross section for the $^{12}C(\pi^+, \pi^+p)^{11}B$ knockout reaction as a function of the invariant mass $\sqrt{s'}$ of the outgoing π^+p pair, following the (coplanar) experimental protocol of Ref. 5. The experimental points are the data of Ref. 5. The curve labeled "free" uses the unmodified π-N t matrix, Eq. (8); the curves labeled "with vertex" and "without vertex" correspond to NL and PL in Fig. 2 and 3—the vertex function referred to is that controlling the off-shell behavior of $\Sigma(k)$ of Eqs. (5)—(7).

tion (DWIA) to describe $(\pi,\pi N)$ knockout on finite nuclei.

The interaction of a pion with a nucleus traditionally has been visualized as a combination of both MS and PR. Multiple scattering theory posits a Schrödinger equation for a particle interacting with scatterers (nucleons) *via* two-body potentials. In this picture, the π-nucleus optical potential is represented as a sum

$$V_{\text{opt}}=\sum_i t^i_{\pi N}+\sum_{i<j}\sum t^i_{\pi N} G t^j_{\pi N}+\cdots \ , \tag{3}$$

where $t^i_{\pi N}$ is the π-N scattering amplitude on isolated nucleon "i,"[6] and G is a Green's function. Alternatively, the presence of the nuclear medium may well modify the π-N scattering amplitude appearing in Eq. (3) from its free value, for example by modifying the fundamental πNN coupling which gives rise to $t^i_{\pi N}$, or by modifying the pion and nucleon propagators in intermediate states. This latter class of effects is pure renormalization. It is easy to see that either MS or PR could shift the resonance in elastic π-nucleus scattering.

The traditional distinction between MS and PR effects is artificial. Nevertheless we feel there is something to be learned from the study of a simple model in which the MS-PR distinction *can* be made. The simpliest model of this kind is a separable representation of the π-N matrix,

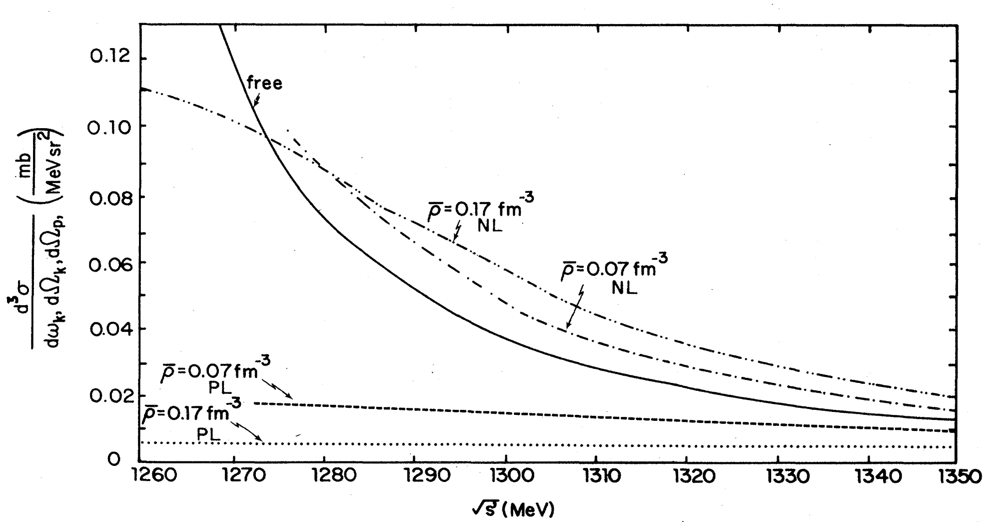

FIG. 2. The fivefold differential cross section for the $^{12}C(\pi^+,\pi^+p)^{11}B$ knockout reaction as a function of the invariant mass $\sqrt{s'}$ of the outgoing π^+p pair, for constant nuclear density, ρ. The labels NL and PL stand for finite-size (nonlocal) and zero-range (pointlike) π-N vertices, as described in Fig. 1 and the text. The label free stands for $\rho=0$.

