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Relativistic treatment of the spin difference functions in inelastic proton nucleus scattering

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Spin observables for inelastic proton nucleus scattering are calculated in plane-wave impulse approximation both nonrelativistically and using the Dirac equation. The relativistic treatment yields nonzero spin difference functions that vanish in the nonrelativistic case. We identify the origin of this important contribution of the relativistic treatment, and stress its universal character. We treat 500 MeV proton scattering from ^{12}C as an example.

I. INTRODUCTION

In recent years a relativistic treatment, based on the Dirac equation, of proton-nucleus scattering has provided a better description of medium energy data than the traditional nonrelativistic models.[1,2] It is not surprising then, due to the very natural way in which the Dirac equation incorporates spin, that the most dramatic effects occur precisely in the spin observables. In elastic scattering, however, the number of independent spin observables is highly constrained, and only by studying inelastic transitions can we uncover the full richness contained in the spin observables.

Our first study of inelastic scattering, using the relativistic approach, was for the collective excitations.[3] Since in the collective case, the dynamics responsible for driving the transition is already present in elastic scattering, these elasticlike reactions did not reveal any new physics not already present in the elastic case. In order to uncover new physics we now focus on reactions for which the transition itself is the fundamental driving mechanism of the reaction, for example 0^+ to $1^{\pm}$ excitation. In the present work we concentrate precisely on these transition dominated excitations. We show that the spin difference function, defined in Ref. 4,

$$\Delta_s \equiv (Q-B)+i(P-A_y) \equiv (D_{qK}+D_{Kq})+i(D_{no}-D_{on}) ,$$

is very sensitive to differences between the relativistic and nonrelativistic treatments and how, even in a simple plane wave impulse approximation (PWIA) treatment, the relativistic formalism already has the necessary features to give $\Delta_s \neq 0$, while in the nonrelativistic case we obtain $\Delta_s = 0$ in plane wave local on-shell impulse approximation. By using symmetry principles to write the transition amplitude, we are able to isolate those pieces which are responsible for a nonzero value of Δ_s. We show how those pieces arise from the lower components of the wave function introduced by the Dirac treatment. A different approach to many of these same questions is presented by Shepard, Rost, and McNeil.[5]

We start in Sec. II by using a general, model independent, formalism to write the $0^+ \to J=1$ transition amplitude. All spin observables can then be easily calculated, and we explicitly display the spin difference function. In Sec. III we perform a PWIA calculation for inelastic scattering using a nonrelativistic model, and we point out the notable absence of those terms necessary for obtaining a nonzero spin difference function. In Sec. IV we repeat the same calculation using a relativistic model and show the appearance of the previously absent terms and their close connection with lower components. In Sec. V we generalize our method to arbitrary J. Our conclusions and some sample calculations for 500 MeV proton incident on ^{12}C are given in Sec. VI.

II. GENERAL FORMS AND SPIN OBSERVABLES

In this section we start by writing the $0^+ \to J=1$ transition amplitude using a completely general, model independent formalism which only assumes a rotational and parity invariant interaction. In Ref. 6 it is shown that the most general transition amplitude that one can write for the $0^+ \to 1^+$ process, respecting the above symmetries, is given by

$$\begin{aligned}\hat{\mathbf{A}}(1^+) = {} & A_n(\boldsymbol{\Sigma}\cdot\hat{\mathbf{n}}) + A_{nn}(\boldsymbol{\Sigma}\cdot\hat{\mathbf{n}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}) \\ & + A_{KK}(\boldsymbol{\Sigma}\cdot\hat{\mathbf{K}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{K}}) + A_{qq}(\boldsymbol{\Sigma}\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}) \\ & + A_{qK}(\boldsymbol{\Sigma}\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{K}}) + A_{Kq}(\boldsymbol{\Sigma}\cdot\hat{\mathbf{K}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}) , \qquad (1)\end{aligned}$$

where $\hat{\mathbf{q}}$ and $\hat{\mathbf{K}}$ are unit vectors in the direction of the momentum transfer $\mathbf{q}=\mathbf{k}-\mathbf{k}'$ and average momentum $\mathbf{K}=\frac{1}{2}(\mathbf{k}+\mathbf{k}')$, respectively, $\hat{\mathbf{n}}=\hat{\mathbf{q}}\times\hat{\mathbf{K}}$, and neglecting the Q value of the reaction they consistute an orthogonal system [to keep the effect of the Q value and maintain orthogonality, $\mathbf{q}$ should be defined as the transverse momentum; $\mathbf{q}=(\mathbf{k}-\mathbf{k}')-\mathbf{K}\mathbf{K}\cdot(\mathbf{k}-\mathbf{k}')/K^2$], $\boldsymbol{\sigma}$ is the spin operator of the projectile, and $\boldsymbol{\Sigma}$ is the polarization (axial) vector of the target defined by

$$\Sigma_M \equiv |\, 1^+ M \rangle \langle 0^+ \,| \ . \qquad (2)$$

The A's are scalar functions of the energy and momentum transfer.

In a completely analogous way we can write the most general form for the 1^- amplitude. Assuming only, as in the 1^+ case, a rotational and parity invariant interaction, the amplitude can be written in terms of the six remaining

TABLE I. Complete set of projectile spin observables for a $0^+ \to J=1$ target excitation in terms of invariant amplitudes.

