Nonradiative versus radiative nuclear excitation in the positron-K-electron annihilation

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We have calculated the cross sections for both nonradiative and radiative processes in the nuclear excitation during positron-K-electron annihilation. Also their relative contributions are compared and discussed. The radiative process cannot explain the discrepancies between experiment and theory.

I. INTRODUCTION

The first prediction of nuclear excitation during positron-K-electron annihilation was made by Present and Chen,[1] who showed, on the basis of semiclassical arguments, that the cross section for the process can be factorized into a cross section for annihilation with emission of a spherical wave converging on the nucleus, multiplied by the probability of the nucleus being excited by the radiation. They used the Born approximation, retaining only the leading term in the αZ expansion, to calculate the cross section for annihilation, and estimated the probability of the nucleus being excited from photoexcitation data. Assuming an $E1$ transition, they estimated a cross section of $\sim 3\times 10^{-26}$ cm^2 for exciting the 1078 keV level in ^{115}In.

Watanabe *et al.*[2–4] followed the approach of Present and Chen and calculated the cross section for positron-K-electron annihilation into $M1$ and $E2$ spherical waves converging on the nucleus. By combining these results with experimental data for photoexcitation they estimated a cross section for excitation of levels in ^{115}In, ^{111}Cd, and ^{176}Lu. Their experimental results are many orders of magnitude greater than the calculated values. Grechukhin and Soldatov[5] have also made estimates of the cross sections for nuclear excitation of levels in ^{115}In during positron-K-electron annihilation and their estimates are also orders of magnitude below the experimental results.

In order to reduce the discrepancies between theory and experiment, Raghavan and Mills[6] suggested that a radiative or inelastic process could make a significant contribution. In this model a photon could be emitted during the process and this would relax the requirement of the nonradiative process where only positrons with the correct kinetic energy could contribute to the resonant excitation of the nuclear level. Many more positrons would be involved in possible excitations, and they proposed that the radiative process could account for the observed cross section.

However, Ljubičić *et al.*[7] have shown that Raghavan and Mills overestimated the magnitude of the radiative process by many orders of magnitude and that the mechanism for nuclear excitation during positron-K-shell annihilation is still an open question. The analyses of Raghavan and Mills and of Ljubičić *et al.* are only semiquantitative, and the purpose of this work is to make a much more quantitative estimate of the nonradiative and radiative processes, and in particular, to compare their respective contributions.

II. ESTIMATE OF THE CROSS SECTIONS

A. Nonradiative process

We assume that nuclear excitation during positron-K-electron annihilation is produced by a dynamical interaction between the electron-positron current and the nuclear electromagnetic transition current, described by the exchange of a virtual photon. The multipole expansion of the photon propagator introduces the nuclear multipole transition operators for transitions between the ground state and the excited nuclear levels. These operators are expected to enter in the respective γ-ray emission probabilities. The amplitude M for such a process is associated with the Feynman diagram shown in Fig. 1 and can be written as

$$M=2\pi e\delta(\varepsilon+\varepsilon_0-\omega')\int d\mathbf{x}\,d\mathbf{y}\overline{\Psi}_{\mathbf{p}}(\mathbf{x})D(\mathbf{x}-\mathbf{y})\hat{J}(\mathbf{y})\Psi_0(\mathbf{x})\ , \qquad (1)$$

where the units and metric of Ref. 8 are used. In Eq. (1) ε and ε_0 are the total energies of the positron and electron, respectively, and ω' is the energy of the nucleus in the final state. Here, Ψ_{p} and Ψ_0 are the wave functions for the positron (with a momentum $\mathbf{p}$) and for the K electron, respectively. J_μ represents the nuclear transition current

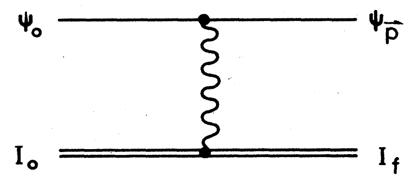

FIG. 1. The Feynman diagram for the nonradiative process.

and D is a photon propagator.

