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Resonances in $^{13}C(^{13}C,\alpha)^{22}Ne$

G. P. Gilfoyle, L. C. Bland,* R. Gilman, M. Carchidi, K. S. Dhuga,† J. W. Sweet,
A. H. Wuosmaa, G. S. F. Stephans,‡ R. W. Zurmühle, and H. T. Fortune
Physics Department, University of Pennsylvania, Philadelphia, Pennsylvania 19104

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Complete angular distributions ($\theta_{c.m.}=9°-90°$) for the reaction $^{13}C(^{13}C,\alpha)^{22}Ne$ in the energy range $E_{c.m.}=6.25-13.38$ MeV have been measured. The data exhibit significant resonancelike behavior and a portion of it has been fitted to extract the amplitudes and phases of the relevant l values with two methods. The first uses a least-squares procedure to fit the angular distributions to a linear sum of Legendre polynomials while the second makes a grid search to find a best fit to an amplitude squared equation.

Since the discovery of the $^{12}C+^{12}C$ "nuclear molecule" in 1960 (Ref. 1) a considerable effort has been made to find similar phenomena in other heavy-ion systems.[2] In many of these searches intermediate oscillations of the type seen in $^{12}C+^{12}C$ were not found initially to accompany the gross structure. More careful investigations later revealed a "rich spectrum of intermediate structure."[3] One particular example was the $^{12}C+^{13}C$ system. Early work found no surprising behavior,[4,5] but later studies of the $^{12}C(^{13}C,\alpha)^{21}Ne$ reaction[6] and a more recent fusion experiment[7] have provided clear evidence of resonances. It has been further suggested that the presence of an extra neutron does not make the $^{13}C+^{12}C$ system fundamentally different from $^{12}C+^{12}C$.

With this history in mind we have turned our attention to a two valence neutron case, $^{13}C+^{13}C$. Fusion studies over a broad energy range have shown little structure in this system.[8–10] In fact, the cross section was found to be significantly greater than in the single and no valence neutron cases. However, the strength of any given exit channel in $^{13}C+^{13}C$ should be reduced by about three orders of magnitude because of the increased density of states.[11] If background and resonant amplitudes are both reduced by a similar factor, a given exit channel could still exhibit resonances even though these resonances would not dominate the fusion cross section. A single exit channel might be a more selective probe of the behavior of the $^{13}C+^{13}C$ system. We have chosen the $^{13}C(^{13}C,\alpha)^{22}Ne$ reaction because its large positive Q value (11.851 MeV) makes it readily distinguishable from reactions on ^{13}C and a ^{12}C impurity, which would produce light particles, and also tends toward matching incoming and outgoing angular momenta. An investigation of the $^{14}C(^{12}C,\alpha)^{22}Ne$ reaction which has the same intermediate and final states, produced results consistent with a statistical analysis.[12] That conclusion, however, was based on data at only a limited number of angles. We have measured angular distributions at 54 energies which cover a wide angular range ($\theta_{c.m.}=9°-90°$) and contain data at 141 angles. We have subjected the data to an extensive statistical analysis that results in different conclusions from those of Ref. 12 that is presented in another paper.[13]

In this paper we present data for the $^{13}C(^{13}C,\alpha)^{22}Ne$ reaction that exhibit a rich array of structure. We have analyzed a portion of these data in two ways in order to extract the l values contributing to the reaction.

Several features of the $^{13}C+^{13}C$ system should be noted. The incoming channel consists of two spin one-half fermions which can couple to a symmetric ($S=1$) or an antisymmetric ($S=0$) part. Thus, even though the two particles are identical, both odd and even l values will contribute to the interaction. It has been shown[14] that only the $M=1,S=1$ and $M=0,S=0$ amplitudes are nonzero. Hence the differential cross section for decay to the ^{22}Ne ground state can be written as:

$$\frac{d\sigma}{d\Omega}=\frac{\pi}{2k^2}\left[\left|\sum_l^{\text{even}}(2l+1)^{1/2}S_{l,l}^{0,l,0}Y_l^0(\theta,\phi)\right|^2+\left|\sum_l^{\text{odd}}(2l+1)^{1/2}S_{l,l}^{1,l,0}Y_l^1(\theta,\phi)\right|^2\right], \quad (1)$$

where $S_{l,l}^{S,J,S'}$ refers to the scattering matrix and all other quantities have their standard definitions. Equation (1) can be parametrized so that:

$$\frac{d\sigma}{d\Omega}=\left|\sum_l^{\text{even}}c_lY_l^0(\theta,\phi)\right|^2+\left|\sum_l^{\text{odd}}c_lY_l^1(\theta,\phi)\right|^2 . \quad (2)$$