$(1^+$ state)	$(1^-$ state)
$\left(\frac{d\sigma}{d\Omega}\right) = \lvert A_n\rvert^2 + \lvert A_{nn}\rvert^2 A_{KK}\rvert^2 + \lvert A_{qq}\rvert^2 + \lvert A_{qK}\rvert^2 + \lvert A_{Kq}\rvert^2$	$\left(\frac{d\sigma}{d\Omega}\right) = \lvert A_K\rvert^2 + \lvert A_q\rvert^2 + \lvert A_{Kn}\rvert^2 + \lvert A_{qn}\rvert^2 + \lvert A_{nK}\rvert^2 + \lvert A_{nq}\rvert^2$
$\left(\frac{d\sigma}{d\Omega}\right) D_{on} = 2[\mathrm{Re}(A_n A_{nn}^*) + \mathrm{Im}(A_{Kq}A_{KK}^* + A_{qq}A_{qK}^*)]$	$\left(\frac{d\sigma}{d\Omega}\right) D_{on} = 2[\mathrm{Re}(A_K A_{Kn}^* + A_q A_{qn}^*) + \mathrm{Im}(A_{nq}A_{nK}^*)]$
$\left(\frac{d\sigma}{d\Omega}\right) D_{no} = 2[\mathrm{Re}(A_n A_{nn}^*) + \mathrm{Im}(A_{KK}A_{Kq}^* + A_{qK}A_{qq}^*)]$	$\left(\frac{d\sigma}{d\Omega}\right) D_{no} = 2[\mathrm{Re}(A_K A_{Kn}^* + A_q A_{qn}^*) + \mathrm{Im}(A_{nK}A_{nq}^*)]$
$\left(\frac{d\sigma}{d\Omega}\right) D_{Kq} = 2[\mathrm{Im}(A_n A_{nn}^*) + \mathrm{Re}(A_{Kq}A_{KK}^* + A_{qq}A_{qK}^*)]$	$\left(\frac{d\sigma}{d\Omega}\right) D_{Kq} = 2[\mathrm{Im}(A_K A_{Kn}^* + A_q A_{qn}^*) + \mathrm{Re}(A_{nq}A_{nK}^*)]$
$\left(\frac{d\sigma}{d\Omega}\right) D_{qK} = 2[\mathrm{Im}(A_{nn} A_n^*) + \mathrm{Re}(A_{KK}A_{Kq}^* + A_{qK}A_{qq}^*)]$	$\left(\frac{d\sigma}{d\Omega}\right) D_{qK} = 2[\mathrm{Im}(A_{Kn} A_K^* + A_{qn} A_q^*) + \mathrm{Re}(A_{nK}A_{nq}^*)]$
$\left(\frac{d\sigma}{d\Omega}\right) D_{nn} = \lvert A_n\rvert^2 + \lvert A_{nn}\rvert^2 - \lvert A_{KK}\rvert^2 - \lvert A_{qq}\rvert^2 - \lvert A_{qK}\rvert^2 - \lvert A_{Kq}\rvert^2$	$\left(\frac{d\sigma}{d\Omega}\right) D_{nn} = \lvert A_K\rvert^2 + \lvert A_q\rvert^2 + \lvert A_{Kn}\rvert^2 + \lvert A_{qn}\rvert^2 - \lvert A_{nK}\rvert^2 - \lvert A_{nq}\rvert^2$
$\left(\frac{d\sigma}{d\Omega}\right) D_{KK} = \lvert A_n\rvert^2 - \lvert A_{nn}\rvert^2 + \lvert A_{KK}\rvert^2 - \lvert A_{qq}\rvert^2 + \lvert A_{qK}\rvert^2 - \lvert A_{Kq}\rvert^2$	$\left(\frac{d\sigma}{d\Omega}\right) D_{KK} = \lvert A_K\rvert^2 + \lvert A_q\rvert^2 - \lvert A_{Kn}\rvert^2 - \lvert A_{qn}\rvert^2 + \lvert A_{nK}\rvert^2 - \lvert A_{nq}\rvert^2$
$\left(\frac{d\sigma}{d\Omega}\right) D_{qq} = \lvert A_n\rvert^2 - \lvert A_{nn}\rvert^2 - \lvert A_{KK}\rvert^2 + \lvert A_{qq}\rvert^2 - \lvert A_{qK}\rvert^2 + \lvert A_{Kq}\rvert^2$	$\left(\frac{d\sigma}{d\Omega}\right) D_{qq} = \lvert A_K\rvert^2 + \lvert A_q\rvert^2 - \lvert A_{Kn}\rvert^2 - \lvert A_{qn}\rvert^2 - \lvert A_{nK}\rvert^2 + \lvert A_{nq}\rvert^2$

$$D_{Ko} = D_{oK} = D_{qo} = D_{oq} = 0$$

$$D_{Kn} = D_{nK} = D_{qn} = D_{nq} = 0$$

operators of the $0^+ \to J=1$ process, i.e.,

$$\hat{\mathbf{A}}(1^-) = A_K(\mathbf{\Sigma}\cdot\hat{\mathbf{K}}) + A_q(\mathbf{\Sigma}\cdot\hat{\mathbf{q}}) + A_{Kn}(\mathbf{\Sigma}\cdot\hat{\mathbf{K}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}) + A_{qn}(\mathbf{\Sigma}\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}) + A_{nK}(\mathbf{\Sigma}\cdot\hat{\mathbf{n}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{K}}) + A_{nq}(\mathbf{\Sigma}\cdot\hat{\mathbf{n}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}) \,, \tag{3}$$

where in this expression the polarization (polar) vector of the target is given by

$$\Sigma_M \equiv |1^- M\rangle\langle 0^+| \,. \tag{4}$$

A great advantage of separating the target and projectile space contributions to the amplitude lies in the calculation of projectile spin observables. In calculating spin observables we adopt the standard definition given by

$$\left[\frac{d\sigma}{d\Omega}\right] D_{\alpha\beta} = \tfrac{1}{2}\,\mathrm{Tr}(\sigma_\alpha \hat{\mathbf{A}} \sigma_\beta \hat{\mathbf{A}}^\dagger) \,, \tag{5}$$

where $\alpha,\beta=(o,n,q,K)$, $\sigma_0\equiv 1$, and $D_{00}=1$ so the unpolarized cross section is

$$\left[\frac{d\sigma}{d\Omega}\right] = \tfrac{1}{2}\,\mathrm{Tr}(\hat{\mathbf{A}}\,\hat{\mathbf{A}}^\dagger) \,. \tag{6}$$

If no property of the final state besides energy, angular momentum, and parity is determined, then a trace, within the target space, of the form $\mathrm{Tr}[(\mathbf{\Sigma}\cdot\mathbf{A})(\mathbf{\Sigma}\cdot\mathbf{B})^\dagger]$ must be performed which by using the definition of $\mathbf{\Sigma}$ can trivially be done, to obtain

$$\mathrm{Tr}[(\mathbf{\Sigma}\cdot\mathbf{A})(\mathbf{\Sigma}\cdot\mathbf{B})^\dagger] = \mathbf{A}\cdot\mathbf{B} \,. \tag{7}$$

This result implies in terms of (1) or (3) that only those amplitudes in which $\mathbf{\Sigma}$ is dotted into the same vector will be able to interfere. The remaining traces, in the space of the projectile, can then be performed in the standard way. Of particular importance in our treatment are spin observables sensitive to differences between the relativistic and the nonrelativistic models and therefore useful in providing insight into the importance of relativity. As we shall see in the following sections these are for the 1^+ case

$$(P-A_y) \equiv (D_{no}-D_{on}) = \left[4\Big/\frac{d\sigma}{d\Omega}\right] \mathrm{Im}(A_{KK}A_{Kq}^* + A_{qK}A_{qq}^*) \,,$$
$$(Q-B) \equiv (D_{qK}+D_{Kq}) = \left[4\Big/\frac{d\sigma}{d\Omega}\right] \mathrm{Re}(A_{KK}A_{Kq}^* + A_{qK}A_{qq}^*) \,, \tag{8}$$

or more compactly the spin difference function

$$\Delta_s = (Q-B) + i(P-A_y) = \left[4\Big/\frac{d\sigma}{d\Omega}\right] (A_{KK}A_{Kq}^* + A_{qK}A_{qq}^*) \,, \tag{9}$$

while for the 1^- case the relevant spin observables are

$$\Delta_s = \left[4\Big/\frac{d\sigma}{d\Omega}\right] (A_{nK}A_{nq}^*) \tag{10}$$

and

$$[(1-D_{nn}) + (D_{qq}-D_{KK})] = \left[4\Big/\frac{d\sigma}{d\Omega}\right] |A_{nq}|^2 \,. \tag{11}$$

The full set of spin observables assuming only the projectile spin is monitored is given in Table I.

We stress that the above results are model independent, they were obtained from the general structure of the amplitudes (1) and (3), which follow just from assuming a rotational and parity invariant interaction. In the following two sections we concentrate on two specific models, viz., the nonrelativistic and relativistic, respectively, in order to evaluate the above amplitudes.