We calculate M up to the leading term in the formal αZ expansion by using the momentum representation in Eq. (1) and we obtain

$$M=2e\sqrt{\pi}(\alpha Z)^{3/2}\frac{\delta(\varepsilon+\varepsilon_0-\omega')}{\mathbf{q}^2-(\omega')^2+2i\alpha Z\omega'}\times\bar{v}(-\mathbf{p})\hat{J}(\mathbf{q})u(0)\ , \tag{2}$$

where v and u are the spinors of the positron and electron, respectively, and the nuclear momentum transfer $\mathbf{q}$ is given in this approximation as

$$\mathbf{q}=\mathbf{p}\ . \tag{3}$$

For nuclear levels with definite spin and parity the nuclear current J_μ can be represented by electric or magnetic multipole terms.[8]

We assume the density of final nuclear states has the form of a Lorentz-shaped function, centered at the energy ω, with a width Γ. These considerations lead us to express the nonradiative cross section for two K electrons, unpolarized positrons, and unoriented nuclei in the form

$$\sigma^{(e,m)}=16\pi\alpha^4Z^3g\frac{\Gamma_0}{\Gamma}\frac{p^{2L-1}}{\omega^{2L+1}}\frac{1}{(p^2-\omega^2)^2+(2\alpha Z\omega)^2}\times[2A^{(e,m)}+(1+\varepsilon)B^{(e,m)}]\ , \tag{4}$$

with the energy constraint

$$|\varepsilon+\varepsilon_0-\omega|<\Gamma/2\ . \tag{5}$$

In Eq. (4),

$$g=(2I_f+1)/(2I_0+1)\ ,$$

where I_f and I_0 are the respective spins of the excited and ground state nucleus. Γ_0 is the ground state transition width of the excited level.

Also for 2^L magnetic transitions,

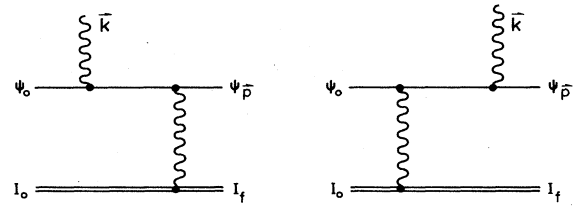

FIG. 2. The Feynman diagrams for the radiative process.

$$A^{(m)}=0\ ,\qquad B^{(m)}=1\ , \tag{6}$$

and for 2^L electric transitions,

$$A^{(e)}=\frac{L}{L+1}(\varepsilon-\omega)\ ,\qquad B^{(e)}=\frac{1}{L+1}\frac{1}{p^2}[(2L+1)\omega^2-Lp^2]\ , \tag{7}$$

where $p=|\mathbf{p}|$.

In the limit $\alpha Z\to 0$ the cross section of Eq. (4) reproduces the results of Present and Chen and Watanabe *et al.*

B. Radiative process

In this subsection we calculate the cross section for the radiative (or inelastic) process proposed by Raghavan and Mills. This higher order process involves the emission of a real photon. Although other higher order inelastic processes are possible, the one considered is the most probable as it only has one interaction vertex more than the nonradiative process. Feynman diagrams for the radiative process are given in Fig. 2. The amplitude $M_{\rm rad}$ corresponding to these diagrams can be written as follows:

$$M_{\rm rad}=2\pi e^2\delta(\varepsilon+\varepsilon_0-\omega'-k)\int d\mathbf{x}\,d\mathbf{y}\,d\mathbf{z}\bar{\Psi}_p(\mathbf{x})[D(\mathbf{x}-\mathbf{y})\hat{J}(\mathbf{y})S(\mathbf{x},\mathbf{z})\hat{a}^{\dagger}(\mathbf{z})+\hat{a}^{\dagger}(\mathbf{z})S(\mathbf{x},\mathbf{z})D(\mathbf{z}-\mathbf{y})\hat{J}(\mathbf{y})]\Psi_0(\mathbf{z})\ . \tag{8}$$

Repeated symbols have the same meaning as in Eq. (1). The energy of the emitted photon is k, and the photon four-potential a_μ is described by the momentum vector $\mathbf{k}$ and the polarization vector $\mathbf{e}$; the propagator of the electron-positron field is represented by S.