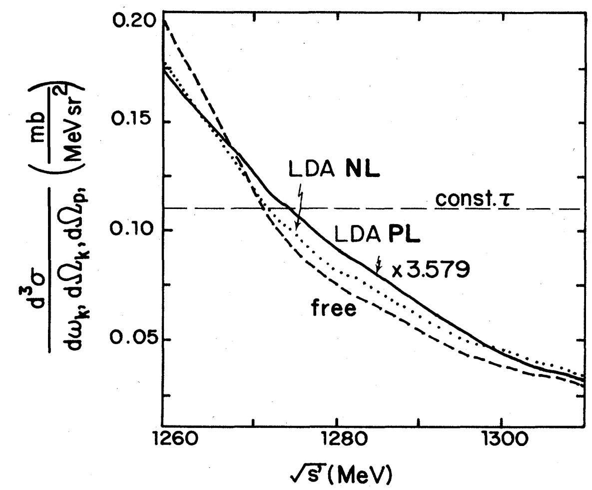

FIG. 3. Local-density approximation (LDA) results for the fivefold differential cross section for nonlocal and pointlike π-N vertices. Also shown are results for free ($\rho=0$) case and for τ of Eqs. (11) and (12) taken to be constant (nonresonant case).

where the fundamental πNN interaction is represented as a vertex amplitude (for our purposes, pure p wave), i.e., a nonlocal coupling,

$$H_{\pi NN}=\frac{G}{2M}\sum_{i=1}^{3}\int d\mathbf{r}\int d\mathbf{r}' N^{+}(\mathbf{r})\sigma\tau_i N(\mathbf{r})\nabla_{r'}\pi_i(\mathbf{r}')\cdot \times V(|\mathbf{r}-\mathbf{r}'|) . \tag{4}$$

The Δ isobar arises from the summation of an infinite series of π-N graphs which diverges when the barycentric variable $\sqrt{s}$ equals the Δ mass. In nuclear matter we expect the πNN vertex function to be modified, and the nucleon propagator to be affected (at least) by Pauli blocking.[7] However, the dominant effect, and the one we focus on here, comes from the renormalization of the free pion propagator $(k^2-m^2)^{-1}$ to

$$\Delta_F(k)=[k^2-m^2-\Sigma(k)]^{-1} , \tag{5}$$

where m is the pion mass and $k=(k^0,\mathbf{k})$ is the four-momentum of the virtual pion. The irreducible self-energy $\Sigma(k)$ is given, to first order in the π-nucleus optical potential, by

$$\Sigma(k)=2k^0\langle \mathbf{k}\,|\,V_{\rm opt}(k^0)\,|\,\mathbf{k}\rangle . \tag{6}$$

When the modified pion propagator (5) is inserted into the expression representing the π-N t matrix as an infinite series of diagrams,[4] we obtain various subsets of the perturbative expansion of the *renormalized* t matrix, $T^{i}_{\pi N}$, depending on how we define the optical potential. The *empirical* π-nucleus optical potential combines effects of both MS and PR. [It would also be possible, in principle, to define an optical potential which exhibits only MS effects by means of the formal expression Eq. (3).] As a matter of convenience we use the pure p-wave part of the empirical optical potential;[8] in principle we could demand self-consistency between this and $T^{i}_{\pi N}$, but in practice we do not. A difficulty we cannot resolve without deeper understanding is how to continue the empirical optical potential off the energy shell; since this continuation determines how strongly $\Sigma(k)$ affects $T^{i}_{\pi N}$, it is necessary to resolve this question in a definite manner. We shall *guess*[9] that the off-shell continuation may be written

$$\langle \mathbf{k}\,|\,V_{\rm opt}(k^0)\,|\,\mathbf{k}\rangle=\mathbf{k}^2 v^2(|\mathbf{k}|)c(k^0)\bar{\rho} , \tag{7}$$

where $v(|\mathbf{k}|)$ is the πNN vertex form factor [normalized to $v(0)=1$], and $\bar{\rho}$ is the local nucleon density.

The free π-N t matrix in this separable model is

$$\langle \mathbf{k}'\,|\,t_{\pi N}(k^0)\,|\,\mathbf{k}\rangle=\mathbf{k}'\cdot\mathbf{k}\,v(|\mathbf{k}'|)v(|\mathbf{k}|)\tau(\sqrt{s}) , \tag{8}$$

where

$$k^0=(s+m^2-M^2)/2\sqrt{s} \tag{9}$$

is the barycentric pion energy expressed in terms of the barycentric π-N total energy $\sqrt{s}$ (M is the nucleon mass, m is the pion mass), and where $\tau(\sqrt{s})$ is given by

$$\tau(\sqrt{s})=\left\{\lambda^{-1}-\tfrac{4}{3}\pi[1+(M^2-m^2)/s]k^0 \times\int_0^\infty \frac{dk''k''^4v^2(k'')}{(k^0)^2-m^2-(k'')^2}\right\}^{-1} . \tag{10}$$