When integrated over all angles, the total cross section is

$$\sigma_{\text{total}}=\sum_l\sigma_l , \quad (3)$$

where

$$\sigma_l=|c_l|^2 . \quad (4)$$

Data for the $^{13}C(^{13}C,\alpha)^{22}Ne$ reaction were collected using two position-sensitive slice detectors[15] and a ^{13}C beam from the University of Pennsylvania tandem Van de Graaff accelerator striking 20 $\mu g/cm^2$ self-supporting ^{13}C targets. Nickel foils in front of the detectors stopped ions heavier than the α's. Energy loss in the target was about 50 keV in the center-of-mass system. In this manner we obtained angular distributions ranging from 9°–90° in the center of mass system and containing data at 141 angles. These typically required only six to eight hours to com-

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plete. The energy range covered was $E_{\text{c.m.}} = 6.250\text{--}13.375$ MeV usually in 125 keV steps.

The angle-integrated cross section is plotted vs $E_{\text{c.m.}}$ in Fig. 1 for the transition to the ^{22}Ne ground state. The excitation function is dominated by prominent peaks at 7.9, 9.8, 11.4, 11.8, and 13.0 MeV. In what follows we restrict our attention to the peak at 9.8 MeV. Angular distributions in the region of this peak are displayed in Fig. 2. Two features of the data are immediately apparent. The angular distribution at $E_{\text{c.m.}} = 10.0$ MeV is dominated by $l = 8$, and the distributions in the two minima (at 9.25 and 10.12 MeV) are quite different. This latter feature suggests that the background under the peak is not smoothly varying.

These data have been fitted in two ways. The first is with a linear sum of Legendre polynomials:

$$\frac{d\sigma}{d\Omega} = \sum_{L=0}^{L_{\max}} a_L P_L(\cos\theta) \; . \tag{5}$$

From this point on, L refers to a term in Eq. (5) while l refers to the angular momentum in the reaction amplitude. Only even L's are needed as the target and projectile are identical. With good data, the values of a_L are unique. The zeroth coefficient, a_0, provides the total cross section:

$$\sigma_{\text{total}} = 4\pi a_0 \; . \tag{6}$$

The largest L needed to fit the data, $L_{\max}$, is determined by the behavior of the reduced χ^2 of the fit and is twice the largest l value contributing to the reaction. We obtained fits at all energies with $\chi^2/\nu \approx 1$ where ν is the number of degrees of freedom. This last coefficient can be used to determine unambiguously the partial cross section for the maximum l value, $l_{\max}$ (recall $\sigma_l = |c_l|^2$). Thus, for $l = l_{\max}$ and l even:

$$|c_l|^2 = a_{L_{\max}} \frac{4\pi}{2l+1} \frac{1}{(ll00\,|\,L_{\max}0)^2} , \quad L_{\max} = 2l \; , \tag{7}$$

and for $l = l_{\max}$ and l odd

$$|c_l|^2 = -a_{L_{\max}} \frac{4\pi}{2l+1} \frac{1}{(ll1-1\,|\,L_{\max}0)(ll00\,|\,L_{\max}0)} ,$$
$$L_{\max} = 2l \; . \tag{8}$$

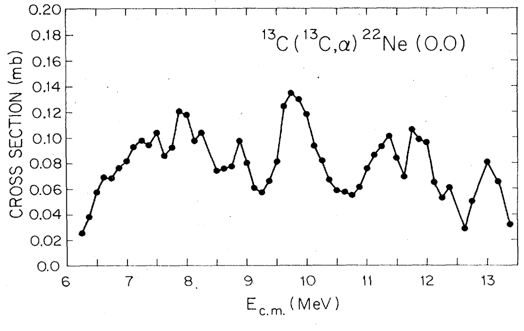

FIG. 1. Excitation function of the angle-integrated cross section for the ^{13}C(^{13}C,α)^{22}Ne(0.0) reaction.