III. NONRELATIVISTIC AMPLITUDE

We consider the excitation of a nucleus, originally in a 0^+ state, to a final JM state with parity π_f by means of its interaction with a proton with initial momentum $\mathbf{k}$ and final momentum $\mathbf{k}'$. The transition amplitude, in a nonrelativistic plane wave impulse approximation (PWIA) treatment, is given by

$$A_{JM} = \langle JM | \sum_{n=1}^{A} e^{i\mathbf{q}\cdot\mathbf{x}_n} t_n | 0^+ \rangle \,, \tag{12}$$

where t_n is the NN t matrix. In the space of the target particles, this operator can be expressed in terms of the unit matrix and the three Pauli matrices, i.e.,

$$t_n = d_0 + \mathbf{d}\cdot\boldsymbol{\sigma}(n) \,. \tag{13}$$

A convenient parametrization of the two-body t matrix, in the c.m. frame, is given by the Wolfenstein representation.[7] In this case the above coefficients, which are operators in the projectile space, can be written explicitly; the scalar operator is given by

$$d_0 = A(q) + iqC(q)(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}) \,, \tag{14}$$

while the axial vector operator by

$$\mathbf{d} = [B(q)\boldsymbol{\sigma} + iqC(q)\hat{\mathbf{n}} + q^2 D(q)(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})\hat{\mathbf{q}} + E(q)(\boldsymbol{\sigma}\cdot\hat{\mathbf{K}})\hat{\mathbf{K}}] \,, \tag{15}$$

where now $\boldsymbol{\sigma}$ is the spin operator of the projectile. By expanding $e^{i\mathbf{q}\cdot\mathbf{x}_n}$ in spherical harmonics and by writing the inner product in (13) in terms of its spherical components, we can isolate the part of the operator which acts on the target particle; we write

$$e^{i\mathbf{q}\cdot\mathbf{x}_n} t_n = \sum_{jmls} 4\pi i^l (-1)^{j+l+s} j_l(qx_n) \times [Y_l(\hat{\mathbf{x}}_n)\sigma_s(n)]_{jm} [Y_l(\hat{\mathbf{q}}) d_s]_{jm}^* \,, \tag{16}$$

where s takes only the values zero or one.

We now choose to express the one-particle nuclear operator in a second quantized formalism

$$\sum_{n=1}^{A} j_l(qx_n)[Y_l(\hat{\mathbf{x}}_n)\sigma_s(n)]_{jm} = \sum_{\substack{j_i m_i \\ j_f m_f}} \langle j_f m_f \mid j_l(qx)[Y_l(\hat{\mathbf{x}})\sigma_s(1)]_{jm} \mid j_i m_i \rangle a^\dagger_{j_f m_f} a_{j_i m_i}$$

$$= -\sum_{j_i, j_f} \frac{\hat{j}_f}{\hat{j}} \langle j_f || j_l(qx)[Y_l(\hat{\mathbf{x}})\sigma_s(1)]_j || j_i \rangle [a^\dagger_{j_f} \tilde{a}_{j_i}]_{jm} , \quad (17)$$

where $\hat{j}=(2j+1)^{1/2}$ and where $a^\dagger_{jm}$ and a_{jm} are particle creation and annihilation operators for orbits of the target, respectively, and the hole creation operator $\tilde{a}_{jm}$, which is related to the annihilation operator by a time reversal transformation, is given by

$$\tilde{a}_{jm} = (-1)^{j-m} a_{j,-m} . \quad (18)$$

In the above expression $[a^\dagger_{j_f} a_{j_i}]_{jm}$ is the only remaining target operator, and we can then take matrix elements between initial and final target states, we obtain

$$\langle JM \mid [a^\dagger_{j_f} a_{j_i}]_{jm} \mid 0^+ \rangle = A_J(j_f, j_i) \delta_{jJ} \delta_{mM} , \quad (19a)$$

where the nuclear structure amplitude $A_J(j_f, j_i)$ is given by

$$A_J(j_f, j_i) = \langle J || [a^\dagger_{j_f} \tilde{a}_{j_i}]_J || 0 \rangle . \quad (19b)$$

We can now write the full transition amplitude in a very simple form, namely a nuclear form factor times an operator in the projectile space

$$A_{JM} = \sum_{ls} G_{Jls}(q)[Y_l(\hat{\mathbf{q}}) d_s]^*_{JM} , \quad (20a)$$

where the nuclear form factor is given by

$$G_{Jls}(q) = \sum_{j_i, j_f} 4\pi i^l (-1)^{J+l+s+1} \frac{\hat{j}_f}{\hat{J}} A_J(j_f, j_i) \langle j_f || j_l(qx)[Y_l(\hat{\mathbf{x}})\sigma_s(1)]_J || j_i \rangle \quad (20b)$$

and

$$\langle j_f || j_l(qx)[Y_l(\hat{\mathbf{x}})\sigma_s(1)]_J || j_i \rangle = \frac{1}{\sqrt{2\pi}} \hat{s} \hat{l}_i \hat{l} \hat{j}_i \hat{J} \langle l0; l_i 0 \mid l_f 0 \rangle \begin{Bmatrix} \frac{1}{2} & l_i & j_i \\ \frac{1}{2} & l_f & j_f \\ s & l & J \end{Bmatrix} \rho_l(q) , \quad (20c)$$

$$\rho_l(q) = \int_0^\infty x^2 dx \, \psi_f^*(x) j_l(qx) \psi_i(x) . \quad (20d)$$

We note that the above expression for the amplitude still depends on the magnetic substate of the target (normally not measured in an experiment); it is more convenient, however, in order to evaluate all spin observables, to treat nuclear and projectile parts on an equal footing by writing the amplitude as an operator in the target and projectile space. This can easily be done since the matrix elements are already known; we write

$$\hat{\mathbf{A}}_J = \sum_{M=-J}^{J} | JM \rangle A_{JM} \langle 0^+ | = \sum_{ls} G_{Jls}(q) \Omega_{Jls} , \quad (21)$$

where the rotational invariant operator Ω_{Jls} is defined as

$$\Omega_{Jls} = \sum_{M=-J}^{J} (-1)^M [Y_l(\hat{\mathbf{q}}) d_s]_{J,-M} \Sigma_{JM} , \quad (22a)$$

and the nuclear polarization tensor operator by

$$\Sigma_{JM} = | JM \rangle \langle 0^+ | . \quad (22b)$$

So far everything has been completely general. We now specialize to the $J=1$ case, but in Sec. V return to the general case.

The operator $[Y_l(\hat{\mathbf{q}}) d_s]_{1,-M}$ in (22a) is now a spherical tensor of rank one, and therefore we should be able to find a vector $\mathbf{T}_{ls}$ such that

$$[Y_l(\hat{\mathbf{q}}) d_s]_{1,-M} = \mathbf{T}_{ls} \cdot \hat{\mathbf{e}}(-M) , \quad (23)$$

where $\hat{\mathbf{e}}(M)$ are the standard circular polarization vectors defined by

$$\hat{\mathbf{e}}(1) = -\frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}}) ,$$

$$\hat{\mathbf{e}}(0) = \hat{\mathbf{z}} , \quad (24)$$

$$\hat{\mathbf{e}}(-1) = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}}) .$$

The fact that $s=0$ or 1 implies a set of four different vectors. These are easily calculated and given by

$$\mathbf{T}_{ls}=\begin{cases}\left[\frac{3}{4\pi}\right]^{1/2} d_0\hat{\mathbf{q}} \quad (l=1,s=0)\\ \left.\begin{array}{l}\frac{1}{4\pi}\mathbf{d} \quad (l=0)\\ \left[\frac{3}{8\pi}\right]^{1/2} i(\hat{\mathbf{q}}\times\mathbf{d}) \quad (l=1)\\ \frac{1}{\sqrt{8\pi}}[\mathbf{d}-3(\mathbf{d}\cdot\hat{\mathbf{q}})\hat{\mathbf{q}}] \quad (l=2)\end{array}\right\} (s=1)\,.\end{cases} \tag{25}$$

