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Dispersive effects in pion-nucleus scattering

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The influence of dispersive, medium effects on pion-nucleus elastic scattering is investigated in a simple model. The pion-nucleus interaction in the impulse approximation is represented by an optical potential whose form is motivated by field-theoretic considerations and the properties of the Δ_{33} resonance. The intermediate Δ_{33} spectrum is described by a mean spectral energy, $E_{\rm ms}$, where the shift $E_{\rm ms}$ represents the average of the Δ_{33}-nucleus mean field, U_Δ, over the nucleon and pion wave functions. We calculate the real part of $E_{\rm ms}$ as a function of mass number A and pion energy ω, assuming that U_Δ is numerically the same as the nucleon-nucleus shell-model potential. We compare our theory to more conventional ones, showing that the mean spectral energy is an important correction for pion-nucleus elastic scattering.

I. INTRODUCTION

The subject of dispersive effects in pion-nucleus scattering refers to the influence of the nuclear medium on the energetics of the intermediate states that form the pion-nucleus scattering amplitude. Deciding how to incorporate dispersive effects is not straightforward either in theory or practice, which is evident from their long history in both multiple scattering theory and also in the many-body approach to nuclear structure and scattering. Today the importance of dispersive effects is acknowledged in many, but not all, of the modern treatments of pion-nucleus scattering. The purpose of the present paper is to propose a practical method for dealing with dispersive effects and to explicate their importance in a model.

One would expect dispersive effects to be most important in theoretical treatments of projectile-nucleus scattering when the elementary amplitude is strongly energy dependent. One of the most dramatic examples occurs for the case of the pion, where the rapid energy dependence in the region from 100 to 300 MeV arises from the Δ_{33} resonance. The half-width of this resonance is 55 MeV. Nuclear energies, whether one considers binding energies, nucleon kinetic energies, or the potential energy associated with the mean field, range from zero to about 50 MeV. The energy dependence of the pion-nucleon amplitude is thus comparable to the energy scales associated with the nucleus. This implies that dispersive effects should be included in the impulse approximation and that they can have important consequences.[1] Understanding the energetics of the nucleon propagating in the nuclear medium while it is interacting with the pion and the interaction of the delta with the core nucleus are questions of understanding the dispersion of the interacting pion-nucleon pair, and is the subject of this work.

The original treatment of the dispersion of the intermediate nucleon in the impulse approximation for the scattering amplitude can be found in the works of Chew and Wick[2] and Chew and Goldberger.[3] The question was how to approximate the propagator $G_A(\omega)$ defined by

$$G_A(\omega)=(\omega^+-K_0-H_A)^{-1}\,, \qquad (1.1)$$

where K_0 is the projectile kinetic energy and H_A is the target Hamiltonian. The presence of H_A in the propagator renders $G_A(\omega)$ a many-body operator and thus unmanageable in practice. In order to have a computationally feasible theory, $G_A(\omega)$ must be approximated by a two-body operator. The essence of the argument found in Chew and Wick[2] and repeated in Chew and Goldberger[3] is simply that at sufficiently high energies K_0 will dominate over H_A and one can neglect H_A.

The impulse approximation for the scattering amplitude was soon replaced by multiple scattering theory for the optical potential. The formal theory for the optical potential was derived in the pioneering work of Watson,[4] motivated by the early pion-nucleus scattering data emerging from the Brookhaven Cosmotron. On the question of approximating the many-body $G_A(\omega)$ by a two-body operator, he quotes the earlier argument of Chew and collaborators.[2,3]

The first application of the Watson multiple scattering theory for the optical potential which clearly demonstrated the validity of this approach to intermediate energy nucleon-nucleus elastic scattering was performed by Kerman, McManus, and Thaler.[5] In their derivation of the multiple scattering theory they also quote Chew and collaborators on the approximate treatment of the propagation of the struck nucleon. In their Appendix IV, they make some important points which we shall stress in this work: that the choice of the nucleon propagator is at the disposal of the theorist, that this choice defines the perturbation theory, and that the criterion which distinguishes between choices is the rate of convergence of the perturbation theory.

Because the nucleon-nucleon interaction is slowly varying with energy for energies greater than about 100 MeV, dispersive effects which modify the intermediate propagation of the struck nucleon were believed to have only a small impact on nucleon-nucleus cross sections at intermediate energies. Interest in these effects thus lay dormant for many years. High precision measurements at intermediate energies and a desire to learn about other specific effects, such as correlations between target nucleons, eventually led to a reexamination of this problem. In this connection, it was pointed out that the lowest-order optical potential[6] can naturally be expressed as a three-body problem, the three bodies being the incident nucleon, the struck nucleon, and the rest of the nucleus. Replacing the rest of the nucleus by a single particle potential leads to a model in which the correct collision kinematics and the effect of the nucleon-core binding energy can be included.

The nucleon-core binding potential has been a topic of much interest in nuclear structure theory and, in particular, in nuclear matter theory.[7] In nuclear matter theory, the choice of the spectrum for the particles (and holes) controls the rate of convergence of the perturbation theory. Although the exact answer is formally independent of the treatment of these dispersive effects, the result of a real calculation, which truncates perturbation theory at a finite order, depends upon the particle spectrum used. For the pion-nucleus problem, we find the parallel result. However, because of the rapid energy dependence of the two-body amplitude, the validity of the predicted cross sections depends much more critically on the choice of the intermediate spectrum chosen for the nucleon and the delta.

The sensitivity of the pion-nucleus interaction to the treatment of the nucleon-residual nucleus interactions has been investigated by several authors. In Refs. 8 and 9 the three-body approach to the pion-nucleus optical potential was implemented in the context of a pion-nucleon potential interaction. In these works, the kinematics of the three-body problem are simplified by the small value of the ratio of the pion mass to the nucleon mass. The conclusion of these works is clear. The nucleon-residual nucleus interaction causes a large change in the pion-nucleus differential cross sections in comparison with results of a standard impulse approximation.