We ignore the inessential complication of the nucleon spin. [The expression Eq. (10) for $\tau(\sqrt{s})$ is derived using off-shell two-body unitarity rather than the Lippmann-Schwinger equation with a separable potential.] When we replace the free pion propagator $(k^2-m^2)^{-1}$ in Eq. (10) by the renormalized propagator (5) (Pauli blocking of the nucleon propagator is neglected on the grounds that the dominant intermediate states have momenta far above the Fermi momentum), we obtain the (approximately) renormalized π-N t matrix

$$\langle \mathbf{k}'\,|\,T_{\pi N}(\sqrt{s},\epsilon,\bar{\rho})\,|\,\mathbf{k}\rangle=\mathbf{k}'\cdot\mathbf{k} \times v(|\mathbf{k}'|)v(|\mathbf{k}|)\tau(\sqrt{s},\epsilon,\bar{\rho}) , \tag{11}$$

where

$$\tau(\sqrt{s},\epsilon,\bar{\rho})=\left\{\lambda^{-1}-\tfrac{4}{3}\pi[1+(M^2-m^2)/s]k^0\int_0^\infty\frac{dk''k''^4v^2(k'')}{(k^0)^2-m^2-(k'')^2[1+\bar{\rho}c(\epsilon)v^2(k'')]}\right\}^{-1} \tag{12}$$

plays the role of a Δ propagator in nuclear matter. The effects of the nuclear environment are included in Eq. (12) *via* the local density approximation (LDA) which neglects the variation of the nuclear density over the volume of the π-N interaction, so that the density ρ in (12) may be replaced by $\rho(\mathbf{R})$ at the point $\mathbf{R}$ which is the barycenter of the π-N subsystem. Note that the energy ϵ refers to the intermediate-state π-N scatterings, and is generally different from $k^0(s')$ defined in (9) above.

In what follows we shall be referring to two choices of the off-shell continuations of the self-energy $\Sigma(k)$: In the curves labeled NL (nonlocal) the off-shell behavior of $\Sigma(k)$ is controlled by the same vertex function $v(|\mathbf{k}|)$ that appears in the numerator of Eq. (12) (and which controls the high-momentum behavior of the integral); whereas in the curves labeled PL (pointlike), $v(|\mathbf{k}|)=1$ is assumed. For economy, we give the vertex functions the usual monopole form[9]

$$v(|\mathbf{k}|)=1/(1+\mathbf{k}^2/\Lambda^2),\quad \Lambda\approx 750\ \text{MeV}/c\ . \tag{13}$$

Note that in principle the strength and range of the πNN coupling could differ from those appropriate to isolated nucleons. About the renormalization of the strength $G_{\pi NN}$ we can say little of interest since obviously any major change in $G_{\pi NN}$ would modify the Δ's parameters by an unknown amount. However, because $v(|\mathbf{k}|)$ differs from unity only at large momenta (or equivalently, at short distances) we expect the range $1/\Lambda$ to be less strongly renormalized than the position and width of the Δ.

From Eq. (11) for the renormalized π-N t matrix, we evaluate the exclusive $(\pi,\pi\text{p})$ knockout amplitude

$$T_{fi}\approx \int d\mathbf{R}(\mathbf{k}'-k^{0\prime}\mathbf{p}'/M)\cdot\{[\mathbf{k}-(ik^0/M)\nabla_R]\Phi(\mathbf{R})\} \times\tau[\sqrt{s'},\epsilon',\rho(R)]e^{i(\mathbf{k}-\mathbf{k}'-\mathbf{p}')\cdot\mathbf{R}}\ , \tag{14}$$

where $\Phi(\mathbf{R})$ is the (bound state) wave function of the proton in the target nucleus. The gradient operates only on Φ. The momenta and energies appearing in Eq. (14) are laboratory quantities; primed (′) means "final state" and unprimed means "initial state." Equation (14) contains the effects of pion distortion in the final state, but neglects them in the initial state. It also neglects the distortion of the final proton wave function. (These omissions are justified since the neglected distortions will affect mainly the overall magnitude of the cross section rather than its shape.) From (14) we obtain the differential cross section

$$\frac{d^3\sigma}{dk^{0\prime}d\Omega_{\mathbf{k}'}d\Omega_{\mathbf{p}'}}=2\pi\frac{k^0k^{0\prime}p^{0\prime}|\mathbf{k}'|\,|\mathbf{p}'|}{|\mathbf{k}|}\langle\,|T_{fi}|\,\rangle^2 \tag{15}$$

for the exclusive knockout reaction, where $\langle\,|T_{fi}|^2\rangle$ is the usual squared, summed, and spin-averaged t matrix.