We note that for $l=1$, $\mathbf{T}_{ls}$ transforms like a polar vector while for $l\neq 1$ it behaves like an axial vector [remember $\mathbf{d}$ is an axial vector (15)]. Furthermore, for the 1^+ state $\boldsymbol{\Sigma}$ is an axial vector which implies that the rotational invariant operator $\Omega_{Jls}=\mathbf{T}_{ls}\cdot\boldsymbol{\Sigma}$ behaves like a scalar for $l\neq 1$ and like a pseudoscalar for $l=1$. Conversely, for the 1^- state $\boldsymbol{\Sigma}$ transforms like a polar vector and therefore the parity properties of Ω_{Jls} are reversed. The requirement that the full amplitude $\hat{\mathbf{A}}_J$ should be a true scalar operator (we are assuming a rotational and parity invariant interaction) is carried by the parity Clebsch $\langle l0,l_i0\,|\,l_f0\rangle$ in the nuclear form factor [Eq. (20)]. For the 1^+ state only even values of l are allowed, while for the 1^- case only odd ones survive. By using the explicit values of $\mathbf{d}$ [Eq. (15)] we obtain the following closed form expression for the 1^+ amplitude as given in Eq. (1), with

$$\begin{aligned}A_n(q)&=iqC(q)G_+(q)\,,\quad G_+(q)\equiv\frac{1}{\sqrt{8\pi}}[\sqrt{2}G_{101}(q)+G_{121}(q)]\,,\\ A_{nn}(q)&=B(q)G_+(q)\,,\quad G_-(q)\equiv\frac{1}{\sqrt{4\pi}}[G_{101}(q)-\sqrt{2}G_{121}(q)]\,,\\ A_{KK}(q)&=[B(q)+E(q)]G_+(q)\,,\\ A_{qq}(q)&=[B(q)+q^2D(q)]G_-(q)\,,\\ A_{qK}(q)&=A_{Kq}(q)=0\,,\end{aligned} \tag{26}$$

and we have used

$$(\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma})=(\boldsymbol{\Sigma}\cdot\hat{\mathbf{n}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}})+(\boldsymbol{\Sigma}\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})+(\boldsymbol{\Sigma}\cdot\hat{\mathbf{K}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{K}})\,. \tag{27}$$

The form of the amplitude conforms, as it should, to the model independent case obtained in Sec. II. The most interesting part of our result is the fact that the two cross term amplitudes A_{qK} and A_{Kq}, allowed by general invariance principles and essential for obtaining spin observable differences [Sec. II, Eqs. (8) and (9)], are absent in a PWIA treatment. We stress, however, that this result holds even in a distorted wave eikonal treatment to first order in the spin orbit distorting potential, and a nonzero value can be obtained only through explicit inclusion of nonlocal effects.[8]

In a completely analogous way we can write the result for the 1^- amplitude, in the form given in Eq. (3), where in the present PWIA treatment the amplitudes are given by

$$\begin{aligned}A_K&=\left[\frac{3}{8\pi}\right]^{1/2} qC(q)G_{111}(q)\,,\\ A_{qn}&=i\left[\frac{3}{4\pi}\right]^{1/2} qC(q)G_{110}(q)\,,\\ A_q&=\left[\frac{3}{4\pi}\right]^{1/2} A(q)G_{110}(q)\,,\\ A_{nK}&=i\left[\frac{3}{8\pi}\right]^{1/2}[B(q)+E(q)]G_{111}(q)\,,\\ A_{Kn}&=-i\left[\frac{3}{8\pi}\right]^{1/2} B(q)G_{111}(q)\,,\quad A_{nq}=0\,.\end{aligned} \tag{28}$$

As in the 1^+ case, the absence of the A_{nq} term has profound consequences for the relations between spin observables. In particular the spin difference functions defined in (10) and (11) are both zero.

IV. RELATIVISTIC AMPLITUDE

In this section we address the same problem as in the preceding one, now, however, within the realm of a relativistic plane wave impulse approximation (RPWIA). The transition amplitude is now given by

$$A_{JM}=u^\dagger(\mathbf{k}')\langle JM\,|\,\sum_{n=1}^{A} e^{i\mathbf{q}\cdot\mathbf{x}_n}\gamma^0\gamma^0(n)\hat{\mathfrak{t}}_n\,|\,0^+\rangle u(\mathbf{k})\,, \tag{29}$$

where $\mathbf{k}$ ($\mathbf{k}'$) is the initial (final) momentum of the projectile,

$$u(\mathbf{k})=\left[\frac{E+m}{2m}\right]^{1/2}\begin{bmatrix}1\\ \frac{\boldsymbol{\sigma}\cdot\mathbf{k}}{E+m}\end{bmatrix},\qquad u(\mathbf{k}')=\left[\frac{E'+m}{2m}\right]^{1/2}\begin{bmatrix}1\\ \frac{\boldsymbol{\sigma}\cdot\mathbf{k}'}{E'+m}\end{bmatrix} \tag{30}$$

are free Dirac spinors for the projectile and $\hat{\mathfrak{t}}_n$ is, aside from numerical factors, the Lorentz invariant nucleon-nucleon amplitude[9] written as

$$\hat{\mathfrak{t}}_n=t_S+t_V\gamma^\mu\gamma_\mu(n)+t_T\sigma^{\mu\nu}\sigma_{\mu\nu}(n)+t_P\gamma^5\gamma^5(n)+t_A\gamma^5\gamma^\mu\gamma^5(n)\gamma^\mu(n)\,. \tag{31}$$

All five amplitudes are functions of the Lorentz invariants s and $t=-q^2$, n refers to operators in the target or nuclear space, and we use the Bjorken-Drell[10] convention for the gamma matrices. By following Ref. 11 we can recast the transition amplitude in a form that conveniently separates the spin dependence from the particular combination of upper and lower components involved; we write

$$\gamma^0\gamma^0(n)\hat{t}_n=\sum_{\nu=1}^{4}\{f^{\nu}+g^{\nu}[\sigma\cdot\sigma(n)]\}\Gamma_\nu\Gamma_\nu(n)\ , \tag{32}$$

where the structure matrices Γ_ν are defined by

$$\Gamma_1=1=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ \Gamma_2=\gamma^0=\begin{bmatrix}1&0\\0&-1\end{bmatrix},$$
$$\Gamma_3=\gamma^5=\begin{bmatrix}0&1\\1&0\end{bmatrix},\ \Gamma_4=\gamma^0\gamma^5=\begin{bmatrix}0&1\\-1&0\end{bmatrix}, \tag{33}$$

and

$$\begin{aligned}f^1&=t_V\ ,\ g^1=-t_A\ ,\\ f^2&=t_S\ ,\ g^2=2t_T\ ,\\ f^3&=t_A\ ,\ g^3=-t_V\ ,\\ f^4&=t_P\ ,\ g^4=2t_T\ .\end{aligned} \tag{34}$$

We note that in (32) the transition amplitude takes a very simple form since only a central and a spin-spin term are present (compare to the full Wolfenstein decomposition). The additional structure in the amplitude comes from the structure matrices Γ_ν which have no dynamical content other than to specify the particular upper and lower component combinations.