All versions of the delta-hole model[10–12] emphasize the dispersive effects. For example, in Ref. 11 the interaction of the delta with the residual nucleus is parametrized by a potential with real and imaginary, central and spin-orbit pieces. It is the medium modifications of the delta propagation, and, in particular, the difference between the nucleon-nucleus and the assumed form of the delta-nucleus interaction that constitutes the second-order, phenomenological part of the model.

Evidence that dispersive effects play an important role in pion-nucleus scattering presents several choices. One can omit this effect, guaranteeing a slowly convergent perturbation series. At the other extreme, one can include the dispersive effects into the first-order optical potential and solve the three-body problem. Even with the kinematic simplifications that result from the light mass of the pion, this is a formidable numerical problem.[8,9] Incorporating the delta-nucleus interaction *à la* the delta-hole model is also numerically formidable[10–12] and limits one to the resonance region.

A third option is to seek an approximation that incorporates the physics but stops short of a full three-body calculation. We have chosen this route and have implemented it so as to incorporate some of the field theoretic aspects that are absent in many of the other theories. A momentum space optical model program has been developed which performs the Fermi averaging integration, includes the dispersive effects considered here, and yet is sufficiently efficient so that we are able to make significant applications and extensions of the theory. Such an approach will allow us to apply our theory to light and heavy nuclei from energies well below to well above the Δ_{33} resonance region and permit inclusion of higher-order terms explicitly in the optical potential. We anticipate future extensions to a global approach that encompasses elastic, single, and double charge exchange and to scattering from nuclei with spin.

In Sec. II we show how the formalism of Refs. 13 and 14 leads one to choose an unperturbed propagator for the nucleon and the delta. By the choice of this propagator one is naturally led to the consideration of dispersive effects. We also discuss how dispersive effects may be incorporated into models which assume potential pion-nucleon interactions, delta models, or field theoretic Chew-Low or bag models. Basically, the propagation of the delta (or pion-nucleon interacting pair) and the surface localization of the pion-nucleus interaction are used to approximate a spatially varying delta or nucleon potential by a constant in space which depends on ω, the energy of the incident pion. This quantity, $E_{\mathrm{ms}}(\omega,A)$, we call the "mean spectral energy" as it determines, on the average, the spectrum of the intermediate states which occur in summing interactions (be they potential or field theoretic in nature) to produce the t matrix in the impulse approximation. The propagator (or "energy denominator") which contains $E_{\mathrm{ms}}(\omega,A)$ we call the "mean spectral propagator."

In Sec. III, we present results for pion-nucleus scattering and demonstrate the importance of including $E_{\mathrm{ms}}(\omega,A)$ in the propagator. As $E_{\mathrm{ms}}(\omega,A)$ shifts the energy at which the t matrix is evaluated, we present differential cross sections at selected energies and also total cross sections and forward scattering amplitudes as a function of energy. We find that $E_{\mathrm{ms}}(\omega,A)$ alters the diffractive character of cross sections near the resonance region and has a non-negligible effect on cross sections below and above the resonance region.

In Sec. IV, we discuss these results and draw conclusions. Since the pion-nucleus interaction is sensitive to the treatment of the dispersive effects, elastic scattering data are useful for constraining their treatment in various models of the pion-nucleus interaction. Data near the forward direction in the nuclear-Coulomb interference region are particularly important, as they allow extraction of the strong forward scattering amplitude.

II. MEAN SPECTRAL PROPAGATOR

Our treatment of the dispersive effects is based on an optical model description of the projectile-nucleus interaction. It incorporates the improvements in the traditional multiple scattering theory that have been stimulated by the availability of pion beams over the last decade. The light mass of the pion has led to an improved understanding of the role played by relativistic kinematics in multiple scattering.[15] This discussion, which originally[16] proceeded under the appellation of "angle transformation," is incorporated as a frame transformation of an off-energy shell, nonlocal, two-body t matrix. The requirement of relativistic quantum field theory that the pion be allowed to propagate both forward and backward in time leads us to view the optical potential as the proper self-energy in a Klein-Gordon equation.[13,14,17,18] In this framework, the multiple scattering approach can be truncated so that the crossing symmetry of the two-body amplitude guarantees the crossing symmetry[13] of the pion-nucleus amplitude. The unitarity relations[19] for the optical potential show that inclusion of the many-pion intermediate states generated by the Klein-Gordon equation yields an optical potential with simple and reasonable reactive content, while truncation of the theory according to the number of pions present at a given time would yield an optical potential with spurious reactive content.

Many of the other commonly available optical model theories of pion-nucleus scattering are built around the fixed scatterer approximation, in which the nucleons and Δ_{33} have infinite mass. In order to go beyond this approximation and systematically include binding effects, dispersive effects, nuclear fermi motion, and the recoil of the Δ_{33} and nucleon, one must employ the momentum space techniques we use here.

In Refs. 13 and 14, a framework is developed for treating the pion-nucleus problem field theoretically, giving an expansion of the optical potential in terms of propagators and meson-baryon vertices. Reference 14 extends Ref. 13 to include a finite nucleon mass and thus more correctly incorporates the target dynamics. When this is done, the nucleon hole propagator is given by

$$G_B(\mathbf{r}',t';\mathbf{r},t)=\Psi_B^*(\mathbf{r}')\exp[-iE_B(t-t')]\Psi_B(\mathbf{r}) , \tag{2.1}$$

where $t-t'$ is the time difference counted in the direction of the hole line and E_B is the binding energy of the hole. The propagator for a finite mass nucleon in a state of momentum $\mathbf{p}$ is given by

$$G_{\mathbf{p}}(\mathbf{r}',t';\mathbf{r},t)=\Psi_{\mathbf{p}}^*(\mathbf{r}')\exp[-iE_{\mathbf{p}}(t'-t)]\theta(t'-t)\Psi_{\mathbf{p}}(\mathbf{r}) , \tag{2.2}$$

where the states $\Psi_{\mathbf{p}}(r)$ satisfy a single particle Schrödinger equation

$$h^p\Psi_{\mathbf{p}}(\mathbf{r})=E_{\mathbf{p}}\Psi_{\mathbf{p}}(\mathbf{r}) . \tag{2.3}$$