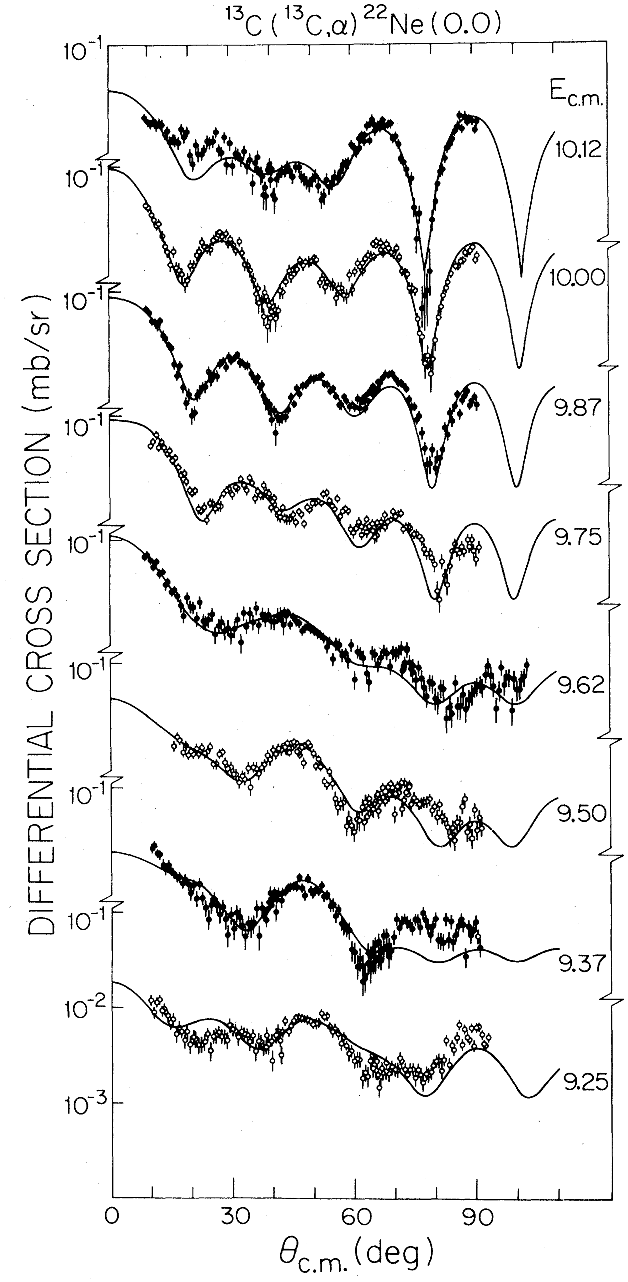

FIG. 2. Angular distributions for the ^{22}Ne ground state for $E_{\text{c.m.}} = 9.25\text{--}10.12$ MeV. The solid curves are the results of a grid search to find a best fit to Eq. (2).

Figure 3 compares σ_{total} and the partial cross sections for $l_{\max}$ over the peak. The maximum l value is 8 on the low-energy side, but near the peak, the $l = 9$ contribution suddenly rises and accounts for more than a third of the total cross section at its peak. We are seeing a prominent 9^- level in ^{26}Mg at an excitation energy of 21.7 MeV with a striking resonancelike shape. For comparison, we note that the incident channel grazing partial wave in this region is about seven or eight.

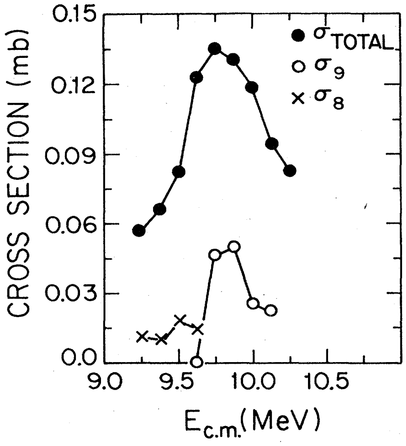

FIG. 3. A comparison of the total cross section and the partial cross sections for the maximum l value contributing to the reaction for $E_{c.m.}=9.25-10.12$ MeV.

The second fitting procedure used a grid search to find a best fit to Eq. (2). The amplitude for the maximum l value was taken from the linear fit and held fixed while the program searched over the appropriate amplitudes and phases of up to three other l values. A critical problem arises because different combinations of l values can give comparable fits. Presumably though, only one of these combinations will possess a regular energy dependence. Thus, having data at closely spaced intervals aids in determining the amplitude fit. Using, as a starting point, the combination of l values obtained at 10.0 MeV that clearly provided the best fit, we varied the parameters slowly from energy to energy in order to get a smooth variation across the peak. We needed l values of 4, 6, 8, and sometimes 9 to fit the data in this energy range. The angular distribution curves obtained in this manner are shown in Fig. 2. We then compared the total cross section extracted from these four l values with the data. As shown in Fig. 4(d), the agreement is excellent—these four l values provide virtually all the cross section. A notable advantage of this technique is that it requires no assumptions about the nature of the background in the reaction.