From here on we proceed as close in spirit as possible to the nonrelativistic case. We start by expanding $e^{i\mathbf{q}\cdot\mathbf{x}_n}$ in spherical harmonics to obtain

$$\begin{aligned}e^{i\mathbf{q}\cdot\mathbf{x}_n}\gamma^0\gamma^0(n)\hat{t}_n&=\sum_{\nu=1}^{4}\sum_{jmls}4\pi i^l(-1)^{j+l+s}h_s^\nu(q)j_l(qx_n)[Y_l(\hat{\mathbf{x}}_n)\sigma_s(n)]_{jm}[Y_l(\hat{\mathbf{q}})\sigma_s]_{jm}^*\Gamma_\nu(n)\Gamma_\nu\ ,\\ h_s^\nu(q)&=f^\nu(q)\ ,\ s=0\\ &=g^\nu(q)\ ,\ s=1\ .\end{aligned} \tag{35}$$

We continue by writing the one-particle target space operator in a second quantized formalism

$$\sum_{n=1}^{A}j_l(qx_n)[Y_l(\hat{\mathbf{x}}_n)\sigma_s(n)]_{jm}\Gamma_\nu(n)=-\sum_{j_i,j_f}\frac{\hat{j}_f}{\hat{j}}\langle\psi_{j_f}||j_l(qx)[Y_l(\hat{\mathbf{x}})\sigma_s(1)]_j\Gamma_\nu(1)||\psi_{j_i}\rangle[a_{j_f}^\dagger\tilde{a}_{j_i}]_{jm}\ . \tag{36}$$

At the end, just as in the nonrelativistic case, the amplitude is written as a nuclear form factor times a projectile space operator, i.e.,

$$A_{JM}=u^\dagger(\mathbf{k}')\left\{\sum_{\nu=1}^{4}\sum_{ls}G_{Jls}^\nu(q)[Y_l(\hat{\mathbf{q}})\sigma_s]_{JM}^*\Gamma_\nu\right\}u(\mathbf{k})\ , \tag{37}$$

or, by introducing the nuclear polarization tensor operator Σ_{JM} (22b) as an operator in the target and projectile space

$$\hat{\mathbf{A}}_J=\sum_{\nu=1}^{4}\sum_{ls}G_{Jls}^\nu(q)u^\dagger(\mathbf{k}')\Omega_{Jls}(\sigma,\Sigma)\Gamma_\nu u(\mathbf{k})\ , \tag{38}$$

where

$$G_{Jls}^\nu(q)=\sum_{j_i,j_f}4\pi i^l(-1)^{J+l+s+1}\frac{\hat{j}_f}{\hat{J}}A_J(j_f,j_i)h_s^\nu(q)\langle\psi_{j_f}||j_l(qx)[Y_l(\hat{\mathbf{x}})\sigma_s(1)]_J\Gamma_\nu(1)||\psi_{j_i}\rangle\ , \tag{39}$$

and the rotational invariant operator Ω_{Jls} was defined in (22a).

Note, however, that in the present form the single particle wave functions are four-component Dirac bound states. The prescription we use for calculating the lower bound state component is as simple as the Dirac formalism allows. We write the Dirac bound state, on completely general grounds for a particle in the presence of central scalar and fourth component of a vector potential, as

$$\psi_{jm}(\mathbf{x})=\begin{bmatrix}u_{jl}(x)&\mathcal{Y}_{jl}^m(\hat{\mathbf{x}})\\ iw_{jl'}(x)&\mathcal{Y}_{jl'}^m(\hat{\mathbf{x}})\end{bmatrix}, \tag{40}$$

where $u_{jl}(x)$ and $w_{jl'}(x)$ are real functions which satisfy a set of coupled differential equations and l' is the other l

with the same j. By assuming, however, that

$$\phi_{jl}^{m}(\mathbf{x})=u_{jl}(x)\mathcal{Y}_{jl}^{m}(\hat{\mathbf{x}})$$

is a known nonrelativistic bound state wave function, the lower component can easily be obtained; it is given by

$$w_{jl'}(r)=\frac{1}{E+m+S(r)-V(r)}\times\left[\frac{d}{dr}+\frac{(1+\kappa)}{r}\right]u_{jl}(r)\,, \tag{41}$$

where

$$\kappa=\pm(j+\tfrac{1}{2})\ \text{ for } l=j\pm\tfrac{1}{2}\,,$$

and where $S(r)$ and $V(r)$ are the central scalar and vector potentials in the Dirac equation for the bound state orbitals.

So far everything has been completely general. Now we specialize to the $J=1$ case. The general case is addressed in Sec. V. In Sec. III we found the structure of the rotational invariant operator Ω_{Jls} by making the appropriate change in (22) and (25), namely $d_s\rightarrow\sigma_s$; we have in the present case

$$\Omega_{J=1,ls}=\left[\frac{3}{4\pi}\right]^{1/2}(\boldsymbol{\Sigma}\cdot\hat{\mathbf{q}})\,,\quad (l=1,s=0)$$

$$\left.\begin{aligned}&=\frac{1}{\sqrt{4\pi}}(\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma})\,,\quad (l=0)\\ &=i\left[\frac{3}{8\pi}\right]^{1/2}\boldsymbol{\Sigma}\cdot(\hat{\mathbf{q}}\times\boldsymbol{\sigma})\,,\quad (l=1)\\ &=\frac{1}{\sqrt{8\pi}}[(\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma})-3(\boldsymbol{\Sigma}\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})]\,,\quad (l=2)\end{aligned}\right\}\ (s=1)\,. \tag{42}$$

We argued then, on the basis of a rotational and parity invariant interaction, that the transition amplitude should behave like a scalar operator, and, therefore, we should rule out the $l=1$ terms from the 1^+ case and the $l\neq1$ from the 1^- case. These assertions were justified by the presence of the parity Clebsch in the nuclear form factor. In the present relativistic case, however, all four terms are allowed to contribute. The previously forbidden "pseudoscalar" terms are now able to contribute since they appear in conjunction with the structure matrices 3 and 4 which imply an amplitude linear in the lower components, carriers of the necessary parity change. The confirmation of these assertions resides again in the parity Clebsch contained in the nuclear form factor. In general, and due to the presence of lower components, we have four types of parity Clebsch's. These are

$$\langle l0;l_i0\,|\,l_f0\rangle \ \text{ and } \ \langle l0;l_i'\,|\,l_f'0\rangle \ \text{ for } \nu=1,2$$

and

$$\langle l0;l_i'0\,|\,l_f0\rangle \ \text{ and } \ \langle l0;l_i0\,|\,l_f'0\rangle \ \text{ for } \nu=3,4\,.$$

We then observe that for the 1^+ case, where $l_i+l_f=$even, the $l=1$ terms will be zero, as before, for $\nu=1,2$ but nonzero for $\nu=3,4$; in a similar way we have that for the 1^- case, where $l_i+l_f=$odd, the $l=$even terms will vanish for $\nu=1,2$ but will be nonzero for $\nu=3,4$. In general we have

	$\nu=1,2$	$\nu=3,4$
(1^+)	$l=0,2$	$l=1$
	$s=1$	$s=0,1$
(1^-)	$l=1$	$l=0,2$
	$s=0,1$	$s=1$

It is not surprising, then, that precisely those previously absent terms will make the interesting contributions to the relativistic amplitude. In particular we note that in sharp contrast to the nonrelativistic case, there exists a spin independent contribution to the unnatural parity 1^+ case.