In the conventional many-body theory, the Hamiltonian h^p is taken to be the same Hamiltonian which produces the bound states $\Psi_B(\mathbf{r})$ and the set $\Psi_{\mathbf{p}}(\mathbf{r})\oplus\Psi_B(\mathbf{r})$ forms an orthonormal, complete set. This leads to a perturbation theory which, when ladder diagrams are summed, has the Bethe-Goldstone reaction matrix as the basic pion-nucleon interaction. Our approach of Refs. 13 and 14 is close to this except that we do not include the Pauli operator in the effective two-body interaction. The Pauli principle is systematically and convergently included in higher orders of the optical potential in our approach.[20]

If the delta has finite mass, then its propagation in a state α is given by

$$G_\alpha^\Delta(\mathbf{r}',t';\mathbf{r},t)=\Psi_\alpha^{\Delta *}(\mathbf{r}')\exp[-iE_\alpha^\Delta(t'-t)] \times\theta(t'-t)\Psi_\alpha^\Delta(\mathbf{r}) , \tag{2.4}$$

with

$$h^\Delta\Psi_\alpha^\Delta=E_\alpha^\Delta\Psi_\alpha^\Delta(\mathbf{r}) . \tag{2.5}$$

Just as one must choose an h^p in Eq. (2.3), one must choose an h^Δ in order to define the unperturbed propagator in a perturbative approach. In potential model theories or the Chew-Low[21,22] theory, the Δ_{33} is not an independent degree of freedom and the dispersive effects are defined in terms of h^p. In the simple delta model[11] or the dynamic combination[23] of this model with the Chew-Low model represented by the "cloudy bag," one must choose both h^p and h^Δ.

Combining the above expressions for the propagators we may evaluate the optical potential. The pion-nucleus optical potential in the impulse approximation can be written as

$$\langle \mathbf{k}_\pi' \mid \Sigma(\omega) \mid \mathbf{k}_\pi \rangle = \sum_B \int d^3\mathbf{k}_n' d^3\mathbf{k}_n \Psi_B^*(\mathbf{k}_n')\langle \mathbf{k}_\pi'\mathbf{k}_n' \mid t(\omega) \mid \mathbf{k}_\pi\mathbf{k}_n \rangle \Psi_B(\mathbf{k}_n) , \tag{2.6}$$

where $\Psi_B(\mathbf{k}_n)$ is a bound state nucleon wave function and $t(\omega)$ is an appropriate field theoretic, off-shell pion-nucleon t matrix. For the following discussion, we need not specify the exact form of the pion-nucleon t matrix. However, near resonance all t matrices can be well approximated as separable, i.e., by the simple delta t matrix. Thus to simplify the presentation we will use the form

$$\langle \mathbf{k}_\pi'\mathbf{k}_n' \mid t(\omega) \mid \mathbf{k}_\pi\mathbf{k}_n \rangle = v^\dagger(\mathbf{k}')\left\langle \mathbf{P}' \left| \frac{1}{\omega^+ + E_B - M_\Delta - P^2/2M_\Delta - U_\Delta(R)} \right| \mathbf{P} \right\rangle v(\mathbf{k}) , \tag{2.7}$$

where ω is the incident pion energy; E_B is the binding energy of the struck nucleon appearing in Eq. (2.1); $\mathbf{k}$ and $\mathbf{P}$ are the relative and center-of-mass momenta, respectively; $\mathbf{R}$ is the center-of-mass coordinate conjugate to $\mathbf{P}$; and $v(\mathbf{k})$ is the form factor (including implicitly the spin and isospin dependence) for forming a delta. The generalization of the following discussion to other models of the pion-nucleon interaction is straightforward. In Eq. (2.7) $U_\Delta(R)$ is the mean field of the Δ_{33} arising from its interaction with the nucleons in the nucleus. The width of the Δ_{33} resonance is included in M_Δ.

As we argued in Sec. I, introducing the nucleon and Δ_{33} mean fields into the pion-nucleus optical potential requires, for practical reasons, an efficient and quantitatively accurate approximation scheme. We will attempt to accomplish this by replacing $U_\Delta(R)$ in Eq. (2.7) by a spatial constant, $E_{\rm ms}(\omega,A)$. Thus, our approach is unlike the delta-hole model in which the full interaction $U_\Delta(R)$ appears in the delta propagator. We outline below a procedure for calculating $E_{\rm ms}(\omega,A)$ so that the difference $U_\Delta(R)-E_{\rm ms}(\omega,A)$ will be small and can be treated as a correction. We propose to incorporate this difference along with other effects at a later stage in the higher-order part of the optical potential.

The quantity $U_\Delta(R)$ is not well known, and determining it is one of the interesting pursuits of contemporary nuclear physics. One expects that $U_\Delta(R)$ closely resembles the nucleon-nucleus shell model potential,[24] $V(R)$. For our calculations we ignore the isovector, spin-orbit, and imaginary part of $U_\Delta(R)$ and take $U_\Delta(R)=V(R)$ where[25]

$$V(R)=V_0\{1+\exp[(R-R_0)/a]\}^{-1}, \tag{2.8}$$

with $V_0=-51$ MeV, $R_0=1.27A^{1/3}$ fm, and $a=0.67$ fm. In a more comprehensive description, for which $U_\Delta(R)$ is identified with specific pieces of the Δ_{33} self-energy,[14] the same methods could be used to determine the corresponding $E_{\rm ms}(\omega,A)$.