As in the calculation of M, the calculation of $M_{\rm rad}$ is simplified by introducing the momentum representation. The result is

$$M_{\rm rad}=2e^2\sqrt{\pi}(\alpha Z)^{3/2}\frac{1}{(2k)^{3/2}}\frac{\delta(\varepsilon+\varepsilon_0-\omega'-k)}{\mathbf{q}^2-\omega'^2+i2\alpha Z\omega'}\bar{v}(-\mathbf{p})[\hat{J}(\mathbf{q})(i\hat{j}-1)e\gamma+fe\gamma(i\hat{t}-1)\hat{J}(\mathbf{q})]u(0)\ , \tag{9}$$

where we have introduced the four-vectors

$$j=[-\mathbf{k},i(1-k)]\ ,\qquad t=[-\mathbf{q},i(1-\omega)]\ . \tag{10}$$

From momentum conservation we have

$$\mathbf{q}=\mathbf{p}-\mathbf{k}\ . \tag{11}$$

In Eq. (9) f is given by

$$f=(|\mathbf{p}|\cos\theta-\varepsilon)^{-1}, \tag{12}$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{p}$.

The calculation of the cross section for unpolarized electrons and positrons can be accomplished with the trace technique used previously.[8] Assuming unpolarized photons and unoriented nuclei, and the same density of final nuclear states as in the nonradiative process, we obtain, for two K electrons, the radiative cross section in the form

$$\frac{d\sigma_{\rm rad}}{d\Omega}=\frac{\alpha^5Z^3}{2\pi}\frac{1}{k|\mathbf{p}|}\frac{1}{(\mathbf{q}^2-\omega^2)^2+(2\alpha Z\omega)^2}$$

$$\times\{-[(1+\varepsilon)j\cdot j+k(2p\cdot j-1)+\omega]S_0^{e,m}+4kR^{e,m}(p,j)+2(1+j\cdot j)I^e(p)$$

$$+f^2[\omega(1-2\bar{p}\cdot t)-(1+\varepsilon)t\cdot t-k]S_0^{e,m}+4f^2(1-\bar{p}\cdot t)I^e(t)+2f^2(1+t\cdot t)I^e(\bar{p})+f\sum_{i=1}^{2}G_i^{e,m}\}, \tag{13}$$

$$G_i^{e,m}=-(\varepsilon+1)[R^{e,m}(j^{(i)},t)+R^{e,m}(t^{(i)},j)]+k[R^{e,m}(t^{(i)},p^{(i)})-R^{e,m}(p,t)]$$

$$+\omega[R^{e,m}(p,j)+R^{e,m}(j^{(i)},p^{(i)})]+(1+p^{(i)}\cdot j)[I^e(t)-I^e(t^{(i)})]$$

$$+(1+t\cdot j)[I^e(p)-I^e(p^{(i)})]+[(1+\varepsilon)t\cdot j-kp^{(i)}\cdot t-\omega p^{(i)}\cdot j]S_i^{e,m}.$$

For a 2^L electric-type transition we found

$$S_0^e=C^e[(2L+1)\omega^2-Lq^2],$$

$$S_1^e=C^eL(q^2-\omega^2),$$

$$S_2^e=C^e\left[L(q^2-\omega^2)+(L-1)\frac{\omega^2}{\mathbf{q}^2}\mathbf{p}^2\sin^2\theta\right],$$

$$R^e(a,b)=\tfrac{1}{2}C^e[\omega^2(L+1)\mathbf{a}\cdot\mathbf{b}+\omega^2(L-1)(\mathbf{a}\cdot\mathbf{q}^0)(\mathbf{b}\cdot\mathbf{q}^0)$$

$$-2\omega qL(b_0\mathbf{a}\mathbf{q}^0+a_0\mathbf{b}\mathbf{q}^0)+2Lq^2a_0b_0],$$