Our results are summarized in Fig. 4. As none of the partial cross sections exhibit a smooth behavior there is no flat background. The contribution from $l=8$ is the most prominent for the amplitudes extracted with this technique and is the largest at the energy at which the angular distribution is clearly dominated by that l value, 10.0 MeV. Another feature is the narrow width for the $l=4$ contribution. One does not expect this behavior in a low l value at such a high energy, but similar structure for $l=4$ has been noted in ^{12}C+^{12}C at a comparable energy.[16]

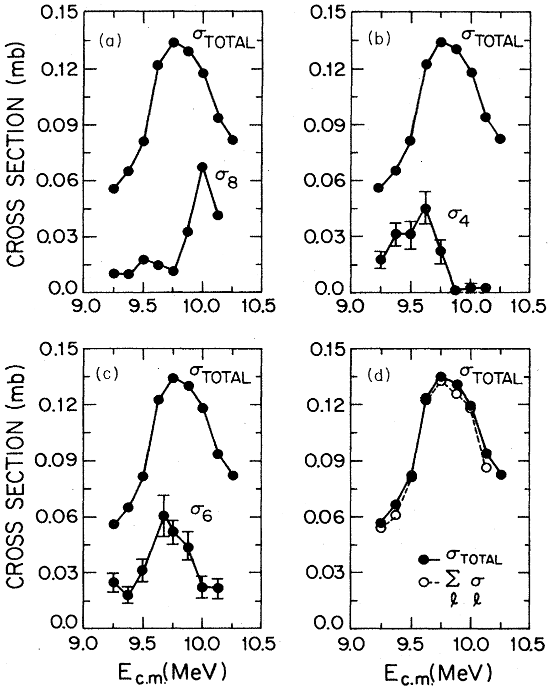

FIG. 4. A comparison of the total cross section and the partial cross sections used in finding a best fit to Eq. (2) (a)—(c). (d) compares the measured total cross section with the sum of the partial cross sections extracted from those fits.

A check on the validity of the $l=8$ parameters is shown in Table I. Whenever the maximum l value is 9 and if there is no $l=7$ contribution, the second to last term in the linear fit, a_{16}, depends only on $|c_9|^2$ and $|c_8|^2$. The agreement between a_{16} from the linear fit and a_{16} calculated from the amplitude squared fit results is excellent. Thus, the amplitude for $l=7$ is zero even though higher l values, 8 and 9, contribute to the reaction. This consistency between the two methods allows us to calculate $|c_8|^2$ and its uncertainty from a_{16} and $|c_9|^2$. The uncertainties in the other partial cross sections were found by varying each amplitude in the fit until the re-

TABLE I. Comparison of a_{16} from Legendre polynomial fit with a calculation of a_{16} based on the parameters from a fit to Eq. (2).

Energy (MeV)	a_{16} from linear Legendre polynomial fit (μb/sr)	a_{16} calculated from a fit to Eq. (2) (μb/sr)
9.750	-8.3 ± 0.9	-0.3 ± 4.0
9.875	6.9 ± 0.8	7.0 ± 3.0
10.00	21.8 ± 0.7	$23\ \pm2$
10.125	12.1 ± 0.6	$13\ \pm5$

duced χ^2 changed by one. Comparison with the other terms in the linear fit is less rigorous since one must add and subtract several large numbers to get a small number except for a_0 as mentioned above. The calculation becomes dominated by the uncertainties in the amplitude squared results.

The partial cross sections extracted with both methods reveal that ^{26}Mg has several broad, overlapping levels at this excitation energy. The behavior of the phase of a resonating l value has a well-known energy dependence relative to a flat background, but the "background" here consists of other l values that vary considerably themselves. Thus, the phase of the $l=8$ shows no simple energy dependence relative to the $l=4$ or $l=6$.

We have performed an extensive statistical analysis of the entire data set in this energy region, including results for the transitions to the first two excited states in ^{22}Ne (1.275 and 3.357 MeV) as well as the ground state. It clearly indicates that a significant portion of the angle-integrated cross section arises from a nonstatistical mechanism. We have also found evidence for correlations among the angle-integrated cross section for these three states, including one in the region of the above partial wave decomposition at the energies where the $l=8$ and $l=9$ contributions are strongest.

In this paper we have succeeded in measuring angular distributions in great detail over a broad energy range. We have identified a sharp 9^- level in ^{26}Mg using the unique and unambiguous linear Legendre polynomial fit technique. We have applied an amplitude squared fitting method successfully and decomposed the reaction into all its constituent l values, namely $l=4$, 6, and 8. The amplitude for $l=7$ has been found to be zero in this region allowing a more precise measurement of a prominent $l=8$ feature. The "standard" assumption that such reactions have a smooth background with few resonating l values has not been borne out in our analysis.

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[*] Present address: Indiana University, Bloomington, IN 47405.

[†] Present address: Los Alamos Meson Physics Facility, Los Alamos, NM 87545.

[‡] Present address: Argonne National Laboratory, Argonne, IL 60439.

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