To determine $E_{\rm ms}(\omega,A)$, we expand the propagator in Eq. (2.7) and keep only the leading correction term in $U_\Delta(R)$,

$$\left\langle \mathbf{P}' \left| \frac{1}{\omega^+ + E_B - M_\Delta - P^2/2M_\Delta - U_\Delta(R)} \right| \mathbf{P} \right\rangle \cong \left\langle \mathbf{P}' \left| \frac{1}{\omega^+ + E_B - M_\Delta - P^2/2M_\Delta} \right| \mathbf{P} \right\rangle + \left\langle \mathbf{P}' \left| \frac{1}{\omega^+ + E_B - M_\Delta - P^2/2M_\Delta} U_\Delta(R) \frac{1}{\omega^+ + E_B - M_\Delta - P^2/2M_\Delta} \right| \mathbf{P} \right\rangle . \tag{2.9}$$

The second term on the right-hand side is the leading correction term in $U_\Delta(R)$ and, if we Fourier transform to coordinate space, it can be written as

$$\langle \mathbf{R}' | \delta G | \mathbf{R} \rangle = \int d^3R_i \frac{e^{i\bar{k}|\mathbf{R}'-\mathbf{R}_i|}}{4\pi|\mathbf{R}'-\mathbf{R}_i|} U_\Delta(\mathbf{R}_i) \frac{e^{i\bar{k}|\mathbf{R}-\mathbf{R}_i|}}{4\pi|\mathbf{R}-\mathbf{R}_i|} \tag{2.10}$$

where $\bar{k}$ is given by

$$\bar{k}=\sqrt{2M_\Delta(\omega^+ - M_\Delta + E_B)} . \tag{2.11}$$

We examine first the modifications to $U_\Delta(R)$ caused by the delta (pion-nucleus center-of-mass) propagation. We define $\tilde{U}_\Delta(R)$ by

$$\tilde{U}_\Delta(R) \int d^3R_i \frac{e^{2i\bar{k}|\mathbf{R}-\mathbf{R}_i|}}{(4\pi|\mathbf{R}-\mathbf{R}_i|)^2} = \langle \mathbf{R} | \delta G | \mathbf{R} \rangle , \tag{2.12}$$

with the right-hand side calculated from Eq. (2.10). The difference between $U_\Delta(R)$ and $\tilde{U}_\Delta(R)$ is a measure of the smearing of $U_\Delta(R)$ by delta (pion-nucleon center-of-mass) propagation.

We calculate $\tilde{U}_\Delta(R)$ from Eq. (2.12) for $T_\pi=164$ MeV. First, we neglect E_B and the width of the delta and take M_Δ to be real. The results are given in Table I. At $T_\pi=164$ MeV, $\omega+M_N$ is slightly larger than M_Δ, and the value of $\bar{k}$ given in Eq. (2.10) is purely real. This produces in Eq. (2.12) a $\tilde{U}_\Delta(R)$ which is complex. We see that there is little smearing of $U_\Delta(R)$ by the delta propagation.

We also repeated the calculation including the width of the delta in M_Δ in Eq. (2.10). This gives $\bar{k}$ both a real and imaginary part. These results are also given in Table I. We see that the smeared potential $\tilde{U}_\Delta(R)$ is now even closer to the original potential, $U_\Delta(R)$.

With this demonstration that the delta (center-of-mass) propagation will not appreciably modify the potential, we look for the spatial constant that most closely approximates the effect of $U_\Delta(R)$. We employ the fact that, for energies in the resonance region, the pion-nucleus interaction is surface dominated. Making use of standard perturbation theory (or the two-potential formula for scattering states) the average correction to the optical potential is given by the matrix element of the correction term with distorted pion waves, $\phi_{\mathbf{k}}^{\pm}(\mathbf{r}_\pi)$,

TABLE I. Values of $U(R)$ and $\tilde{U}(R)$ for 165 MeV pions on ^{40}Ca.

R (fm)	U (MeV)	$\tilde{U}(\Gamma=0)$ (MeV)	$\tilde{U}(\Gamma=94$ MeV) (MeV)
2.0	−49.5	−50.0 −1.67i	−49.6 +0.12i
4.0	−31.9	−33.5 +1.35i	−31.9 +0.41i
5.0	−13.9	−13.6 −0.58i	−13.9 −0.11i
5.8	− 5.21	− 4.10−0.51i	− 5.18−0.16i

$$\langle \delta\Sigma \rangle = \left[\frac{1}{\omega^+ + E_B - M_\Delta - \langle P^2 \rangle / 2M_\Delta} \right]^2 \times \sum_B \int d^3r'_\pi d^3r_\pi d^3r_N \phi_{\mathbf{k}}^{-*}(\mathbf{r}'_\pi)\psi_B^*(\mathbf{r}_N) v^\dagger(\mathbf{r}'_\pi - \mathbf{r}_N) U_\Delta(\mathbf{r}_N) v(\mathbf{r}_\pi - \mathbf{r}_N)\phi_{\mathbf{k}}^+(\mathbf{r}_\pi)\psi_B(\mathbf{r}_N) , \tag{2.13}$$

where we have assumed for simplicity an s-wave pion-nucleon interaction and have eliminated the delta propagation from the integration in accordance with our result of the previous paragraph. The form factor $v(\mathbf{r})$ is the Fourier transform of $v(\mathbf{k})$ in Eq. (2.7). This form factor is of short range in coordinate space[21,22] and it will thus serve to set $\mathbf{r}'_\pi \cong \mathbf{r}_N$ and $\mathbf{r}_\pi \cong \mathbf{r}_N$. Neglecting this additional smearing gives

$$\langle \delta\Sigma \rangle = \left[\frac{1}{\omega^+ + E_B - M_\Delta - \langle P^2 \rangle / 2M_\Delta} \right]^2 \times \int d^3R \phi_{\mathbf{k}}^{-*}(\mathbf{R})\phi_{\mathbf{k}}^+(\mathbf{R})\rho(R) U_\Delta(R) . \tag{2.14}$$

We are led to define the mean spectral energy by

$$E_{\text{ms}}(\omega_k, A) \int d^3R \phi_{\mathbf{k}}^{-*}(\mathbf{R})\phi_{\mathbf{k}}^+(\mathbf{R})\rho(R) = \int d^3R \phi_{\mathbf{k}}^{-*}(\mathbf{R})\phi_{\mathbf{k}}^+(\mathbf{R})\rho(R) U_\Delta(R) . \tag{2.15}$$

The mean spectral propagator, $G_{\text{ms}}(\omega)$, may then be defined by

$$G_{\text{ms}}(\omega) \equiv \frac{1}{\omega^+ + E_B - M_\Delta - P^2/2M_\Delta - E_{\text{ms}}(\omega, A) + i\eta} . \tag{2.16}$$

This propagator and the full delta propagator including $U_\Delta(R)$ will clearly give the same average change in the optical potential, $\langle \delta\Sigma \rangle$, to second order in $E_{\text{ms}}(\omega, A)$, assuming $\tilde{U}_\Delta(R) = U_\Delta(R)$. By virtue of its appearance in the full propagator we hope to include some of the effect of $U_\Delta(R)$ from higher-order terms in the expansion of Eq. (2.8). We propose to use the delta propagator defined in Eq. (2.16). One could also allow the mean spectral energy to depend on other quantities such as the total momentum $\mathbf{P}$ or the partial wave of the delta in order to better approximate the effect of $U_\Delta(R)$.