$$I^e(a)=qLC^e[\omega\mathbf{a}\cdot\mathbf{q}^0-qa_0],$$

$$C^e=-\frac{2\pi}{L+1}g\Gamma_0\frac{q^{2(L-1)}}{\omega^{2L+1}},$$

and for a 2^L magnetic-type transition we found

$$S_0^m=C^m,$$

$$S_1^m=0,$$

$$S_2^m=-C^m\frac{\mathbf{p}^2}{\mathbf{q}^2}\sin^2\theta,$$

$$R^m(a,b)=\tfrac{1}{2}C^m[\mathbf{a}\mathbf{b}-(\mathbf{a}\cdot\mathbf{q}^0)(\mathbf{b}\cdot\mathbf{q}^0)],$$

$$C^m=-2\pi g\Gamma_0\frac{q^{2L}}{\omega^{2L+1}},$$

where

$$q=|\mathbf{q}|,$$

$$\mathbf{q}^0=\frac{\mathbf{q}}{q},$$

$$j^{(1)}=j^{(2)}=[0,0,k,i(k-1)],$$

$$p=[\mathbf{p},i\varepsilon],$$

$$p^{(1)}=[0,-|\mathbf{p}|\sin\theta,-|\mathbf{p}|\cos\theta,-i\varepsilon],$$

$$p^{(2)}=[0,|\mathbf{p}|\sin\theta,-|\mathbf{p}|\cos\theta,-i\varepsilon],$$

$$\bar{p}=[0,0,|\mathbf{p}|\cos\theta,i\varepsilon],$$

$$t^{(1)}=[0,|\mathbf{p}|\sin\theta,|\mathbf{p}|\cos\theta-k,i(\omega-1)],$$

$$t^{(2)}=[0,-|\mathbf{p}|\sin\theta,|\mathbf{p}|\cos\theta-k,i(\omega-1)].$$

Ω represents the photon solid angle, and from energy conservation

$$\varepsilon+\varepsilon_0=k+\omega. \tag{14}$$

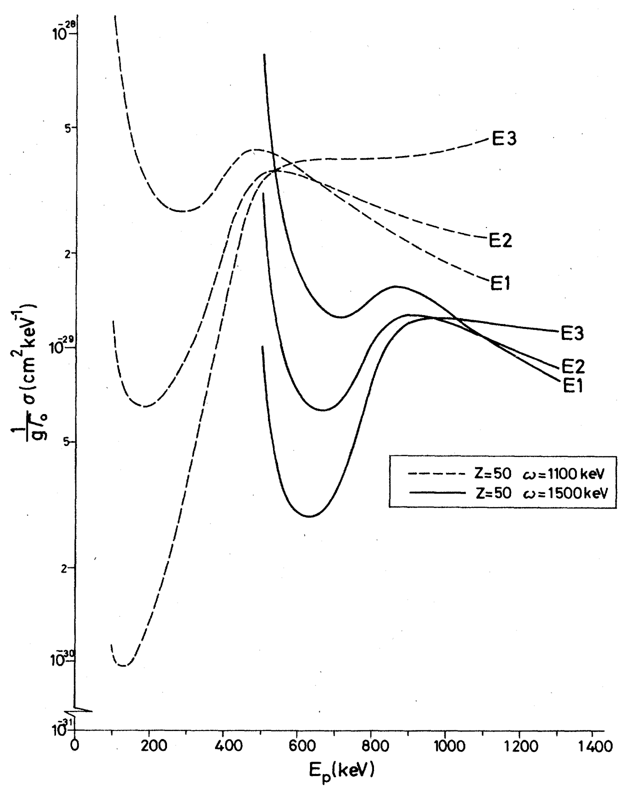

FIG. 3. The dependence of the total cross section for the radiative process on the positron kinetic energy for electric multipole transitions.