In Table I we give sets of angles and energies, for fixed nuclear recoil momentum $|\mathbf{P}|=|\mathbf{k}-\mathbf{k}'-\mathbf{p}'|$ and fixed p-wave factor H at which it would be appropriate to perform measurements similar to those of Ref. 5. The recoil momentum $|\mathbf{P}|$ is set to coincide with the peak of a $1p$-shell nucleon wave function (in momentum space) in order to minimize the variation of the data resulting from the bound-nucleon wave function, since nuclear structure is not the focus of this experiment. It is necessary to make the p-wave factor H as large as possible in order to maximize the counting rate. Conversely, if this factor is given its maximum possible value, there is little variation possible for $\sqrt{s'}$. The value chosen, $H=4\times 10^4$ (MeV/c)^2, is the largest consistent with a reasonable range of $\sqrt{s'}$. Note that the experimental protocol was extended by varying the energy of the incoming pion in order to increase the range of variation of $\sqrt{s'}$.

We calculated the fivefold differential cross section $d^3\sigma/dk^{0\prime}d\Omega_{\mathbf{k}'}d\Omega_{\mathbf{p}'}$ of Eq. (15), where $k^{0\prime}$ is the total energy of the outgoing pion, and $\mathbf{k}'$ and $\mathbf{p}'$ are the momenta of the outgoing pion and proton, respectively. Also, $d\Omega_{\mathbf{k}'}$ and $d\Omega_{\mathbf{p}'}$ are the corresponding elements of solid angle. To determine the importance of the density variation (within the local density approximation introduced in I), we calculated the differential cross section first for fixed nuclear density $\bar{\rho}$ (assuming various values for $\bar{\rho}$), and then in LDA. The fixed-density results are shown in Fig. 2, and the LDA results in Fig. 3, as functions of $\sqrt{s'}$. In Fig. 2 we show two forms of the results for each $\bar{\rho}$. These forms differ according to the assumed model of the off-shell extension of nuclear π-N scattering in intermediate states, as explained above and in I. The curves labeled NL and PL refer to the two choices of off-shell continuation of the pion self-energy $\Sigma(k)$ described above. In Fig. 3 (LDA results) we also distinguish between the two kinds of off-shell behavior in intermediate states. In both figures we exhibit for reference the impulse approximation using the unmodified (free) π-N t matrix from our model (whose parameters were adjusted to give the position and width of the isolated Δ).

The range of $\sqrt{s'}$ produced by varying the beam energy and various angles under the most experimentally advantageous circumstances begins well above the free Δ mass, 1232 MeV, because of the kinematic constraints we have had to impose. Thus, although the $\approx$100 MeV range of $\sqrt{s'}$ achieved in our proposed protocol improved considerably on the $\approx$30 MeV range of $\sqrt{s'}$ of the experiment of Ziock *et al.*[5] the new experiment would measure the upper tail of the resonance and therefore would be less sensitive to small shifts in its position. It will be seen from Fig. 2 that there is a striking difference in magnitude between the curves labeled NL and those labeled PL indicating that the latter are far more strongly renormalized than the former. The same is evident in Fig. 3, with the PL curve lying much lower (on an absolute scale) than the NL curve. Does this mean that the revised experiment can distinguish a shifted from an unshifted Δ? We feel that only in the event that the absolute cross sections can be determined to 10% accuracy or better, will it be possible to make useful statements about the nuclear renormalization of the Δ. (Of course we do not imply that the *theory* presented here is capable of anything like 10% accuracy. Its schematic character and neglect of distortion allow, at best, agreement within a factor of 2 or so.) The reason for this is that there is insufficient difference in the *shapes* of the two LDA curves (NL and PL), or for that matter, between them and the impulse approximation ("free"), for the shapes to make a reliable signature. The alert reader will note that for some ranges of $\sqrt{s'}$ there are

TABLE I. Angles and energies for revised experimental protocol in exclusive $(\pi,\pi p)$ experiments. Units: MeV for energies, MeV/c for momenta, degrees for angles.