The mean spectral energy, Eq. (2.15), is an intuitive result; it is the potential $U_\Delta(R)$ averaged with the probability of finding a nucleon at R, $\rho(R)$, and the probability of finding the pion at $\mathbf{R}$, $\phi_{\mathbf{k}}^{-*}(\mathbf{R})\phi_{\mathbf{k}}^+(\mathbf{R})$. In Fig. 1 we show a typical plot of $\rho(R)$, $W_k(R)$, and $U_\Delta(R)$, where our choice for $U_\Delta(R)$ is given in Eq. (2.8) and where the weight function $W_k(R)$ is defined by

$$W_k(R) \equiv R^2 \rho(R) \sum_l (2l+1) \phi_{lk}^{-*}(R) \phi_{lk}^+(R) . \tag{2.17}$$

We see that $W_k(R)$ is a smooth function which is peaked in the surface of the nucleus where the interaction is localized. We also see that $U_\Delta(R)$ is reasonably linear over the important range where $W_k(R)$ is nonzero, thus giving support to approximating it by its average over this region. *Because $U_\Delta(R)$ has a larger half-radius than $\rho(R)$, $U_\Delta(R)$ is still large where $W_k(R)$ peaks and will lead to a large value for $E_{\text{ms}}(\omega, A)$.* Note that $W_k(R)$ is complex, which means that $E_{\text{ms}}(\omega, A)$ is also complex, even when $U_\Delta(R)$ is real. We shall ignore $\text{Im}E_{\text{ms}}(\omega, A)$ in the following discussion because this adds to the (half) width of the Δ_{33} resonance, which is already quite large. Eventually, $\text{Im}E_{\text{ms}}(\omega, A)$ should be considered along with estimates of $\text{Im}U_\Delta(R)$.

We have calculated $E_{\text{ms}}(\omega, A)$ for ^{16}O, ^{40}Ca, and ^{208}Pb

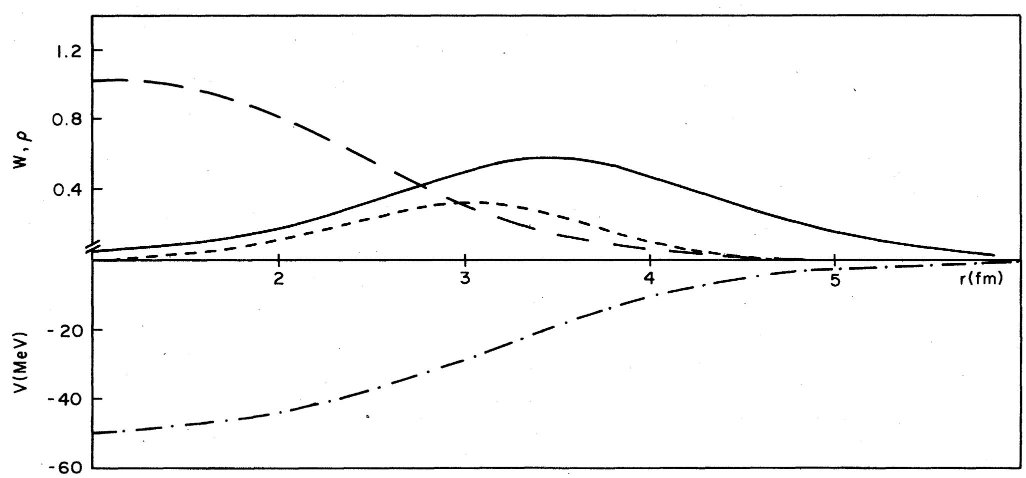

FIG. 1. Quantities that are needed to calculate the mean spectral energy $E_{\text{ms}}(\omega, A)$ for ^{16}O at $T_\pi = 164$ MeV. The solid curve is $\text{Re}W_k(R)$, the short dashed curve $\text{Im}W_k(R)$, the long dashed curve $\rho(R)$, and the dot-dashed curve $U_\Delta(R)$. $W_k(R)$ and $\rho(R)$ are read on the upper scale (arbitrary units); $U_\Delta(R)$ is read on the lower curve.

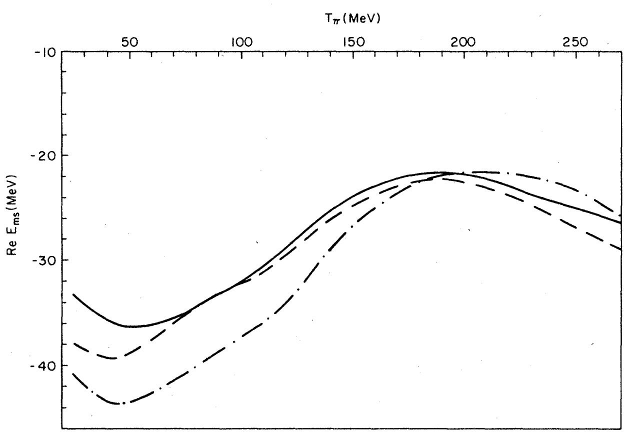

FIG. 2. The real part of the mean spectral energy $E_{\text{ms}}(\omega,A)$ for ^{16}O (the solid curve), ^{40}Ca (the dashed curve), and ^{208}Pb (the dot-dash curve) as a function of the pion laboratory kinetic energy.

using pion distorted waves based on the program[26] PIRK and the lowest-order optical potential constructed from the free pion-nucleon scattering amplitude and realistic densities. In Fig. 2 we plot $E_{\text{ms}}(\omega,A)$ for these three nuclei. We see that $E_{\text{ms}}(\omega,A)$ is nearly independent of A and is a smooth function of ω. At resonance, the pions are limited to the far nuclear surface near the 10% density point, and thus $E_{\text{ms}}(\omega,A)$ is smaller in magnitude than at the other energies where the pion can penetrate further into the nucleus and thus sample $U_\Delta(R)$ where it is stronger. The dependence on ω is so smooth because, as noted earlier, $U_\Delta(R)$ extends out relatively far into the nuclear surface. The insensitivity of $E_{\text{ms}}(\omega,A)$ to A and ω gives indirect evidence that replacing $U_\Delta(R)$ by a spatial constant, which depends upon the target and the pion energy, is a reasonable approximation to have made.