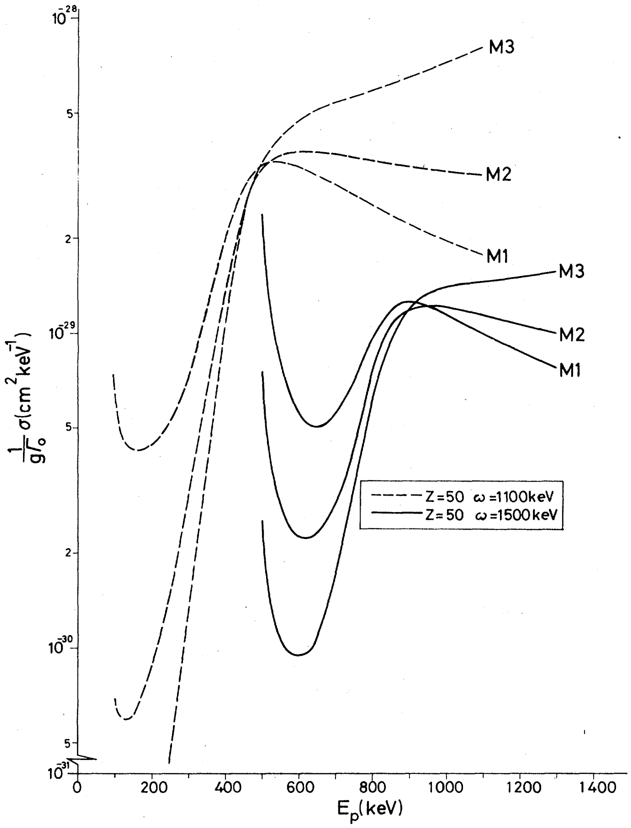

FIG. 4. The dependence of the total cross section for the radiative process on the positron kinetic energy for magnetic multipole transitions.

Figures 3 and 4 illustrate the dependence of the total cross section for the radiative process on the positron kinetic energy for the electric and magnetic multipole transitions.

III. DISCUSSION

In our calculations we neglect the effect of the nuclear Coulomb field on the positron motion and on the intermediate electron-positron states. Furthermore, the K-electron binding is only included through the zero-momentum contribution of the wave function. These approximations can be expected to be reasonable for small Z and large positron kinetic energies. Because of Coulomb repulsion, the inclusion of the nuclear Coulomb field would reduce the probabilities of both processes under consideration, as in one-quantum positron-electron annihilation.[9] However, an estimate of the ratio of the radiative to nonradiative probabilities should be reasonably accurate, as there is a cancellation of the similarly neglected dynamic effect in the two processes. Calculations of internal Compton coefficients[10] and double internal bremsstrahlung in electron capture[11] support this conclusion.

The energy constraint of Eq. (5) on the positron energy is relaxed in the radiative process [see Eq. (14)], and it should be noted that the cross section for the radiative process is nonresonant in character. This is due to the presence of the photon in the final state which results in a summation over the resonance structure of the excited nuclear level. As a consequence of this a rough estimate of the ratio of the radiative and nonradiative cross sections will include not only α, but also the nuclear level width Γ.[7]

We have calculated the ratios of the radiative to nonradiative processes for excited states of ^{115}In. Assuming the positron spectrum from a ^{64}Cu source, we find the ratio of radiative to nonradiative probabilities to be 4.8×10^{-2} for the 1078-keV level, and 5.7×10^{-3} for the 1464-keV level. These results are in good agreement with the rough estimate of $\sim10^{-2}$ made by Ljubičić *et al.*[7]

IV. CONCLUSIONS

Because of the nonresonance character of the radiative process, the approach we have made, assuming the multipole expansion of the exchanged virtual photon, can be expected to be basically correct. The source of the possible errors lies in simplified calculations we have performed by partially neglecting the influence of the external nuclear Coulomb field on the electron-positron motion. These calculations are expected to be valid for low Z and high positron energies. However, the ratio of radiative to nonradiative probabilities is much more realistic, if in the calculations for the latter process the external Coulomb field is taken into account in the same way as for the radiative process. Because our estimates for the nonradiative cross sections are always smaller than the measured values, we can conclude from the numerical results from the preceding section that the radiative process we are considering in this paper cannot explain the discrepancies between the experiment and theory in the nuclear excitation by positron-K-electron annihilation.

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