After some straightforward spin algebra we can display our results:

(i) For the 1^+ case, in terms of the general structure given in Eq. (1), we obtain

$$\begin{aligned}
A_n&=\left[\frac{3}{8\pi}\right]^{1/2}\left[\frac{iK}{m}\right]G_{111}^{3}(q)\,,\quad A_n=\frac{iqK}{2m(E+m)}[G_{\bar{1}01}^{-}(q)+G_{\bar{1}21}^{-}(q)]\,,\\
A_{nn}=A_{KK}&=\left[\frac{3}{8\pi}\right]^{1/2}\left[\frac{q}{2m}\right]G_{\bar{1}11}^{3}(q)\,,\quad A_{nn}=[G_{101}^{1,2}(q)+G_{121}^{1,2}(q)]+\frac{q^2}{4m(E+m)}[G_{\bar{1}01}^{-}(q)+G_{\bar{1}21}^{-}(q)]\,,\\
A_{qq}&=\left[\frac{3}{4\pi}\right]^{1/2}\left[\frac{q}{2m}\right]G_{110}^{4}(q)\,,\quad A_{KK}=[G_{101}^{2,1}(q)+G_{121}^{2,1}(q)]\,,\\
A_{qK}&=\left[\frac{3}{4\pi}\right]^{1/2}\left[\frac{K}{m}\right]G_{\bar{1}10}^{3}(q)\,,\quad A_{qq}=[G_{101}^{1,2}(q)-2G_{121}^{1,2}(q)]\,,\\
A_{Kq}&=-\left[\frac{3}{8\pi}\right]^{1/2}\left[\frac{K}{m}\right]G_{\bar{1}11}^{4}(q)\,,\quad A_{qK}=A_{Kq}=0\,.
\end{aligned} \tag{43}$$

(ii) While for the 1^- case, where the amplitude is given by Eq. (3), we obtain

$$\begin{aligned}
A_K &= \left[\frac{K}{m}\right] G^3_+(q) , \quad A_K = \frac{qK}{2m(E+m)} G_{111}(q) , \\
A_q &= \left[\frac{q}{2m}\right] G^4_-(q) , \quad A_q = \left[G^{2,1}_{110}(q) - \frac{q^2}{4m(E+m)} G^-_{110}(q) \right] , \\
A_{Kn} &= -\left[\frac{iq}{2m}\right] G^3_+(q) , \quad A_{Kn} = (-i)\left[G^{1,2}_{111}(q) + \frac{q^2}{4m(E+m)} G^-_{111}(q) \right] , \\
A_{qn} &= \left[\frac{iK}{m}\right] G^4_-(q) , \quad A_{qn} = -\frac{iqK}{2m(E+m)} G^-_{110}(q) , \\
A_{nK} &= \left[\frac{iq}{2m}\right] G^3_+(q) , \quad A_{nK} = iG^{2,1}_{111}(q) , \\
A_{nq} &= -\left[\frac{iK}{m}\right] G^4_+(q) , \quad A_{nq} = 0 ,
\end{aligned} \tag{44}$$

and we have defined

$$\begin{aligned}
G^\mu_+(q) &\equiv \frac{1}{\sqrt{8\pi}}[\sqrt{2}G^\mu_{101}(q) + G^\mu_{121}(q)] , \quad \alpha_{Jls} \equiv \left[\frac{(3J-l+1)(1+\delta_{lJ})}{8\pi(s+1)} \right]^{1/2} , \\
G^\mu_-(q) &\equiv \frac{1}{\sqrt{4\pi}}[G^\mu_{101}(q) - \sqrt{2}G^\mu_{121}(q)] , \\
G^{\mu,\nu}_{Jls}(q) &\equiv \alpha_{Jls}\left[G^\mu_{Jls}(q) + \left[\frac{E}{m}\right] G^\nu_{Jls}(q) \right] , \\
G^-_{Jls}(q) &\equiv \alpha_{Jls}[G^1_{Jls}(q) - G^2_{Jls}(q)] .
\end{aligned} \tag{45}$$

The left-hand side contains those amplitudes linear in the lower components and therefore absent from a nonrelativistic treatment. In contrast, the right-hand side contains those amplitudes which arise only from upper-upper and lower-lower coupling and are, therefore, in the spirit of a nonrelativistic approach. They do not add any new features not already present in the nonrelativistic answer, in particular the cross term amplitudes A_{Kq} and A_{qK} of the 1^+ case as well as A_{nq} in the 1^- case are zero for the right-hand sides, as before. But they are present on the left-hand side. The full transition amplitude, of course, consists of the coherent sum of both contributions.

The new and remarkable features of the RPWIA approach have, as we mentioned before, their origin in the linear coupling of the lower components. The appearance of nonzero terms, so important for obtaining spin observable differences, is simple and natural. They arise from a consistent relativistic treatment of the upper and lower components on equal footing, no *ad hoc* assumptions are necessary, and no exchange effects are needed. We further mention that from the possible four Lorentz invariant amplitudes (pseudoscalar and axial in the spin independent case and vector and tensor in the spin dependent one) only two of them, namely the axial and tensor, are responsible in feeding these new terms. In particular the axial amplitude drives the A_{qK} term, while the tensor does the same for the A_{Kq} and A_{nq} terms.

V. GENERAL CASE

In the present section we extend our previous results to the general J case. It might seem difficult to obtain results close in spirit to the $J=1$ case, since some of our previous assertions [e.g., Eq. (23)] are only valid in that particular case. We will prove, however, that by working in a particular frame, namely one in which the momentum transfer points along the z axis "q frame," we will be able to recover the same structure in the amplitude as in the $J=1$ case, and with it all its previous conclusions. We focus our attention in the rotational invariant operator, defined in (22) by

$$\Omega_{Jls} = \sum_{n=-J}^{J} (-1)^M [Y_l(\hat{\mathbf{q}}) d_s]_{J,-M} \Sigma_{JM} , \tag{22a}$$

with

$$\Sigma_{JM} = |JM\rangle\langle 0^+| ,$$

and with parity properties given by

$$\pi\Omega_{Jls}\pi^{-1} = (-1)^{l+\pi_f}\Omega_{Jls} ,$$

where π_f is the parity of the final state. As before, those operators that change sign under parity will not contribute in the nonrelativistic case but will contribute in the relativistic case by appearing in conjunction with terms linear in the lower components.

	$\nu=1,2$	$\nu=3,4$
Natural parity	$l=J$ $s=0,1$	$l=J\pm1$ $s=1$
Unnatural parity	$l=J\pm1$ $s=1$	$l=J$ $s=0,1$

If we now choose to work in the q frame we can write

$$[Y_l(\hat{\mathbf{q}})d_s]_{J,-M}=\left[\frac{2l+1}{4\pi}\right]^{1/2} \times\langle l0;s,-M\mid J,-M\rangle d_{s,-M} . \quad (46)$$

The presence of the Clebsch-Gordan coefficient in the above expression and the fact that s takes only the values 0 or 1 implies that only three magnetic substates, namely 1, 0, and -1, can be populated at all no matter how large the value of J is; this in turn allows us to write

$$\Omega_{Jls}=\left[\frac{2l+1}{4\pi}\right]^{1/2}\sum_{M=-1}^{1}(-1)^M\langle l0;s,-M\mid J,-M\rangle \times d_{s,-M}\Sigma_{JM} . \quad (47)$$

The fact that only three values of the polarization tensor are needed, and inspired by our previous $J=1$ discussion, we define the target space operator

$$\mathbf{\Sigma}_J=\sum_{M=-1}^{1}(-1)^M\hat{\mathbf{e}}(-M)\Sigma_{JM} , \quad (48)$$

which, as before, also satisfies

$$\mathrm{Tr}[(\mathbf{\Sigma}_J\cdot\mathbf{A})(\mathbf{\Sigma}_J\cdot\mathbf{B})^\dagger]=\mathbf{A}\cdot\mathbf{B} . \quad (49)$$

We stress, however, that the above operator does not behave as a vector since its components do not transform in the proper way, in particular they do not even close under rotations; in fact its rotational properties are known and given by

$$U(R)\Sigma_{JM}U^\dagger(R)=\sum_{M'=-J}^{J}D^J_{M'M}(R)\Sigma_{JM'} . \quad (50)$$