III. PION-NUCLEUS SCATTERING

In this section, we examine the effect of the dispersion correction on pion-nucleus elastic scattering cross sections. The optical potential is calculated according to Eq. (2.6). The mean spectral propagator prescribes that the t matrix be taken of the form

$$\langle \mathbf{k}'_\pi \mathbf{k}'_n \,|\, t(\omega) \,|\, \mathbf{k}_\pi \mathbf{k}_n \rangle = \delta(\mathbf{P}'-\mathbf{P}) \langle \mathbf{k}' \,|\, t(\omega_{\text{c.m.}}) \,|\, \mathbf{k} \rangle \;, \tag{3.1}$$

where the off-shell pion-nucleon t matrix is to be evaluated at the energy $\omega_{\text{c.m.}}$ given by

$$\omega_{\text{c.m.}} = \omega + E_B - E_{\text{ms}}(\omega,A) - P^2/2(M_N+\omega) \;, \tag{3.2}$$

where we have replaced M_Δ in the recoil term by $M_N+\omega$ so that the results will remain valid at energies away from resonance. For the numerical results in this section we take the t matrix from Ref. 21. We also perform the fermi averaging over the momentum of the struck nucleon exactly utilizing a technique developed in Ref. 27. Similar results could be obtained utilizing the expansion technique of Ref. 28 but the exact evaluation of the fermi averaging as in Ref. 27 takes only moderately longer than the expansion approach. By performing the fermi integration, we treat the operator character of P^2 (i.e., the delta recoil) without approximation.

An important practical difference between the potential models (or the simple delta model of Ref. 11) and the field theoretic approach is that the pion-nucleon form factor has a much higher momentum cutoff in the field theoretic approach. The low momentum cutoff of the potential models has been shown[22] to arise from an improper treatment of the s- and u-channel nucleon poles in the pion-nucleon amplitudes in these models. This higher momentum cutoff has two effects on pion-nucleus cross sections. First, it results in a slightly decreased radius[18] of the optical potential resulting from the smaller range of the pion-nucleus interaction. Second, the fermi integration will extend over a larger range of momentum. This alters the effective energy[27] at which the pion-nucleon t matrix is evaluated. Since the nucleon-nucleus interaction also alters the energy at which the two-body interaction is evaluated, the quantitative importance of these dispersive corrections is not completely independent of the choice of the pion-nucleon form factor. There are also other qualitative differences[18] between our approach through the Klein-Gordon equation and one utilizing an optical poten-

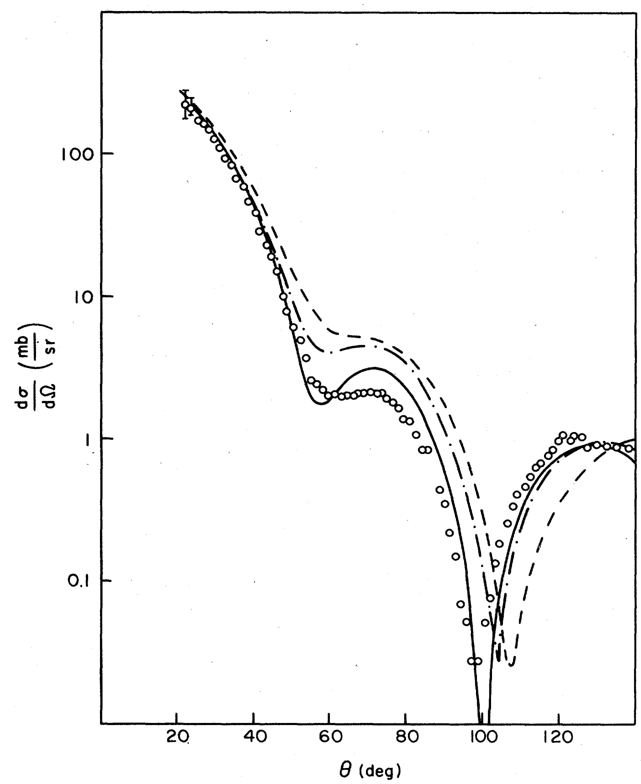

FIG. 3. The elastic differential cross section for elastic scattering of π^+ from ^{16}O at a laboratory kinetic energy of 114 MeV. The data are from Ref. 31. The solid curve is the result of using the mean spectral propagator, the dashed curve uses the three-body energy denominator, and the dash-dot curve sets both the binding energy E_B and the mean spectral energy $E_{\text{ms}}(\omega,A)$ equal to zero. The Coulomb interaction is included.

tial in a Schrödinger equation, especially at energies well below resonance.

In Figs. 3—5 we depict elastic differential cross sections for π^+ scattering from ^{16}O at 114, 162, and 240 MeV. The solid curve is the full calculation utilizing the mean spectral propagator defined in Eq. (2.15) with the mean spectral energy as depicted in Fig. 2. The single particle wave functions and binding energies are from Ref. 29. The dashed curve is the result of using the "three-body" energy denominator[30,31] which derives from the mean spectral propagator by setting $E_{ms}(\omega,A)$ equal to zero. Because $E_{ms}(\omega,A)$ tends to cancel E_B in the propagator, we also show results (the dot-dash curve) for the case when both $E_{ms}(\omega,A)$ and E_B are set to zero. The data are from Ref. 32.