$T_\pi=300$ $T_{\pi'}=225$ $P=125$ $H=40\,000$ (MeV/c)^2			
θ_π	θ_p	Φ	$\sqrt{s'}$
45.0	34.8	169.9	1262

$T_\pi=325$ $T_{\pi'}=225$ $P=125$							
$H=40\,000$ (MeV/c)^2				$H=-40\,000$ (MeV/c)^2			
θ_π	θ_p	Φ	$\sqrt{s'}$	θ_π	θ_p	Φ	$\sqrt{s'}$
46.8	37.3	174.1	1275				
48.3	39.1	166.2	1280				
56.3	49.0	157.3	1310	72.7	30.4	167.2	1310
57.6	50.5	157.6	1315	73.8	32.6	163.3	1315
58.8	52.0	158.2	1320	74.9	34.7	161.4	1320
60.1	53.5	159.2	1325	76.1	36.7	160.6	1325
61.3	54.9	160.5	1330	77.2	38.6	160.7	1330
62.6	56.3	162.2	1335	78.3	40.5	161.4	1335
63.8	57.7	164.5	1340	79.4	42.2	162.7	1340
65.0	59.1	167.4	1345	80.5	44.0	164.8	1345
66.2	60.5	171.9	1350	81.6	45.7	167.8	1350
				82.7	47.3	172.6	1355

$T_\pi=350$ $T_{\pi'}=225$ $P=125$							
$H=40\,000$ (MeV/c)^2				$H=-40\,000$ (MeV/c)^2			
θ_π	θ_p	Φ	$\sqrt{s'}$	θ_π	θ_p	Φ	$\sqrt{s'}$
51.0	41.0	171.1	1297				
53.5	43.8	165.0	1307	69.2	27.1	176.2	1307
54.8	45.1	163.6	1312	70.3	29.1	165.7	1312
56.0	46.5	162.8	1317	71.4	31.0	161.7	1317
57.3	47.8	162.4	1322	72.4	32.8	159.5	1322
58.5	49.1	162.5	1327	73.5	34.5	158.2	1327
59.7	50.3	162.8	1332	74.6	36.2	157.7	1332
60.9	51.6	163.5	1337	75.7	37.8	157.6	1337
62.1	52.8	164.6	1342	76.8	39.4	158.0	1342
63.2	54.0	166.1	1347	77.8	40.9	158.7	1347
64.4	55.2	168.2	1352	78.9	42.4	159.9	1352
65.6	56.4	171.1	1357	80.0	43.8	161.5	1357
66.7	57.6	177.0	1362	81.1	45.2	163.6	1362
				82.1	46.6	166.3	1367

$T_\pi=375$ $T_{\pi'}=225$ $P=125$							
$H=40\,000$ (MeV/c)^2				$H=-40\,000$ (MeV/c)^2			
θ_π	θ_p	Φ	$\sqrt{s'}$	θ_π	θ_p	Φ	$\sqrt{s'}$
54.6	43.1	178.3	1319	69.1	29.4	165.8	1319
55.8	44.3	173.0	1324	70.2	31.0	162.3	1324
57.0	45.4	171.0	1329	71.2	32.5	160.2	1329
58.2	46.6	169.9	1334	72.3	34.0	158.9	1334
59.4	47.7	169.5	1339	73.3	35.5	158.2	1339
60.5	48.8	169.6	1344	74.4	36.9	157.9	1344
61.7	49.9	170.2	1349	75.4	38.2	158.0	1349
62.8	51.0	171.4	1354	76.5	39.5	158.5	1354
63.9	52.0	173.4	1359	77.5	40.8	159.2	1359
65.1	53.1	177.7	1364	78.5	42.1	160.3	1364

two sets of angles which give the same value of $\sqrt{s}\,'$, $|P|$, and $|H|$, but for which H has opposite signs. Because the LDA is to some extent a distorted-wave approximation, and because the momentum $\mathbf{p}=\mathbf{k}'+\mathbf{p}'-\mathbf{k}$ of the plane-wave Born approximation must be replaced with a gradient operator, it is by no means obvious that both sets of angles should yield the same exclusive differential cross section. However, as we noted in I, distortion effects on the shape of the differential cross section vs $\sqrt{s'}$ are actually not very large for exclusive (π,πN) knockout, and we see this confirmed in the fact that our LDA calculations with either set of angles give almost the same cross sections when $\sqrt{s'}$, $|\mathbf{P}|$, and $|H|$ are the same. Typical differences are $\leq 1\%$ for the NL (finite-size vertex) curve, and $\leq 10\%$ for the PL (pointlike vertex) curve in Fig. 3.

Finally, one of the things that convinced us that the data of Ziock *et al.*[5] were useless for detecting shifts in the mass or width of the Δ, was that when we assumed the π-N t matrix had no variation at all with $\sqrt{s'}$ (that is, was nonresonant) but depended only on H from Eq. (2), we obtained as good fits to the data as when the resonance was present.[4] To convince the reader that the experimental protocol we propose herein does not suffer from this defect, we included the curve labeled "const. τ" (that is, nonresonant p-wave π-N scattering) to show that for the new protocol the theory is flat, as it should be.

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