Nevertheless, we shall see that $\mathbf{\Sigma}_J$ plays the same role in the amplitude as the polarization vector in the $J=1$ case does, and therefore all conclusions will follow in exactly the same way. In particular, by evaluating Ω_{Jls} in the q frame, with $d_s=\sigma_s$, we obtain

$$\begin{aligned}
\Omega_{JJ0}&=\left[\frac{2J+1}{4\pi}\right]^{1/2}(\mathbf{\Sigma}_J\cdot\hat{\mathbf{q}}) ,\\
\Omega_{JJ1}&=i\left[\frac{2J+1}{8\pi}\right]^{1/2}\mathbf{\Sigma}_J\cdot(\hat{\mathbf{q}}\times\boldsymbol{\sigma}) ,\\
\Omega_{J,J-1,1}&=\left[\frac{J+1}{8\pi}\right]^{1/2}\left\{(\boldsymbol{\sigma}\cdot\mathbf{\Sigma}_J)+\left[\left[\frac{2J}{J+1}\right]^{1/2}-1\right]\times(\mathbf{\Sigma}_J\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})\right\} ,\\
\Omega_{J,J+1,1}&=\left[\frac{J}{8\pi}\right]^{1/2}\left\{(\boldsymbol{\sigma}\cdot\mathbf{\Sigma}_J)-\left[\left[\frac{2(J+1)}{J}\right]^{1/2}+1\right]\times(\mathbf{\Sigma}_J\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})\right\} .
\end{aligned} \quad (51)$$

Aside from numerical factors, this is just the $J=1$ result (42). From here on, then, everything follows in exactly the same way as before: The amplitude will only contain the appropriate numerical changes, while the structure of the spin observables (Table I) will remain unchanged.

VI. EXAMPLES AND CONCLUSIONS

We have seen that certain "mixed" amplitudes [e.g., A_{Kq} and A_{qK} of Eq. (1) and A_{nq} of Eq. (3)] vanish in a nonrelativistic treatment of $0^+\rightarrow1^\pm$ nuclear excitation by protons in a plane wave treatment when off-shell effects and exchange effects are omitted, but do not vanish in the

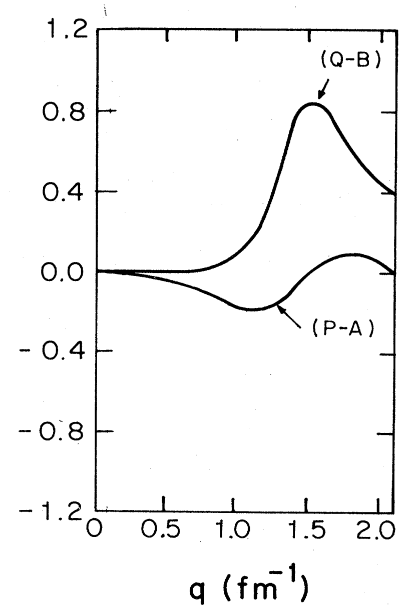

FIG. 1. A relativistic plane wave impulse approximation (RPWIA) calculation of the spin difference functions $(P-A)$ and $(Q-B)$ for 500 MeV protons exciting the 1^+ (12.71 MeV) $T=0$ state in ^{12}C as a function of momentum transfer. The maximum momentum transfer shown corresponds to a c.m. angle of 23.48°. The interaction strengths and structure parameters are discussed in the text.

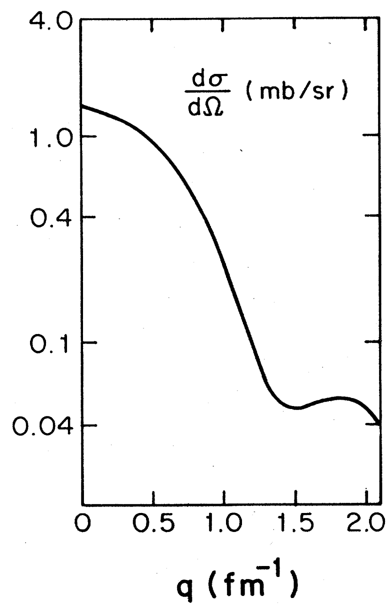

FIG. 2. Same as Fig. 1 for the inelastic cross section.

corresponding relativistic treatment using the Dirac equation. These nonvanishing terms are essential for spin observables such as $P-A$. In the relativistic treatment the situation is saved by the lower or "small" components that have opposite parity from the upper components. Hence, the new terms are linear in this lower component. Earlier attempts to obtain a nonvanishing $P-A$ in the $0^{+}\rightarrow 1^{+}$ reaction in a nonrelativistic theory have invoked exchange.[12] We have seen that the Dirac treatment leads to differences between P and A even without exchange. These two explanations are complementary since exchange is expected to contribute at low energy but to decrease in importance with increasing energy, while by contrast the significance of the relativistic contribution should grow with energy. A complete treatment should combine both effects as well as off-shell effects in the elementary NN t matrix.

The formal discussion of spin observables we have given is instructive, but the ultimate test of the theory must be confrontation with data. Since the purpose of this paper is frankly pedagogic, we will not attempt detailed comparisons, but it is important to see what general size we obtain for the spin observables. As a model, we study the 1^{+} (12.71 MeV $T=0$) and the 1^{+} (15.11 MeV $T=1$) states in ^{12}C excited by 500 MeV protons.[13] For the $T=1$ case both $P-A$ and $Q-B$ are essentially zero in the Dirac plane wave treatment. This is a "dynamical zero" and arises from the very small size of spin orbit terms in the $T=1$ nucleon-nucleon force at 500 MeV. For the $T=0$ state the situation is quite different. Figure 1 shows the RPWIA (relativistic plane wave impulse approximation) calculation of $P-A$ and of $Q-B$ for 500 MeV protons on ^{12}C exciting the 1^{+} $T=0$ state. In the absence of a relativistic shell model, the transition form factor (39) was based on nonrelativistic nuclear wave functions. We assume a pure single particle excitation from a $p^{3/2}$ to a $p^{1/2}$ nuclear orbital with the upper components for these orbitals given by harmonic oscillator wave functions with a characteristic length of 1.5 fm. The lower components were obtained through Eq. (41) using the free space relation. We stress that although a realistic calculation should include the strong scalar and vector potentials in the distortion as well as in the calculation of lower components (41), nonvanishing spin observable differences arise just from the structure of the Dirac equation (i.e., presence of lower components) and independently of the strong potentials. The Lorentz invariant NN amplitudes we used were calculated from Arndt[14] phase shifts and the relativistic transformation developed in Ref. 9. We see that $P-A$ and $Q-B$ are sizable. (Recall that