The first and most general conclusion one can draw from these curves is that the inclusion of dispersive effects is important in pion-nucleus physics, and that the way in which dispersive effects are treated can alter predicted differential cross sections significantly. This is certainly true in the resonance region and also below resonance as can be seen in the results at 114 MeV. The role of the dispersive correction above resonance is still significant but relatively less important than at the lower energies.

We also plot in Fig. 6 the total, total elastic, and total inelastic cross sections for π^+ scattering from ^{16}O as a function of energy. The data for the total cross section are from Ref. 33. In Fig. 7 the real and imaginary parts of the forward scattering amplitude $f(0)$ are depicted. The points are the forward scattering amplitude extracted

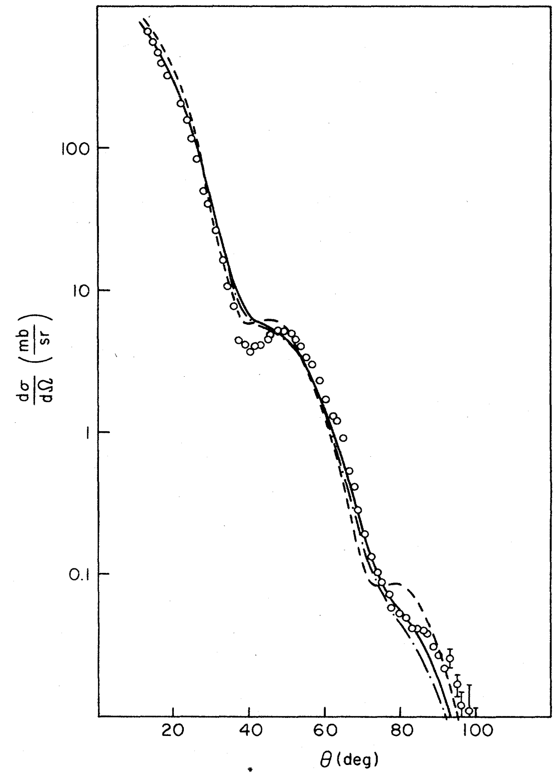

FIG. 5. The same as Fig. 4 except the energy is 240 MeV.

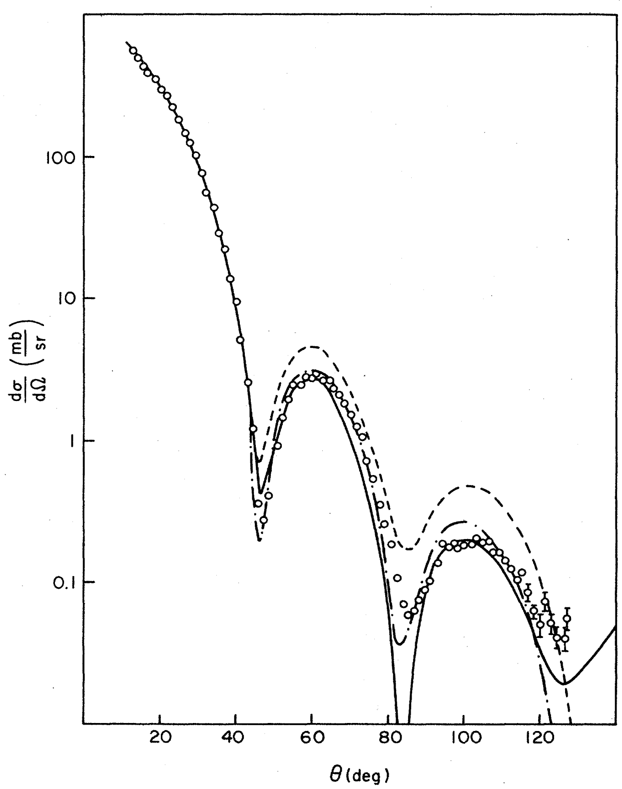

FIG. 4. The same as Fig. 3 except the energy is 163 MeV.

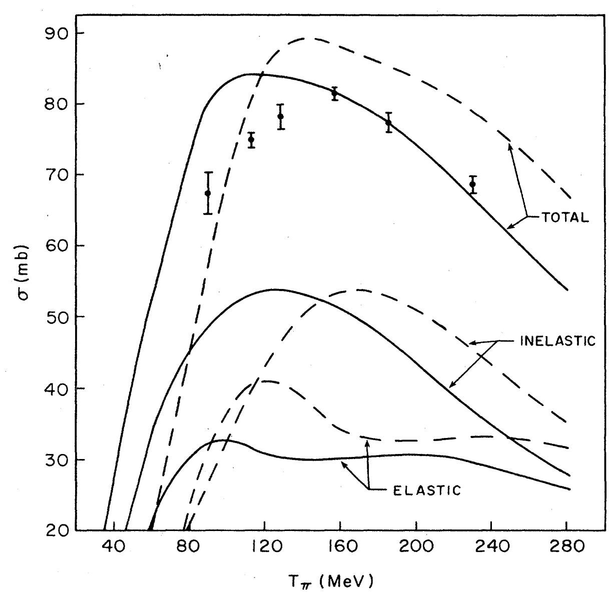

FIG. 6. The total, total elastic, and total inelastic cross section for π^+ scattering from ^{16}O as a function of pion kinetic energy. The solid curves are the result of using the mean spectral propagator and the dashed curves result from the three-body energy denominator. The Coulomb interaction is turned off. The data are from Ref. 33.

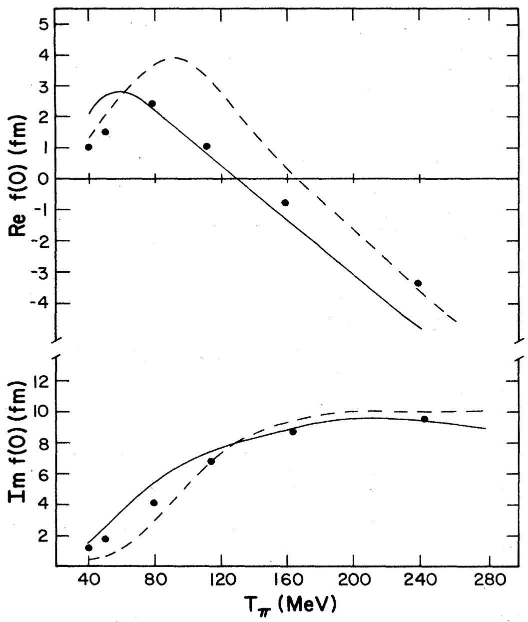

FIG. 7. The real and imaginary parts of the strong, forward scattering amplitude for π^+ elastic scattering from ^{16}O as a function of pion laboratory kinetic energy. The solid curves are the results of using the mean spectral propagator and the dashed curves result from the use of the three-body energy denominator. The Coulomb interaction is turned off. The points are values of $f(0)$ that are extracted in Ref. 34 from data.

from data in Ref. 34. Near resonance ($T_\pi \sim 160$ MeV) we note the following. First, the forward differential cross section is not particularly sensitive to dispersive effects. This is because the imaginary part of the optical potential is dominant at this energy and is determining the differential cross section in this angular region. When one is exactly at the peak of the resonance, the imaginary part of the amplitude is stationary. This fact, combined with the fact that the nucleus is already very black, explains why the positions of the minima are not particularly sensitive to dispersive corrections. The depths of the minima, however, are strongly dependent on their treatment. As one can see from Re$f(0)$ in Fig. 7, the real part of the scattering amplitude is rapidly passing through zero for $T_\pi \sim 125$ MeV. When one turns on the Coulomb potential, the net real amplitude for positive pions passes through zero at $T_\pi \sim 160$ MeV, which is the energy at which the angular distribution looks most diffractive. As the energy is varied away from resonance the minima are filled in by an amount equal to the square of the real part of the total amplitude. For these reasons, Coulomb-nuclear interference measurements and angular distributions are particularly sensitive indicators of dispersive effects.

At the energies below resonance, the dispersive effects play the largest role. In this region both the real and imaginary parts of the two-body amplitudes are rapidly energy dependent. This can be seen in the curves of $f(0)$ vs energy, in the total cross section curves, and in the differential cross sections at 114 MeV. At energies above the resonance the sensitivity to the treatment of dispersive effects is somewhat less than at resonance or below. The total cross sections and the forward differential cross section do show a sensitivity to the dispersive effects as the imaginary part of the two-body amplitude is strongly energy dependent in this region.

From Fig. 7, one sees that the experimental determination of Re$f(0)$ for several nuclei over an extended energy region would be of great use in understanding the role of dispersive effects in a quantitative way. This can be accomplished by taking data in the far forward angular region and extracting the real part of the strong amplitude by its interference with the known Coulomb amplitude. Because the energy dependence of Re$f(0)$ is nearly linear for $T_\pi > 120$ MeV (unlike the energy dependence of the depth of the diffractive minima), one need measure Re$f(0)$ at only three or four energies in this region. Data on nuclei with N not equal to Z would help to understand the isovector pieces of the pion-nucleus interaction.

An amplitude analysis of the extent data on ^{16}O and ^{40}Ca was performed in Ref. 34 to determine the strong forward scattering amplitude. The results for ^{16}O are plotted in Fig. 7. We see that the data confirm the peaking of Re$f(0)$ at low energy and indicate a need for the mean spectral energy, $E_{ms}(\omega,A)$.

IV. CONCLUSIONS

The half-width of the Δ_{33} resonance is 55 MeV, a number which is comparable to typical nuclear energies. It has long been known[1] that nuclear medium effects which can alter the energy of the pion-nucleon amplitude would have significant effects on pion-nucleus cross sections. The dispersive effect of the nucleon-nucleus interaction acting on the nucleon while it is interacting with the pion has been examined in the context of a three-body potential model in Refs. 8 and 9. We study in a field theoretic model an alternative and much simpler way of treating the delta-nucleus potential which enables us to incorporate its effects on elastic scattering while simultaneously incorporating full relativistic kinematics and performing exactly the fermi averaging integral (which includes an exact treatment of the delta recoil).

This approach, as it presently stands, is to be augmented by a second-order optical potential. Our approach is sufficiently flexible to allow a simultaneous treatment of elastic scattering, and single and double charge exchange along the lines of Ref. 35. Because the lowest-order potential is calculated theoretically in the present approach, the phenomenological energy shift of Ref. 35 would not be needed. This will make possible a study of the systematics of the second-order processes and should prove helpful in sorting out the underlying dynamics of the higher-order physics. Microscopic calculations of the second-order terms may be made following the theory of Ref. 14.

The mean spectral propagator is an efficient way to include in the impulse approximation modifications caused by the Δ_{33}-nucleus mean field. We are looking at several possible improvements on this treatment. In Eq. (2.14) we use the short range (in coordinate space) pion-nucleon form factor to motivate our approximation. We did not, however, use the p-wave structure of the coupling. If we keep the difference between an s-wave and p-wave coupling to intermediate states, we would expect $E_{\text{ms}}(\omega,A)$ to be different for pion-nucleon s waves and p waves. We have also not yet included the spin-orbit part of the delta-nucleus interaction. We are presently developing techniques for incorporating this potential. Finally, the use of a propagator which includes the delta-nucleus interaction will, we expect, have important consequences for inelastic scattering, including charge exchange reactions. Since inelastic scatterings can result from differences in amplitudes (as in charge exchange) or can be particularly sensitive to the relative strengths of various partial waves (which are altered differently by energy shifts in the t matrix), we expect to see interesting modifications caused by the use of an improved, lowest-order delta propagator.

We have found that the dispersive effects have a substantial impact on the cross sections, both differential and total, which are predicted by theories. These dispersive effects alter the energy at which the two-body amplitude is evaluated in the impulse approximation. Their treatment is intimately connected with the choice of the unperturbed propagator in perturbation theory, and hence the definition of what is meant by first order and second order. We have proposed a simple way of incorporating a major part of these dispersive effects into the first-order optical potential without the necessity of solving a three-body problem or using the delta-hole model through the use of a mean spectral energy. We have noted that additional measurements of the real part of the strong, forward scattering amplitude, $f(0)$, would provide valuable experimental input into our understanding of the role of the delta-nucleus interaction.

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