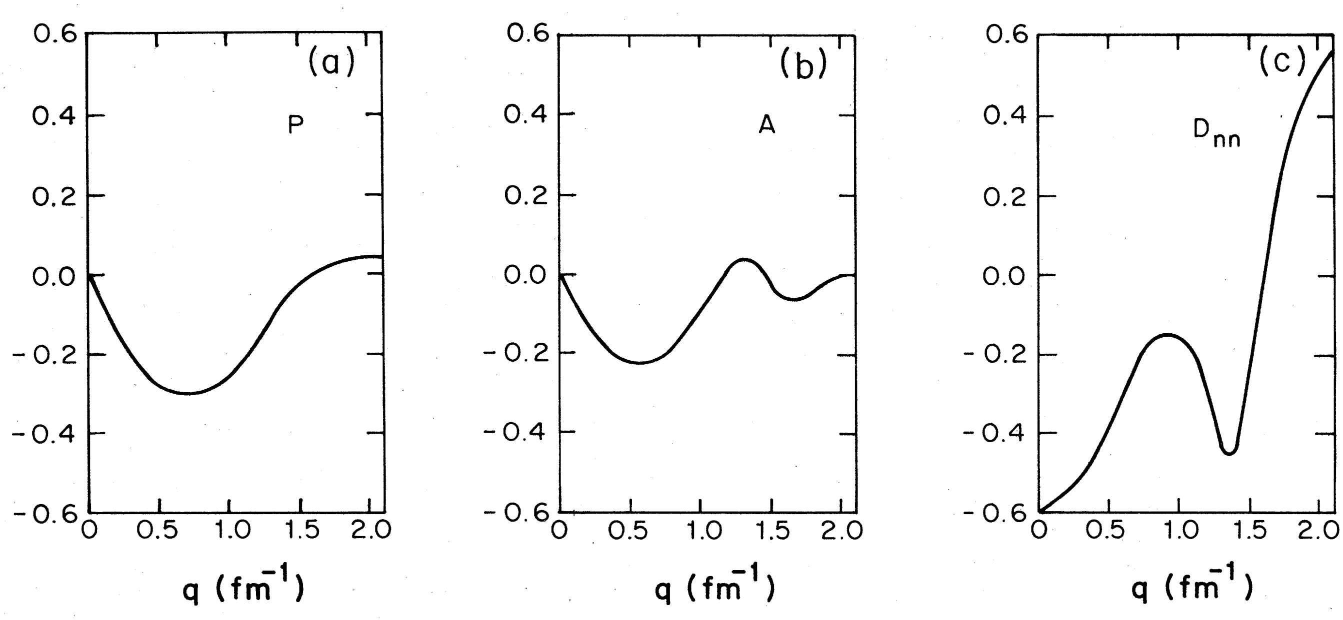

FIG. 3. Same as Fig. 1 for the remaining spin observables as labeled.

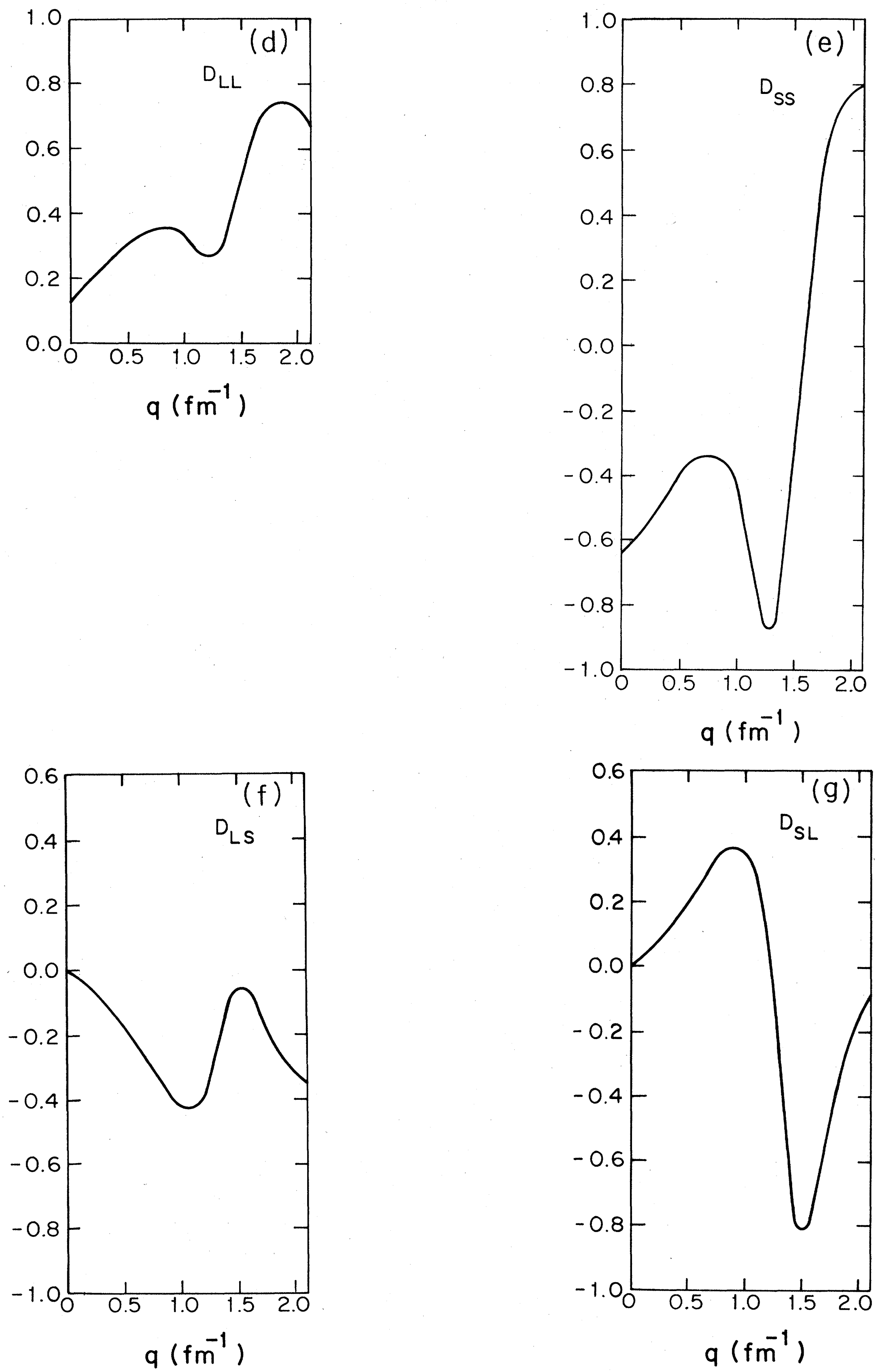

FIG. 3. (*Continued*).

both are bounded by 2.) Although the effects of distortion can change these shapes in significant ways, particularly at the larger angles, the qualitative futures of the spin difference observables remain. In particular, one can see from the RPWIA cross section shown in Fig. 2 that the structure in Fig. 1 is *not* coming from the structure in the cross section but is intrinsic to $P-A$ and $Q-B$. Other spin observables are shown in Fig. 3 in the RPWIA. The same geometric and dynamical input is used as in Fig. 1. Again we see considerable structure in the spin observables. Preliminary studies indicate that the addition of distortion and of strong potentials does not qualitatively change the spin observables but does lead to modest improvement with the data. But, of course, most of these observables are also nonvanishing in a nonrelativistic treatment. [See Eq. (26).]

Many of the same spin observables that we have studied here, and in particular the spin differences, have been studied for the purpose of detailed comparison with data by Sparrow *et al.*[4] This study includes the effects of distortion, but is at 150 MeV where the competition with exchange is probably still significant.

In conclusion, we have shown how a relativistic treatment of inelastic scattering gives spin observables (particularly $P-A$ and $Q-B$) that are zero in the corresponding nonrelativistic treatment. This is even true in a plane wave impulse approximation, and, although distortion changes the details, it does not affect the underlying dynamic origin of the effects. From this we conclude that careful theoretical and experimental studies of these spin observables may serve as a sensitive test to distinguish between relativistic effects, off-shell effects, and exchange effects. Because of the importance of distortion in detailed studies, and because distortion is largely a geometric question, experimental studies should include elastic and inelastic cross sections along with the spin observables.

ACKNOWLEDGMENTS

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